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Kinematic ages of OB associations

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ABSTRACT

Astrometric data from the *HIPPARCOS* mission include measurements of early-type stars in nearly all OB associations within 1 kpc from the Sun, and are available as of 1996. In anticipation thereof we studied the determination of kinematic ages and initial sizes of OB associations. These studies have traditionally been done using proper motion data. We investigate their reliability by generating synthetic data, using self-consistent *N*-body simulations of an OB association. We consider two classical methods for deriving the kinematic age. First, the proper motions of the stars are traced back in time to the smallest configuration in the past, which is assumed to correspond to the initial state of the association. Secondly, the proper motion in a certain direction is plotted versus the corresponding coordinate, and then the linear expansion coefficient is measured to derive the kinematic age. We find that the first method always leads to underestimated ages. All age estimates converge to ~4 Myr. The second method can lead to overestimated as well as underestimated ages, depending on the chosen coordinate direction and the magnitude of the effects of virtual expansion caused by radial motion. The first method also provides an estimate of the initial size of the OB association, which is always overestimated. We conclude that the longstanding discrepancy between the kinematic and nuclear ages for OB associations can be attributed to underestimates of the kinematic age.

Key words: astrometry – stars: kinematics – Galaxy: kinematics and dynamics – open clusters and associations: general.

1 INTRODUCTION

Since the discovery that O and B stars are not randomly distributed on the sky but instead are concentrated in isolated groups, it has become clear that these groups must be very young. Ambartsumian (1947) found that the stellar mass density in these groups, subsequently called OB associations, is very low, usually less than 0.1 $M_\odot\,pc^{-3}$. Investigations by Bok (1934) had already shown that stellar groups are unstable against Galactic tidal forces if their mass density is lower than ~0.1– 0.3 $M_\odot\,pc^{-3}$, so that the observed OB associations must be young. The additional discovery that OB associations can always be found near star-forming regions (see e.g. Blaauw 1964) and in the vicinity of giant molecular clouds led to the recognition of OB associations as important sites for the study of star formation processes.

The most reliable membership determinations for OB associations are based on proper motion studies. Associations have small internal velocity dispersions (e.g. Mathieu 1986), so that the streaming motion of the association as a whole, combined with the solar motion, results in a motion of the members towards a convergent point on the sky (see e.g. Bertiau 1958). Membership determinations have been carried out by various investigators (see Blaauw 1964, 1991 and references therein).

Once membership has been established, accurate proper motion data can be used to investigate the internal kinematics of OB associations. The low mass densities cause the associations to be unbound, so that they expand during their entire lifetime. This suggests a simple model for the evolution of an OB association based on the notion of linear expansion (Blaauw 1946, 1964). In this model the association members all move away from their birthplace without experiencing any forces. The expansion causes an increase in the component of proper motion as a function of the corresponding coordinate. The slope of the resulting relation is called the expansion coefficient, and its reciprocal is a measure of the *kinematic* age of the association (Blaauw 1964).

The kinematic age is of interest because its value is determined independently from stellar evolution models (from which the *nuclear* age follows). Differences between the kinematic and nuclear ages may provide important constraints on gas removal time-scales in young stellar clusters.

The linear expansion model assumes that the associations have a small initial size (as compared with their present size). This led Blaauw (1978, 1983) to consider an alternative method for determining the kinematic age. The individual motions of the stars can be traced back until they reach a smallest configuration at some past

Table 1. Ages of OB associations (in Myr).

Association	expansion coefficient	tracing of motions	nuclear
Upper Scorpius		4.5[1]	5–6[2]
Orion OB1a	$4.5 \pm 30\%$[3]		11.4 ± 1.9[4]
Orion OB1b		2.2–4.9[5]	1.7 ± 1.1[4]
Perseus OB2	1.3[6]	1[7]	6–8[8]
Cepheus OB3a	0.5[9]		8–12[8]
Lacerta OB1b	2.5[6]		12–20[8]

[1]Blaauw (1978, 1991), [2]de Geus, de Zeeuw & Lub (1989), [3]Lesh (1968), [4]Brown, de Geus & de Zeeuw (1994), [5]Blaauw (1961), [6]Lesh (1969), [7]Blaauw (1983), [8]de Zeeuw & Brand (1985), [9]Garmany (1973).

time. In this way one may find the kinematic age as well as the initial configuration of the association.

Kinematic ages have been determined with both methods, and the results are listed in Table 1 together with determinations of nuclear ages of the same associations (based on colour–magnitude diagrams in combination with isochrone fitting). It is clear that in all cases (except Orion OB1b) the kinematic age is smaller than the nuclear age of the association. This is a known fact, but it is still an open question whether the difference in ages is real or indicative of a misunderstanding of the physics involved in the estimate of the nuclear age, or of the reliability of the kinematic age determinations (due, for example, to imprecise proper motion data).

Detailed investigations of OB associations have been hampered by a lack of accurate data for stars of spectral type later than ~B5. To remedy this problem the SPECTER consortium, formed at Leiden, proposed the observation by *HIPPARCOS* of over 10 000 candidate members of OB associations within 1 kpc from the Sun (see de Zeeuw, Brown & Verschueren 1994). Astrometric data have been taken for spectral types as late as G.

In order to prepare for the release of *HIPPARCOS* data we investigated the accuracy of kinematic ages by performing *N*-body simulations of OB associations. Synthetic associations are evolved in time in the Galactic potential and projected on to the sky. Subsequently we 'observe' these associations and determine the kinematic ages as described above. The results are then compared with the real ages of the associations (the 'real' age being the time elapsed since the start of the simulation). The method of tracing the motions of the members of the associations is also used to check whether one can actually derive their initial configuration.

The outline of this paper is as follows. In Section 2 we describe how we generated synthetic associations and briefly outline the *N*-body code that was used. The analysis of the simulations in terms of kinematic ages and initial configurations is presented in Section 3. In Section 4 we discuss the results. Finally, in Section 5 we summarize our conclusions and suggest future work.

2 SYNTHETIC ASSOCIATIONS: THE *N*-BODY CODE AND INITIAL CONDITIONS

We first discuss the classification of stellar groups and then define what we mean by an OB association. From there we discuss the *N*-body code and the initial conditions that are used in the simulations.

2.1 The classification of stellar groups

Definitions of clusters and associations have been summarized by Lada & Lada (1991). A cluster is defined as a group of 10 or more physically related stars whose mass density is large enough to render the group stable against tidal disruption by the parent galaxy and the passage of interstellar clouds. The tidal limit in our Galaxy is 0.1 $M_\odot\,pc^{-3}$ (Bok 1934), while stability against disruption by clouds requires densities larger than 1 $M_\odot\,pc^{-3}$ (Spitzer 1958). An association is defined as a loose group of at least 10 members whose stellar density is less than 1 $M_\odot\,pc^{-3}$.

OB associations are characterized by low stellar densities and a large spatial extent. They survive as recognizable stellar groups for a short time only (~25 Myr, see Blaauw 1991). This is because they are gravitationally unbound and thus quickly disperse into the stellar background population. These characteristics do not imply that the association started out as a very loose grouping of stars. For instance, a group of 500 stars distributed homogeneously in a sphere of radius 0.5 pc with a velocity dispersion of 2 $km\,s^{-1}$ in one coordinate direction, a power-law initial mass function, $\xi(\log m) \propto m^{-1.7}$, and a total mass of 1000 $M_\odot$ is gravitationally unbound [see equation (2) below; note that the density is ~1900 $M_\odot\,pc^{-3}$]. Such a group expands to the dimensions of a typical OB association within 5–10 Myr.

Thus a mass density criterion does not serve to distinguish adequately between associations and clusters. We choose therefore to define OB associations as stellar groups that are left unbound after the process of gas removal from the protostellar group is completed. This means that an association can start out with a high stellar density, as in the example above, but it will quickly expand and become like the OB associations observed presently. The process of gas removal itself is not taken into account in our simulations.

2.2 The *N*-body code and the simulation procedure

We use the NBODY2 code of Aarseth (1995), which employs the Ahmad & Cohen (1973) neighbour scheme. By default this code scales all variables to so-called standard units recommended by Heggie & Mathieu (1986). This is not suitable for unbound systems, such as our associations, and we do not apply the scaling. We also discard the routine for removal of stars that have escaped from a bound system.

The synthetic association is placed in the Galactic plane and rotates around the Galactic Centre. Fig. 1 shows the coordinate system that is used in the *N*-body part of the simulations. This coordinate system is attached to the association and rotates around the Galactic Centre with angular frequency ω. The X-axis is always pointed away from the Galactic Centre and lies in the Galactic plane. The Y-axis also lies in the Galactic plane perpendicular to the X-axis in the direction of rotation, i.e. clockwise. The Z-axis is perpendicular to the Galactic plane and pointed upwards.

