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κ-symmetry, supersymmetry and intersecting branes

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Abstract

We present a new form of κ-symmetry transformations for D-branes in which the dependence on the Born–Infeld field strength is expressed as a relative rotation on the left- and right-moving fields with opposite parameters. Then, we apply this result to investigate the supersymmetry preserved by certain intersecting brane configurations at arbitrary angles and with non-vanishing constant Born–Infeld fields. We also comment on the covariant quantization of the D-brane actions. © 1997 Elsevier Science B.V.

1. Introduction

It is well known that the covariant formulation of superstrings [1] and supermembranes [2] is based upon a special fermionic gauge symmetry on the world-volume which is called κ-symmetry. Upon gauge-fixing this κ-symmetry, the global target space supersymmetry combines with a special field-dependent κ-transformation into a global world-volume supersymmetry. This world-volume supersymmetry guarantees the equality of bosonic and fermionic degrees of freedom on the world-volume. This close relationship between κ-symmetry and supersymmetry can be applied to determine the fraction of space-time supersymmetry preserved by certain single bosonic string and

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membrane configurations (see, e.g., Ref. [3]). A recent development has been the construction of κ-symmetric non-linear effective actions and/or equations of motion for D-branes [4–7] and the M5-brane [8–10] complementing the κ-symmetric superstring and M2-brane [1,2]. In these new cases, there is again a close relation between κ-symmetry and supersymmetry which leads to an equality of bosonic and fermionic degrees of freedom. We will apply this relation to investigate the supersymmetry preserved by certain single bosonic D-brane and M-brane configurations.

Apart from the single-brane configurations, in many applications of superstring dualities a central role is played by intersecting-brane configurations that preserve an, in general smaller, fraction of the vacuum space-time supersymmetry. The allowed intersections depend on the world-volume field content of the branes involved in the intersection [11]. The effective action of an intersecting configuration is expected to be a non-Abelian generalization of the single brane actions. In the linearized limit, this action becomes that of a coupled system with (non-Abelian) vector, tensor and matter multiplets. For example, if the branes involved in the intersection are D-branes, the effective theory is a Yang–Mills theory coupled to matter. In the "Abelian" limit the effective action of an intersecting brane configuration reduces to a non-linear action similar to that of a single brane.

All known κ-symmetry transformations of brane actions take the form

$$\delta\theta = (1 + \Gamma)\kappa, \tag{1}$$

where θ is a space-time spinor depending on the world-volume coordinates σ, $\kappa(\sigma)$ is the parameter of the κ-transformation and Γ is a hermitian traceless product structure, i.e.

$$\mathrm{tr}\,\Gamma = 0, \qquad \Gamma^2 = 1. \tag{2}$$

The expression for Γ depends on the embedding map X from the world-volume of the brane into space-time, and for D-branes is non-linear in

$$\mathcal{F} = F - B, \tag{3}$$

where F is the Born–Infeld (BI) 2-form field strength and B is the background NS–NS 2-form gauge potential.

In this paper we shall show that the non-linear dependence of Γ on $\mathcal{F}$ can be expressed as

$$\Gamma = e^{-a/2} \Gamma'_{(0)} e^{a/2}, \tag{4}$$

where $a = a(\mathcal{F})$ contains all the dependence on the BI field and $\Gamma'_{(0)}$ (which depends only on X) is also a hermitian traceless product structure (i.e. $\mathrm{tr}\,\Gamma'_{(0)} = 0$ and $\left(\Gamma'_{(0)}\right)^2 = 1$). In this new form of Γ, the proof that Γ is a hermitian traceless product structure is straightforward. As another application we shall use (4) to investigate the supersymmetry preserved by intersecting brane configurations.

The classical D-brane actions have in addition to world-volume κ-symmetry also a 32-component space-time supersymmetry. As we shall see for single bosonic brane-probe configurations,[2] the fraction of the supersymmetry preserved is determined by the number of solutions of the following equation:[3]

$$(1 - \Gamma)\epsilon = 0, \tag{5}$$

where ϵ is the space-time supersymmetry parameter. For brane probes this is the only supersymmetry condition that arises. However, for supergravity configurations with branes as sources (that is, for BI Dp-brane actions coupled to the supergravity action), the above condition must be complemented with the usual Killing spinor equation of the supergravity theory. In all cases that we know of the supergravity Killing spinor equation implies the above condition.[4]

Several methods can be used to find the fraction of supersymmetry preserved by intersecting brane configurations. In this paper we shall apply (5) to investigate the supersymmetry preserved by such configurations. For this we shall introduce the projection (5) for each brane involved in the intersection and then we shall examine the compatibility of all the projections. One of the advantages of this method is that all intersections can be treated in a unified way. To simplify the computation, we shall first assume that all the branes involved in the intersection are probes propagating in the $D = 10$ Minkowski space-time. In this case, we shall find that one can take the BI fields associated with the D-branes and M-branes to be constant rather than zero. For vanishing BI fields, we shall reproduce all the known results for the allowed supersymmetric intersecting brane configurations.

Next we shall briefly comment on D-branes in their appropriate supergravity background. The matching of the supergravity solution to the source necessitates that the BI field of the brane must vanish if the supergravity solution does not contain a non-vanishing NS–/NS 2-form gauge potential.

There are some limitations to the above method for determining the fraction of supersymmetry preserved by intersecting brane configurations. One is that we are considering Abelian BI-type effective actions despite the fact that the full effective theory is expected to be non-Abelian. The non-Abelian case corresponds to configurations of coincident branes. Such configurations will not be considered in this paper. We have also ignored parts of the effective action; for example in intersections involving D-branes, we have not taken into account the matter multiplets that are associated with open strings ending at two different D-branes involved in the intersection. Nevertheless, the results of our paper apply in the "Abelian" limit of the full theory.

The organization of this paper is as follows. In Section 2 we shall review the action and κ-symmetry transformations of Dp-branes. In Section 3 we derive the new form of

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[2] In this paper we define brane probes as solutions of the world-volume action for fixed target space background.

[3] The same condition has been derived in the boundary state formalism [12,13].

[4] Apparently supersymmetric solutions always have supersymmetric sources. It would be interesting to have a general and rigorous proof of this.

the κ-transformations given in Eqs. (1) and (4). In Section 4 we discuss the condition (5) for supersymmetric configurations. In Section 5 we shall investigate the conditions for intersecting D-brane probes to preserve a fraction of space-time supersymmetry. In Section 6 we extend our results to include M-branes. In Section 7 we comment on the supersymmetry preserved by supergravity/brane configurations and in Appendix A we shall comment on the covariant quantization of D-brane actions.

