Prog. Theor. Phys. Vol. 72, No. 2, August 1984, Progress Letters

Kaluza-Klein Inflation

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(Received April 27, 1984)

Dynamical evolution of the Kaluza-Klein space-time is studied using higher dimensional Einstein equation with dust matter. The difference of the topology between the usual space and the internal space gives rise to the segregation of these subspaces. Furthermore the contraction of the internal space causes the inflation of the usual space.

From the standard big-bang model of the universe we obtain excellent results, the explanation of 3K background radiation and of primodial helium abundance in the universe. This model has, however, fundamental problems, so-called "Horizon problem" and "Flatness problem". These problems originate from two basic assumptions:

[I] The universe has 3-dimensional homogeneous isotropic space.

[II] The matter contained in it is isentropic fluid (or dust) which satisfy the energy conditions.

These are very natural at a later epoch in the evolution of the universe, but these may not hold at an early epoch.

To solve the Horizon and Flatness problems, we must violate assumptions [I] and/or [II] in an early time of the universe. As the model which violates condition [I], we may mention the Mixmaster model in the sense of Misner.[1] On the other hand, the vacuum energy behaving like the cosmological constant which violates condition [II] is introduced in the inflationary cosmological model based on the phase transition in Grand Unified Theories.[2]

By the way, many authors are interested in the generalized Kaluza-Klein theories in relation to the supersymmetric theories.[3] One of the most important points of these theories is why the internal space is so small ($\sim$Planck length) in contrast to the usual space. According to these theories, the internal space is compactified spontaneously whose size is determined by the vacuum expectation value of the gauge field contained therein. Candelas and Weinberg proposed that the direct product of Minkowski space-time and n-dimensional static sphere whose rudius is order of the Planck length is realised on account of the quantum effect of the matter fields in curved space-time.[4] On the other hand, many authors study the dynamical aspects of the space-time of this type to account for the smallness of the internal space in comparison to the usual space.[5] In this paper, we consider the $(4+n)$-dimensional space-time (as it were, the models of this line violate condition [I]) and in particular, focus on the dynamical evolution of the space determined by the classical Einstein equation,*) but neither the classical expectation value of the gauge field used in Ref. 3) nor quantum effects are considered. We imply that the difference of topology (this terminology contains not only the difference of the spatial curvature but also of the dimensions) between the usual space and internal space causes the segregation of these spaces. Furthermore it is shown that the contraction of the internal space gives rise to the inflation of the usual space. The horizon problem can be solved by this mechanism.

For simplicity, we assume some ansatzs:

A1) Metric is $(4+n)$-dimensional extended Robertson-Walker type characterized by two scale factors, i.e.,

*) Recently Sahdev and Okada have studied respectively a similar problem. Of the two studies, Sahdev's is directly related to ours. His approach is mainly numerical one, but our concern is the segregation mechanism induced by the difference between the usual space and the internal one. Moreover the time evolutions in typical regions are also clarified.

$$g_{MN}=\begin{bmatrix} -1 & & \\ & f^2(t)g_{ij} & \\ & & h^2(t)\gamma_{\alpha\beta} \end{bmatrix}, \quad (1)$$

$(M, M=0, 1, \cdots, 4+n;\ i, j=1, 2, 3;$
$\alpha, \beta=4, 5, \cdots, 4+n)$

where $g_{ij}[\gamma_{\alpha\beta}]$ is the metric of $3[n]$-dimensional usual [internal] space.

A2) the evolution of space-time obeys the higher dimensional Einstein equation

$$R_{MN}-\frac{1}{2}Rg_{MN}=-T_{MN}. \quad (2)$$

A3) Initial conditions of usual space and internal space are the same ($f(t_i)=h(t_i)$, $\dot{f}(t_i)=\dot{h}(t_i)$), but the spatial curvature need not equal each other.

The last assumption comes from the point of view that the universe may be so hot that all interactions are unified in the early time.

According to the metric form of Eq. (1), Eq. (2) and its combinations are reduced to

$$\begin{cases} 3\frac{\dot{f}^2+k}{f^2}+\frac{n(n-1)}{2}\frac{\dot{h}^2+1}{h^2}+3n\frac{\dot{f}\dot{h}}{fh}=T^0{}_0, & (3) \\ \frac{\ddot{f}}{f}+2\frac{\dot{f}^2+k}{f^2}+n\frac{\dot{f}\dot{h}}{fh}=-T^1{}_1+\frac{1}{n+2}g^1{}_1T, & (4) \\ \frac{\ddot{h}}{h}+(n-1)\frac{\dot{h}^2+1}{h^2}+3\frac{\dot{f}\dot{h}}{fh} =-T^4{}_4+\frac{1}{n+2}g^4{}_4T, & (5) \end{cases}$$

$$\left(T=\sum_{M=0}^{4+9}T_M{}^M\right)$$

where ($k=-1, 0, +1$) is spatial curvature of the usual space, and n is dimension of the internal space. Here and below the dot denotes the differentiation with respect to t. Being interested in the unification of gravity and non-Abelian gauge fields we restrict ourselves to the case of that the internal space is S_n. It might be natural to consider the pressure of the matter in $(4+n)$-dimensional space-time. In the anisotropic space-time, however, the presence of pressure terms of the energy-momentum tensor makes the situation complicated. In this paper we assume the dust matter, so the energy-momentum tensor reduces to

$$T^0{}_0=\rho \quad \text{other components}=0. \quad (6)$$

There is no pressure terms and we do not take account of the excitation of the off-diagonal components of g_{MN} which become the gauge particles in the later time. This assumption and energy conservation law lead to

$$\rho=\frac{C}{f^3h^n}, \quad (7)$$

where C is a constant. If we adopt ansatz A3), initial conditions $f(t_i)=h(t_i)$ and $\dot{f}(t_i)=\dot{h}(t_i)$ determine the value of C.