An important role is played by the external potential, provided by the Galaxy, that acts on the association. The Galactic orbit of the association is assumed to be circular with small perpendicular oscillations (see Aarseth 1994). This permits a linearized expansion of the smooth external potential and gives rise to the following

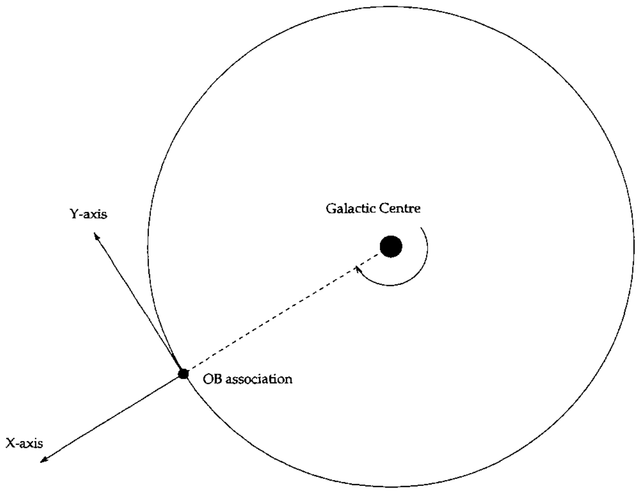

Figure 1. The coordinate system used in the *N*-body simulations. The system rotates with the OB association around the Galactic Centre. The directions of the X- and Y-axes are indicated. The Z-axis points upwards.

equations of motion:

$$\ddot{x} = F_x + 4A(A-B)x + 2\omega\dot{y},$$
$$\ddot{y} = F_y - 2\omega\dot{x}, \qquad (1)$$
$$\ddot{z} = F_z + \frac{\partial K_z}{\partial z} z.$$

The first term on the right-hand side is the *N*-body part of the force acting on the star. A and B are Oort's constants and $(\partial K_z/\partial z)$ is the vertical force gradient due to the Galactic potential, which can be replaced by a constant for small z. For details on this description of the Galactic potential see e.g. Hayli (1967) and Icke (1982).

The description of the Galactic tidal field leads to the epicyclic approximation for the motion in the Galactic plane. The motion in the vertical direction is a harmonic oscillation. The epicyclic frequency that we used is $\kappa = 1.13 \times 10^{-15}\,\mathrm{s}^{-1}$ and the vertical frequency is $\nu = 2.52 \times 10^{-15}\,\mathrm{s}^{-1}$. The epicycles described by the stars in the XY-plane are all elongated in the Y-direction, with axial ratio $X/Y \approx 0.7$ (see e.g. Binney & Tremaine 1987).

After the association has been evolved for a certain amount of time, it is placed at a certain distance and in a certain direction on the sky, and all positions and velocities in the coordinate system of Fig. 1 are transformed to a similar coordinate system centred on the Sun. From there the transformation to positions on the sky, proper-motions and radial velocities is performed. Details of the transformation to observables are described in the Appendix. The astrometric data are in Galactic coordinates. We do not include the differential rotation of the Galaxy as a whole in the observed proper motions. This is a complicating factor that will be discussed in Section 4.

In the next subsection we describe how the synthetic associations are generated. Only the 35 brightest (i.e. most massive) members of the association are 'observed'. This is done to simulate the fact that in most existing studies only the bright O and B stars in an association were observed. Observational errors are added to the astrometric data and the kinematic age and initial configuration are determined as described in Section 3. The results are then compared with the real age and initial configuration.

2.3 Initial conditions

As mentioned in Section 2.1, we choose to model OB associations as a group of stars that is unbound after the process of gas removal is completed. We are not after a precise modelling of OB associations. We simply want to assess the reliability and accuracy of kinematic ages and derived initial configurations. We therefore choose to start with very simple initial conditions. We do not include binaries or stellar mass loss in the simulations. We discuss the consequences of using more realistic initial conditions in Section 4.

The stars are distributed homogeneously throughout a spherical volume. The masses of the stars are selected randomly from a power-law initial mass function (IMF). The IMF is $\xi(\log m)\mathrm{d}\log m \propto m^{-1.7}\mathrm{d}\log m$, where the exponent in the power law is consistent with a recent determination of the IMF for Orion OB1 by Brown et al. (1994). The initial velocities of the stars are assumed to be independent for the X-, Y- and Z-directions and are distributed according to Gaussians with dispersions $\sigma_{x,0}$, $\sigma_{y,0}$, and $\sigma_{z,0}$. We also allow for a possible initial streaming motion (with respect to Galactic rotation) of the association, which is reflected as a systematic offset in the stellar velocity distribution.

If the stars in the spherical volume are to form an unbound system then the radius R, the total mass M_{tot} and the velocity dispersion σ must satisfy

$$R > 1.72 \times 10^{-3} \frac{M_{\mathrm{tot}}}{\sigma^2}, \qquad (2)$$

where R is in pc, M_{tot} is in $\mathrm{M}_\odot$, and σ is in $\mathrm{km\,s^{-1}}$. Equation (2) is an analytical approximation based on the expectation value for the masses and mutual distances of the stars in the association. To generate the initial conditions, appropriate values for the three variables in equation (2) have to be chosen.

The observed internal velocity dispersion in clusters and associations is rather low. The velocity dispersions in open clusters are of the order of $1\,\mathrm{km\,s^{-1}}$ (e.g. Mathieu 1985a). The observed average residual velocities in associations in one coordinate are 1.7 to $6.5\,\mathrm{km\,s^{-1}}$(Blaauw 1991). For Gaussian velocity distributions this corresponds to velocity dispersions of 2.1–$8.1\,\mathrm{km\,s^{-1}}$. These numbers are probably inflated by the presence of undetected spectroscopic binaries (see e.g. Mathieu 1985b) and thus form an upper limit. We decided to take initial velocity dispersions (in one coordinate) of 1, 2, 3 and 4 $\mathrm{km\,s^{-1}}$. This allows us to study the influence of the velocity dispersion.

It is not known what size an OB association typically has when it emerges from its parental molecular cloud. It may be as small as the Trapezium cluster or, as Blaauw (1978, 1991) suggested, the initial size may be of the order of 15–45 pc. The important property for us is that the stellar group is not gravitationally bound. As shown by the example in Section 2.1, the initial size can be very small as long as the velocity dispersion is high enough or the total mass low enough. We investigate associations with initial radii of 2 and 15 pc in order to cover both possibilities.

The stellar content of OB associations is only well-known for nearby systems, such as Sco OB2, and only for spectral types B5 and earlier. The number of members in this spectral type range varies between ~20 and ~50. To be consistent with this we require our synthetic associations to have about 50 stars more massive than 4 $\mathrm{M}_\odot$ (corresponding to B5 stars). We take 90 $\mathrm{M}_\odot$ as the upper mass limit in the IMF. The lower mass limit in OB associations is not known at present, because of the lack of knowledge of membership. We arbitrarily take a lower mass limit of 1 $\mathrm{M}_\odot$. If we then generate 500 stars from the IMF given above we get the required number of early-type stars. The total mass in the synthetic associations is 1150 $M_{\odot}$. This implies that an association with an initial radius of 2 pc and a velocity dispersion of 1 km s^{-1} in each direction is just unbound.

Finally, upon transformation of the positions and velocities to proper motions and coordinates on the sky, the reflection of the solar motion is added to the proper motions. The solar motion that we use is 16.5 km s^{-1} in the direction $(\ell, b) = (53°, 25°)$ (see e.g. Binney & Tremaine 1987). All synthetic associations are placed at a distance of 160 pc and in the direction $\ell = 0°$. The distance corresponds to the distance of the Upper Scorpius subgroup of Sco OB2 (de Geus et al. 1989), which is the nearest well-studied association. The results will indicate what is the best that can be achieved for the nearby associations.

3 CAN WE RELIABLY DETERMINE KINEMATIC AGES AND INITIAL CONFIGURATIONS?

If an association were to expand solely due to its own internal velocity dispersion and experienced no forces, the stars would move away on straight tracks. The kinematic age could then be determined by simply tracing back the motions of the stars or by using the linear expansion model. However, any influence on the stars that causes their trajectory to deviate from a straight line will induce errors in the kinematic age determination. In the case of the tracing back of the motions of the stars, the smallest configuration will be reached earlier and the age is then underestimated. The derived initial size will be overestimated. In the case of the linear expansion model the data will scatter about the expected straight line relation. This will generally lead to a different expansion coefficient from that expected from the age of the association. It is the aim of the investigations described below to find out what the magnitude of the errors is, and whether the derived ages can be corrected reliably after the fact.

