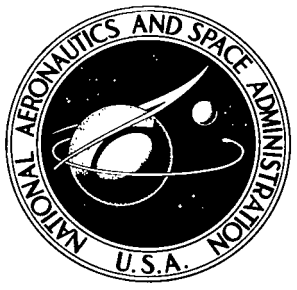
NASA TECHNICAL NOTE

NASA TN D-8024

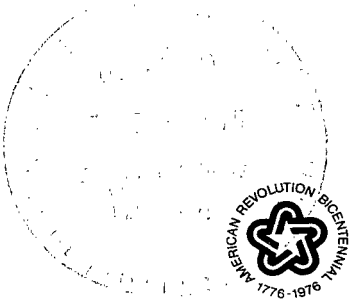


NASA TN D-8024

KALMAN FILTER ESTIMATION OF HUMAN PILOT-MODEL PARAMETERS

James R. Schiess and Vincent R. Roland

Langley Research Center
Hampton, Va. 23665



NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • NOVEMBER 1975

1 Report No
NASA TN D-8024

2. Government Accession No.

3. Recipient's Catalog No.

4. Title and Subtitle
KALMAN FILTER ESTIMATION OF HUMAN PILOT-MODEL PARAMETERS

5. Report Date
November 1975

6. Performing Organization Code

7. Author(s)
James R. Schiess and Vincent R. Roland

8. Performing Organization Report No.
L-10041

10. Work Unit No.
506-25-99-01

9. Performing Organization Name and Address
NASA Langley Research Center
Hampton, Va. 23665

11. Contract or Grant No.

13. Type of Report and Period Covered
Technical Note

12 Sponsoring Agency Name and Address
National Aeronautics and Space Administration
Washington, D.C. 20546

14. Sponsoring Agency Code

15. Supplementary Notes

16 Abstract

The parameters of a human pilot-model transfer function are estimated by applying the extended Kalman filter to the corresponding retarded differential-difference equations in the time domain. Use of computer-generated data indicates that most of the parameters, including the implicit time delay, may be reasonably estimated in this way. When applied to two sets of experimental data obtained from a closed-loop tracking task performed by a human, the Kalman filter generated diverging residuals for one of the measurement types, apparently because of model assumption errors. Application of a modified adaptive technique was found to overcome the divergence and to produce reasonable estimates of most of the parameters.

17. Key Words (Suggested by Author(s))
Kalman filter
Pilot model
Adaptive technique

18. Distribution Statement
Unclassified — Unlimited

Subject Category 64

19. Security Classif. (of this report)
Unclassified

20. Security Classif. (of this page)
Unclassified

21. No. of Pages
42

22. Price*
$3.75

*For sale by the National Technical Information Service, Springfield, Virginia 22161

KALMAN FILTER ESTIMATION OF HUMAN PILOT-MODEL PARAMETERS

James R. Schiess and Vincent R. Roland
Langley Research Center

SUMMARY

The parameters of a human pilot-model transfer function are estimated by applying the extended Kalman filter to the corresponding retarded differential-difference equations in the time domain. Use of computer-generated data indicates that most of the parameters, including the implicit time delay, may be reasonably estimated in this way. When applied to two sets of experimental data obtained from a closed-loop tracking task performed by a human, the Kalman filter generated diverging residuals for one of the measurement types, apparently because of model assumption errors. Application of a modified adaptive technique was found to overcome the divergence and to produce reasonable estimates of most of the parameters.

INTRODUCTION

The mathematical modeling of human dynamics in specific tasks is of considerable interest to control engineers. For example, a model which accurately predicts a pilot's response in the pitch control of a spacecraft is beneficial in the analysis of the handling qualities of the vehicle during the development stages of the vehicle. Studies of the pilot models in general have been reported in the literature (refs. 1 to 4).

In order to use any of these models, it is necessary to know various parameters which occur in the model. Investigations of applying model-matching techniques to determine the best parameters from pilot-response data have been reported previously (ref. 5). Since the human pilot-model equations classify as retarded differential-difference equations in the time domain, the application of linear estimators to this type of equation by Koivo and Stoller (refs. 6 and 7) and by Kwakernaak (ref. 8) is also of interest. The work by Kwakernaak (ref. 8) on the linear problem concerns an estimate of the state only and no parameters. Although an exact solution for the optimal filter is obtained, this approach presents practical computational difficulties. The findings of Koivo (ref. 6) present an estimator (not necessarily optimal) both for the state and for particular parameters, but does not include the time delay.

The present paper investigates an application of a form of the extended Kalman filter to the time-domain representation of a particular pilot model in a single-control compensatory

tracking task. The time delay is included as one of the parameters to be determined. The time delay is retained as an implicit parameter rather than as an explicit parameter (obtained if a Padé approximation is applied) for exactness. When applied to the experimental pilot-response data, the initial results of the Kalman filter indicate difficulties in obtaining satisfactory estimates. An analysis of the mathematical model and techniques for improving the estimates are presented.

SYMBOLS

$c(t)$	plant output, V
$D(t)$	input-disturbing function, V
$E\{\}$	expected value of $\{\}$
$e(t)$	system error, V
$\underline{f}(\underline{y}(t),\underline{y}(t-\theta),D(t-\theta),t)$	13 X 1 column vector representing right-hand side of differential-difference equations
$H(t_{k+1})$	measurement coefficient matrix at t_{k+1}
I	identity matrix
K	plant static gain, sec^{-2}
K_1	pilot static gain
$K(t_{k+1})$	Kalman filter gain at t_{k+1}
$P(t_{k+1} \mid t_k)$	state covariance matrix at t_{k+1} before processing measurement at t_{k+1}
$P(t_{k+1} \mid t_{k+1})$	state covariance matrix at t_{k+1} after processing measurement at t_{k+1}
$Q(t_{k+1})$	process noise covariance matrix
q	integer defined by $q = \theta/\Delta t$

$\bar{q}(k)$	variance computed in adaptive procedure
$\bar{q}(k+1)$	estimated process noise variance at t_{k+1}
$R\left(t_{k+1}\right)$	measurement covariance matrix at t_{k+1}
$R_p(s)$	second-order transfer function in pilot remnant
s	Laplace operator, sec^{-1}
t	time, sec
t_{k+1}	k plus first time t, sec
Δt	time increment, sec
u(t)	deterministic pilot function, V
$\underline{w}(t)$	13 X 1 process noise vector
$w_p(t)$	white noise input, V/sec^2
$Y_p(s)$	deterministic pilot transfer function
$\underline{y}(t)$	13 X 1 state column vector
$y_4(t)$	fourth component of $\underline{y}(t)$ defined in equation (6), V
α_1, α_2	coefficients in pilot remnant transfer function
$\Gamma(k+1)$	process noise coefficient matrix at t_{k+1}
$\delta(t)$	stick motion, V
$\eta(t)$	pilot remnant, V
θ	pilot time delay, sec

τ_1, τ_2 pilot lag-time constants, sec

τ_3 pilot lead-time constant, sec

$\phi(t_{k+1}, t_k)$ state transition matrix from t_k to t_{k+1}

Superscripts:

$\hat{}$ best estimate

$\cdot$ first derivative with respect to time

$\cdot\cdot$ second derivative with respect to time

T matrix transpose

-1 matrix inverse

Subscript:

k denotes evaluation at time t_k

PROBLEM DESCRIPTION

This analysis considers the problem of identifying the parameters in an assumed input-output relationship describing the human pilot from data measured in a tracking task in which the pilot provides feedback control. Because of process noise which is represented by the pilot remnant and measurement noise in the data accumulation, the estimation process considers the problem of minimizing the effect of the noise. A well-established method of reducing the error in such an estimation process is the Kalman filter. This estimator considers both process and measurement noise and provides the best estimation of state and parameter vectors in the sense that the expected value of the sum of the squares of the error in the measurement is minimized. In addition, this filter provides a covariance matrix which indicates the quality of the estimates.

A block diagram of the pilot airplane model used in this analysis is given in figure 1. The model consists of a closed-loop single-control compensatory tracking task. The dynamics of the controlled elements are of the acceleration type (K/s^2). Such a type is found in the attitude control of a space vehicle by control jets. The dashed lines enclose the portion of

the model representing the pilot. The transfer function representing the deterministic portion of the pilot model $Y_p(s)$ is given (ref. 4) as

$$Y_p(s) = \frac{u(s)}{e(s)} = \frac{K_1(\tau_3 s + 1)e^{-s\theta}}{(\tau_1 s + 1)(\tau_2 s + 1)} \tag{1}$$

The parameter K_1 is a pilot static gain and θ is an effective time delay made up of transport delays and high-frequency neuromuscular lags. The constant τ_3 is a time constant associated with the low-frequency characteristics of the neuromuscular system (ref. 4). The parameters τ_1 and τ_2 are pilot-adjusted parameters.

The pilot-remnant function used in this analysis is a second-order noise filter which converts the white noise input $w_p(t)$ to colored noise input $\eta(t)$. The form of this filter is

$$R_p(s) = \frac{1}{s^2 + \alpha_1 s + \alpha_2} \tag{2}$$

The remnant $\eta(t)$ represents noise produced by the pilot. Such noise is caused by high-frequency neuromuscular lag and lead terms and low-frequency neuromuscular lag terms.

For the current analysis the measurement types used are the plant output $c(t)$ and the stick motion $\delta(t)$. The latter is the sum of the deterministic pilot function and the pilot remnant.

The functions of time which represent the aircraft dynamics, the deterministic pilot-transfer function, and the pilot-remnant function, respectively, are given by the differential-difference equations

$$\ddot{c}(t) = K\,\delta(t) \tag{3}$$

$$\tau_1\tau_2\,\ddot{u}(t) + (\tau_1 + \tau_2)\,\dot{u}(t) + u(t) = K_1\left[\tau_3\dot{e}(t - \theta) + e(t - \theta)\right] \tag{4}$$

$$\ddot{\eta}(t) + \alpha_1\,\dot{\eta}(t) + \alpha_2\,\eta(t) = w_p(t) \tag{5}$$

These equations indicate two possible difficulties in applying the Kalman filter: (1) because of the way in which θ occurs, they represent a hereditary system (i.e., one represented by differential-difference equations); and (2) the equations are nonlinear in the combination of the variables and parameters. The problem of nonlinearity can be overcome by applying

the extended Kalman filter to the nonlinear equations. The problem of applying the Kalman filter to a hereditary system is considered in the section on "Filter Equations."

For convenience in the application of the Kalman filter, equations (3) to (5) are transformed to a state space representation by using the following definitions for the time-dependent variables $y_1(t)$ to $y_6(t)$:

$$\left.\begin{aligned} y_1(t) &= c(t) \\ y_2(t) &= \dot{c}(t) \\ y_3(t) &= u(t) \\ y_4(t) &= \tau_2\,\dot{u}(t) + u(t) - \frac{K_1}{\tau_1}\,\tau_3 e(t-\theta) \\ y_5(t) &= \eta(t) \\ y_6(t) &= \dot{\eta}(t) \end{aligned}\right\} \tag{6}$$

The variable $y_4(t)$ is defined in this way to simplify the final state equations. With these definitions and with the use of the fact that $e(t) = D(t) - c(t)$, the state space form of the pilot-model equations is given by equations (7) to (12)

$$\dot{y}_1(t) = y_2(t) \tag{7}$$

$$\dot{y}_2(t) = K\left[y_3(t) + y_5(t)\right] \tag{8}$$

$$\dot{y}_3(t) = \frac{1}{\tau_2}\,y_4(t) - \frac{1}{\tau_2}\,y_3(t) + \frac{K_1\tau_3}{\tau_1\tau_2}\left[D(t-\theta) - y_1(t-\theta)\right] \tag{9}$$

$$\dot{y}_4(t) = -\frac{1}{\tau_1}\,y_4(t) + \frac{K_1}{\tau_1}\left(1 - \frac{\tau_3}{\tau_1}\right)\left[D(t-\theta) - y_1(t-\theta)\right] \tag{10}$$

$$\dot{y}_5(t) = y_6(t) \tag{11}$$

$$\dot{y}_6(t) = -\alpha_1\,y_6(t) - \alpha_2\,y_5(t) + w_p(t) \tag{12}$$

It should be observed at this point that equations (7) to (12) are linear in the y's. Thus, if $\underline{y}(t)$ is defined as the six-dimensional column vector whose components are $y_1(t)$ to $y_6(t)$, A is the 6 X 6 constant matrix containing the coefficients of the right-hand sides of these equations, $\underline{b}$ is the six-dimensional column vector containing the coefficients of $D(t - \theta) - y_1(t - \theta)$, and $\underline{w}(t)$ is the vector of white noise input. Equations (7) to (12) can then be written in vector form as

$$\dot{\underline{y}}(t) = A\underline{y}(t) + \underline{b}\left[D(t - \theta) - y_1(t - \theta)\right] + \underline{w}(t) \tag{13}$$

Equation (13) is in the form of a single-input, time-invariant linear control system to which the Kalman filter is immediately applicable. (See, for example, ref. 9.)