2. D-branes and κ-symmetry

To make our discussion self-contained we briefly review here the basics of κ-symmetry. For a more detailed discussion and our notation we refer to Ref. [6]. Let G, B and ϕ be the space-time metric, the NS–NS 2-form gauge potential and the dilaton, respectively. The bosonic Dp-brane is described by a map X from the world-volume $\Sigma_{(p+1)}$ into the $d = 10$ space-time $\mathcal{M}$ and by a 2-form BI field strength F on $\Sigma_{(p+1)}$; $dF = 0$ so $F = dV$, where V is the 1-form BI gauge potential. The bosonic part of the effective action of a Dp-brane is

$$
I_p = - \int d^{p+1}\sigma \left[e^{-\phi} \sqrt{|\det(g_{ij} + \mathcal{F}_{ij})|} + C \, e^{\mathcal{F}} + m I_{CS} \right] , \tag{6}
$$

where

$$
g_{ij} = \partial_i X^\mu \partial_j X^\nu G_{\mu\nu} , \tag{7}
$$

is the metric on $\Sigma_{(p+1)}$ induced by the map X, $(\mu, \nu = 0, \ldots, 9)$ are the space-time indices and $\mathcal{F}_{ij}$ $(i = 1, \ldots, (p+1))$ is the modified 2-form field strength defined in (3) (B in $\mathcal{F}$ is the pull-back of the NS–NS 2-form gauge potential B with X). The second term in (6) is a WZ term where

$$
C = \sum_{r=0}^{10} C^{(r)} \tag{8}
$$

is a formal sum of the RR gauge potentials $C^{(r)}$. It is understood that after expanding the potential only the $(p+1)$-form is retained.[5] The last term is only present for even p (the IIA case) [17]. Its coefficient m is the cosmological constant of massive IIA supergravity and I_{CS} is given in [18].

To construct supersymmetric Dp-brane actions, we replace the maps X ($\{X^\mu\}$) with supermaps $Z = (X, \theta)$ ($\{Z^M\}$) and the various bosonic supergravity fields with the corresponding superfields of which they are the leading component in a θ-expansion. The frame index A of the supervielbein decomposes under the action of the $D = 10$ Lorentz group as follows:

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[5] Again here we have used the same symbols to denote the space-time gauge potential and its pull-back with the map X.

$$A = \begin{cases} (a,\alpha) & a = 0,\ldots,9, & \alpha = 1,\ldots,32, & \text{for IIA}, \\ (a,I,\alpha) & a = 0,\ldots,9, & I = 1,2, & \alpha = 1,\ldots,32, & \text{for IIB}, \end{cases} \tag{9}$$

where a is a $d = 10$ vector index and α is a $d = 10$ spinor index in the Majorana representation. This notation allows to treat the IIA and IIB theories in a unified way but it is understood that in the IIB case chiral projection operators should be inserted in appropriate places to reduce the Majorana spinor indices to Majorana–Weyl ones. The induced metric for both IIA and IIB D-branes is

$$g_{ij} = E_i^{\ a} E_j^{\ b} \eta_{ab}, \tag{10}$$

where

$$E_i^A = \partial_i Z^M E_M^{\ A}, \tag{11}$$

and η_{ab} is the Minkowski (frame) metric. In what follows, we shall assume that $\det\{g_{ij}\} \neq 0$, unless otherwise stated.

The action (6) (including the fermions) is invariant under the κ-transformations [6]

$$\begin{cases} \delta_\kappa Z^M E_M^{\ a} = 0, \\ \delta_\kappa Z^M E_M^{\ \alpha} = [\bar{\kappa}(1 + \Gamma)]^\alpha, \\ \delta_\kappa V_i = E_i^{\ A} \delta E^B B_{BA}, \end{cases} \tag{12}$$

with parameter κ.

The expression for Γ for any Dp-brane is [6]

$$\Gamma = \frac{\sqrt{|g|}}{\sqrt{|g + \mathcal{F}|}} \sum_{n=0}^{\infty} \frac{1}{2^n n!} \gamma^{j_1 k_1 \ldots j_n k_n} \mathcal{F}_{j_1 k_1} \ldots \mathcal{F}_{j_n k_n} J_{(p)}^{(n)}, \tag{13}$$

where $g = \det\{g_{ij}\}$, $g + \mathcal{F}$ is shorthand for $\det(g_{ij} + \mathcal{F}_{ij})$ and

$$J_{(p)}^{(n)} = \begin{cases} (\Gamma_{11})^{n + \frac{p-2}{2}} \Gamma_{(0)}, \\ (-1)^n (\sigma_3)^{n + \frac{p-3}{2}} i\sigma_2 \otimes \Gamma_{(0)}. \end{cases} \tag{14}$$

The matrix $\Gamma_{(0)}$ is given by

$$\Gamma_{(0)} = \frac{1}{(p + 1)! \sqrt{|g|}} \epsilon^{i_1 \ldots i_{(p+1)}} \gamma_{i_1 \ldots i_{(p+1)}}. \tag{15}$$

Finally, the 32×32 matrices γ_i are defined as

$$\gamma_i = E_i^{\ a} \Gamma_a, \tag{16}$$

where $\{\Gamma_a; a = 0,\ldots 9\}$ are the space-time gamma matrices. For later use, we note that

$$(\Gamma_{(0)})^2 = (-1)^{(p-1)(p-2)/2}. \tag{17}$$

A crucial property of the κ-rules, which also plays an important role in the actual proof of κ-invariance of the Dp-brane actions, is that they must eliminate half of the

fermionic degrees of freedom. To see this, we remark that the total number of bosonic physical degrees of freedom are 8; $10 - (p + 1)$ are due to the scalars X and $p - 1$ are due to the BI 1-form gauge potential V. In order to make the number of physical bosonic degrees of freedom equal to the number of fermionic ones, the κ-transformations must eliminate exactly half of the fermionic degrees of freedom. The properties of the matrix Γ defined in (1) ensure that this is indeed the case: from $\Gamma^2 = 1_{32 \times 32}$ it follows that all the eigenvalues of Γ are $+1$ or -1. From the tracelessness it follows that it has as many $+1$ as -1 eigenvalues, that is, it has 16 of each, so the projector $\frac{1}{2}(1 + \Gamma)$ has 16 zero eigenvalues and 16 eigenvalues equal to $+1$. This guarantees that κ-symmetry reduces the 32 components of θ to 16. Due to the fact that the kinetic term of θ in the BI action is linear in time derivatives there is a second class constraint which reduces further the components of θ from 16 to 8. Therefore the number of bosonic and fermionic physical degrees of freedom are equal on the world-volume. The details of the invariance of the action (6) under κ-transformations and the proof that Γ has the required properties are given in, e.g., Ref. [6].