There are two effects that intrinsically affect the expansion of the association: *N*-body interactions and the Galactic tidal field. The *N*-body interactions are only important in the denser associations and only in the initial expansion phases. By contrast, the Galactic tidal field influences the stellar orbits throughout the lifetime of the association.

Two effects that can cause an apparent expansion (or contraction) are the reflection of the solar motion in the motions of the stars, and the streaming motion of the association as a whole. The associations located close to the Sun have a large angular extent on the sky and the reflection of the solar motion depends on the position within the association. This causes an apparent difference in proper motions over the extent of the association. If the solar motion is not accounted for, this difference will be interpreted as an expansion or contraction.

The same effect occurs if the association as a whole has a streaming motion with respect to Galactic rotation. Any non-zero radial motion with respect to the line of sight through the centre of the association causes an apparent expansion or contraction. We refer to the latter effect as virtual expansion. One can correct for this effect by assuming that the radial velocities of the stars arise solely as a result of the radial component of the streaming motion. Then for each star one (de)projects the radial velocity on to the line of sight through the centre of the association. Subsequently for each star the proper motion corresponding to this radial streaming motion is calculated and subtracted from the observed proper motion. Details are given in the Appendix. After this is done the expansion effects that are left are real. Note that it does not matter that the assumption that the radial velocity of each star is only due to a radial streaming motion is flawed. It is the differences in proper motion that matter. The extra virtual proper motions introduced by this assumption cancel each other on opposite sides of the line of sight through the centre of the association, after subtraction of the observed mean motion on the sky. This method requires knowledge of the distances to the stars in order to compute the virtual proper motions. The correction thus takes into account the angular size as well as the depth of the association. It follows that one can never prove on the basis of proper motions alone that an association is expanding.

Finally, observational errors play an important role. Errors in the proper motion have the same effect as the gravitational forces acting on the association. The stellar trajectories will deviate from pure linear expansion. Errors in the distances to the stars will lead to errors in the corrections for solar motion and virtual expansion.

3.1 Measuring the kinematic age

The tracing back of the motions of the stars is simply done by reversing the proper motions of the association members. The motions are traced back in time until a smallest configuration on the sky is reached. The associations are unbound and thus expand during their entire lifetime. Therefore, any parameter that depends on the distances of the members to each other (as measured on the sky), or to the centre of the association, will serve as a measure of the size. After some experimentation we decided to use the sum of the distances between the stars in the association as a measure for the size. This parameter varies smoothly as the motions are traced back and goes through a well-defined minimum. The results are similar if the distances to the centre of the association are used.

When the linear expansion model is employed, the proper motion in a certain coordinate is plotted versus the coordinate. The data will lie along a straight line, the slope of which is the expansion coefficient. Its reciprocal is the kinematic age. In the simulations this is implemented by automatically fitting a straight line to the synthetic observations. This is done with a robust fitting method (by minimizing the absolute deviation: see e.g. Press et al. 1992, p. 698) that gives small weights to outlying points.

3.2 The initial configuration

We are primarily interested in finding out whether the size of the smallest configuration found while tracing the motions back in time corresponds to the real initial size. We calculate the dispersion σ in distances to the centre of the association (on the sky) and take 2.5σ as a measure for the size (radius) of the configuration. The dispersion is calculated in a certain direction, such as in ℓ or b. In the case of a Gaussian distribution of distances, 98.8 per cent of the stars in the association would lie within this limit. In Fig. 2 we show that indeed this parameter is a good measure of the size of the association.

3.3 Results

In order to get statistically relevant results and insight into the reliability of the methods used to determine ages and initial configurations, we carried out Monte Carlo simulations. Each simulation of an OB association is performed 50 times with

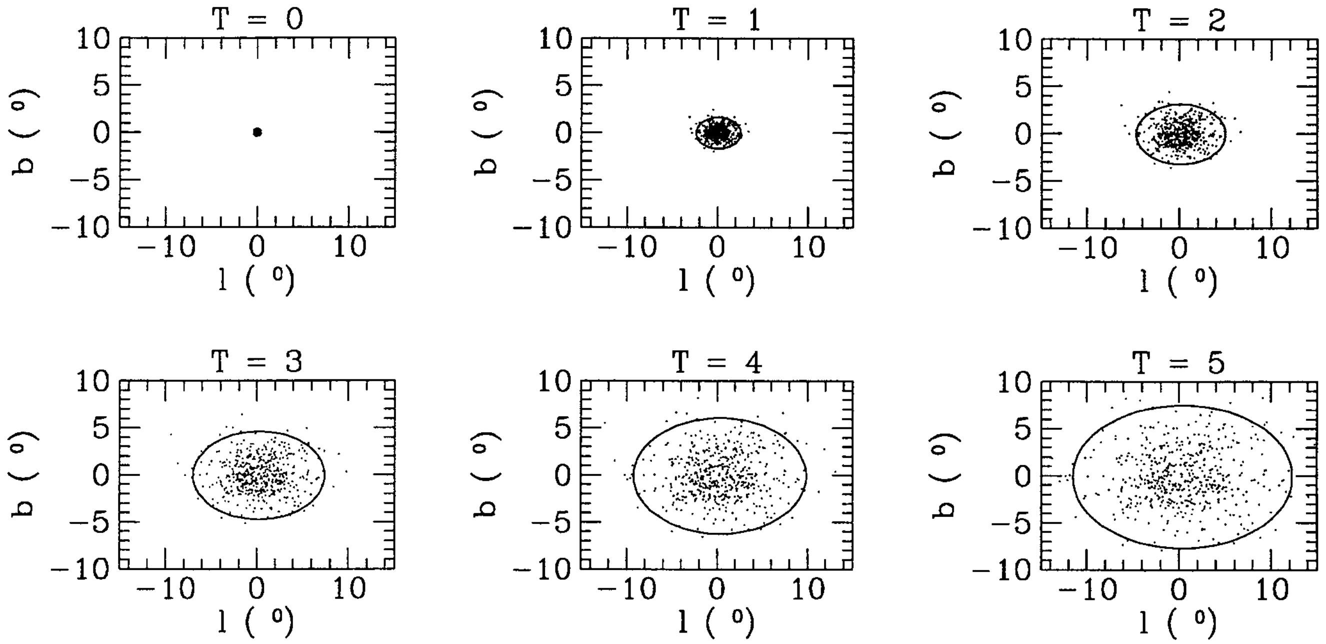

Figure 2. The evolution of a synthetic OB association containing 500 stars (with masses distributed as described in Section 2.3). The initial radius is 1 pc and the initial velocity dispersions are $\sigma_{x,0} = \sigma_{y,0} = 3\,\mathrm{km\,s^{-1}}$ and $\sigma_{z,0} = 2\,\mathrm{km\,s^{-1}}$. The association is located at a distance of 160 pc. No solar or streaming motion is added. The time indicated above the panels is in Myr. The ellipse shows the $\pm 2.5\sigma$ limits in ℓ and b.

different realizations of the initial conditions. For each observed set of data we in turn generate the observational errors 50 times. All results are then averaged before comparison with the real parameters of the association.

The kinematic ages and initial sizes are determined for associations of ages 0, 2, 4, 6, 8 and 10 Myr. The observational errors are taken to be 0, 1, 2 and 3 milliarcsec $\mathrm{yr^{-1}}$ (mas $\mathrm{yr^{-1}}$) for the proper motion in one coordinate direction (these numbers translate to 0, 0.8, 1.5 and 2.3 $\mathrm{km\,s^{-1}}$ at 160 pc). The error of 1 mas $\mathrm{yr^{-1}}$ is comparable to what *HIPPARCOS* provides (Lindegren 1995). We discuss the results by presenting figures in which the estimated kinematic age of the synthetic associations is plotted versus the real age. Again, the associations consist of 500 stars and the observations are only analysed for the 35 most massive (brightest) stars; in line with previous investigations.

3.3.1 *Tracing back the proper motions*

We start with the kinematic ages derived from tracing back the motions of the stars. In Fig. 3 we show the simplest case. No streaming motion is added and the solar motion is also ignored. So the only influences on the proper motions of the stars are N-body effects, the Galactic tidal field and the observational errors. As expected, in all cases with observational errors the kinematic ages that are found are on average smaller than the real age.