In addition to the six variables of equations (7) to (12), it is also of interest here to estimate seven of the parameters in these equations. The estimation may be accomplished by augmenting the state equations with the system

$$\dot{y}_i(t) = 0 \qquad (i = 7, 8, \ldots, 13) \tag{14}$$

where $y_7(t)$ through $y_{13}(t)$ are defined as the parameters K_1, τ_1, τ_2, τ_3, θ, α_1, and α_2, respectively. With $\underline{y}(t)$ extended to a 13-dimensional vector with the newly defined components, the right sides of equations (7) to (12) and (14) are now nonlinear functions of the vector $\underline{y}(t)$. The general form of these equations is given by

$$\dot{\underline{y}}(t) = \underline{f}\left[\underline{y}(t), \underline{y}(t - \theta), D(t - \theta), t\right] + \underline{w}(t) \tag{15}$$

where $\underline{f}\left[\underline{y}(t), \underline{y}(t - \theta), D(t - \theta), t\right]$ is a 13-dimensional column vector defined by the right sides of equations (7) to (12) and (14), and $\underline{w}(t)$ is a 13-dimensional column vector which is all zero except for the sixth component which is $w_p(t)$. The time delay θ is left explicit in equation (15) for later reference.

In the vector notation, the measurements are given by

$$\underline{m}(t) = H(t)\,\underline{y}(t) + \underline{V}(t) \tag{16}$$

where $\underline{m}(t)$ is a two-dimensional column vector,

$$H(t) = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$\underline{V}(t) = \begin{bmatrix} V_1(t) \\ V_2(t) \end{bmatrix}$$

where $V_1(t)$ and $V_2(t)$ represent white noise and are assumed to be uncorrelated with each other and with the process noise $w_p(t)$. In equation (16), the first measurement is $\delta(t)$, the second is $c(t)$.

In subsequent sections of this report the stability of the closed-loop system is discussed. Because the time delay occurs implicitly in equations (7) to (12), the effect of particular values of this parameter on the system stability cannot be readily discerned. Instead, if the first order Padé approximation to the exponential is used in equation (1), and if equations (2) and (3) and the new equation (1) (resulting from use of first-order Padé approximation) are transformed to the time domain, the resulting equations are of the form

$$\dot{\underline{y}}(t) = A' y(t) + b' u(t) + \underline{w}(t)$$

where θ occurs explicitly in the new A' and b'. The approximate asymptotic stability of the system can then be determined by examining the eigenvalues of A'. Thus, in this report, the system is considered stable if all the eigenvalues of A' have negative real parts.

FILTER EQUATIONS

The filter equations used for this study are the discrete extended Kalman filter equations as given by Jazwinski (ref. 9). The use of these equations overcomes the difficulty which arises from the fact that the state equations are nonlinear when the parameters are treated as state variables. It should be pointed out that the extended Kalman filter actually is a nonlinear filter; for example, the Kalman filter is identical to the nonlinear discrete invariant embedding algorithm of Sage and Melsa (ref. 10) when the measurement equation is linear as in the case under consideration here.

In general, with any discrete filter the state is predicted to time t_{k+1} through the application of the state equations to the best estimate of the state at the previous time t_k. Since the pilot-model equations contain the state at $t_k - \theta$, an appropriate value of the state at that time must be chosen. The choice is made by assuming that the time increment Δt between measurements is constant and that the delay θ is an integral multiple of this increment. Thus, let q be the integer defined by

$$q = \frac{\theta}{\Delta t} \tag{17}$$

Then the state is evaluated at time $t_k - \theta$ by using the best estimate of the state for the time $t_{k-q} = t_k - q\,\Delta t$. Since the state at t_{k-q} is known at t_k, the delayed state can be treated simply as a known forcing function in the state equations. In a notation similar to that of Jazwinski (ref. 9) the extended Kalman filter is given in equations (18) to (22) with q defined by equation (17)

$$\underline{\hat{y}}\left(t_{k+1} \mid t_k\right) = \underline{\hat{y}}\left(t_k \mid t_k\right) + \int_{t_k}^{t_{k+1}} \underline{f}\left[\underline{\hat{y}}\left(t_k \mid t_k\right),\ \hat{y}\left(t_{k-q} \mid t_{k-q}\right),\ D\left(t_{k-q}\right),\ t\right] dt \tag{18}$$

$$P\left(t_{k+1} \mid t_k\right) = \phi\left(t_{k+1}, t_k\right)\, P\left(t_k \mid t_k\right)\, \phi^T\left(t_{k+1}, t_k\right) + Q\left(t_{k+1}\right) \tag{19}$$

$$K\left(t_{k+1}\right) = P\left(t_{k+1} \mid t_k\right)\, H^T\left(t_{k+1}\right) \left[H\left(t_{k+1}\right)\, P\left(t_{k+1} \mid t_k\right)\, H^T\left(t_{k+1}\right) - R\left(t_{k+1}\right)\right]^{-1} \tag{20}$$

$$\underline{\hat{y}}\left(t_{k+1} \mid t_{k+1}\right) = \underline{\hat{y}}\left(t_{k+1} \mid t_k\right) + K\left(t_{k+1}\right) \left[\underline{m}\left(t_{k+1}\right) - H\left(t_{k+1}\right)\, \underline{\hat{y}}\left(t_{k+1} \mid t_k\right)\right] \tag{21}$$

$$\begin{aligned} P\left(t_{k+1} \mid t_{k+1}\right) = {} & \left[I - K\left(t_{k+1}\right)\, H\left(t_{k+1}\right)\right]\, P\left(t_{k+1} \mid t_k\right) \left[I - K\left(t_{k+1}\right)\, H\left(t_{k+1}\right)\right]^T \\ & + K\left(t_{k+1}\right)\, R\left(t_{k+1}\right)\, K^T\left(t_{k+1}\right) \end{aligned} \tag{22}$$

In these equations, $\underline{\hat{y}}\left(t_{k+1} \mid t_k\right)$ is the best estimate of the state at t_{k+1} based on measurements through time t_k; $P\left(t_{k+1} \mid t_k\right)$ is the covariance matrix of $\underline{\hat{y}}\left(t_{k+1} \mid t_k\right)$ and $K\left(t_{k+1}\right)$ is the gain matrix. The matrix $\phi\left(t_{k+1}, t_k\right)$ is the state transition matrix which is obtained by integrating with respect to time the state equations (eq. (15)) differentiated with respect to the state variables. The matrices $Q\left(t_{k+1}\right)$ and $R\left(t_{k+1}\right)$ are the diagonal covariance matrices of the process and measurement noise, respectively.

The effectiveness of the filter equations in estimating the state variables can be determined by examining the diagonal elements of the covariance matrix $P\left(t_k \mid t_k\right)$, since these elements are the variances of the estimated state variables. Further, the sum of the squares of the measurement residuals may be treated as a cost function in order to measure the overall effectiveness of the filtering process.

RESULTS FROM COMPUTER-GENERATED DATA

In order to observe the behavior of the pilot-model state equations and to uncover difficulties associated with applying the extended Kalman filter to these state equations, the Kalman filter was first applied to data generated with the differential-difference equations.

All of the computations required for this study, including the generation of the simulated data and the filtering of both the simulated and experimental data, were performed on the Control Data series 6000 series computers at Langley Research Center.

In generating the data, reasonable initial conditions for the state variables and parameters were chosen; the state equations were numerically integrated to obtain a "true" time history of the variables. The "true" variables were then used to calculate a series of measurements containing an additive random noise of zero mean and a variance of 0.00092 volt^2. For the simulated data presented here, the static gain K was 10 seconds^{-2}, and the variance of the process noise was 59 600 volts^2/seconds^4. The value of 59 600 volts^2/seconds^4 for the process noise was obtained experimentally so that the ratio of the mean-squared pilot remnant to the mean-squared stick motion is about 0.5, as is indicated by current pilot-model theory (ref. 5). The parameter values along with the initial values of the variables are given in the first column of table I; for these values, the system is stable. (The first-order Padé approximation is stable.)

The disturbing function $D(t)$ was chosen to be $0.25 \sin t$. The state integrated to the final time of 10.25 seconds is found in the first column of table II. Using these data, measurements containing the additive noise were generated at the rate of 100 measurements per second.

The values used to initiate the estimation procedure are given in the second column of table I. The parameter values are simply guesses; the variable values were mainly derived from the data according to the following scheme: the second measurement (the plant output $c(t)$) gives a direct measure of the first variable; since the first measurement (the stick motion $\delta(t)$) is the sum of the variables $u(t)$ and $\eta(t)$, the variable $u(t)$ was arbitrarily chosen to be zero and $\eta(t)$ evaluated with the measurement value; the second variable $\dot{c}(t)$ is evaluated by computing the first-order divided difference of the plant output $c(t)$ based on two consecutive values of $c(t)$; the fourth variable was arbitrarily set to zero and the sixth evaluated by using the first-order divided difference of the measured stick motion based on consecutive values of $\delta(t)$.

For the parameter variances, each standard deviation was set equal to 10 percent of the corresponding parameter value. Since the plant output is both a state variable and a measurement, it is reasonable to choose the initial variance of this variable to be equal to the variance of the corresponding measurement (0.00092 volt^2). Since the stick motion is the sum of the deterministic pilot output and the pilot remnant, it is true that for the variances

$$E\left\{[u(t) + \eta(t)]^2\right\} = E\left\{[\delta(t)]^2\right\} = 0.00092 \text{ volt}^2 \tag{23}$$

However, expanding the left expectation yields

$$E\left\{[u(t) + \eta(t)]^2\right\} = E\left\{u^2(t) + 2u(t)\,\eta(t) + \eta^2(t)\right\}$$

$$= E\left\{u^2(t)\right\} + 2E\left\{u(t)\,\eta(t)\right\} + E\left\{\eta^2(t)\right\} \qquad (24)$$

where in the last step the linearity of the expectation has been used. If, to simplify the analysis, it is assumed the two variables are uncorrelated, then the second term on the right in equation (24) is zero. Finally, by assuming the two variable variances are equal and by using equation (23), the result is that the initial variances for $u(t)$ and $\eta(t)$ are 0.00046 volt^2. The remaining variances were arbitrarily chosen to be 0.1.

The results given in table II as case 1 indicate that $c(t)$ and $\dot{c}(t)$ have been well estimated but that the remaining variables are in error by at least 8 percent. The estimates of $u(t)$ and $\eta(t)$ illustrate that although the error in the stick motion (which is the sum of these two variables) is small, the errors in the individual estimates may be large offsetting values. Further, an examination of the correlation coefficients, which can be derived by normalizing the covariance matrix, indicated that the correlation coefficient for $u(t)$ and $\eta(t)$ is -0.991. On the other hand, the error in $y_4(t)$ is quite large; this large error presumably results from defining $y_4(t)$ in terms of $u(t)$ (see eqs. (6)) which contains a 46-percent error. Finally, the variable $\dot{\eta}(t)$ is poorly estimated since in the state equations this variable is a function of $\eta(t)$ and of the two poorly estimated parameters α_1 and α_2.

An examination of the state equations (eqs. (7) to (12)) shows potential difficulties in estimating the parameters. These difficulties are verified by the case 1 results. First, the pilot static gain K_1 always occurs in combination with τ_1 and τ_2. Although these latter two parameters occur separately elsewhere, the manner in which K_1 occurs suggests difficulty in separating the three parameters. The case 1 results show that the estimates of these three parameters are less accurate than the initial guesses; the results also indicate that separation of these parameters may be difficult. Also, the delayed term coefficients $\frac{K_1\tau_3}{\tau_1\tau_2}$ and $\frac{K_1}{\tau_1}\left(1 - \frac{\tau_3}{\tau_1}\right)$ are less accurate than the initial guess as a result of the poor estimates obtained for the parameters.

In the case of time delay θ the estimate has changed only slightly, possibly because of the poor estimates obtained for the coefficients of the delayed terms in equations (9) and (10). There is also only a slight change in α_2. The poor estimate obtained for α_1 is evidently because of the poor values obtained for $\eta(t)$ and $\dot{\eta}(t)$; all these values occur in equation (12).

In table III the variable and parameter standard deviations and the root-mean-squared (rms) measurement residuals are presented. For case 1 the standard deviations for α_1 and α_2 have changed only slightly from the initial values (0.31623). This fact, along with the small changes in the estimates of α_1 and α_2, suggests that these parameters are not "observable" in this particular problem; that is, it is not possible to determine the values of these parameters by observing the behavior of the measurements. However, actual proof of observability in nonlinear problems is difficult to obtain. Instead, a separate case, identical to case 1 except that α_1 and α_2 were not estimated, was attempted. The resulting estimates for all the other variables and parameters were not significantly different (less than 0.8 percent) from those given for case 1 in table II; thus, retaining α_1 and α_2 as constant unestimated but erroneous values does not significantly bias the other estimates. Therefore, α_1 and α_2 need not be estimated; however, in all the cases studied these parameters were estimated in order to retain any influence they may have exerted on the other estimates.