3. κ-symmetry revisited

The main task in this section is to show that the product structure Γ associated with the Dp-brane κ-transformation law can be written as given in (4). The proof is inspired by the work of [12,13] and is similar for the IIA and IIB Dp-branes. Because of this we shall present the IIA case in detail and only the main points of the proof for the IIB case. We begin by first rewriting the IIA product structure Γ as

$$\Gamma = \frac{\sqrt{|g|}}{\sqrt{|g + \mathcal{F}|}} \, \mathrm{se}^{\frac{1}{2} \mathcal{F}_{jk} \gamma^{jk} \Gamma_{11}} \Gamma'_{(0)} \,, \tag{18}$$

where

$$\mathrm{se}^{\frac{1}{2} \mathcal{F}_{jk} \gamma^{jk}} := \sum_{n=0}^{\infty} \frac{1}{2^n n!} \gamma^{j_1 k_1 \cdots j_n k_n} \mathcal{F}_{j_1 k_1} \ldots \mathcal{F}_{j_n k_n} \,, \tag{19}$$

so "se" stands for the skew-exponential function (i.e. the usual exponential function with skew-symmetrized indices of the gamma matrices at every order in the expansion so the expansion has effectively only a finite number of terms), and

$$\Gamma'_{(0)} = (\Gamma_{11})^{\frac{p-2}{2}} \Gamma_{(0)} \,. \tag{20}$$

It is worth noting that

$$\left(\Gamma'_{(0)} \right)^2 = 1 \,. \tag{21}$$

To continue, we introduce a world-volume $(p + 1)$-bein, e, i.e. $g_{ik} = e^{\underline{i}}_i e^{\underline{k}}_k \eta_{\underline{ik}}$, where $\underline{i}, \underline{k} = 0, \ldots, p$ are world-volume frame indices. Then we rewrite Γ as

$$\Gamma = \frac{1}{\sqrt{|\eta + \mathcal{F}|}} \, \mathrm{se}^{\frac{1}{2} X_{\underline{ik}} \gamma^{\underline{ik}} \Gamma_{11}} \Gamma'_{(0)} \,, \tag{22}$$

where $\mathcal{F}$ in the determinant is in the frame basis. Then without loss of generality, we use a world-volume Lorentz rotation to write $\mathcal{F}$ as

$$\mathcal{F} \equiv \tfrac{1}{2} \mathcal{F}_{\underline{ik}} e^{\underline{i}} \wedge e^{\underline{k}} = \tanh \phi_0 \, e^{\underline{0}} \wedge e^{\underline{\ell}} + \sum_{r=1}^{\ell} \tan \phi_r \, e^{\underline{r}} \wedge e^{\underline{\ell+r}}, \tag{23}$$

where $\{\phi_0, \phi_r; r = 1, \ldots, \ell\}$, $\ell = [p/2]$, are "angles" and $\{e^{\underline{i}}\} = \{e^{\underline{0}}, e^{\underline{s}}; s = 1, \ldots, p\}$ is a Lorentz basis. Using this, we have

$$\sqrt{|\eta + \mathcal{F}|} = \left(-1 + \tanh^2 \phi_0\right)^{1/2} \prod_{r=1}^{\ell} \left(1 + \tan^2 \phi_r\right)^{1/2}$$

$$= \frac{1}{\cosh \phi_0 \, \prod_{r=1}^{\ell} \cos \phi_r}. \tag{24}$$

Substituting this in Γ, we get

$$\Gamma = \left(\cosh \phi_0 \prod_{s=1}^{\ell} \cos \phi_s\right) \mathrm{se}^{\tanh \phi_0 \gamma^{\underline{0\ell}} \Gamma_{11}} \left(\prod_{r=1}^{\ell} \mathrm{se}^{\tan \phi_r \gamma^{\underline{r\ r+\ell}} \Gamma_{11}}\right) \Gamma'_{(0)}. \tag{25}$$

From the definition of se, we can rewrite Γ as

$$\Gamma = \left(\cosh \phi_0 + \sinh \phi_0 \, \gamma^{\underline{0\ell}} \Gamma_{11}\right)$$

$$\times \left[\prod_{r=1}^{\ell} \left(\cos \phi_r + \sin \phi_r \gamma^{\underline{r\ r+\ell}} \Gamma_{11}\right)\right] \Gamma'_{(0)}, \tag{26}$$

which in turn can be expressed as

$$\Gamma = e^{\phi_0 \gamma^{\underline{0\ell}} \Gamma_{11}} \left(\prod_{r=1}^{\ell} e^{\phi_r \gamma^{\underline{r\ r+\ell}} \Gamma_{11}}\right) \Gamma'_{(0)} \tag{27}$$

$$= \exp\left\{\phi_0 \gamma^{\underline{0\ell}} + \sum_{r=1}^{\ell} \phi_r \gamma^{\underline{r\ r+\ell}} \Gamma_{11}\right\} \Gamma'_{(0)}. \tag{28}$$

In the last step we have used the fact that $(\gamma^{\underline{0\ell}} \Gamma_{11})^2 = 1$ while $(\gamma^{\underline{r\ r+\ell}} \Gamma_{11})^2 = -1$. It is clear from this that the product structure Γ can be written as

$$\Gamma = e^{\frac{1}{2} Y_{\underline{ik}} \gamma^{\underline{ik}} \Gamma_{11}} \Gamma'_{(0)}, \tag{29}$$

where

$$Y \equiv \tfrac{1}{2} Y_{\underline{ik}} e^{\underline{i}} \wedge e^{\underline{k}} = \phi_0 \, e^{\underline{0}} \wedge e^{\underline{\ell}} + \sum_{r=1}^{\ell} \phi_r e^{\underline{r}} \wedge e^{\underline{\ell+r}}. \tag{30}$$

Although we have shown this equation in a particular Lorentz frame, it holds in any Lorentz frame. The relation between $\mathcal{F}$ and Y is now

$$\mathcal{F} = \text{"tan"}Y, \tag{31}$$

where "tan" is defined by Eq. (23) in the special Lorentz frame. The explicit expression of the function "tan" is in general frames more complicated but it can be always be found by going to the special frame as an intermediate step.

Now let us turn to examine the product structure Γ associated with IIB Dp-branes. In this case the product structure Γ can be written as

$$\Gamma = \frac{\sqrt{|g|}}{\sqrt{|g + \mathcal{F}|}} s e^{-\frac{1}{2}\mathcal{F}_{\underline{ik}}\sigma_3 \otimes \gamma^{\underline{ik}}} \, \Gamma'_{(0)}, \tag{32}$$

where

$$\Gamma'_{(0)} = (\sigma_3)^{\frac{p-3}{2}} i\sigma_2 \otimes \Gamma_{(0)}, \tag{33}$$

is an $\mathcal{F}$-independent traceless product structure.

Following a similar computation as for the IIA Dp-branes, we find that

$$\Gamma = e^{-\frac{1}{2}Y_{\underline{ik}}\sigma_3 \otimes \gamma^{\underline{ik}}} \, \Gamma'_{(0)}, \tag{34}$$

where $\mathcal{F}$ and Y are again related as in Eq. (31).