If there are no observational errors the kinematic age is an overestimate for velocity dispersions larger than 1 $\mathrm{km\,s^{-1}}$ (the solid lines in Fig. 3 lie above the dashed-dotted lines). This is caused by the fact that the tracing process is not stopped as soon as the initial size of the association is reached, but continues until the minimum size is reached. One has to keep in mind that the model for determining the ages essentially assumes a zero initial size in the ideal case. At the lowest velocity dispersion the observational errors are disastrous and one will never find ages of more than ~2 Myr. At this velocity dispersion the initial N-body interactions in the association cause deviations from linear expansion. For an association that is just unbound this means that the kinematic age will be in error even if there are no observational errors. This is clearly visible in Fig. 3. For the two highest velocity dispersions the derived kinematic ages are within 10 per cent of the real ages, if the observational errors are of the order of 1 mas $\mathrm{yr^{-1}}$ (i.e. less than 30 per cent of the initial velocity dispersion).

In Fig. 4 we show the results when the solar motion and a streaming motion are included. The initial streaming motion is 5 $\mathrm{km\,s^{-1}}$ in X and Y and $-4\,\mathrm{km\,s^{-1}}$ in Z. The association is placed at $\ell = 0°$ so the X-direction is towards the Sun. The mean motion on the sky is subtracted by transforming the proper motions to the centre of mass frame (on the sky) of the association. The solar motion is corrected using the distance to the association of 160 pc. From Fig. 4 it is clear that above a real age of 4 Myr all age determinations converge to an age of ~4 Myr. Only for very young associations with a high velocity dispersion can one determine the age with any accuracy. The reason the ages are determined as badly as they are is that there is no correction for the effect of virtual expansion, and the above correction for solar motion is erroneous.

Because the association has a significant depth along the line of sight, the correction for solar motion and virtual expansion should be done using the individual distance to each star. If we do this with error-free distances then indeed the age determinations improve markedly for the association with a velocity dispersion larger than 1 $\mathrm{km\,s^{-1}}$ and low (~1 mas $\mathrm{yr^{-1}}$) observational errors. Above a real age of ~8 Myr the kinematic ages remain too small. In practice the distances will contain errors. The median error in the parallax measurements of *HIPPARCOS* is 1.45 mas (Lindegren 1995). This corresponds to an error in the distance of ~20 per cent at 160 pc. In Fig. 5 we show the results if the observational errors on the distances are included. For velocity dispersions of 2 $\mathrm{km\,s^{-1}}$ or less, the kinematic age provides no indication whatsoever of the real age. This is caused by the distance errors which are large with respect to the depth of the associations. For higher velocity dispersions the

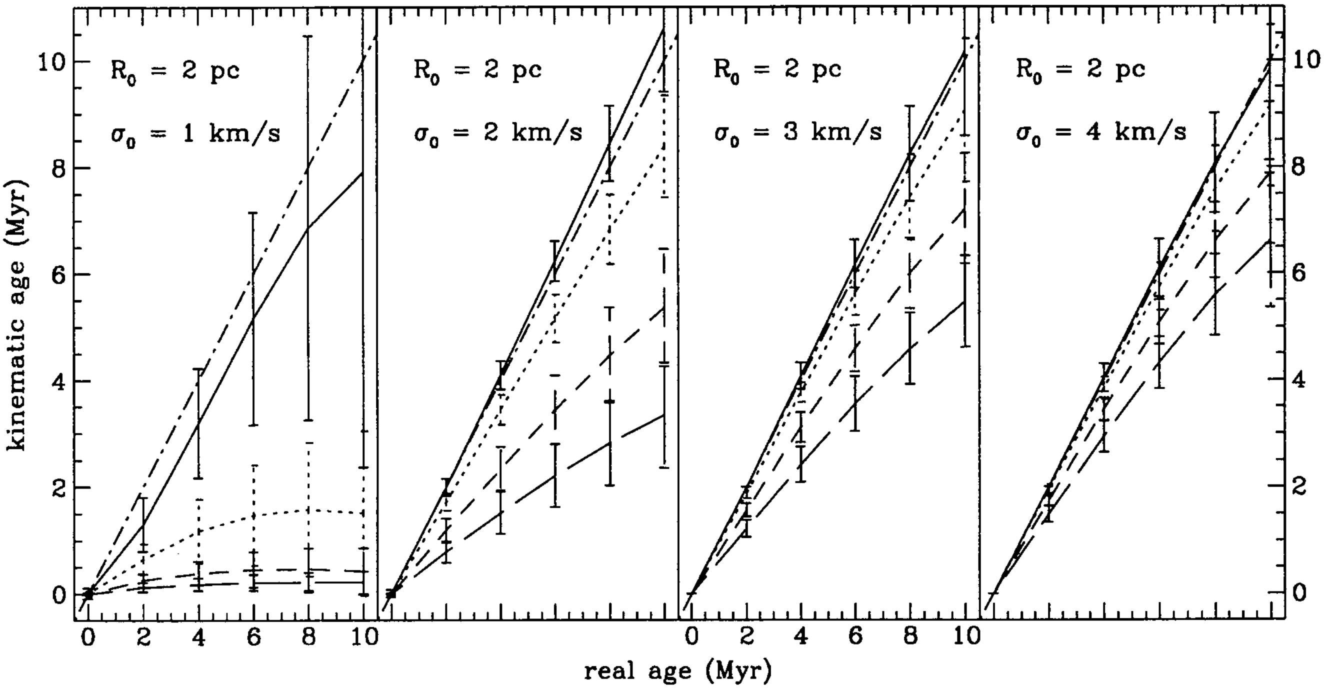

Figure 3. The kinematic age found from tracing the motions of the association members back in time, versus the real age for different initial velocity dispersions. The initial radius is 2 pc and the distance to the association is 160 pc. The number of stars is 500. The 35 brightest (most massive) stars are included in the determination of the kinematic age. The lines correspond to observational errors of 0, 1, 2 and 3 mas yr^{-1} (solid, dotted, short-dashed and long-dashed line, respectively). These numbers translate to 0, 0.8, 1.5 and 2.3 km s^{-1} at 160 pc. The dot-dashed line indicates perfect agreement. The error bars indicate the spread due to different realizations of the initial conditions of the simulated OB associations. Solar and streaming motions are not included.

kinematic ages are very inaccurate and unreliable. The age determinations again converge towards the same value of ~4 Myr. Note that observational errors on the radial velocities (needed for the correction for virtual expansion) are not taken into account.

We have indicated in all figures the spread in the derived kinematic ages. This spread is due to different realizations of the initial conditions for the synthetic association. It indicates the statistical uncertainty of an individual age determination. The spreads are all large, which means that it is not possible to correct the derived kinematic age reliably by means of the relations shown above.

The results for associations with an initial radius of 15 pc are not shown here. They are very similar to the results for associations with an initial radius of 2 pc. The age estimates also converge to ~4 Myr for the older associations.

3.3.2 *Ages from the expansion coefficient*

We now turn to the ages derived from the expansion coefficient. The ages are essentially a projected size divided by a proper motion difference over the association. The oscillatory motion of the stars in the Z-direction leads to smaller velocities as they move away from the Galactic plane. This in turn leads to a smaller projected size than expected on the basis of the initial velocities. If only the effects of the Galactic tidal field are taken into account, the age from the expansion coefficient that one finds can be estimated as $\sim z/\dot{z} = \tan(\nu t)/\nu$ (with ν being the vertical frequency). Thus we expect the ages to be overestimated by up to 30 per cent for real ages up to 10 Myr. This is indeed borne out by the results.

The evolution in the Galactic plane of an expanding association in the presence of the Galactic tidal field has been described by Blaauw (1952b). The time-scale of the epicyclic motions in XY is $2\pi/\kappa = 176$ Myr. Hence on the time-scales relevant to our simulations we do not expect that the associations will become significantly elongated as shown in Blaauw (1952b) for time-scales longer than 30 Myr.

For an association located at $(\ell, b) = (0°, 0°)$ it is the motions in Y that determine the proper motions in ℓ. For the same reasons as for the motions in Z we expect ages to be overestimated in that case, due to the oscillations in Y. The age overestimates can now be approximated as $y/\dot{y} = \tan(\kappa t)/\kappa$. We expect ages to be overestimated by up to 4 per cent for real ages up to 10 Myr.

Because of the different time-scales involved in the motion in Z and XY, we show the results for the ℓ- and b-components of the proper motions separately.

The results shown are not the averages of the different simulations, but the medians. We choose the median because the distribution of expansion coefficients has a large spread owing to outliers. These arise because the expansion coefficient is much more sensitive to the specific Monte Carlo realization of synthetic data. The spread indicated in the figures is now the inner quartile range instead of the rms, for the same reason.