One other notable item given in table III for case 1 is the rms residual for the stick motion $\delta(t)$. The rms residual should be approximately equal to the standard deviation of the noise added to this measurement (0.03033 volt). The smaller value obtained by the Kalman filter is undoubtedly a result of the fact that this measurement is the sum of the variables $u(t)$ and $\eta(t)$. Since these variables have nearly equal variances and a large negative correlation, the correlation tends to produce the small measurement variance. This tendency is clearly seen in the following relationship where Var(x) = Variance of x (any variable) and p is the correlation coefficient:

$$\mathrm{Var}[u(t) + \eta(t)] = \mathrm{Var}[u(t)] + \mathrm{Var}[\eta(t)] + 2p\left\{\mathrm{Var}[u(t)]\ \mathrm{Var}[\eta(t)]\right\}^{1/2}$$

Another possible reason for the poor parameter estimates is the standard deviation of $\dot{\eta}(t)$ which is much larger than the standard deviations of the parameters; this relationship is seen in table III where the standard deviations at $t = 10.25$ are given. The large standard deviation for $\dot{\eta}(t)$ is a direct result of the large process-noise standard deviation. Since the variance of $\dot{\eta}(t)$ is so large, this variable seems to absorb too much of the stick-motion residual. In order to test this possibility the initial parameter standard deviations were increased to 100 percent of the parameter values in case 2.

The results of case 2 in table II indicate some improvement in the estimates of K_1, τ_2, τ_3, and α_1; however, the estimates of τ_1, θ, and α_2 are less accurate. Further, the coefficients of the delayed terms are not significantly better, evidently because of the degraded time-delay estimate. Therefore, increasing the parameter variances generally did not improve the estimates.

Additional experiments in choosing the variances were attempted. None of the experiments resulted in generally good estimates. Although using a smaller process-noise variance produces a smaller mean-squared pilot-remnant to mean-squared stick-motion ratio, the remaining simulated cases discussed here use data generated with a smaller process-noise variance. In this manner the effect of a smaller variance on the estimates was investigated. Except for this smaller variance, the initial parameter and variable values are identical to those used before. For case 3 the process-noise variance was chosen to be 576 volts^2/seconds^4; compared to the previous data, the ratio of the mean-squared pilot remnant to the mean-squared stick motion for the new data is only 0.04.

The results given in tables II and III as case 3 were produced by using 10 percent of the parameter values for the initial standard deviation, extending the time interval for processing the measurements to 15 seconds, and iterating the filter process three times over this interval. Of the various attempts at filtering this data, case 3 gave the best estimates of the parameters. The variable estimates are not given in table II since a change in the process-noise variance significantly changed the time histories of the variables. However, the estimates of all the variables are better than in case 1 or 2; for example, the error in u(t) is only 10 percent. The estimates of K_1, τ_1, τ_3, and α_1 are better than the previous two cases. Although the estimate of τ_2 is poor, the improvement in the other three deterministic pilot parameters resulted in much better estimates of the delayed term coefficients. The degraded estimate of τ_2 emphasizes the apparent difficulty in separating these four parameters.

The small changes in θ and α_2 suggest that these variables are not observable for some reason. An "observable" parameter here means that a change in the parameter results in an observable change in one or more of the measured quantities. The use of large parameter variances in a case not presented here did produce a significant change in the estimated time delay thus indicating that θ is to some extent observable. However, for the same case, the estimate of α_2 was not significantly different from the value given in case 3. More significant, perhaps, was the fact that the resulting parameter estimates yielded an unstable system. As a final test of the observability of θ, a new set of measurements was generated; this set differed from the data for case 3 in that only the input disturbance D(t) was changed. For the new data, D(t) was composed of two sine functions having distinct amplitudes and frequencies. The purpose of this test was to determine if the observability of θ depends on the characteristics of the disturbing function. The time delay estimated from the new data (0.0402) differed only slightly from the case 3 estimate. Thus, the ability to estimate θ depends on other factors. The observability of α_2 is questionable.

Figures 2 and 3 present the time histories of the stick-motion and plant-output residuals for case 3. Although all the parameters are not correctly estimated, these figures indicate that the measurement residuals are generally random as is desired. In the case of

the plant output, the residuals in figure 3 are of the same magnitude as the noise; this finding is further exemplified by the rms residual given in table III. In addition, figure 2 and the rms stick-motion residual in table III illustrate the fact that the estimates of $\eta(t)$ and $u(t)$ contain a significant portion of the measurement noise.

For the final case presented, the process-noise variance was reduced to 25 volts^2/seconds^4. The object was to process data with process noise comparable to that of the experimental data, even though the ratio of the average pilot remnant to the average stick motion is less than 0.01. Other than this smaller variance, all the parameter and variable values and the measurement-noise variance are the same as in the previous cases. The disturbing function is the same function used in case 3. The estimates given in table II as case 4 are the result of iterating the filtering process three times. Since this estimation problem is nonlinear, it is reasonable to iterate the estimation process in order to obtain satisfactory parameter estimates. In this context "iteration" means using parameter estimates and the corresponding variances at the final time of one pass through the filter equations as initial values for the next pass through the filter.

The results of case 4 show improvement in the estimates of K_1, τ_1, τ_3, θ, and α_1. In particular, the much better estimate of θ indicates that the ability to estimate this parameter is mainly related to the magnitude of the process noise and not to the disturbing function. Further, the improved estimate of α_1 is also evidently a result of the smaller process noise, whereas the slight degradation of the α_2 estimate suggests this parameter is unobservable. The unobservability is further confirmed by the fact that α_2 has the smallest correlations of all the parameters (specifically, the correlations between the parameters and the three variables ($c(t)$, $u(t)$, and $\eta(t)$) defining the measurements). The degraded estimate of τ_2 indicates the apparent difficulty in separating the four deterministic pilot parameters. Even with this poor estimate, the two resulting delayed-term coefficients are generally good.

Comparison of the standard deviations in table III with the estimation errors derived from the estimates in table II provides a means for determining the ability of the standard deviations to predict the actual variable and parameter errors. In almost every instance the estimate errors are less than three standard deviations; in case 4, which is of major interest here, over half of the estimate errors are less than one standard deviation. The one exception to this statement is the parameter τ_2 which was poorly estimated. Therefore, the standard deviations give a good measure of the errors in the estimates.

Based on the results obtained by processing simulated data, certain conclusions have been reached. Reasonably accurate estimates of the variables $c(t)$, $u(t)$, and $\eta(t)$ may be obtained, although separation of the stick motion $\delta(t)$ into good estimates of $u(t)$ and $\eta(t)$ may be difficult. If the process noise is approximately the same order of magnitude as the pilot-remnant parameters α_1 and α_2, then good estimates of all the

parameters (except α_2 and possibly one of the deterministic pilot-model parameters) may be obtained. Further, the ability to estimate the time delay depends on the magnitude of the process noise and not on the disturbing function. Finally, the standard deviations provided by the filter give a good measure of the estimate errors.

RESULTS BASED ON EXPERIMENTAL DATA

The experimental data used in this analysis were obtained from closed-loop compensatory tracking tasks conducted by Langley Research Center personnel with engineers and test pilots used as subjects. A description of this research has been reported by Adams (ref. 5); in the particular cases discussed here, an engineer was the subject.

The variances of the measurements used in this experiment were obtained by setting the disturbance $D(t)$ to zero, removing the pilot from the loop, and recording the measurements (which therefore should consist only of noise). The resulting value for both measurement variances was 0.00092 volt^2. The variance of the process noise was obtained by a power spectral-density analysis of a model of the pilot remnant used by Adams (ref. 5). The resulting variance values were 19.8 volts^2/seconds^4 and 5.9 volts^2/seconds^4, respectively, for the first and second sets of experimental data presented here. The same engineer was the subject for both experiments,

In order to obtain initial conditions for the estimation procedure a modification of the method presented in the previous section was used. The variables $c(t)$ and $\dot{c}(t)$ were evaluated from the measurements as previously described. However, the pilot remnant was evaluated using a mathematical model given by Adams (ref. 5). This value, along with the measured stick motion, provides the initial value for the deterministic pilot output $u(t)$. The derivative of the pilot remnant is obtained by numerically differencing pilot-remnant values at consecutive points; the variable $y_4(t)$ is evaluated from equations (6). The parameter values are based on those given by Adams (ref. 5) and yield a stable system. For the variable variances, the same initial values used for case 1 of the simulated data were employed. Thus, the variances for the variables $c(t)$, $u(t)$, and $\eta(t)$ were 0.00092 volt^2, 0.00046 volt^2, and 0.00046 volt^2, respectively; for the remaining variables, the value 0.1 was used. For the parameter variances, each standard deviation was set to 10 percent of the corresponding parameter value. The initial conditions for the first set of experimental data are given in column one of table IV. The estimates obtained after processing 1500 data points may be found as case 1 in the second column of table IV. The statistical parameters are in column one of table V. Most of the parameter estimates differ considerably from the corresponding initial values, particularly the negative values of K_1, α_1, and α_2. Further, these parameters represent an unstable system. The most revealing information is contained in the rms residual of the plant output, which is almost two orders of magnitude

larger than the known measurement noise standard deviation (0.0303 volt). The time histories of the stick-motion and plant-output residuals are given in figures 4 and 5. The behavior of the residuals in figure 5 suggests that the plant-output errors contain a signal as a result of errors in the mathematical model. Only the plant-output residuals are consistently larger than the measurement standard deviation whereas the corresponding variable c(t) standard deviation is small (0.0063 in table V). Therefore, it appears that the Kalman gain on this variable initially decreased rapidly as the measurement residuals decreased; the gain could not increase to compensate for later large measurement residuals. Since this phenomenon did not arise during the processing of the computer-generated data, the divergence of the plant-output residuals is a result either of processing very accurate measurements for a short period or of the presence of an unknown process noise (ref. 11). This situation is further aggravated by model biases caused by incorrect parameter values. The net result is an adjustment of the parameters; such adjustment continues until the system becomes unstable.

As indicated by reference 11, all of the possible causes of the divergence may be treated by assuming the existence of a process noise of appropriate magnitude. The adaptive technique described in the appendix was chosen to calculate automatically the required process-noise statistics. Application of the original adaptive technique to this problem proved satisfactory. However, since the stick-motion residuals did not diverge in case 1, the modified technique described in the appendix was found to be sufficient to treat the plant-output divergence. In the modified technique only excessive plant-output residuals (that is, those exceeding the one-sigma value of 0.0303 volt) are used to adjust directly the Kalman filter gain for only the plant-output variable c(t). Changes in other gains and variables are the direct result of a normal application of the Kalman filter. In this way the modified technique produces a compromise between the results of the Kalman filter and the original adaptive technique.

The results of applying the modified technique are given as case 2 in tables IV and V. The rms plant-output residual is approximately the correct value while the rms stick-motion residual is little changed from that of case 1. Comparison of figures 4 and 5 with figures 6 and 7 further illustrates that only the plant output is significantly affected and that these residuals are more random. The new parameter estimates are clearly more reasonable than those in case 1. The system evaluated with these estimates is also stable. Table V indicates that by increasing the Kalman gain on only the plant output, many of the standard deviations have also increased to account for the uncertainty introduced into the modified estimation procedure by the fictitious process noise.

Since the study using computer-generated noise indicated that iterating the Kalman filter over the data improved the parameter estimates, this iterative approach was also applied to the experimental data. The case 2 parameter estimates and variances are used to start the second iteration of the filter equations at 5.3 seconds. In the tables, case 3 gives the

estimates at the end of the third iteration. Although K_1, τ_2, τ_3, θ, and α_1 appear to be converging to constant values, τ_1 appears to be decreasing rapidly. For this particular problem all the parameters should be positive. The poor estimate of τ_1 undoubtedly results from the difficulty in separating the deterministic pilot-model parameters; a similar difficulty was observed in the case of the simulated data. Also, as seen previously, the observability of α_2 is highly questionable; in case 3, α_2 appears to be converging to a large positive number at best. The stability of case 3 is verified by the eigenvalues in table VI.

Since the initial parameter values used in cases 1 to 3 should be good starting values for the estimation process, the only quantities which can be adjusted to improve the estimates of case 3 are the initial parameter variances. With this idea in mind, iteration of the Kalman filter was reinitiated using the original parameter values and new standard deviations equal to 3.2 percent of the parameter values. The results after three iterations are given as case 4 in tables IV and V.