Now, observing that $\Gamma'_{(0)}$ anticommutes with the gamma matrices that appear in the exponential in the expression for Γ we can write

$$\Gamma = e^{-a/2}\Gamma'_{(0)} e^{a/2}, \tag{35}$$

as in Eq. (4) of the introduction, where, as we have just shown,

$$a = \begin{cases} -\frac{1}{2}Y_{jk}\gamma^{jk}\Gamma_{11}, & \text{IIA}, \\ \frac{1}{2}Y_{jk}\sigma_3 \otimes \gamma^{jk}, & \text{IIB}. \end{cases} \tag{36}$$

As an application of the new expression for Γ we remark that it is straightforward to show that $\Gamma^2 = 1$ and $\text{tr}\,\Gamma = 0$ using the above-mentioned property of the exponential, the cyclic properties of the trace and the analogous properties of $\Gamma'_{(0)}$.

4. Supersymmetry

We would like to derive here the condition (5) of the introduction for the fraction of supersymmetry preserved by a single brane from the κ-symmetry transformation (1). We remark that the known κ-symmetry transformations of all M, IIA, IIB, and heterotic branes have the same form, so the result of this derivation applies to all these cases. Here we consider the type II case. Since we are interested in *bosonic* configurations that preserve a fraction of the space-time supersymmetry (i.e. we set $\theta = 0$ for these

configurations), it is enough to examine the supersymmetry transformation of the θ field up to terms linear in θ. The supersymmetry and κ-symmetry transformations of θ are

$$\delta\theta = (1 + \Gamma)\kappa + \epsilon, \tag{37}$$

where ϵ is the space-time supersymmetry parameter. Assuming a gauge-fixing condition for κ-symmetry of the form

$$\mathcal{P}\theta = 0, \tag{38}$$

where $\mathcal{P}$ is a (field-independent) projection, $\mathcal{P}^2 = \mathcal{P}$. The remaining non-vanishing components of θ are given by $(1 - \mathcal{P})\theta$ and the transformation (37) becomes a global supersymmetry transformation. The condition for preserving the gauge-fixing condition $\mathcal{P}\delta\theta = (1 - \mathcal{P})\delta\theta = 0$ is now equivalent to having unbroken supersymmetry. Therefore, the condition for unbroken supersymmetry is

$$\delta\theta = (1 + \Gamma)\kappa + \epsilon_{\text{unbr}} = 0, \tag{39}$$

which in turn implies the condition $(1 - \Gamma)\epsilon_{\text{unbr}} = 0$ of the introduction.

For the IIA case, a convenient gauge-fixing condition is

$$(1 + \Gamma_{11})\theta = 0. \tag{40}$$

To obtain more explicit expressions we go to a (chiral) basis in which Γ_{11} is diagonal split the index α into the pair (α_1, α_2) with opposite chiralities, $\alpha_1, \alpha_2 = 1, \ldots, 16$ so

$$\Gamma_{11} = \begin{pmatrix} \delta^{\alpha_1}{}_{\beta_1} & \\ & -\delta^{\alpha_2}{}_{\beta_2} \end{pmatrix}, \qquad \theta^\alpha = \begin{pmatrix} \theta^{\alpha_1} \\ \theta^{\alpha_2} \end{pmatrix}. \tag{41}$$

and similarly for κ^α and ϵ^α. In this basis the above gauge-fixing condition is simply $\theta^{\alpha_1} = 0$. Since Γ_{11} anticommutes with Γ, in a basis that Γ_{11} is diagonal, Γ is off-diagonal, i.e.

$$\Gamma = \begin{pmatrix} & \Gamma^{\alpha_1}{}_{\beta_2} \\ \Gamma^{\alpha_2}{}_{\beta_1} & \end{pmatrix}. \tag{42}$$

Preserving the gauge-fixing condition $\theta^{\alpha_1} = 0$ in this basis $\delta\theta^{\alpha_1} = 0$ implies

$$\kappa^{\alpha_1} + \Gamma^{\alpha_1}{}_{\beta_2}\kappa^{\beta_2} + \epsilon^{\alpha_1} = 0. \tag{43}$$

This leads to the (world-volume) supersymmetry transformation

$$\delta\lambda^{\alpha_2} = -\Gamma^{\alpha_2}{}_{\beta_1}\epsilon^{\beta_1} + \epsilon^{\alpha_2}, \tag{44}$$

where $\lambda = (1 - \mathcal{P})\theta$. For supersymmetric bosonic configurations $\delta\lambda^{\alpha_2} = 0$ as well, which is precisely the condition $(1 - \Gamma)\epsilon = 0$.

For the IIB case it is convenient to choose as a gauge-fixing condition

$$\left(1 + \sigma_3 \otimes 1_{32 \times 32}\right) \theta = 0, \tag{45}$$

where θ is a doublet of chiral space-time spinors. Again it is convenient to go to a basis in which $\sigma_3 \otimes 1_{32 \times 32}$ is diagonal, so

$$\sigma_3 \otimes 1_{32 \times 32} = \begin{pmatrix} \delta^{1,\alpha}{}_{1,\beta} & \\ & -\delta^{2,\gamma}{}_{2,\delta} \end{pmatrix}, \qquad \theta = \begin{pmatrix} \theta^{1,\alpha} \\ \theta^{2,\beta} \end{pmatrix}, \tag{46}$$

and similarly for κ^A, ϵ^A. However, now we have to take into account the (positive) chirality of the spinors. Thus, which the choice of Γ_{11} matrix (41) we split the spinors $\theta^{1,\alpha}, \theta^{2,\alpha}$ as in Eq. (41) and set to zero the negative chirality components $\theta^{1,\alpha_2}, \theta^{2,\alpha_2}$ so

$$\theta^{1,\alpha} = \begin{pmatrix} \theta^{1,\alpha_1} \\ 0 \end{pmatrix}, \qquad \theta^{2,\alpha} = \begin{pmatrix} \theta^{2,\alpha_1} \\ 0 \end{pmatrix}. \tag{47}$$

(The same applies to the spinors κ, ϵ.)

In this basis, the gauge-fixing condition is simply $\theta^{1,\alpha_1} = 0$. Again, $\sigma_3 \otimes 1$ anticommutes with Γ. Therefore, in the above basis that $\sigma_3 \otimes 1$ is diagonal, Γ is off-diagonal as in the IIA case, [6]

$$\Gamma = \begin{pmatrix} & \Gamma^{\delta_1}{}_{\beta_1} \\ \Gamma^{\gamma_1}{}_{\alpha_1} & \end{pmatrix}. \tag{48}$$

The supersymmetry transformation is given by

$$\delta \lambda^{2,\alpha_1} = \epsilon^{2,\alpha_1} - \Gamma^{\alpha_1}{}_{\beta_1} \epsilon^{1,\beta_1}, \tag{49}$$

where $\lambda = (1 - \mathcal{P})\theta$.

It is instructive to compare this supersymmetry transformation with the one of the supersymmetric $d = 10$ Maxwell theory in Minkowski space. For this, we have to linearize the 9-brane supersymmetry transformation in terms of the BI field. This leads to

$$\delta \lambda^{2,\alpha_1} = \epsilon^{2,\alpha_1} - \epsilon^{1,\alpha_1} - F_{ij} [\gamma^{ij}]^{\alpha_1}{}_{\beta_1} \epsilon^{1,\beta_1}, \tag{50}$$

which reproduces the supersymmetry transformation of the usual Maxwell theory with parameter $\epsilon^{2,\beta}$ when $\epsilon^{1,\alpha} = \epsilon^{2,\alpha}$ as well as Volkov–Akulov-type supersymmetries.