In Fig. 6 we show the results for an initial radius of 2 pc in the simplest case, where no solar or initial streaming motion is included. The spreads are not indicated for $\sigma_0 = 1$ km s^{-1}. They are very large and would confuse the figure. In effect, the expansion coefficient for associations with such a small velocity dispersion has no relation to the real age. In this low-dispersion case, deviations from linear expansion are caused by observational errors that are large with respect to the internal motions, or by the initial N-body interactions (see Section 3.3.1).

Fig. 6(a) shows that at higher velocity dispersion indeed the derived ages are overestimates. However, they are fairly

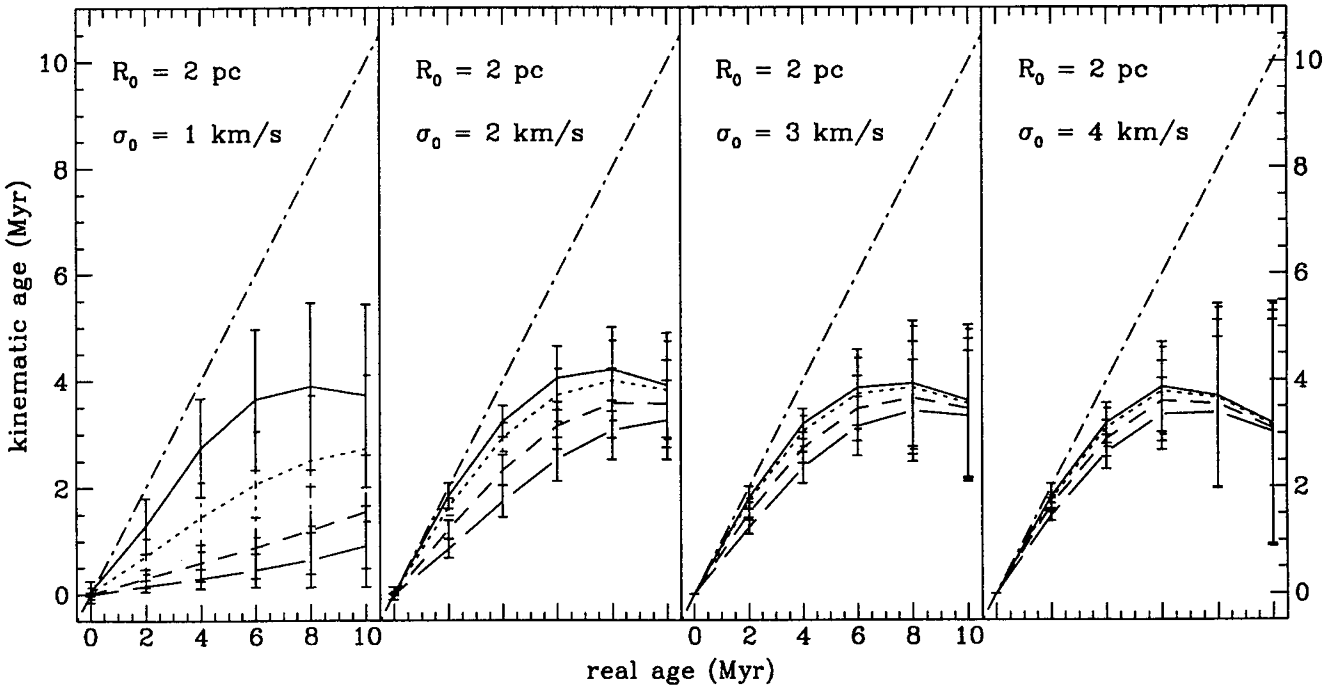

Figure 4. The same as Fig. 3. However, now the effects of solar and streaming motion are included. The initial streaming motion is $5\,\mathrm{km\,s^{-1}}$ in X and Y and $-4\,\mathrm{km\,s^{-1}}$ in Z. The solar motion is corrected for by using the distance to the association of 160 pc for each star. Note that all age determinations eventually converge to ~4 Myr.

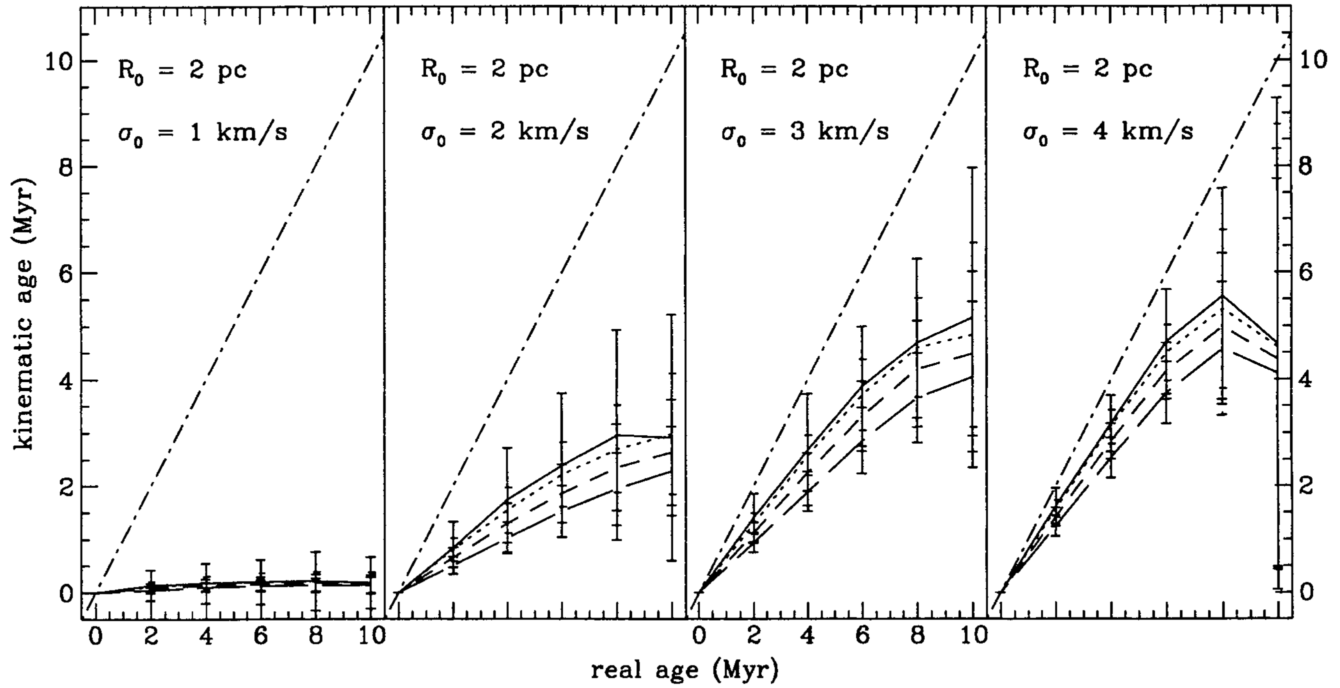

Figure 5. The same as Fig. 4. The solar motion and virtual expansion are both corrected for by using individual distances. The errors on the distances are taken to be 20 per cent.

accurate for the ℓ-direction. The errors vary from 3 to 20 per cent for $\sigma_0 = 2\,\mathrm{km\,s^{-1}}$ and from ~1 to ~7 per cent for velocity dispersions of 3 and $4\,\mathrm{km\,s^{-1}}$. For very young associations an additional factor that leads to overestimated ages is the finite size that the association has when the expansion starts. The linear expansion model assumes a zero initial size for the association.

That the time-scales for tidal effects are shorter in Z can clearly be seen in Fig. 6(b), where the results are shown for the b-direction. Now the ages are much more overestimated for the older associations. The