All of the parameters except α_2 have essentially converged to constants in case 4; a few additional iterations would only refine the values in the third or fourth place. For this case, all the parameters including τ_1 are reasonable; the smaller initial variances have reduced the tendency of any one parameter to diverge. Even α_2 appears that it would converge to a more reasonable, although large, value if enough iterations were attempted. As in the previous cases, the system evaluated with these parameter values is stable; the system eigenvalues are given in table VI.

Examination of the correlations for case 4 shows that $u(t)$ and $\eta(t)$ have the expected large negative correlation (-0.97) and that only the correlations between $u(t)$ and $y_4(t)$ (0.85), between $y_4(t)$ and $\eta(t)$ (-0.82), and between $u(t)$ and τ_3 (-0.81) are very significant. The $\left(u(t),y_4(t)\right)$ and $\left(u(t),\tau_3\right)$ correlations are induced by equation (9); the $\left(y_4(t),\eta(t)\right)$ correlation is a result of the $(u(t),\eta(t))$ and $\left(u(t),y_4(t)\right)$ correlations.

Comparison of the parameter standard deviations for cases 3 and 4 verifies the convergence of case 4. Since the initial standard deviations of cases 3 and 4 are, respectively, 10 percent and 3.2 percent of the initial parameter values, the case 4 initial standard deviations are 32 percent of those of case 3. However, from table V, the parameter standard deviations of case 4 are 35 percent to 73 percent of those of case 3. The smaller standard deviations suggest that the standard deviations of case 4 are attaining steady values which are bounded below by the Cramer-Rao bound. This fact was further confirmed by examination of the behavior of the standard deviations on the third iteration; during this iteration the standard deviations changed much less than on previous iterations.

The second set of experimental data was obtained with the same engineer as test subject; essentially the same closed-loop task was performed but with a different disturbing function. The measurement variances are identical to those for the first experiment; however,

since the disturbing function was different, the process-noise variance of the pilot remnant changed to 5.9 volts^2/seconds^4. The initial conditions for the estimation problem, given in table VII, were obtained from the measured data in the same manner as they were computed for the first experiment. The state variances for each case are identical to the variances chosen for the corresponding case of the first experiment. The initial parameters, which can be found in reference 5, yield a stable system. The initial parameter standard deviations are 10 percent of the parameter values.

The results of case 1, in which the unmodified extended Kalman filter was applied to the data, are given in table VII and in figures 8 and 9. As with the previous data set, the plant-output residuals are excessively large and appear to contain a signal. This situation is apparently caused by the same errors that occurred in the case of the first experimental data set. Because of the erroneous parameter estimates, particularly the estimate of K_1, the system has become unstable.

Application of the complete adaptive technique to the data again indicated the use of the modified adaptive technique. Therefore, in case 2 one iteration of the modified technique was used; this technique uses only the plant-output residuals and only the predicted variance of the plant output is directly altered. Figures 10 and 11 and the rms residuals in table VIII indicate that the divergence of the plant-output residuals has been corrected with little effect on the stick-motion residuals. Although the parameter estimates appear reasonable, the system of equations based on these estimates is unstable, and therefore inadequate to describe the physical system.

As in the case of the previous data, the approach to preventing the system from becoming unstable is to restrict the changes in the parameter estimates by reducing the variances. In case 3, therefore, 3.2 percent of the parameter values was used for the corresponding initial standard deviations. The parameters given in table VII are the result of two iterations of the Kalman filter. Although these values appear reasonable, the system actually became unstable during the course of the second iteration. (See table VI.) Thus, the estimation procedure was abandoned for this set of standard deviations.

In case 4, 1 percent of the parameter values was used for the initial standard deviations. The values in tables VII and VIII are the results obtained after four iterations of the Kalman filter. The system based on these values is stable. (See table VI.) With the exception of α_2 the estimates have essentially converged. Although the estimates and standard deviations of τ_1 and τ_2 are sufficiently similar to suggest these parameters are highly correlated, such is not the case. The two parameters have very similar, but extremely small, correlations with several other variables and parameters; thus, since the correlation coefficient for τ_1 and τ_2 is only -0.0043, the two parameters only appear to be correlated. With the exception of the usual high correlation between $u(t)$ and $\eta(t)$, the greatest correlations are between $u(t)$ and $y_4(t)$ (0.79) and between $u(t)$ and τ_3 (-0.86).

Therefore, the modified technique provides reasonable estimates of the state and of most of the parameters when applied to experimental data if a careful selection of the initial variances is made. In particular, reasonable estimates of the deterministic pilot-model parameters can be made.

CONCLUDING REMARKS

An attempt has been made to estimate the parameters of a human pilot model by using the extended Kalman filter. The chosen human pilot model, originally written in transfer function form, was transformed to a differential-difference equation in the time domain; in this form the pilot model was amenable to treatment by the extended Kalman filter, although the time delay occurs implicitly.

In studies utilizing computer-generated data, it was found that the deterministic pilot-model parameters, including the time delay, could be reasonably estimated if the magnitude of the process noise was sufficiently small, although the remnant to stick-motion ratio was realistically too small. The only difficulties appear to be separation of delayed term coefficients into component parameters and correct estimation of a parameter in the pilot remnant.

In the case of experimental data, it was found that the Kalman filter produced diverging plant-output residuals, apparently because of erroneous assumptions made on the mathematical model. Application of a modified adaptive technique was found to overcome this divergence problem. Careful selection of initial parameter variances produced reasonable estimates of the deterministic pilot-model parameters and maintained the stability of the system of equations.

Therefore, the extended Kalman filter is recommended as a technique for estimating the parameters of the dynamical plant model and of the deterministic pilot model in a human tracking-task situation.

Langley Research Center
National Aeronautics and Space Administration
Hampton, Va. 23665
July 9, 1975

APPENDIX

AN ADAPTIVE TECHNIQUE THAT COMPENSATES FOR MODEL ERRORS

In many estimation problems, errors in the state model arise from the use of simplified models or models containing erroneous parameter values. As a result of using such models, the estimation procedure fails to track the measurements properly and eventually diverges. Various techniques for alleviating this divergence phenomenon have been proposed (ref. 12). For the purposes of the current study a technique which could be easily automated was desired.

The technique selected for this research is basically the scheme presented by Jazwinski in references 9 and 13. The philosophy of this technique is based on the fact that as long as the measurement residuals do not exceed the one standard deviation level, the Kalman filter is not altered. However, should the residuals exceed this level, a fictitious plant noise is computed from the measurement residuals. The fictitious noise is calculated in such a fashion that as the measurement residuals increase, the fictitious plant noise increases. The effect of the fictitious noise is to increase the predicted covariance matrix of equation (19) and, consequently, the filter gain. Thus, as the measurement residuals increase, the gain increases to "open" the filter. (That is, the filter uses more of the residuals to correct the predicted state.) If the residuals decrease to less than one standard deviation, the fictitious noise is not computed and the covariance matrix and gains reduce.

With the use of this technique, it is necessary to rewrite two of the filter equations. Equation (18) is replaced by

$$\underline{\hat{y}}\left(t_{k+1}\middle|t_k\right) = \underline{\hat{y}}\left(t_k\middle|t_k\right) + \int_{t_k}^{t_{k+1}} \underline{f}\left[\underline{\hat{y}}\left(t_k H_k\right),\ \underline{\hat{y}}\left(t_{k-q}\middle|t_{k-q}\right),\ D\left(t_{k-q}\right),\ t\right]dt + \Gamma(k+1)\, w(k+1) \tag{A1}$$

where $\Gamma(k+1)$ is a 13×1 coefficient matrix and $w(k+1)$ is a zero mean Gaussian sequence with variance

$$E\left\{w(k)\, w(j)\right\} = \bar{q}(k)\delta_{kj}$$

With this change, equation (19) becomes

$$P\left(t_{k+1}\middle|t_k\right) = \phi\left(t_{k+1},t_k\right) P\left(t_k\middle|t_k\right) \phi^T\left(t_{k+1},t_k\right) + Q\left(t_{k+1}\right) + \bar{q}(k+1)\, \Gamma(k+1)\, \Gamma^T(k+1) \tag{A2}$$

APPENDIX

The key to the adaptive process is to select the variance $\bar{q}(k+1)$ (eq. (A2)) which reflects the previously stated philosophy. This selection is accomplished by estimating $\bar{q}(k+1)$ from the measurement residuals; then $\bar{q}(k+1)$ can be averaged over several measurement times to provide a statistically significant value.

The variance $\bar{q}(k+1)$ is estimated by maximizing the probability-density function of the residual with respect to $\bar{q}(k+1)$; details of the derivation may be found in reference 9. The result of this derivation is that

$$\bar{q}(k+1) = \begin{cases} \frac{1}{d}\left[\bar{\Gamma}^2(k+1) - E\left\{\bar{\Gamma}^2(k+1) \,\middle|\, \bar{q}(k) = 0\right\}\right] & \text{(If positive)} \\ 0 & \text{(Otherwise)} \end{cases} \tag{A3}$$

where

$$\bar{\Gamma}(k+1) = \frac{1}{N}\sum_{j=1}^{N} \frac{\left\{m_j(t_{k+1}) - \left[H(t_{k+1})\ \hat{y}(t_{k+1}|t_k)\right]_j\right\}}{\left[R_{jj}(t_{k+1})\right]^{1/2}} \tag{A4}$$

is the average normalized predicted residual at t_{k+1};

$$E\left\{\Gamma^2(k+1) \,\middle|\, \bar{q}(k) = 0\right\} = \frac{1}{N} + \frac{1}{N^2}\sum_{j=1}^{N}\sum_{i=1}^{N} \frac{H_j(t_{k+1})\ P(t_{k+1}|t_k)\ H_j^T(t_{k+1})}{\left[R_{jj}(t_{k+1})\right]^{1/2}\left[R_{ii}(t_{k+1})\right]^{1/2}} \tag{A5}$$

and

$$d = \frac{1}{N^2}\sum_{j=1}^{N}\sum_{i=1}^{N} \frac{H_j(t_{k+1})\ \Gamma^T(k+1)\ \Gamma(k+1)\ H_i^T(t_{k+1})}{\left[R_{jj}(t_{k+1})\right]^{1/2}\left[R_{ii}(t_{k+1})\right]^{1/2}} \tag{A6}$$

In equations (A4) to (A5), $m_j(t_{k+1})$ and $\left[H(t_{k+1})\ \hat{y}(t_{k+1}|t_k)\right]_j$ indicate the jth component of the corresponding vectors, $R_{jj}(t_{k+1})$ and $R_{ii}(t_{k+1})$ are the jth and ith diagonal elements of $R(t_{k+1})$, and $H_j(t_{k+1})$, and $H_i(t_{k+1})$ are the jth and ith rows of $H(t_{k+1})$. In these equations, N is the number of measurements at time t_{k+1}; for the pilot-model problem, N = 2. The variance $\bar{q}(k+1)$ was then averaged over the latest 20 time points for this particular application.

The major difficulty in applying this adaptive technique to a particular problem is in engineering an appropriate coefficient matrix $\Gamma(k+1)$. Experimentation with the pilot model indicated the vector

$$\Gamma(k+1) = [0.707,\ 0,\ 0.5,\ 0,\ 0.5,\ 0,\ 0,\ \ldots,\ 0]^T$$

provided adequate results. However, since the results of case 1 based on the unmodified Kalman filter showed that only the plant-output residuals diverged, the use of the stick-motion residuals in the adaptive technique and the modification of the corresponding gains (i.e., the gains for the deterministic pilot output and the pilot remnant) should be unnecessary. Therefore, the technique was modified to use only the plant-output residuals; the coefficient matrix $\Gamma(k + 1)$ was altered to influence only the plant-output estimate. The former modification requires restricting the summations in equations (A4) to (A6) to only those terms involving the plant output; the latter was accomplished by defining $\Gamma(k + 1)$ by $\Gamma(k + 1) = [0.707, 0, 0, 0, 0, \ldots, 0]^T$. These modifications were used in cases 2 to 4 for both sets of experimental data.