Finally, we note that the conditions (40) and (45) are *covariant* gauge-fixing conditions for the κ-symmetry. This is rather different from the type IIA/IIB fundamental string which is plagued with a well-known covariant quantization problem. The reason why this distinction between the type IIA/IIB fundamental string and the Dp-branes occurs is explained in more detail in Appendix A.

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[6] Here we have restricted already Γ to the positive-chirality subspace.

5. Supersymmetric D-brane probes

Let us consider a single D-brane probe propagating in $d = 10$ Minkowski space-time. The field equations of the probe are

$$\partial_i \left\{ \sqrt{|g + F|} \left[(g + F)^{-1} \right]^{(ij)} \partial_j X^M \right\} = 0 \,,$$

$$\partial_i \left\{ \sqrt{|g + F|} \left[(g + F)^{-1} \right]^{[ij]} \right\} = 0 \,. \tag{51}$$

A solution of these equations is

$$\begin{cases} X^i = \sigma^i \,, & i = 0, 1, \ldots, p \,, \\ X^m = y^m \,, & m = p + 1, \ldots, 9 \,, \\ F_{ij} = c_{ij} \,, \end{cases} \tag{52}$$

where y^m are the positions of the probe and c_{ij} are constant.

As we have seen in the previous section, the condition for the above configuration to be supersymmetric is

$$\left(1 - e^{-a/2} \Gamma'_{(0)} e^{a/2} \right) \epsilon = 0 \,, \tag{53}$$

where

$$a = \begin{cases} -\frac{1}{2} Y_{jk} \gamma^{jk} \Gamma_{11} & \text{IIA,} \\ \frac{1}{2} Y_{jk} \sigma_3 \otimes \gamma^{jk} & \text{IIB.} \end{cases} \tag{54}$$

Viewing the Dp-brane as a $(p + 1)$-dimensional Minkowski subspace of $d = 10$ Minkowski space-time, it is clear due to the properties of Γ this configuration preserves $1/2$ of the supersymmetry of the $d = 10$ Minkowski vacuum.

Next suppose that two D-brane probes with non-vanishing but constant BI field are placed in the $d = 10$ Minkowski space-time with product structures Γ and $\tilde{\Gamma}$. It is rather involved to find the fraction of the supersymmetry preserved by a generic such configuration. Below we shall examine some special cases.

5.1. Orthogonal intersections

Suppose that two D-branes, with product structure Γ and $\tilde{\Gamma}$, respectively, are intersecting orthogonally, and that both BI field strengths are zero. Because of the latter hypothesis, $a = 0$, and so

$$\begin{cases} \Gamma = \Gamma'_{(0)} \,, \\ \tilde{\Gamma} = \tilde{\Gamma}'_{(0)} \,. \end{cases} \tag{55}$$

Viewing the two D-branes as $(p + 1)$- and $(q + 1)$-dimensional Minkowski subspaces of the $d = 10$ Minkowski space-time, one can introduce an orthonormal basis $\{e_a; a =$

$0, \ldots, 9\}$ in the $d = 10$ target space adopted to these two D-branes, i.e. the orthonormal basis is chosen by extending an orthonormal basis along the common directions of the intersection first along the relative transverse directions of the intersection and then along the overall transverse directions of the intersection, in the terminology of [11]. Using the $d = 10$ gamma matrices adopted to this orthonormal basis, $\Gamma'_{(0)}$ and $\tilde{\Gamma}'_{(0)}$ can be expressed as a product of gamma matrices and therefore

$$\Gamma\tilde{\Gamma} = \pm\tilde{\Gamma}\Gamma, \tag{56}$$

If Γ and $\tilde{\Gamma}$ commute, they can be diagonalized simultaneously and their product $\Gamma\tilde{\Gamma}$ is also a product structure. If $\Gamma \neq \tilde{\Gamma}$, then $\Gamma\tilde{\Gamma}$ is traceless, tr$\Gamma\tilde{\Gamma} = 0$, in which case the amount of supersymmetry preserved is $1/4$. Examples of orthogonally intersecting D-brane configurations preserving $1/4$ of the supersymmetry are those with four or eight relative transverse directions in agreement with [14,15]. If $\Gamma = \tilde{\Gamma}$, then the two Dp-branes are parallel and the fraction of supersymmetry preserved is $1/2$.

However, if Γ and $\tilde{\Gamma}$ anticommute, imposing (53) separately for each D-brane leads to the breaking of *all* space-time supersymmetry (however, see also Ref. [16]).

5.2. Branes intersecting at angles

Another special case is that of two intersecting Dp-branes at an arbitrary angle in $d = 10$ Minkowski space-time with the 2-form BI field vanishing [12,13,19].[7] For each D-brane involved in the configuration, we can associate a $d = 10$ Lorentz frame; we may assume without loss of generality that the two orthonormal frames coincide along the directions of the intersection. For this, we use the assumption that each brane is identified with a Minkowski subspace of the $d = 10$ Minkowski space-time to choose an orthonormal basis for the world-volume directions and then extend this basis to an orthonormal basis for the whole $d = 10$ Minkowski space-time. If $\{e_a; a = 1, \ldots, 10\}$ is the Lorentz frame associated with the first D-brane and $\{\tilde{e}_a; a = 1, \ldots, 10\}$ is the Lorentz frame associated with the second D-brane, there is a Lorentz transformation Λ such that

$$\tilde{e}_a = e_b \Lambda^b{}_a. \tag{57}$$

This in turn implies that the gamma matrices $\{\Gamma_a; a = 1, \ldots, 10\}$ in the frame $\{e_a; a = 1, \ldots, 10\}$ are related to the gamma matrices $\{\tilde{\Gamma}_a; a = 1, \ldots, 10\}$ in the frame $\{\tilde{e}_a; a = 1, \ldots, 10\}$ as follows:

$$\tilde{\Gamma}_a = \Gamma_b \Lambda^b{}_a = S^{-1} \Gamma_a S, \tag{58}$$

where S is an element in $Spin(1, 9)$ that depends on Λ. As in the previous case of parallel or orthogonal D-branes,

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[7] For recent results on supergravity solutions that are related to branes at angles, see Refs. [20–24].