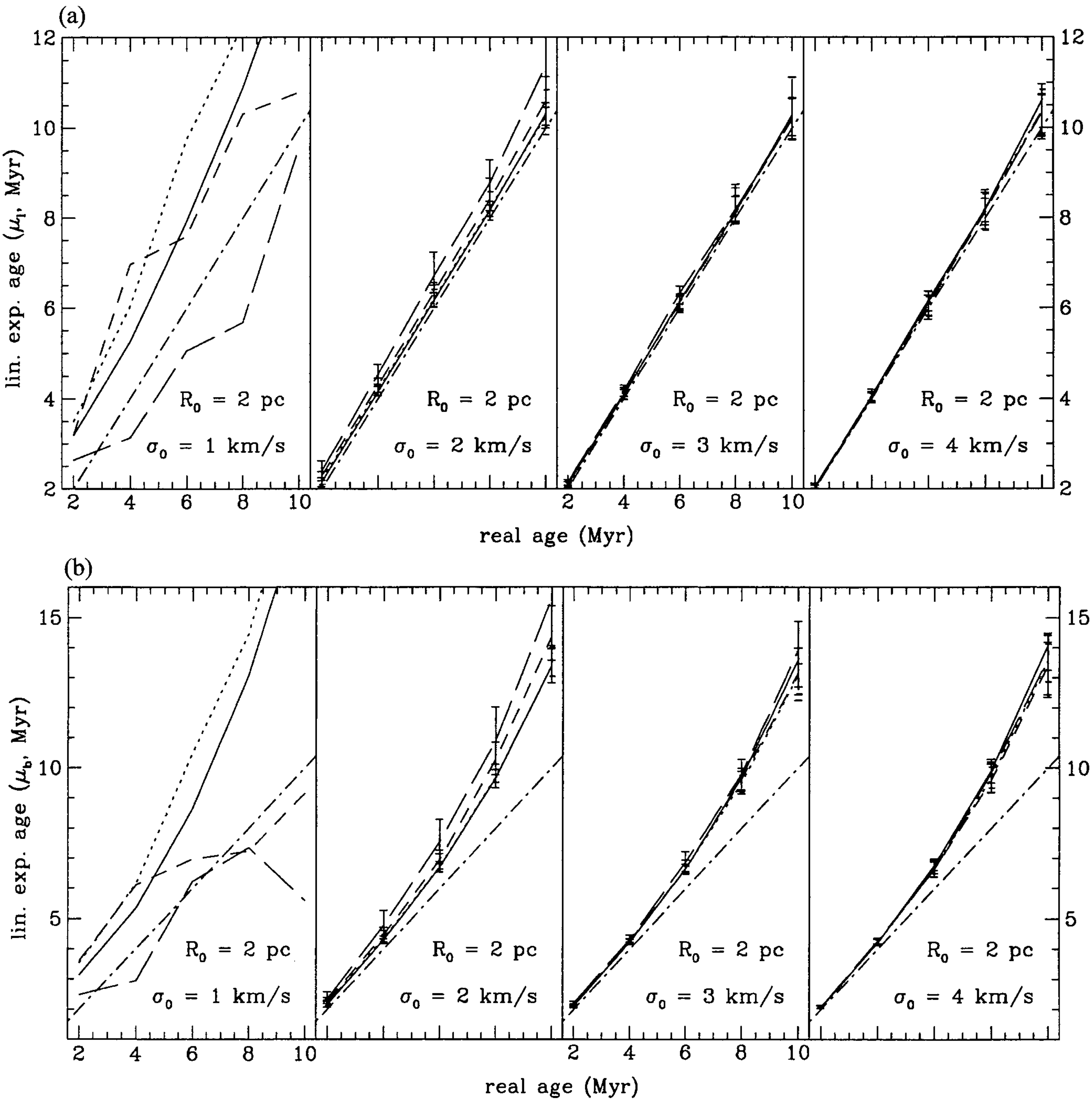

Figure 6. (a) The kinematic age derived from the expansion coefficient (the linear expansion age) versus the real age for different initial velocity dispersions. The results are shown for the ℓ-direction for an association with an initial radius of 2 pc, which is located at a distance of 160 pc. The parameters of the association and the simulations are the same as in Fig. 3. Note that now the median of the results is indicated and that the spreads are now given by the inner quartile range. The spreads are not shown for $\sigma_0 = 1\ \mathrm{km\,s^{-1}}$ in order to avoid confusion. The lines have the same meaning as in Fig. 3. (b) As (a), but now the results are shown for the b-direction.

errors go up to 40–50 per cent. This difference is important, because it means that one has to be careful about the choice of coordinate direction when the expansion coefficient is derived.

For associations with a high velocity dispersion (for which only the effects of the Galactic tidal field are important), the errors on the derived ages are consistent with the estimates given above.

If the solar motion and an initial streaming motion are included we expect again the effects of virtual expansion to play a role. Fig. 7 shows the results if the solar motion is added as well as a streaming motion of $5\ \mathrm{km\,s^{-1}}$ in X and Y and $-4\ \mathrm{km\,s^{-1}}$ in Z. The effects of these motions are corrected for by using individual distances to the stars. The errors in the distances are again taken to be 20 per cent. If no corrections are applied the ages are always underestimated. This is mainly due to the virtual expansion, which causes apparently large proper motion differences over the size of the association. The results shown in Fig. 7 indicate that the ages derived from proper motions in b remain overestimated. The ages derived from proper motions in l are accurate for young associations (up to ~6 Myr). For

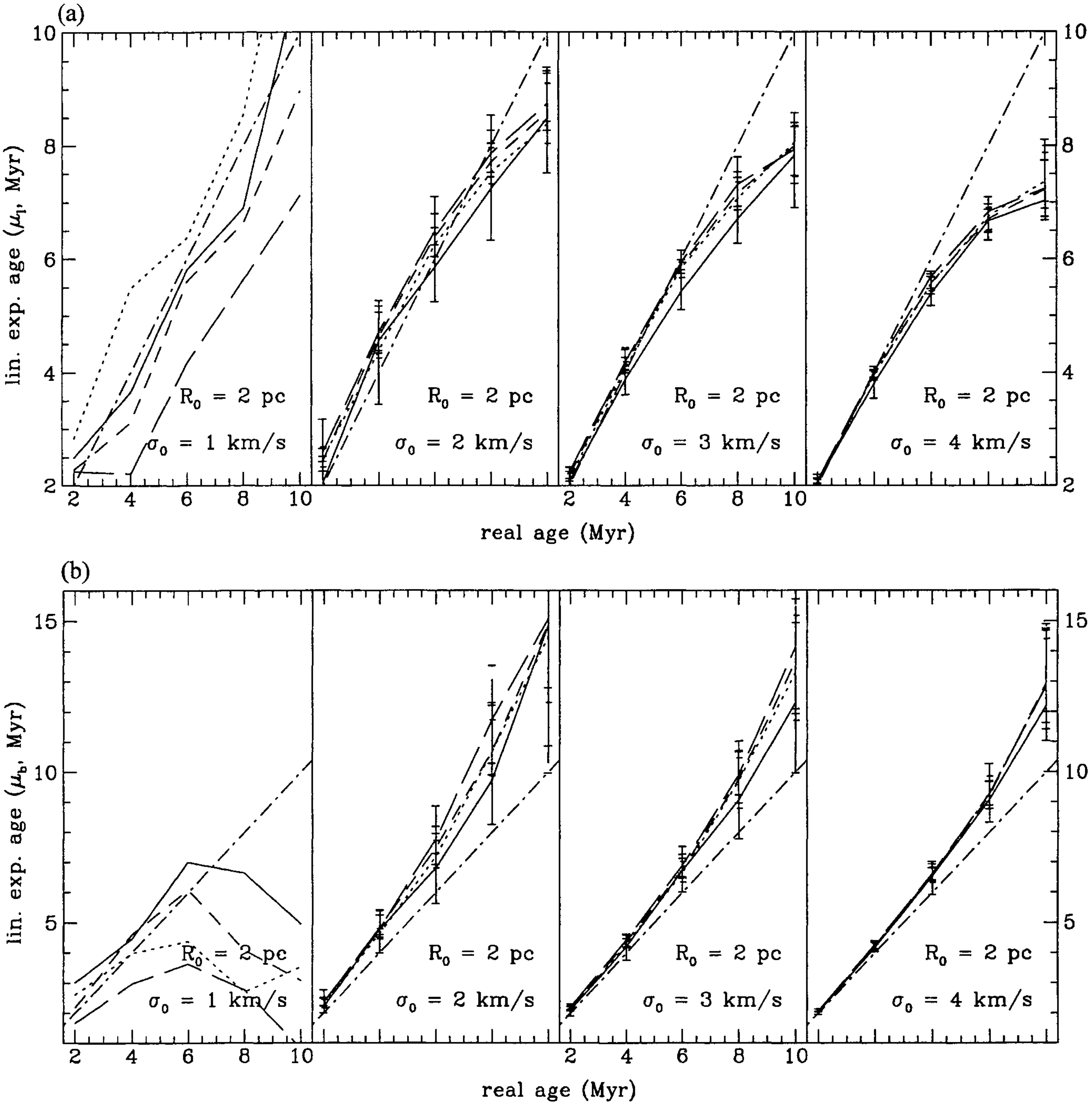

Figure 7. (a) As Fig. 6(a), but now the effects of solar and streaming motion are included. The initial streaming motion is 5 km s^{-1} in X and Y and -4 km s^{-1} in Z. The effects of these motions are corrected for by using individual distances to the stars. The errors on the distances are taken to be 20 per cent. (b) As (a), but now the results for the b-direction are shown.

older associations the virtual expansion effects lead to underestimated ages.