REFERENCES

1. McRuer, Duane T.; and Jex, Henry R.: A Review of Quasi-Linear Pilot Models. IEEE Trans. Human Factors in Electron., vol. HFE-8, Sept. 1967, pp. 231-249.
2. Irving, Gary W.: Adaptive and Optimizing Models of the Human Operator in Manual Control Systems – A Review and Unification. J. Syst. Eng., vol. 1, no. 2, Apr. 1969, pp. 151-195.
3. Adams, James J.; and Bergeron, Hugh P.: Measurements of Human Transfer Function With Various Model Forms. NASA TN D-2394, 1964.
4. Magdaleno, R. E.; and McRuer, D. T.: Experimental Validation and Analytical Elaboration for Models of the Pilot's Neuromuscular Subsystem in Tracking Tasks. NASA CR-1757, 1971.
5. Adams, James J.; and Bergeron, Hugh P.: A Synthesis of Human Response in Closed-Loop Tracking Tasks. NASA TN D-4842, 1968.
6. Koivo, A. J.; and Stoller, R. L.: On Least Squares Estimation in Nonlinear Dynamical Systems With Time Delay. Preprint of Technical Papers, Ninth Joint Automatic Control Conference (Ann Arbor, Mich.), June 1968, pp. 116-122.
7. Stoller, R. L.; and Koivo, A. J.: Estimation in Nonlinear Systems With Transport Delay. Paper No. W9-1, 1971 IEEE Conference on Decision & Control, Dec. 1971, pp. 149-153.
8. Kwakernaak, H: Optimal Filtering in Linear Systems With Time Delays. IEEE Trans. Automat. Contr., vol. AC-12, no. 2, Apr. 1967, pp. 169-173.
9. Jazwinski, Andrew H.: Stochastic Processes and Filtering Theory. Academic Press, Inc., 1970.
10. Sage, Andrew P.; and Melsa, James L.: System Identification. Academic Press, Inc., 1971.
11. Fitzgerald, Robert J.: Divergence of the Kalman Filter. IEEE Trans. Automat. Contr., vol. AC-16, no. 6, Dec. 1971, pp. 736-747.
12. Leondes, Cornelius T.; and Pearson, Jack O.: Kalman Filtering of Systems With Parameter Uncertainties – A Survey. Int. J. Control, vol. 17, no. 4, Apr. 1973, pp. 785-801.
13. Jazwinski, Andrew H.; and Hipkins, Conrad: J-Adaptive Estimation With Estimated Noise Statistics. NASA CR-132886, 1973.

TABLE I.- INITIAL CONDITIONS FOR SIMULATED CASES

Quantity	True value (t = 0.25)	Initial estimate (t = 0.25)
c(t), V	-0.9590	-0.9496
$\dot{c}$(t), V/sec	-5.6363	-.5900
u(t), V	.6237	0
y_4(t), V	-.6880	0
η(t), V	.4105	1.0109
$\dot{\eta}$(t), V/sec	2.5484	2.48
K_1	1.0	1.1
τ_1	.1	.09
τ_2	.1	.12
τ_3	.5	.6
θ	.05	.04
α_1	4.427	4.0
α_2	10.0	10.5
$\frac{K_1\tau_3}{\tau_1\tau_2}$	50.0	61.1
$\frac{K_1}{\tau_1}\left(1 - \frac{\tau_3}{\tau_1}\right)$	-40.0	-69.25

TABLE II.- ESTIMATION RESULTS BASED ON SIMULATED DATA

Quantity	True value (t = 10.25)	Case 1 estimates (t = 10.25)	Case 2 estimates (t = 10.25)	Case 3 estimates (t = 15.25)	Case 4 estimates (t = 15.25)
c(t), V	-1.0692	-1.0602	-1.0602	------	------
$\dot{c}$(t), V/sec	-3.0421	-3.0225	-3.0223	------	------
u(t), V	2.2984	3.3659	3.2007	------	------
y_4(t), V	-.9398	-2.4520	-2.8760	------	------
η(t), V	.0734	-.9941	-.8289	------	------
$\dot{\eta}$(t), V/sec	10.9367	11.8296	12.5505	------	------
K_1	1.0	1.1425	1.1415	1.0488	0.9843
τ_1	.1	.0865	.0831	.0936	.0954
τ_2	.1	.1154	.1060	.1249	.1435
τ_3	.5	.6245	.5782	.5720	.5282
θ	.05	.0399	.0265	.0401	.0435
α_1	4.427	4.0014	4.0650	4.0980	4.3534
α_2	10.0	10.4644	7.6231	10.4935	10.5491
$\frac{K_1\tau_3}{\tau_1\tau_2}$	50.0	71.48	74.93	51.32	37.98
$\frac{K_1}{\tau_1}\left(1 - \frac{\tau_3}{\tau_1}\right)$	-40.0	-82.15	-81.84	-57.27	-46.81

TABLE III.- STANDARD DEVIATIONS AND ROOT-MEAN-SQUARE (rms) MEASUREMENT RESIDUALS OF SIMULATED-DATA CASES

Quantity	Case 1	Case 2	Case 3	Case 4
Standard deviation of $c(t)$	0.0063	0.0063	0.0063	0.0063
Standard deviation of $\dot{c}(t)$	.0201	.0201	.0201	.0201
Standard deviation of $u(t)$	.3173	2.5077	.0580	.1604
Standard deviation of $y_4(t)$	.5370	4.9018	.1584	.4374
Standard deviation of $\eta(t)$	.3187	2.5079	.0677	.1628
Standard deviation of $\dot{\eta}(t)$	240.04	240.01	24.3396	5.7711
Standard deviation of K_1	.0969	.8868	.0966	.0648
Standard deviation of τ_1	.0081	.0753	.0082	.0063
Standard deviation of τ_2	.0106	.1003	.0108	.0100
Standard deviation of τ_3	.0532	.5038	.0530	.0368
Standard deviation of θ	.0040	.0353	.0040	.0056
Standard deviation of α_1	.3999	3.9432	.3999	.3983
Standard deviation of α_2	1.0484	10.2003	1.0488	1.0475
rms residual, stick motion	.00023	.00002	.00101	.00654
rms residual, plant output	.0320	.0320	.0317	.0240

TABLE IV.- RESULTS BASED ON FIRST EXPERIMENTAL-DATA SET

Quantity	Initial values (t = 5.3)	Case 1 estimates (t = 20.3)	Case 2 estimates (t = 20.3)	Case 3 estimates (t = 20.3)	Case 4 estimates (t = 20.3)
c(t), V	-0.5	-9.9201	-4.0711	-4.0711	-4.0709
$\dot{c}$(t), V/sec	-6.248	-8.4231	-20.9579	-21.0600	-20.9791
u(t), V	-.128	-.5683	-1.0824	-1.3206	-1.0170
$y_4(t)$, V	.05	-.5711	-3.2801	-5.3395	-3.5209
η(t), V	.1	2.7453	3.2530	3.4880	3.1870
$\dot{\eta}$(t), V/sec	1.5615	5.5469	15.5910	12.3238	17.2188
K_1	.8	-.1463	.5661	.4474	.6354
τ_1	.1	.3527	.0986	.0417	.1099
τ_2	.1	.1669	.1034	.1046	.1094
τ_3	.6	.0713	.4790	.4980	.4855
θ	.05	.0544	.0706	.0826	.0655
α_1	4.427	-.9218	3.4763	2.8634	3.9206
α_2	10.0	-36.755	12.6328	16.5106	10.9260

TABLE V.- STANDARD DEVIATIONS AND ROOT-MEAN-SQUARE (rms) MEASUREMENT RESIDUALS FOR FIRST EXPERIMENTAL-DATA SET

Quantity	Case 1	Case 2	Case 3	Case 4
Standard deviation of $c(t)$	0.0063	0.0297	0.0297	0.0297
Standard deviation of $\dot{c}(t)$	.0201	.1002	.0994	.1006
Standard deviation of $u(t)$	.1638	.2452	.2006	.1089
Standard deviation of $y_4(t)$	.1243	.4678	.5326	.2068
Standard deviation of $\eta(t)$	.1664	.2466	.2022	.1122
Standard deviation of $\dot{\eta}(t)$	5.2829	5.4000	5.3325	5.2471
Standard deviation of K_1	.0358	.0682	.0542	.0226
Standard deviation of τ_1	.0086	.0092	.0050	.0030
Standard deviation of τ_2	.0025	.0093	.0086	.0030
Standard deviation of τ_3	.0259	.0528	.0488	.0171
Standard deviation of θ	.0026	.0016	.0011	.0008
Standard deviation of α_1	.3475	.3290	.2772	.1235
Standard deviation of α_2	.9637	.9197	.8735	.3063
rms residual, stick motion	.0116	.0107	.0101	.0108
rms residual, plant output	2.8557	.0339	.0359	.0340

TABLE VI.- EIGENVALUES OF FIRST ORDER PADÉ MODEL FOR SELECTED EXPERIMENTAL-DATA CASES

First data set, case 3	First data set, case 4	Second data set, case 3	Second data set, case 4
(-4.754, 0)	(-29.363, 0)	(-31.952, 0)	(-36.575, 0)
(-0.635, 2.624)	(-15.780, 0)	(-14.438, 0)	(-15.615, 0)
(-0.635, -2.624)	(-3.191, 0)	(-4.577, 0)	(-4.159, 0)
(-1.432, 3.803)	(-0.229, 3.297)	(0.026, 2.973)	(-2.186, 2.310)
(-1.432, -3.803)	(-0.229, -3.297)	(0.026, -2.973)	(-2.186, -2.310)
(-25.861, 6.886)	(-1.960, 2.661)	(-2.116, 2.546)	(-0.130, 3.342)
(-25.861, -6.886)	(-1.960, -2.661)	(-2.116, -2.546)	(-0.130, -3.342)

TABLE VII.- RESULTS BASED ON SECOND EXPERIMENTAL-DATA SET

Quantity	Initial values (t = 5.0)	Case 1 estimates (t = 20.0)	Case 2 estimates (t = 20.0)	Case 3 estimates (t = 20.0)	Case 4 estimates (t = 20.0)
c(t), V	-0.0155	-2.4981	-1.5373	-1.5375	-1.5376
$\dot{c}$(t), V/sec	-4.6819	-2.2670	-4.2194	-4.2693	-4.3483
u(t), V	-.0811	-.1618	.0603	-.0003	-.0692
y_4(t), V	.3583	.3185	-.3228	-.5054	-.6790
η(t), V	.05	1.4936	1.2697	1.3307	1.4012
$\dot{\eta}$(t), V/sec	.0	3.8801	5.6356	5.9007	5.9616
K_1	.8	-.1364	.5488	.6854	.7629
τ_1	.1	.1068	.1132	.1095	.1034
τ_2	.1	.0850	.1147	.1094	.1035
τ_3	.4	.2930	.2944	.3438	.3818
θ	.05	.05568	.0646	.0613	.0536
α_1	4.427	4.3639	4.0958	4.2329	4.3721
α_2	10.0	.0199	13.9415	10.9624	10.1136

TABLE VIII.- STANDARD DEVIATIONS AND ROOT-MEAN-SQUARE (rms) MEASUREMENT RESIDUALS FOR SECOND EXPERIMENTAL-DATA SET

Quantity	Case 1	Case 2	Case 3	Case 4
Standard deviation of $c(t)$	0.0063	0.0246	0.0246	0.0247
Standard deviation of $\dot{c}(t)$	.0201	.0706	.0714	.0724
Standard deviation of $u(t)$	.0687	.0331	.0142	.0053
Standard deviation of $y_4(t)$	.1626	.0708	.0344	.0135
Standard deviation of $\eta(t)$	.0734	.0418	.0293	.0262
Standard deviation of $\dot{\eta}(t)$	3.1752	3.1645	3.1599	3.1580
Standard deviation of K_1	.0581	.0716	.0248	.0079
Standard deviation of τ_1	.0093	.0095	.0031	.0001
Standard deviation of τ_2	.0096	.0095	.0031	.0001
Standard deviation of τ_3	.0373	.0366	.0124	.0040
Standard deviation of θ	.0035	.0024	.0013	.0004
Standard deviation of α_1	.3357	.3174	.1332	.0436
Standard deviation of α_2	.9587	.9337	.3136	.0997
rms residual, stick motion	.0164	.0163	.0165	.0169
rms residual, plant output	1.3194	.0373	.0368	.0360

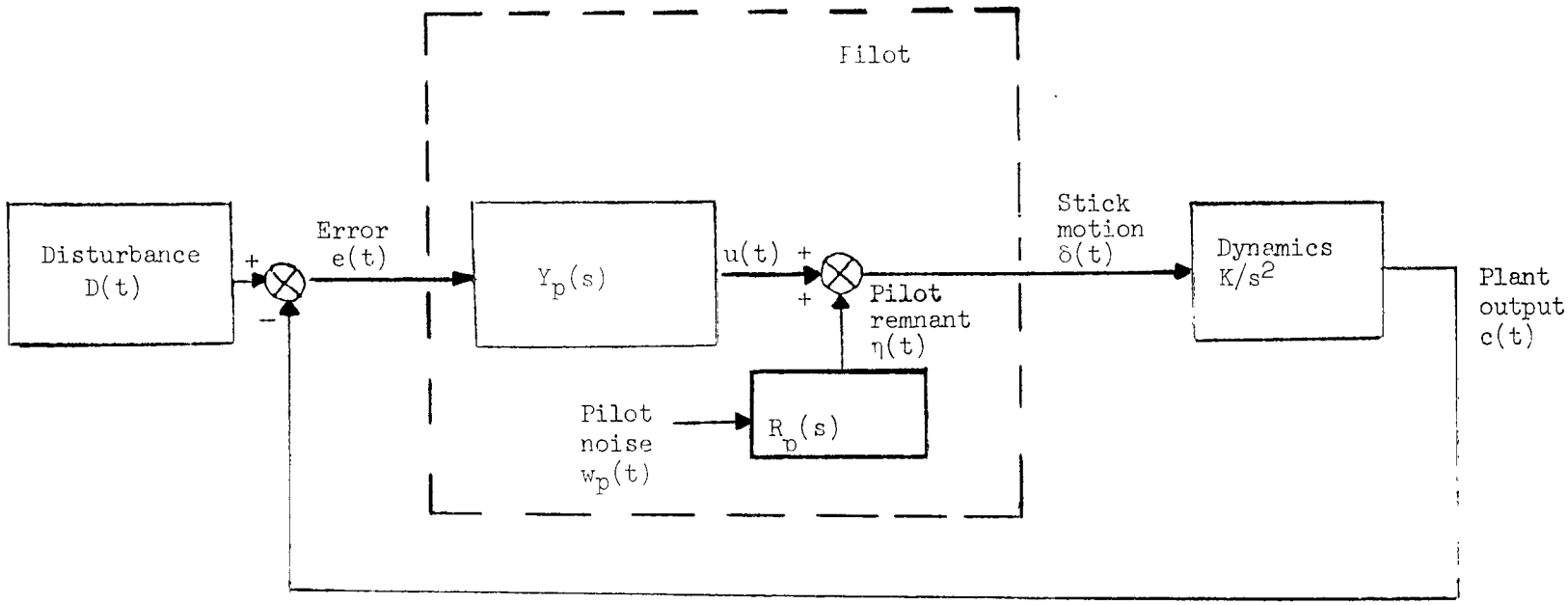


Figure 1.- Block diagram of pilot dynamics.