$$\Gamma = \Gamma'_{(0)} , \tag{59}$$

$$\tilde{\Gamma} = \tilde{\Gamma}'_{(0)} . \tag{60}$$

However, in this case Γ is a product of the gamma matrices associated with the $\{e\}$ basis while $\tilde{\Gamma}$ is a product of the gamma matrices associated with the $\{\tilde{e}\}$ basis. Using (58), the latter product structure written in the $\{e\}$ basis is

$$\tilde{\Gamma}_{(\tilde{e})} = S^{-1}\tilde{\Gamma}_{(e)}S , \tag{61}$$

where the subscript denotes the basis with respect to which $\tilde{\Gamma}$ is expressed and $\tilde{\Gamma}_{(e)}$ is again a product of gamma matrices in the $\{e\}$ basis. Dropping the subscript and expressing both supersymmetry projection operators in the $\{e\}$ basis, we get

$$\Gamma\epsilon = \epsilon , \tag{62}$$

$$S^{-1}\tilde{\Gamma}S\epsilon = \epsilon . \tag{63}$$

The case $\Gamma = \tilde{\Gamma}$ and $S \neq 1$ was studied in [12,13,19] where it was shown that the fraction of the supersymmetry preserved is $k/32$, where k is the number of singlets of the matrix S acting on the spinors ϵ that have the property, $\Gamma\epsilon = \epsilon$. We remark that such intersecting at an angle configuration of two Dp-branes is not associated with Lorentz rotations Λ of the world-volume coordinates of a single Dp-brane. This is because from the definition of the product structures $\Gamma = \tilde{\Gamma}$ and $S = 1$, so condition (63) is not independent from condition (62) and the supersymmetry preserved is $1/2$. Therefore the interesting cases involve Lorentz rotations of the $d = 10$ space-time that are not Lorentz rotations of the world-volume coordinates of a single Dp-brane. In fact the relevant Lorentz rotations are those of the relative transverse coordinates of the intersecting configuration, in the terminology of [11]. Examples of Lorentz rotations that have singlets acting on $SO(1,9)$ spinors are those that lie in the subgroups $SU(n)$, $1 \leqslant n \leqslant 3$, $Sp(2)$, G_2 and $Spin(7)$ of $SO(1,9)$.

Next suppose that $\Gamma \neq \tilde{\Gamma}$ and $S \neq 1$, then since both Γ and $\tilde{\Gamma}$ are products of $d = 10$ gamma matrices Γ and $\tilde{\Gamma}$ either commute or anti-commute. If they commute, there is a basis that can be simultaneously diagonalized, i.e.

$$\Gamma = (\delta^{\alpha_1}{}_{\beta_1}, \delta^{\alpha_2}{}_{\beta_2}, -\delta^{a_1}{}_{b_1}, -\delta^{a_2}{}_{b_2}) ,$$
$$\tilde{\Gamma} = (\delta^{\alpha_1}{}_{\beta_1}, -\delta^{\alpha_2}{}_{\beta_2}, \delta^{a_1}{}_{b_1}, -\delta^{a_2}{}_{b_2}) . \tag{64}$$

In this basis, only the spinors $\epsilon = (\epsilon^{\alpha_1}, \epsilon^{\alpha_2}, 0, 0)$ satisfy (62). Substituting this ϵ into (63), we get

$$S^{\beta_2}{}_{\alpha_1}\epsilon^{\alpha_1} + S^{\beta_2}{}_{\alpha_2}\epsilon^{\alpha_2} = 0 , \tag{65}$$

$$S^{a_2}{}_{\alpha_1}\epsilon^{\alpha_1} + S^{a_2}{}_{\alpha_2}\epsilon^{\alpha_2} = 0 . \tag{66}$$

If $\det(\{S^{\beta_2}{}_{\alpha_2}\}) \neq 0$, the first equation can be solved for ϵ^{α_2} and after substitution into the second equation we get

$$A^{a_2}{}_{\alpha_1}\epsilon^{\alpha_1} = 0 , \tag{67}$$

where

$$A^{a_2}{}_{\alpha_1} = S^{a_2}{}_{\alpha_1} - S^{a_2}{}_{\alpha_2}(S^{-1})^{\alpha_2}{}_{\beta_2} S^{\beta_2}{}_{\alpha_1} \,. \tag{68}$$

Thus the fraction of the supersymmetry preserved is $k/32$ where k is the number of zero eigenvalues of the matrix A. Note that, since $\operatorname{tr} \Gamma \tilde{\Gamma} = 0$, A is an 8×8 square matrix. The case $\det(\{S^{\beta_2}{}_{\alpha_2}\}) = 0$ can be treated in a similar way.

Now if Γ and $\tilde{\Gamma}$ anticommute, there is a basis such that

$$\Gamma = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \,,$$

$$\tilde{\Gamma}(e) = \begin{pmatrix} 0 & U \\ U & 0 \end{pmatrix} \,, \tag{69}$$

where 1 is a 16×16 unit square matrix and U is a diagonal 16×16 matrix with $U^2 = 1$. Note that there is a matrix V such that

$$\tilde{\Gamma} = V^{-1}DV \,, \tag{70}$$

where D is a diagonal matrix with $D^2 = 1$ and $\operatorname{tr} D = 0$. Since now D and Γ commute, we can examine this case by repeating the steps of the previous case after setting $S \to VS$. In particular, the fraction of the supersymmetry preserved is $k/32$ where k are the number of zero eigenvalues of the matrix A defined in (68), after replacing S with VS.

5.3. Branes intersecting at angles with BI fields

It remains to investigate the case of intersecting D-branes with non-vanishing constant BI field F. As we have seen in the previous section the effect that a non-vanishing BI field has on the supersymmetry projection of a D-brane is to rotate it. In this respect the situation is similar to the one examined above but there is an important difference. *The rotation induced by the BI field on the D-brane product structure is a relative rotation of the left- and right-moving fields.* More explicitly, we deduce from the form of the exponential in (29) (IIA) and (34) (IIB) that the dependence of the BI field is induced by a Lorentz rotation that acts differently on the two 16-component (left- and right-moving) κ-symmetry parameters. In the IIA case this is due to the fact that the exponential has a Γ_{11} that multiplies the standard generator of Lorentz rotations in the spinor representation. In the IIB case the different behaviour of the left- and right-moving fields is due to the presence of the σ_3 matrix in the exponential.

Intuitively, it is clear that the dependence on the BI field cannot be written as a Lorentz rotation that acts the same on the left- and right-moving fields. We recall that the BI field is a non-linear generalization of the Maxwell field on the world-volume of the D-branes. Now if the only effect that it has is to induce a Lorentz rotation, it would mean that by changing Lorentz frame one could set the BI field equal to zero. This would have been a contradiction since the Maxwell equations are Lorentz invariant

and if the Maxwell field is non-zero in one Lorentz frame it is non-zero in any Lorentz frame.