We briefly mentioned the effects of the initial size of the association on the expansion age. In Fig. 8 we show an example of the results that we obtain if the initial size of the association is 15 pc. The ages are now overestimated much more and the spreads due to different realizations of the synthetic data are also larger. The same holds for the results for b. If the solar motion and initial streaming motion are included and corrected for, the ages remain very inaccurate and the spreads become larger still. In ℓ the ages are then underestimated for the older associations (the underestimates starting at progressively lower age as the initial velocity dispersion goes up). These results illustrate the importance, for the linear expansion model, of the assumption that associations start out small as compared with their present size.

3.3.3 *The initial configuration*

We now briefly discuss the initial sizes of the OB associations that are derived from the tracing back of the stellar motions. In Fig. 9 we show the results in the simplest case, where no solar or streaming motion is included. The 2.5σ size in the ℓ-direction is plotted versus

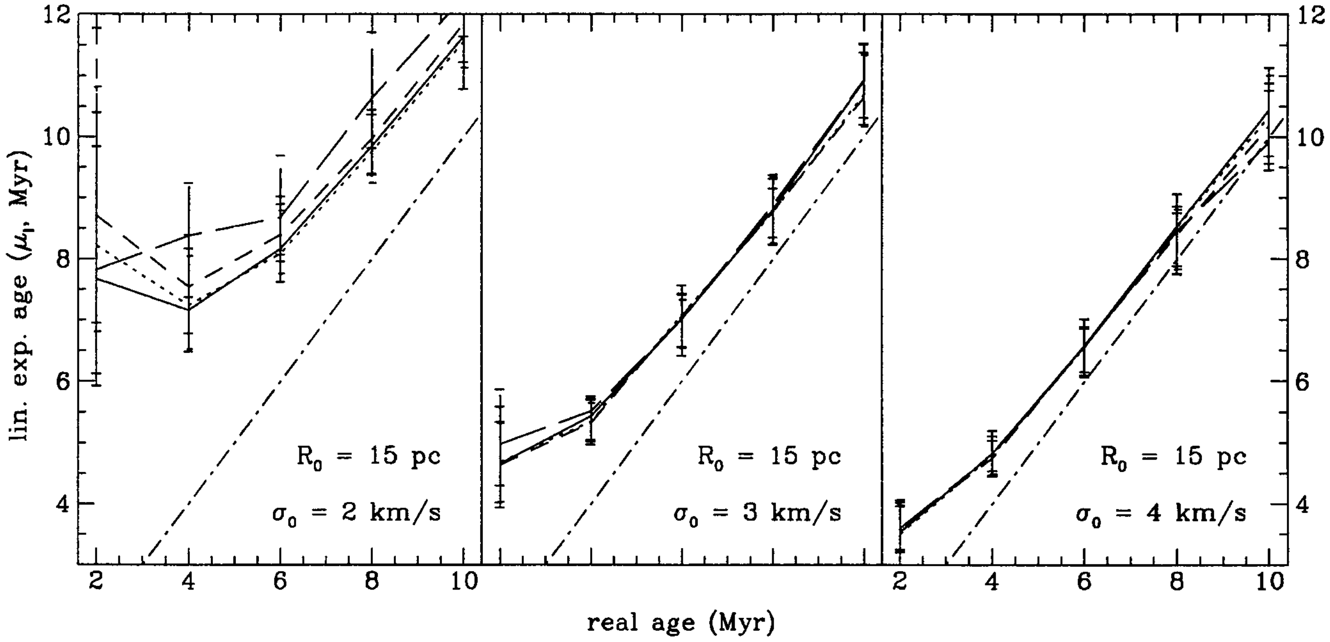

Figure 8. As Fig. 6(a) for an association with an initial radius of 15 pc. The results for $\sigma_0 = 1\,\mathrm{km\,s^{-1}}$ are not shown. They are essentially random.

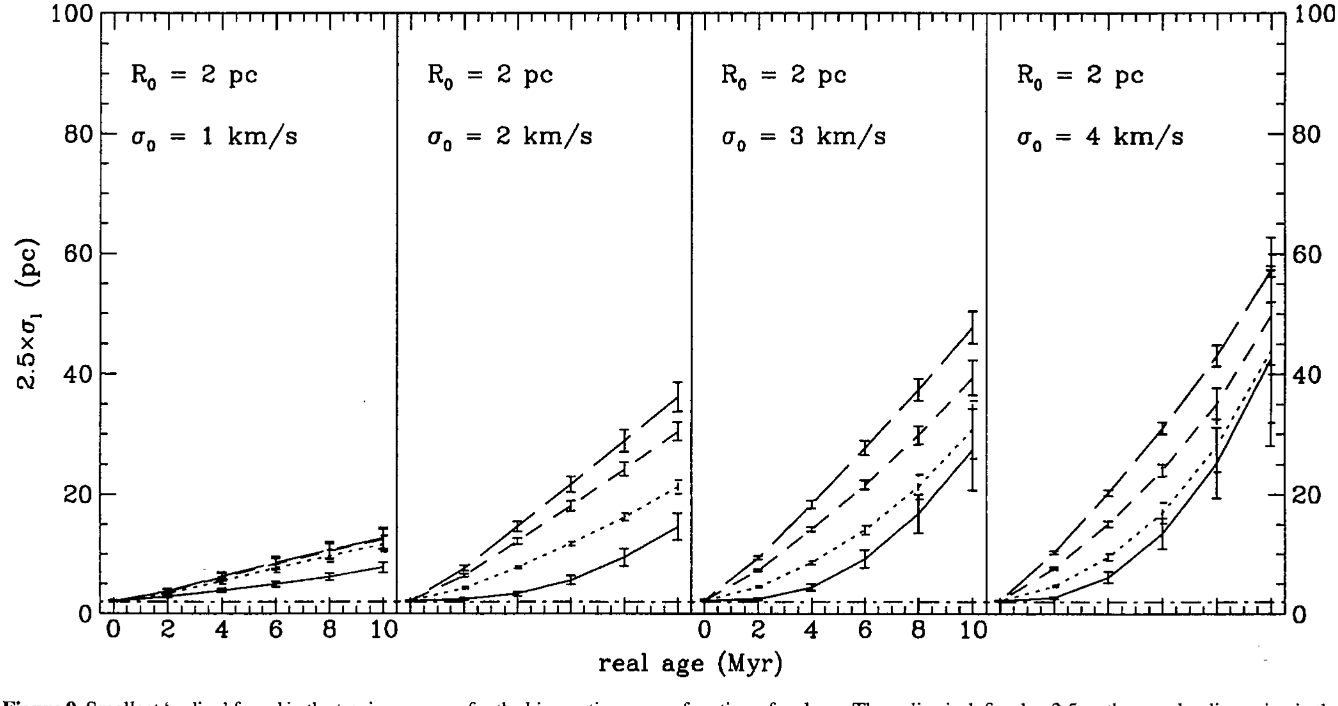

Figure 9. Smallest 'radius' found in the tracing process for the kinematic age as a function of real age. The radius is defined as $2.5\sigma_\ell$, the angular dispersion in the ℓ-direction. This is transformed to parsecs for a distance of 160 pc. The initial radius is 2 pc. The number of stars is 500. The 35 brightest (most massive) stars are included in the determination of the initial size. The lines correspond to observational errors 0, 1, 2 and 3 mas $\mathrm{yr^{-1}}$ (solid, dotted, short-dashed and long-dashed line, respectively). These numbers translate to 0, 0.8, 1.5 and $2.3\,\mathrm{km\,s^{-1}}$ at 160 pc. The error bars indicate the spread due to different realizations of the initial conditions of the simulated OB associations. The horizontal dot-dashed line indicates the real initial radius. Solar and streaming motions were not included.

the real age of the associations. It is immediately apparent from this figure that even in the absence of observational errors the initial size is always grossly overestimated for ages of more than ~2 Myr.

We show in Fig. 10 what happens if solar and streaming motions are included. The streaming motion is $5\,\mathrm{km\,s^{-1}}$ in X and Y and $-4\,\mathrm{km\,s^{-1}}$ in Z. Corrections are applied for the effects of both motions, and individual distances to the stars are used. The errors in these distances are again taken to be 20 per cent. It is clear that the derived initial sizes have no relationship whatsoever to the real initial size.