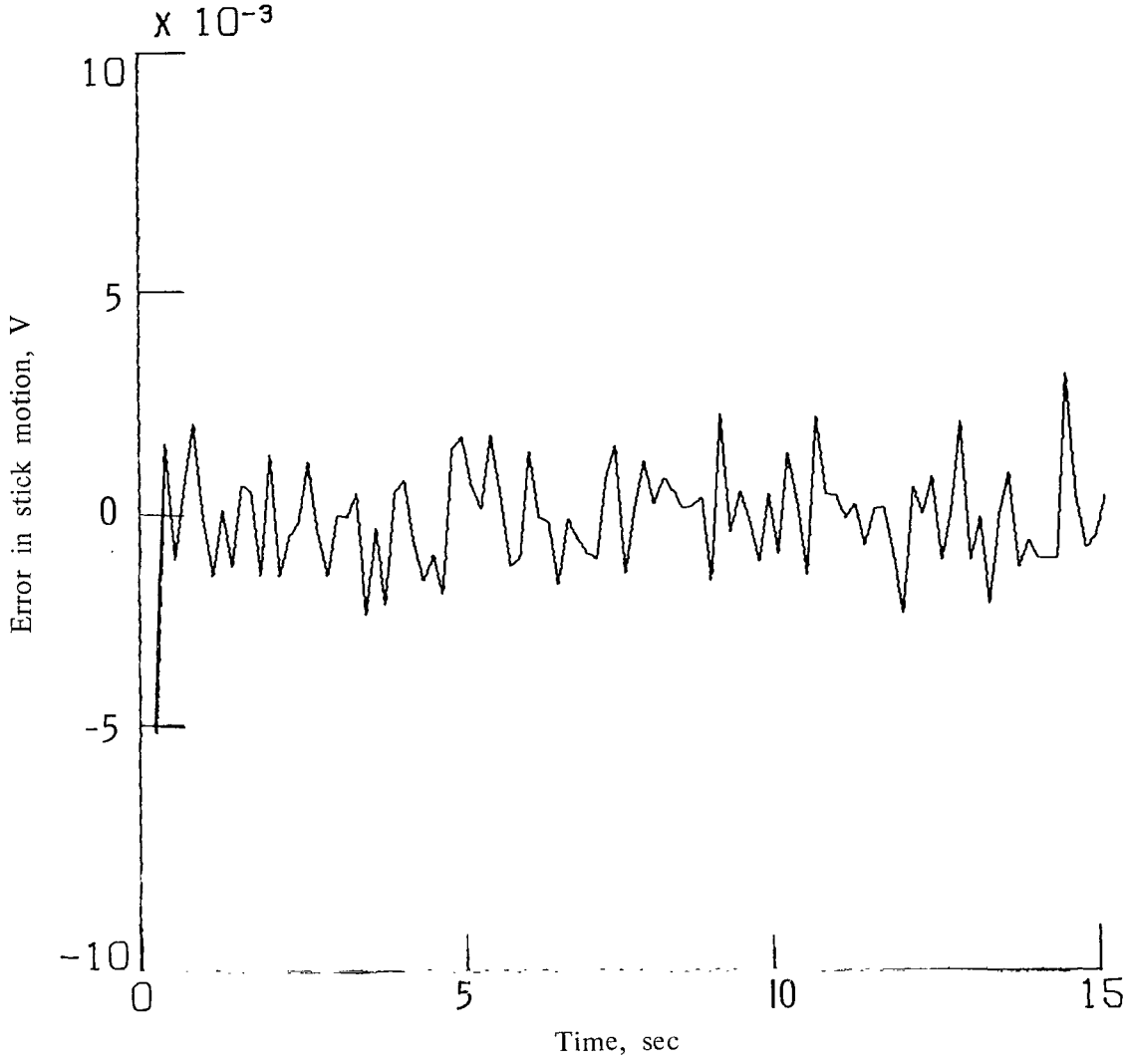


Figure 2.- Stick-motion error plotted against time for simulated data, case 3

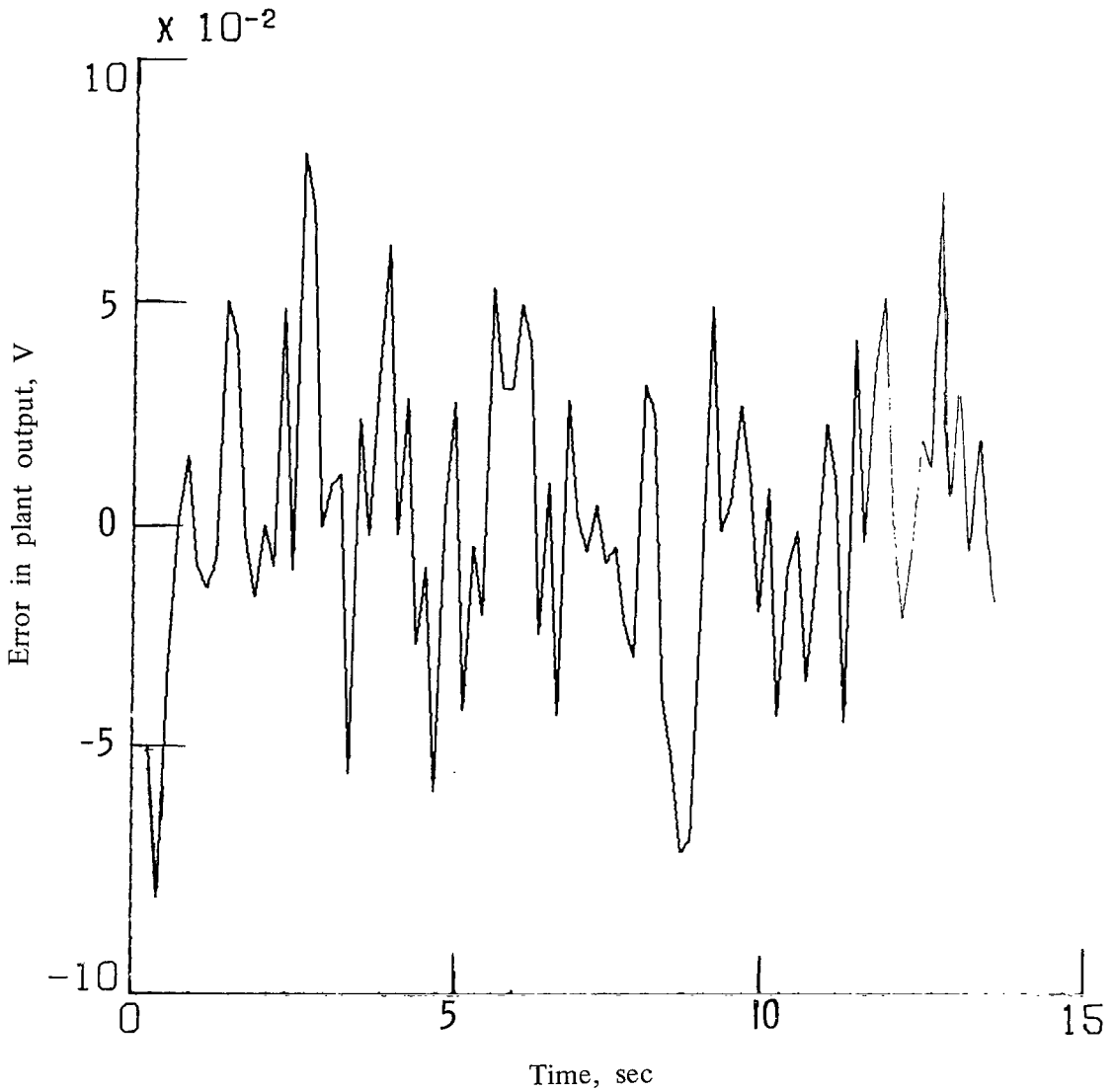


Figure 3.- Plant-output error plotted against time for simulated data, case 3.

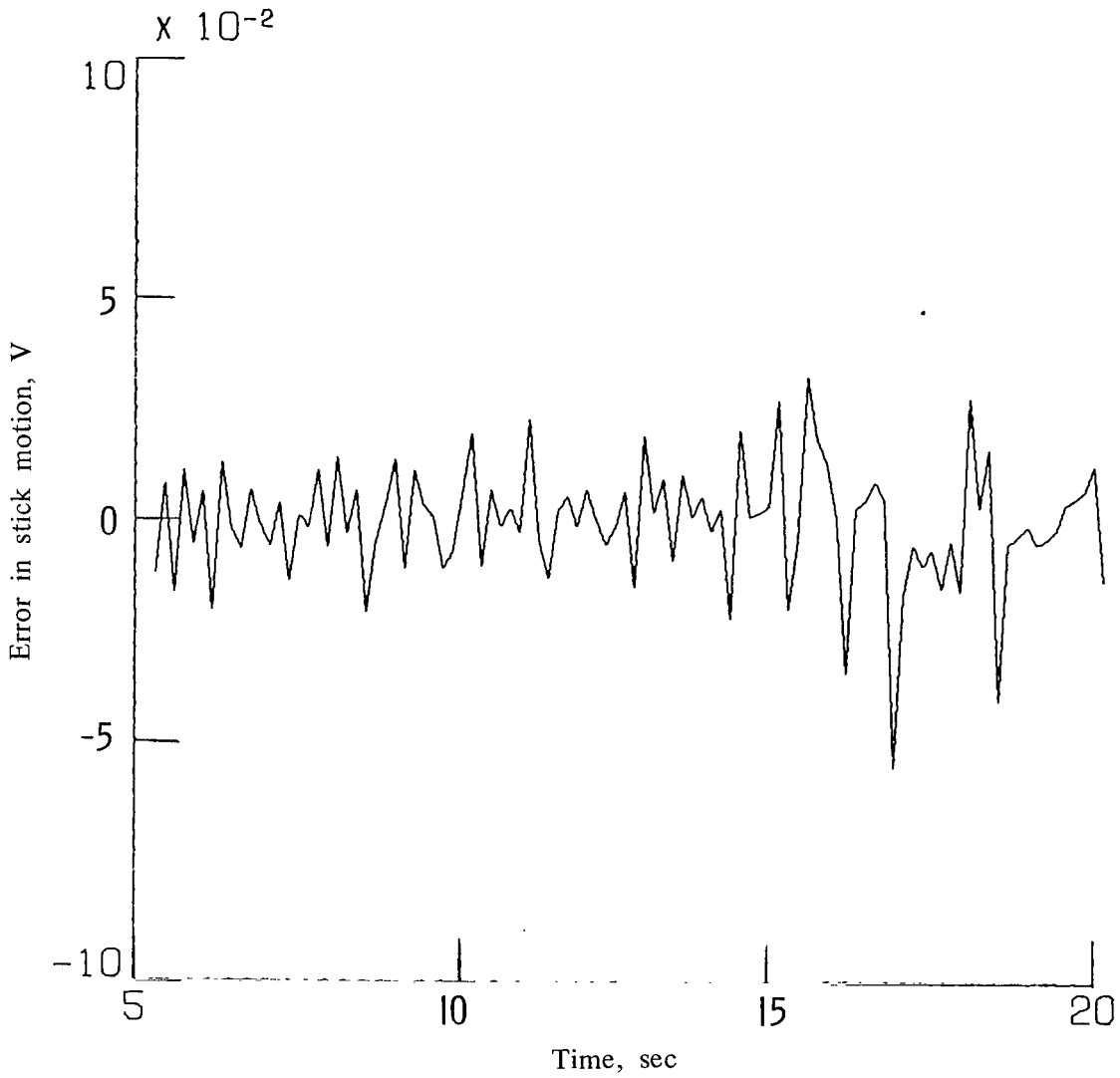


Figure 4.- Stick-motion error plotted against time for case 1 of first experimental-data set.