Nevertheless, the supersymmetry conditions for two intersecting D-branes in $d = 10$ Minkowski space-time can be written as

$$\Gamma\, e^{a/2}\epsilon = e^{a/2}\epsilon\,,$$
$$e^{-\tilde{a}/2}\tilde{\Gamma}\, e^{\tilde{a}/2}\epsilon = \epsilon\,. \tag{71}$$

Now if the two D-branes intersect at a angle, then as before we introduce two $d = 10$ Lorentz frames one for each D-brane. Then there is a $d = 10$ Lorentz transformation, Λ, as in (61), that relates the $d = 10$ gamma matrices adopted to one frame to the gamma matrices adopted to the other frame. Rewriting (71) in the same basis, we get

$$\Gamma'_{(0)}e^{a/2}\epsilon = e^{a/2}\epsilon\,,$$
$$S^{-1}\,e^{-\tilde{a}/2}\tilde{\Gamma}'_{(0)}e^{\tilde{a}/2}S\epsilon = \epsilon\,, \tag{72}$$

where S is induced by Λ and $\Gamma'_{(0)}$ and $\tilde{\Gamma}'_{(0)}$ are expressed in the same basis. Next, let us set

$$\eta = e^{a/2}\epsilon\,. \tag{73}$$

Then (72) can be rewritten as

$$\Gamma'_{(0)}\eta = \eta\,, \tag{74}$$
$$T^{-1}\tilde{\Gamma}'_{(0)}T\eta = \eta\,, \tag{75}$$

where

$$T = e^{\tilde{a}/2}S\,e^{-a/2}\,. \tag{76}$$

To investigate the fraction of supersymmetry preserved by two D-branes intersecting at angles with non-vanishing BI fields, we remark that (74), (75) is the same as (62), (63) after setting

$$T \to S\,. \tag{77}$$

Therefore the methods developed in the previous subsection to examine the fraction of supersymmetry preserved by two intersecting D-branes at an angle without BI fields also apply to this case.

6. Supersymmetric M-brane probes

The κ-symmetry transformation for the M5-brane, in the form given in [8], is

$$\delta\theta = (1 + \Gamma)\kappa\,, \tag{78}$$

where

$$\Gamma = \Gamma_{(0)} \left[1 - \frac{1}{2 \cdot 3!} h_{ijk} \gamma^{ijk} \right] , \tag{79}$$

$$\Gamma_{(0)} = \frac{1}{6! \sqrt{|g|}} \epsilon^{i_1 \ldots i_6} \gamma_{i_1} \ldots \gamma_{i_6} . \tag{80}$$

The metric g is the induced metric and h is a self-dual 3-form world-volume field. Observe that Γ and $\Gamma_{(0)}$ are traceless hermitian product structures.

As we have already mentioned in Section 2, the supersymmetry preserved by a M5-brane probe is

$$(1 - \Gamma)\epsilon = 0 , \tag{81}$$

where ϵ is the supersymmetry parameter. As in the case of D-branes, Γ can be written as

$$\Gamma = e^{-a} \Gamma_{(0)} = e^{-\frac{1}{2}a} \Gamma_{(0)} e^{\frac{1}{2}a} , \tag{82}$$

where

$$a = -\frac{1}{2 \cdot 3!} h_{ijk} \gamma^{ijk} . \tag{83}$$

Note that $a^2 = 0$ due to the self-duality of h.

Although the product structure Γ is easily written in the form given above, the dependence on h has no straightforward geometric interpretation as a kind of rotation since the exponential in (82) is cubic in the world-volume gamma matrices. Nevertheless, one can repeat the analysis of Section 3 to find the fraction of supersymmetry preserved by a configuration of intersecting M5-brane probes. A single M5-brane probe preserves $1/2$ of the supersymmetry of $D = 11$ vacuum with or without a non-vanishing h.

To investigate the supersymmetry preserved by two intersecting M5-brane probes at an angle with constant field h, we can again proceed as in the case of D-branes. For this, we introduce two Lorentz frames, $\{e\}$ and $\{\tilde{e}\}$ adopted to each M5-brane involved in the intersection, and a $D = 11$ Lorentz transformation, Λ, such that $\tilde{e}_a = e_b \Lambda^b{}_a$. Then, there is a $S \in Spin(1,10)$, $S = S(\Lambda)$, such that

$$\tilde{\Gamma}_{(0)} = S^{-1} \Gamma_{(0)} S . \tag{84}$$

Using this, we can write both supersymmetry conditions in the same $D = 11$ gamma matrix basis as follows:

$$\Gamma_{(0)} e^{a/2} \epsilon = e^{a/2} \epsilon , \tag{85}$$

$$S^{-1} e^{-\tilde{a}/2} \Gamma_{(0)} e^{a/2} S \epsilon = \epsilon . \tag{86}$$

Next setting

$$\eta = e^{a/2} \epsilon , \tag{87}$$

the supersymmetry conditions can be rewritten as

$$\Gamma_{(0)}\eta = \eta\,, \tag{88}$$

$$T^{-1}\Gamma_{(0)}T\eta = \eta\,, \tag{89}$$

where

$$T = e^{\tilde{a}/2}S\,e^{-a/2}\,, \tag{90}$$

which is of the form (74), (75). Because of this, the general analysis for D-branes applies in this case as well and so we shall not repeat it here.

As an example let us consider the intersection of two M5-branes on a string [20]. Let us suppose that $a = \tilde{a} = 0$. The rotation Λ involved in this case is an element of $Sp(2) \subset SO(1,10)$. The spinor representation of $SO(1,10)$ decomposed as representation of $Sp(2)$ has 6 singlets and the fraction of the supersymmetry preserved by such configuration is $3/16$.

This method of finding the fraction of supersymmetry preserved by intersecting M5-branes configurations can be easily extended to intersecting configurations involving M2-branes as well. For example, the supersymmetry conditions for a M2-brane/M5-brane intersecting configuration at an angle are

$$\Gamma_{(0)}\epsilon = \epsilon\,,$$
$$S^{-1}\,e^{-\tilde{a}/2}\,\tilde{\Gamma}_{(0)}\,e^{a/2}S\epsilon = \epsilon\,, \tag{91}$$

where

$$\Gamma_{(0)} = \frac{1}{3!\sqrt{|g|}}\epsilon^{i_1 i_2 i_3}\gamma_{i_1}\gamma_{i_2}\gamma_{i_3} \tag{92}$$

is the product structure associated with the M2-brane, and $\tilde{\Gamma}_{(0)}$ is the product structure associated with the M5-brane as given in (79), (80) (both expressed in the same basis). Finally, the supersymmetry conditions for a M2-brane/M2-brane intersecting configuration at an angle are

$$\begin{cases} \Gamma_{(0)}\epsilon = \epsilon\,, \\ S^{-1}\Gamma_{(0)}S\epsilon = \epsilon\,, \end{cases} \tag{93}$$

where $\Gamma_{(0)}$ is the product structure associated with one of the M2-branes.

