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ERRATUM

Killing forms on quaternion-Kähler manifolds

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The aim of this note is to fill a gap in the proof of Theorem 6.1 in [1]—stating that every Killing p-form ($p \geq 2$) on a compact quaternion-Kähler manifold M^{4m} ($m \geq 2$) is parallel. This gap, pointed out by Liana David, is due to two wrong coefficients in the formulas at the middle of page 329. The correct equations read

$$pd^+u = d(L^-u), \qquad (p-1)d^cu = d(Ju + 3u), \qquad pd^-u = d(\Lambda^+u) - \delta^c u,$$
$$(p-1)\delta^+u = -2d(Cu), \quad (p-1)\delta^c u = -2d(\Lambda^+u) - 2d^-u, \quad (p-3)\delta^-u = -2d(\Lambda u).$$

The remaining part of the proof works verbatim for $p > 3$, but an extra argument is needed for $p = 2$ and $p = 3$.

The case $p = 2$. From the six equations above, one obtains the vanishing of d^+u, δ^-u, and $\delta^+u + d^cu + 3du$, but no longer that of δ^cu and d^-u. Correspondingly, the proof of Lemma 6.3 fails. Fortunately, an ad hoc argument shows that $du = 0$ in this case. Indeed, the third equation of the system (7) shows that du is an eigenform of $2C - J$ for the eigenvalue 9. On the other hand, Lemma 5.1, Lemma 5.2, and the decomposition

$$\Lambda^3(H \otimes E) = H \otimes [\Lambda_0^{2,1}E \oplus E] \oplus \mathrm{Sym}^3 H \otimes [\Lambda_0^3 E \oplus E]$$

show that the eigenvalues of $2C - J$ on the four summands of $\Lambda^3 M$ are 3, $4m + 5$, 15, and $4m + 11$, respectively. For $m \geq 2$, none of them equals 9.

The case $p = 3$. The proof works well in this case, except that one does not obtain $\delta^-u = 0$ in Lemma 6.2. However, this relation is not needed until the point (b) at the bottom of

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page 331. In order to rule out that case, one has to replace the argument given there with the fact that for $p = 3$ and $m \geq 2$, the inequality $p \geq 2m + 1$ is impossible.

Remark 1 The assumption that $m \geq 2$ is essential. For $m = 1$, the quaternionic projective space $\mathbb{H}P^1 \simeq S^4$ carries non-parallel Killing 2-forms and 3-forms (cf. [2]).

References

1. Moroianu, A., Semmelmann, U.: Twistor forms on quaternion-Kähler manifolds. Ann. Glob. Anal. Geom. **28**, 319–335 (2005)
2. Semmelmann, U.: Conformal Killing forms on Riemannian manifolds. Math. Z. **245**, 503–527 (2003)