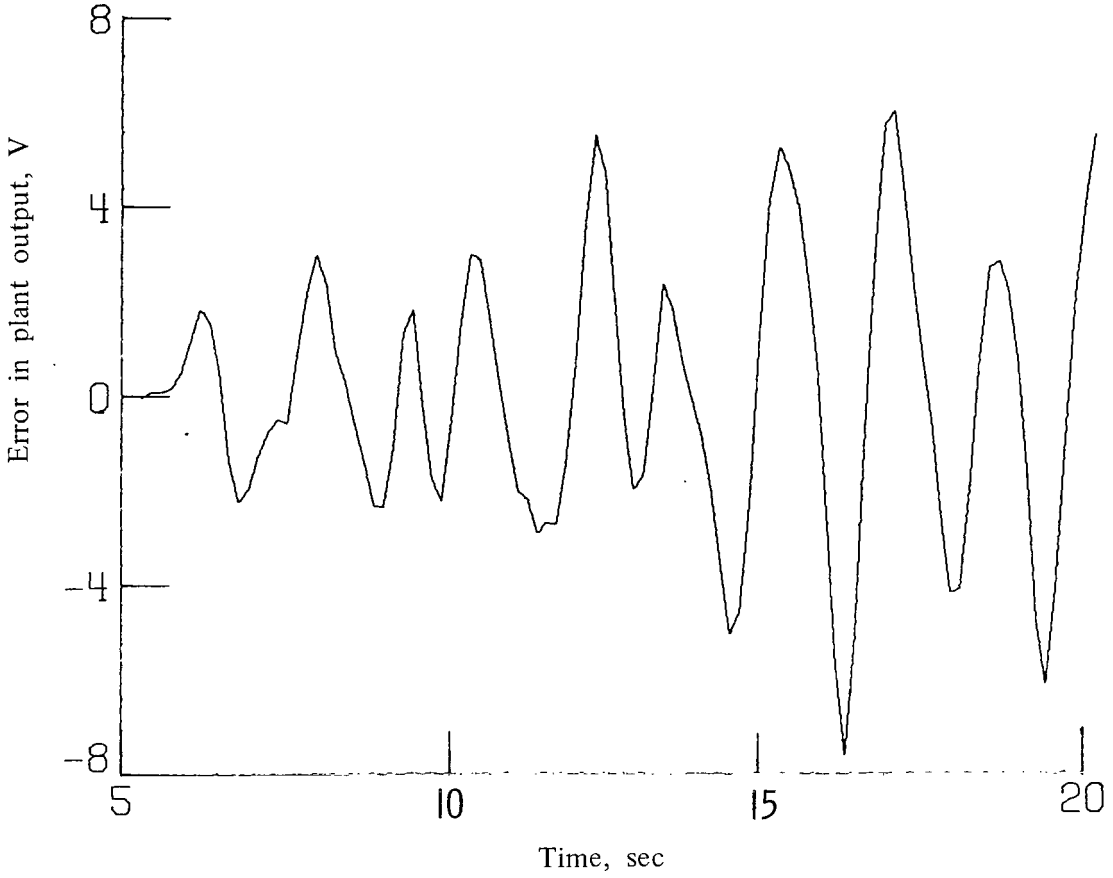


Figure 5.- Plant-output error plotted against time for case 1 of first experimental-data set.

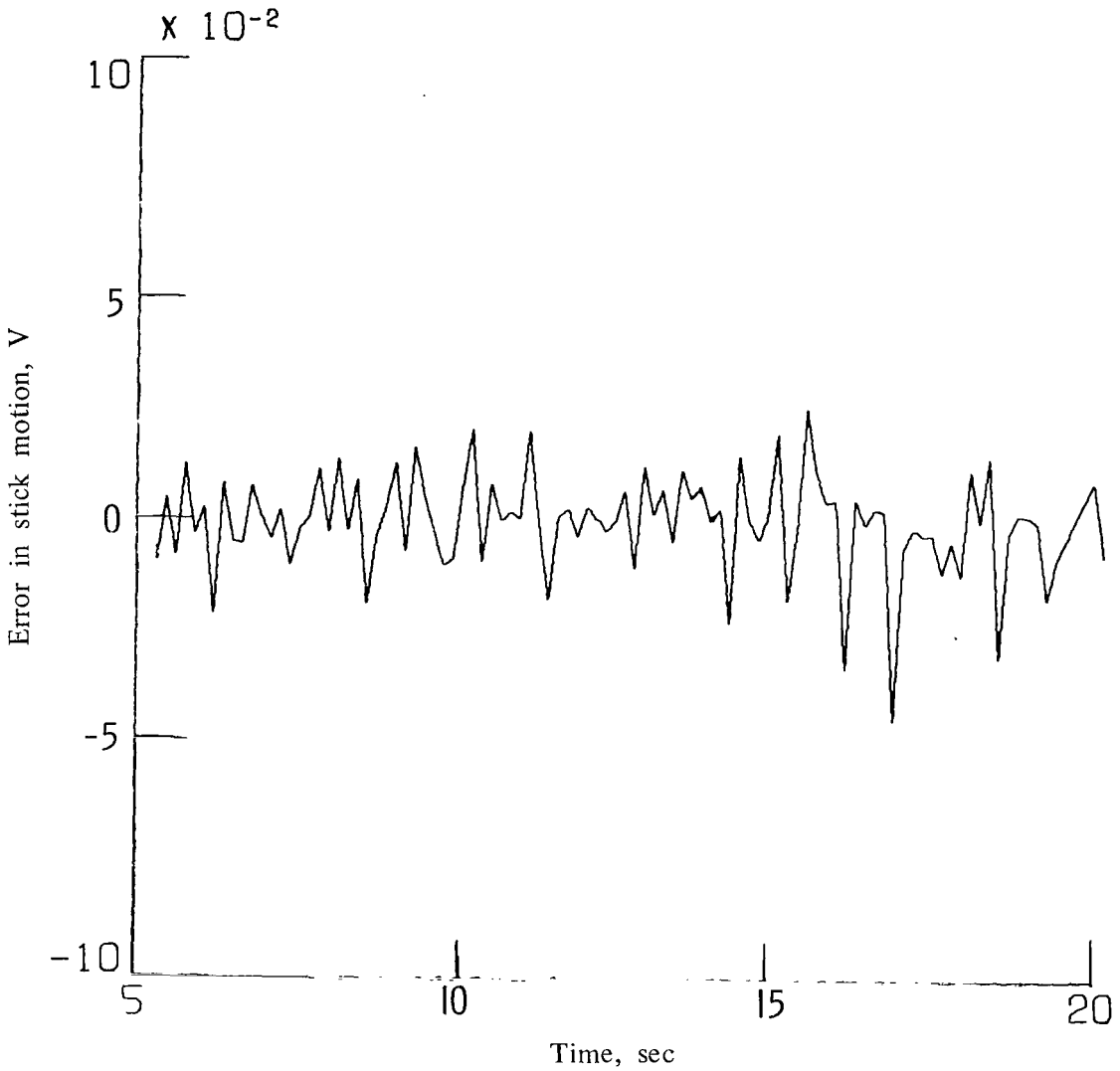


Figure 6.- Stick-motion error plotted against time for case 2 of first experimental-data set.

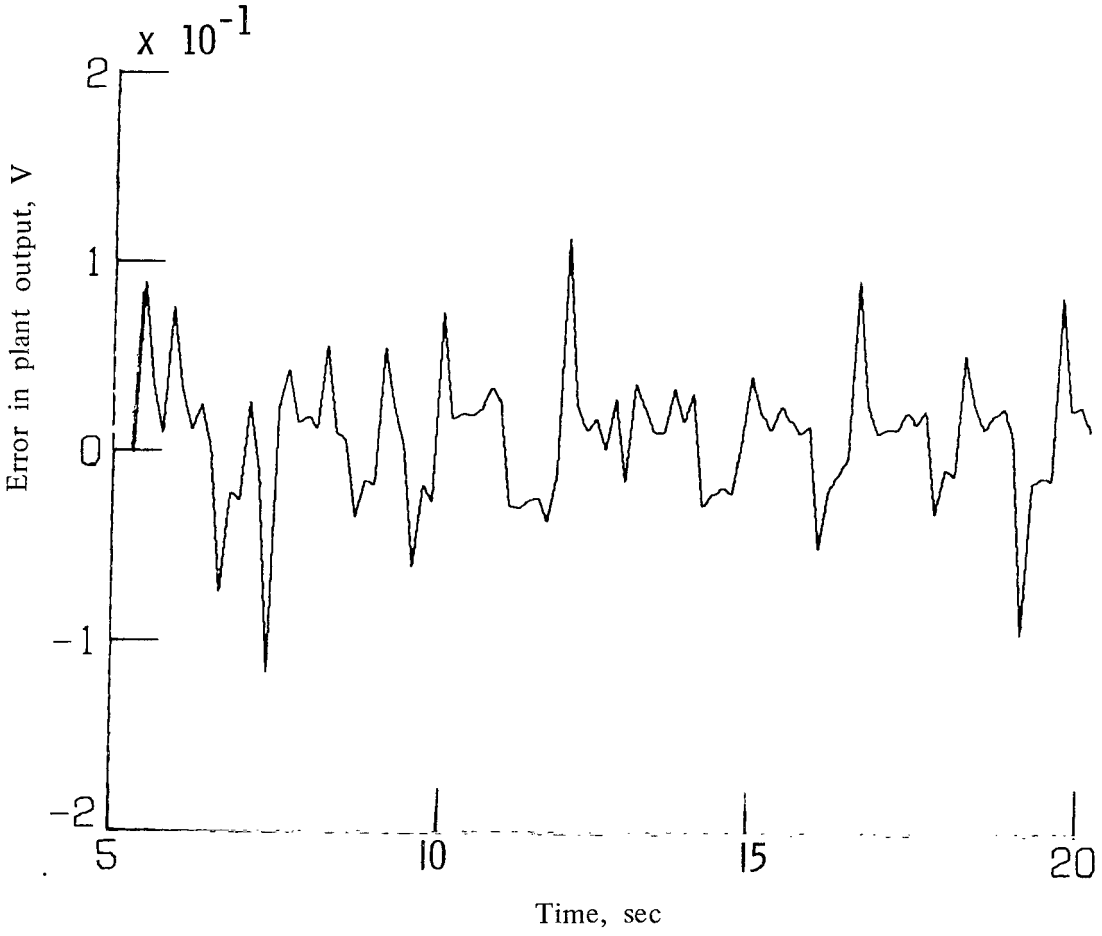


Figure 7.- Plant-output error plotted against time for case 2 of first experimental-data set.