7. Supergravity backgrounds

We briefly consider the coupled D-brane/supergravity equations. The supergravity solution corresponding to Dp-branes in the string frame is

$$\begin{cases} ds^2 = H^{-\frac{1}{2}}ds^2(\mathbb{E}^{(1,p)}) + H^{\frac{1}{2}}ds^2(\mathbb{E}^{9-p}) \,, \\[2mm] e^{\phi} = H^{\frac{3-p}{4}} \,, \\[2mm] F_{p+2} = \omega(\mathbb{E}^{(1,p)}) \wedge dH^{-1} \,, \end{cases} \tag{94}$$

where ω is the volume form of $\mathbb{E}^{(1,p)}$ and

$$H = H(y - y_0) \tag{95}$$

is a harmonic function of $\mathbb{E}^{(9-p)}$. The embedding X of the world-volume into the space-time is specified by identifying the world-volume coordinates of the Dp-brane with $\mathbb{E}^{(1,p)}$, i.e.

$$\begin{cases} X^i = \sigma^i \,, \\[2mm] X^m = y_0^m \,, \end{cases} \tag{96}$$

where y_0 is the position of the harmonic function H. It is straightforward to see that the BI field must vanish by examining the field equation of the NS–NS 2-form gauge potential.

It remains to compare the supergravity Killing equation with the world-volume su-supersymmetry condition (5). The solution of the Killing spinor equation is

$$\epsilon = H^{-\frac{1}{8}}\xi \,, \tag{97}$$

for constant ξ and

$$(1 - \Pi)\xi = 0 \,, \tag{98}$$

where

$$\Pi = \begin{cases} (\Gamma_{11})^{\frac{p-2}{2}} \Gamma_0 \dots \Gamma_p \,, & \text{IIA} \,, \\[2mm] (\sigma_3)^{\frac{p-3}{2}} i\sigma_2 \otimes \Gamma_0 \dots \Gamma_p \,, & \text{IIB} \,. \end{cases} \tag{99}$$

Using the solution (94) and (96), we find that $\Gamma = \Pi$.

It is natural to extend the above analysis to the case of intersecting brane configurations. For orthogonally intersecting ones, we find that the BI field must vanish for each brane separately. This is also the case for all intersecting brane configurations (even for those that intersect at angles) with vanishing NS–NS 2-form gauge potential.

Finally, as for the single brane the conditions for supersymmetry derived from the supergravity Killing spinor equations are compatible with those found in Section 5.

Acknowledgements

Appendix A. κ-symmetry and covariant quantization

The κ-symmetry is an example of an *infinitely reducible* gauge symmetry because the sequence of shifts $\kappa_0 \to (1 - \Gamma)\kappa_1$, $\kappa_1 \to (1 + \Gamma)\kappa_2 \ldots$ leaves the κ-transformations (12) unchanged. This is a property of all branes. However, as far as the covariant quantization is concerned, there is a distinction between the type II fundamental string and the D-branes. This is because the only covariant gauges that we can pick are

$$\begin{cases} \Gamma_{11}\theta = \pm\theta \,, & \text{IIA} \,, \\ \sigma_3 \otimes \mathbb{1}_{32\times32}\theta = \pm\theta \,, & \text{IIB} \,. \end{cases} \tag{A.1}$$

Under κ-symmetry the variation of the action is given by the following expression [4–6]

$$\delta_\kappa S = \int d^{p+1}\sigma\, \delta_\kappa\bar{\theta}\Delta = \int d^{p+1}\sigma\, \bar{\kappa}(1 + \Gamma)\Delta = 0 \,. \tag{A.2}$$

Here $\Delta = (1 - \Gamma)\Psi$. This variation can be presented as consisting of two parts: one due to the variation of $\bar{\theta}\mathcal{P}$ and the other, due to variation of $\bar{\theta}(1 - \mathcal{P})$:

$$\begin{aligned} \delta_\kappa S &= \delta_\kappa^1 S + \delta_\kappa^2 S \\ &= \int d^{p+1}\sigma\, \bar{\kappa}(1 + \Gamma)\mathcal{P}\Delta + \int d^{p+1}\sigma\, \bar{\kappa}(1 + \Gamma)(1 - \mathcal{P})\Delta \,. \end{aligned} \tag{A.3}$$

Assume that we choose the gauge $\bar{\theta}\mathcal{P} = 0$ and do not vary $\bar{\theta}\mathcal{P}$ anymore. The variation of the action under the transformations of the remaining part of θ, which is given by $\delta\bar{\theta}(1 - \mathcal{P})$ should not vanish anymore. This would mean that the gauge symmetry is gauge fixed, the action does not have a symmetry anymore. Thus we have to find whether

$$\delta_\kappa S_{\text{g.f.}} = \delta_\kappa^2 S = \int d^{p+1}\sigma\, \bar{\kappa}(1 + \Gamma)(1 - \mathcal{P})\Delta \tag{A.4}$$

vanishes or not. We observe that

$$\delta_\kappa S_{\text{g.f.}} = \int d^{p+1}\sigma\, \bar{\kappa}(1 + \Gamma)(1 - \mathcal{P})(1 - \Gamma)\Psi \,. \tag{A.5}$$

This explains why the issue of commutativity (non-commutativity) of the projector $\mathcal{P}$ with Γ becomes so important. Indeed, if they commute,

$$[\mathcal{P}, \Gamma] = 0, \qquad (A.6)$$

the action still has a local symmetry since

$$\delta_\kappa S_{\text{g.f.}} = \int d^{p+1}\sigma \, \bar{\kappa}(1+\Gamma)(1-\mathcal{P})(1-\Gamma)\Psi$$
$$= \int d^{p+1}\sigma \, \bar{\kappa}(1+\Gamma)(1-\Gamma)(1-\mathcal{P})\Psi = 0, \qquad (A.7)$$

and therefore $\bar{\theta}\mathcal{P} = 0$ is not an admissible gauge condition. However, if they anticommute,

$$\{\mathcal{P}, \Gamma\} = 0, \qquad (A.8)$$

then

$$\delta_\kappa S_{\text{g.f.}} = \int d^{p+1}\sigma \, \bar{\kappa}(1+\Gamma)(1-\mathcal{P})(1-\Gamma)\Psi$$
$$= -\int d^{p+1}\sigma \, \bar{\kappa}(1+\Gamma)(1+\Gamma)\mathcal{P}\Psi \neq 0, \qquad (A.9)$$

the action is not gauge-symmetric anymore, the gauge-fixing condition is admissible.

For the fundamental GS string the κ-symmetry is given by

$$\delta\bar{\theta} = \bar{\kappa}(1+\Gamma), \qquad \Gamma = \Gamma_{(0)} \quad \text{at} \quad p = 1. \qquad (A.10)$$

This expression for Γ is proportional to $\Gamma^a\Gamma^b$ and therefore it commutes with $\mathcal{P} = \frac{1}{2}(1 + \Gamma_{11})$ in the IIA case and with $\mathcal{P} = \frac{1}{2}(1 + \sigma_3 \otimes 1)$ in the IIB case. These would be Lorentz covariant gauges for the fundamental string, and as we see here, they are not acceptable. This is the well known covariant quantization problem of the IIA/IIB fundamental string.

On the other hand, we have found that for all D-branes the relevant Γ anticommute with Lorentz covariant gauge-fixing projectors above. The gauges are acceptable since the remaining action is not gauge symmetric. This explains the existence of the covariant gauges for D-branes which were found in Ref. [5].

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