From ansatz A3), when the spatial curvature terms can be neglected, i.e.; $1/h^2$, $k/f^2 \ll \rho$, it is

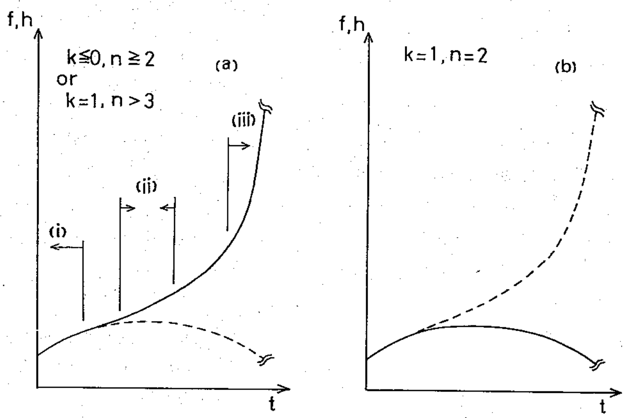

Fig. 1. The behaviour of the scale factors $f(t)$ (the usual space) and $h(t)$ (the internal space) in the linear scales. The solid line represents $f(t)$, and the broken line $h(t)$.
(a) In the case $k<0$, $n>2$, i.e.,(flat or open usual space) $M_3\otimes S_n(n>2)$, the internal space contracts and the usual space inflates. In the case $k=1$, $n>3$, i.e., $S_3\otimes S_n(n>3)$, the situation is similar to the above case. In region (i) ($4+n$ dimensional Friedmann era), $f(t)$ and $h(t)$ are described by Eq. (9). In region (ii) (exponential era), $f(t)$ and $h(t)$ are described by Eq. (12), and in region (iii) by Eq. (14) (time reversal Kasner era).
(b) In the case $k=1$, $n=2$, i.e., $S_3\otimes S_2$, the 3-dimensional space contracts.

valid in a very early time, Eqs. (3) is reduced to

$$\frac{(n+3)(n+2)}{2}\left(\frac{\dot{f}}{f}\right)^2=\rho\,. \tag{8}$$

The solution of Eq. (8) with initial conditions A3) is as follows:

$$f(t)\sim h(t)\sim t^{2/3+n}\,. \tag{9}$$

Expanding $f(t)$ and $h(t)$, since ρ decreases faster than the spatial curvature terms, the latter terms become important. In this epoch $\ddot{h}$ becomes smaller than $\ddot{f}$ because the inequality

$$(n-1)\frac{1}{h^2}>2\frac{k}{f^2} \tag{10}$$

is valid if $k\leqq 0$ or $n>3$. Then $\dot{f}$ becomes greater than $\dot{h}$, so f becomes greater than h. If the situation $f>h$ occurs, inequality (10) acts more strongly. In this way, the spatial curvature terms act as a trigger of the separation of the usual space and the internal space. From Eqs. (3) and (5), we can see that the situation $\dot{h}=0$ ($h=h_{\max}$) and $\dot{f}>0$ is possible. In this region from Eqs. (3) and (4) we obtain

$$\frac{\ddot{f}}{f}+\frac{2n+1}{n+2}\frac{\dot{f}^2+k}{f^2}=\frac{n(n-1)}{2(n+2)}\frac{1}{h^2_{\max}}\,. \tag{11}$$

It allows

$$f(t)=\begin{Bmatrix}(1/\beta)\sinh(\beta t)\\ \exp(\beta t)\\ (1/\beta)\cosh(\beta t)\end{Bmatrix}\quad \text{for } k=\begin{Bmatrix}-1\\ 0\\ +1\end{Bmatrix}, \tag{12}$$

where

$$\beta=\frac{1}{h_{\max}}\sqrt{\frac{n(n+1)}{6(n+1)}}\,. \tag{13}$$

When $h(t)$ is decreasing, Eq. (4) suggests that f is increasing. Because $\dot{f}\dot{h}/fh$ acts as positive cosmological term. If k/f^2, $1/h^2$ and ρ can be neglected in comparison to other terms appear in Eqs. (3)~(5), they have (time reversal) Kasner type solution:

$$\begin{cases}f(t)\sim(t_0-t)^p\,,\\ h(t)\sim(t_0-t)^q\,,\end{cases}$$

$$p=\frac{1}{n+3}\left\{1-\sqrt{\frac{n(n+2)}{3}}\right\},$$

$$q=\frac{1}{n+3}\left\{1+\sqrt{\frac{3(n+2)}{n}}\right\}. \tag{14}$$

Numerical calculation (see Fig. 1) confirms this evolution.

From the numerical integration, we can conclude that the internal space (S_n) separate from the usual space and less expand by means of the difference of the topology (spatial curvature and / or dimension) between them if $k<0$ or $n>3$, and the contraction of the internal space induces the inflation of the usual space. Furthermore, this behaviour is not affected very much of small change of the initial conditions ($f\neq h$, $\dot{f}\neq\dot{h}$).

Since the final stage of this model is described by the time reversal Kasner solution, we cannot avoid the singularity in the future direction. We expect optimistically, however, this situation will be saved if we take into account the effects of quantized matter in curved space-time[6] or other mechanisms. The investigation of these effects will be published elsewhere.

The author would like to thank Professor H. Nariai for continuous encouragement and reading this manuscript. He is also grateful to Dr. K. Tomita for useful discussion.

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