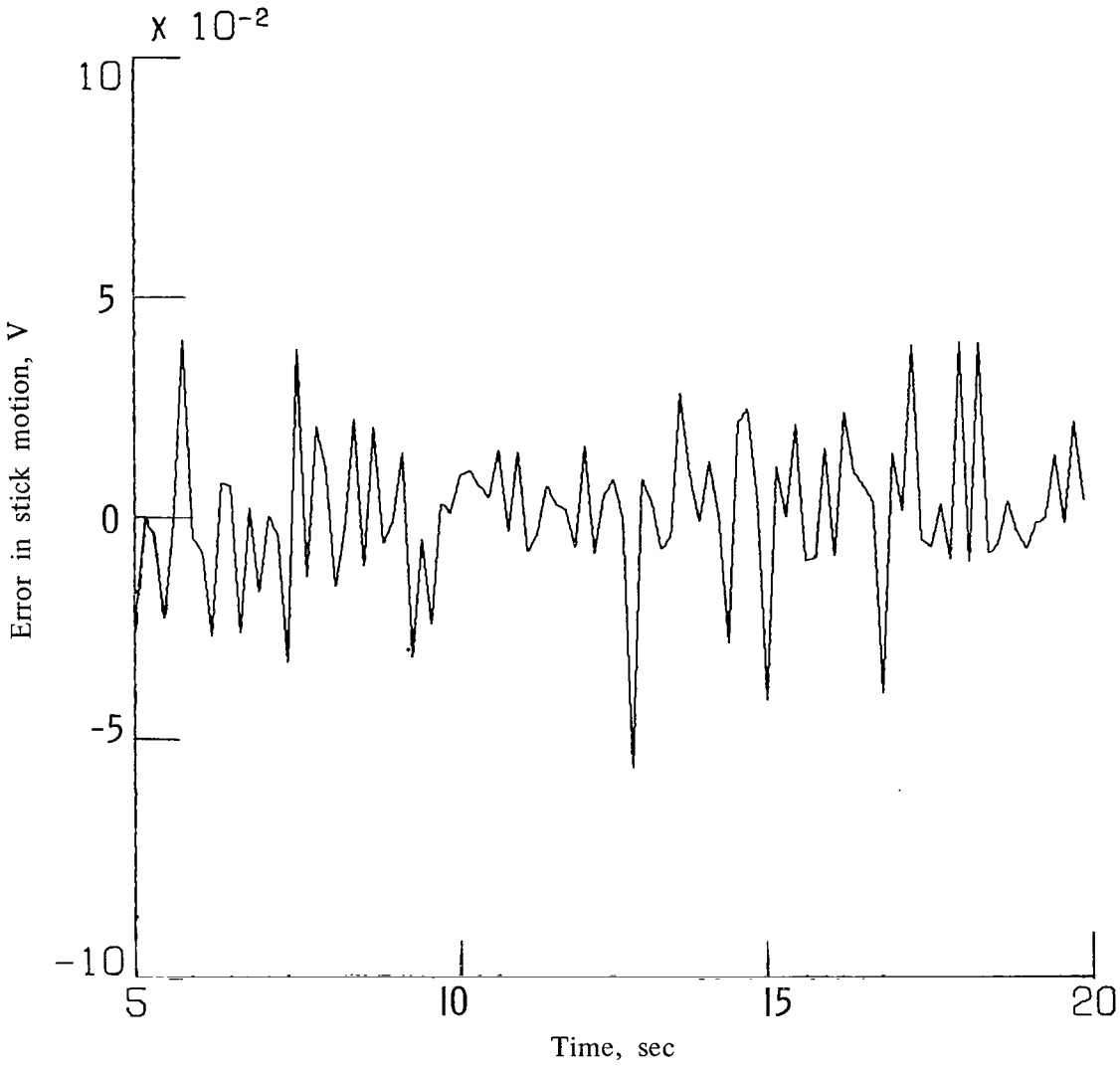


Figure 8.- Stick-motion error plotted against time for case 1 of second experimental-data set.

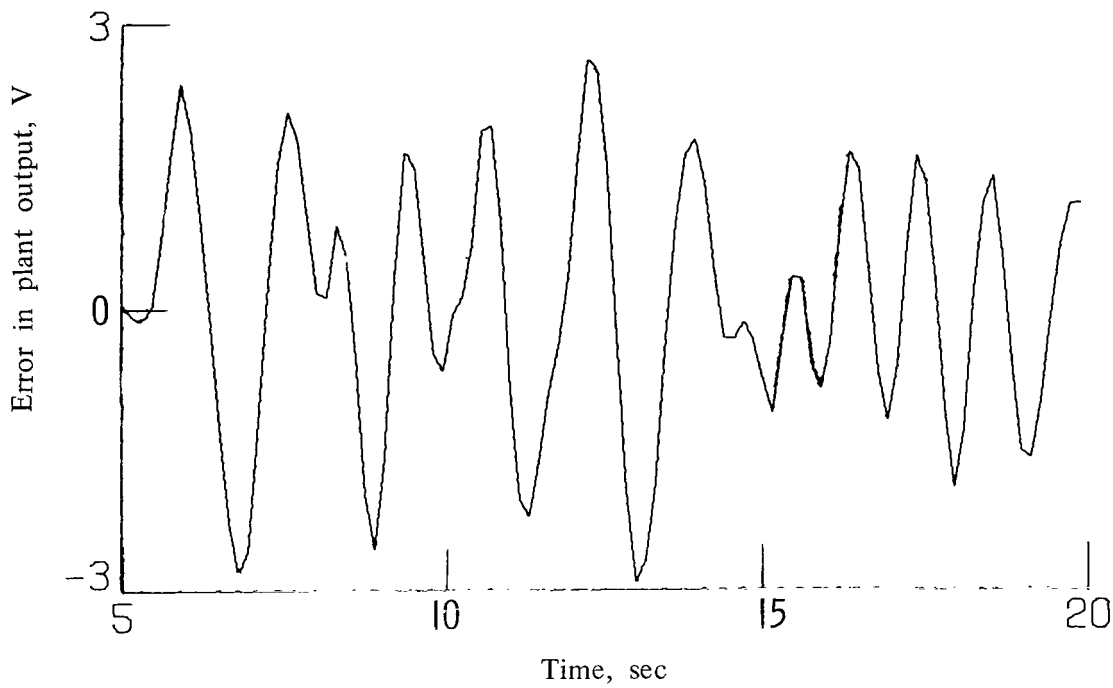


Figure 9.- Plant-output error plotted against time for case 1 of second experimental-data set.

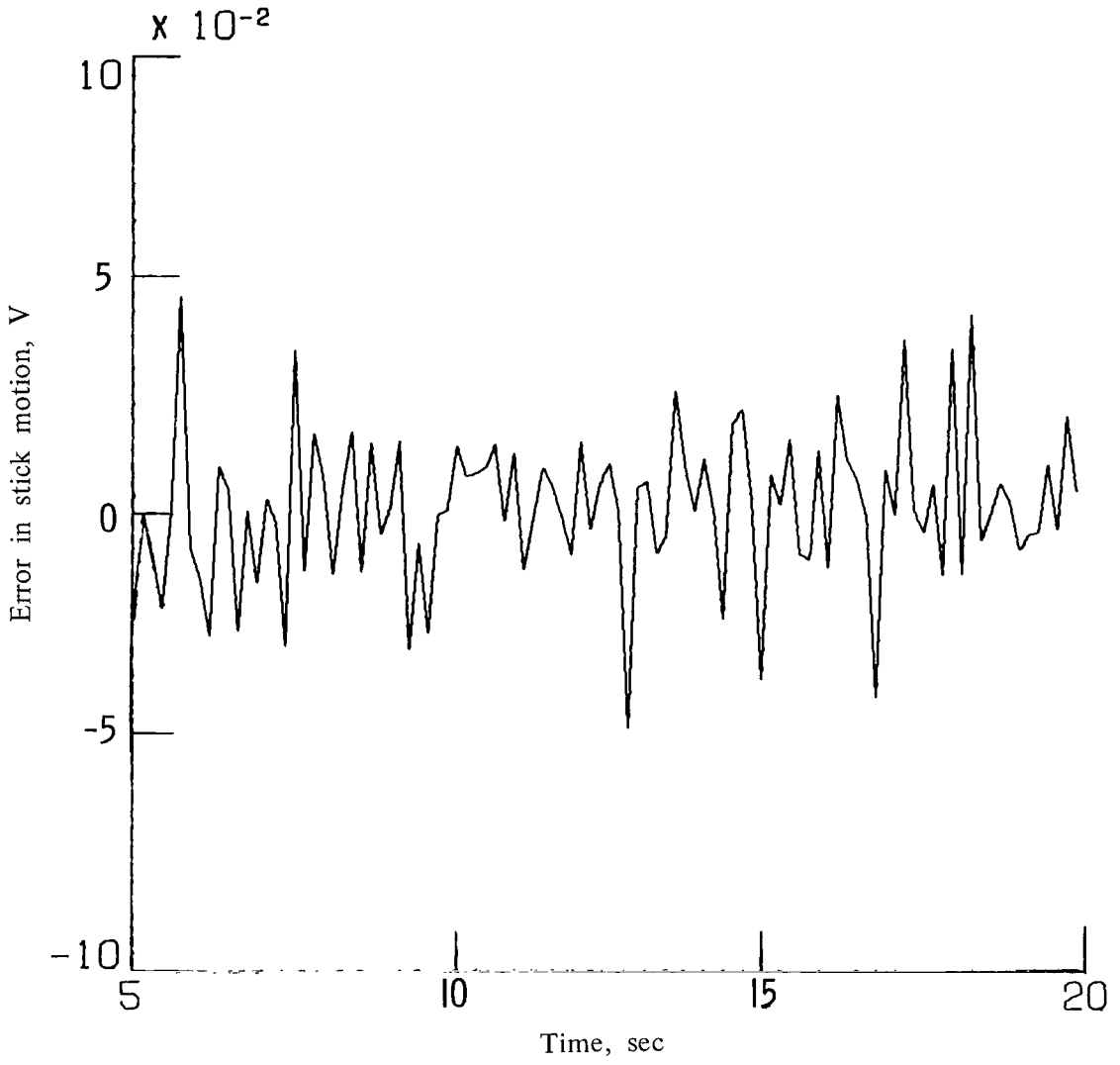


Figure 10.- Stick-motion error plotted against time for case 2 of second experimental-data set.

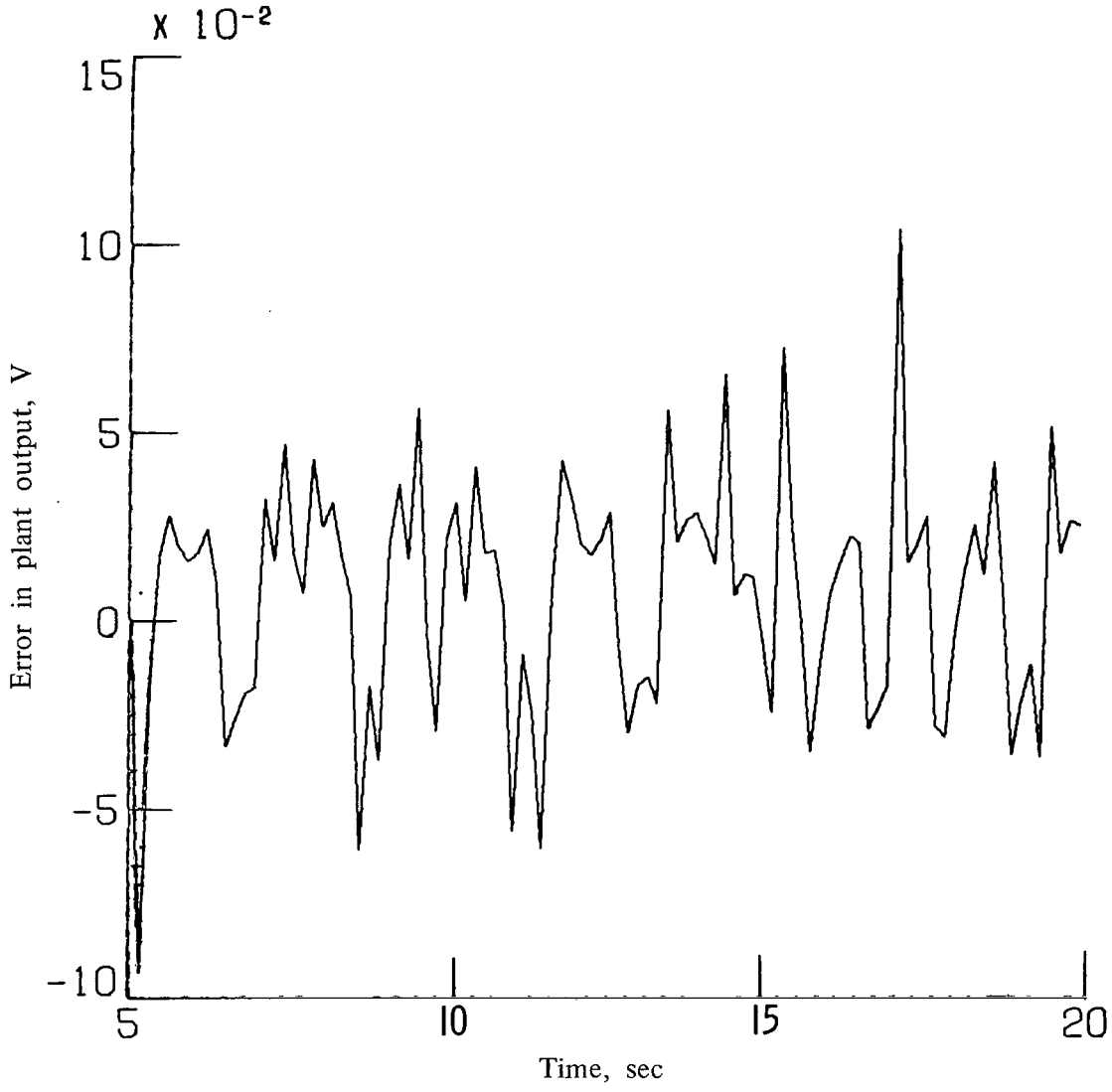


Figure 11.- Plant-output error plotted against time for case 2 of second experimental-data set.

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons. Also includes conference proceedings with either limited or unlimited distribution.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include final reports of major projects, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION OFFICE

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546