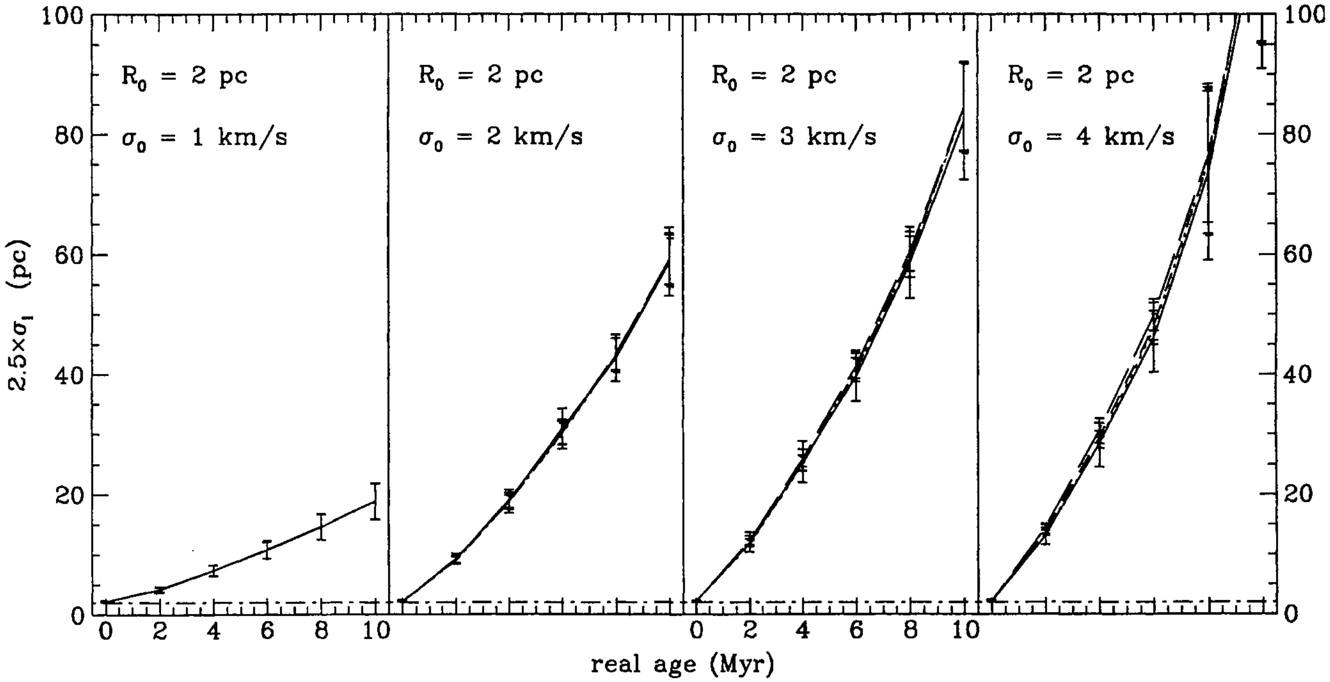

Figure 10. As Fig. 9. Now the solar motion and a streaming motion are included. The initial streaming motion is 5 km s^{-1} in X and Y and -4 km s^{-1} in Z. The effects of both motions are corrected for by using the individual distances to the stars. The errors on these distances are taken to be 20 per cent.

For associations with an initial radius of 15 pc, the initial crossing time is large and thus the size of the association will not change much for very young systems or low velocity dispersions. For associations younger than ~3 Myr the derived initial sizes are never overestimated by more than 40 per cent, if no solar or streaming motions are included. However, if the latter motions are included the results are essentially the same as for the 2-pc associations.

4 DISCUSSION

The results in the previous section show that deriving kinematic ages and initial sizes for OB associations is highly uncertain at best. In the process of tracing back the motions of the stars on the sky, any deviation of the stellar motions from a straight line will lead to underestimated ages and overestimated initial sizes. If the expansion coefficient is used, and corrected for the effects of solar motion and virtual expansion, the ages derived from proper motions in b are always overestimates. The expansion coefficient ages derived from proper motions in ℓ can be underestimated, depending on the magnitude of the effects of virtual expansion. In both cases there are large statistical spreads in the results, which make it impossible to correct the derived age.

On the observational side one is limited by the accuracy of the proper motions that can be obtained as well as by the accuracy of the distances. Good distances are required if a correction for the effects of solar motion and virtual expansion is to be performed.

At a more fundamental level one is limited by the fact that the linear expansion model is a poor approximation to reality. For small initial configurations and a low velocity dispersion the N-body interactions between the stars will severely disturb their linear motion. This will lead to large errors in the age determination, even in the absence of observational errors.

For associations with high velocity dispersions the Galactic tidal field causes deviations of the stellar motions from linear expansion. A correction for the effects of the tidal field requires tracing back the *space* motions of the stars as opposed to the projected motions. The space motions should be traced back with the tidal field of the Galaxy included. This has been done previously for moving groups located near the Sun (Yuan 1977; Yuan & Waxman 1977). These calculations also included the effects of the spiral arms of the Galaxy. The associations are located further away from the Sun than the moving groups, and it remains to be investigated what observational accuracies are needed in order to trace back space motions.

Ages derived from the expansion coefficient are listed in Table 1 for the associations Per OB2, Cep OB3a and Lac OB1b. These associations are located at 360, 720 and 530 pc, respectively (de Zeeuw et al. 1994). The velocity dispersions for these associations, derived from proper motions, are ~3.6, ~4.8 and ~3.1 mas yr^{-1}, respectively. These numbers are derived from the data in Lesh (1968) for Per OB2 and Lac OB 1b, and from Garmany (1973) for Cep OB3a. The observational errors are ~2 mas yr^{-1} for Per OB2 and Lac OB1b, and ~3–4 mas yr^{-1} for Cep OB3a. These numbers suggest that the kinematic ages for these associations are reliable. However, the dispersions derived from radial velocities are much lower. For Per OB2 this dispersion is ~5 km s^{-1} (derived from data in Blaauw 1952a), which translates to ~3 mas yr^{-1} at 360 pc. For Cep OB3a the corresponding numbers are ~8 km s^{-1} (from Garmany 1973) and ~2.5 mas yr^{-1}, while for Lac OB1b these are 2 km s^{-1} (Blaauw 1991) and ~0.8 mas yr^{-1}. The dispersions from radial velocities are comparable to the observational errors on the proper motions. Except for Per OB2, the same results (lower velocity dispersions that are comparable to or smaller than the errors) are found if one uses proper motion data from the PPM catalogue (Röser & Bastian 1989) and the Carlsberg Automatic Meridian Circle catalogue (Fabricius, Morrison & Helmer 1994). It thus remains an open question whether the velocity dispersions in these associations are indeed large compared with the observational errors. Therefore the kinematic ages listed in Table 1 should be treated with caution.

We have kept our simulations very simple. We have not taken into account the effects of gas removal prior to the emergence of the association from its parental molecular cloud. The influence of the remnants of the molecular gas, located near the newly formed association, on the dynamics has been ignored. We have also ignored effects of passing interstellar clouds, spiral arms and the rotation of the Galaxy as a whole. Our simulations do not include the presence of a binary population in the association. The proper motion of a binary may contain a component due to orbital motion. If not corrected for it will effectively act as an observational error on the true motion of the binary system. The inclusion of Galactic rotation would necessitate the correction of the observed proper motions for the effects of changing from a rotating frame of reference to an inertial frame. This would introduce extra error sources. The other effects mentioned lead to larger discrepancies between the linear expansion model and physical reality. Including all these will certainly not improve kinematic age determinations.

Note that the effects of stellar evolution have been ignored in the simulations. The two most important effects ignored are mass loss and evolution of stars away from the main sequence. The first effect is important only for the early-type stars in the association over the time-scales discussed. However, the mass of the stars will only be important in the initial stages of the evolution of the association, when *N*-body interactions still play a role. The terminal orbits of the stars will not be affected by their mass, hence the results of the simulations will not depend on the effects of mass loss. The second effect of stellar evolution is that the most massive stars will have evolved beyond the supernova stage, and these stars will then be lost for kinematic investigations. In any one association this will affect only a couple of stars and for the remaining stars the results of the simulations will still be the same.

In generating our initial conditions we assigned velocities to the stars by assuming a Gaussian velocity distribution. It may be physically more appropriate to assign energies to the stars according to a Gaussian distribution, and subsequently to derive the velocities. This would lead to a smaller velocity dispersion for the most massive stars. These are exactly the ones that were used in deriving the ages. We have already shown that smaller velocity dispersions only lead to larger errors.

Associations are not likely to have formed as a single protostellar cluster in a molecular cloud. Observations of star-forming regions show that there are several clusters forming in a giant molecular cloud at the same time. Often these are strung out along the edge of a cloud. After gas removal the young clusters may expand and merge to form an OB association. If this is the case the kinematic age determinations discussed in this paper will certainly not improve. The effects of these more realistic initial conditions on kinematic age estimates for OB associations have been discussed by Blaauw (1983). An interesting question is whether, by tracing back the motions of the stars, one can still recognize the locations of the different original clusters that merged to form the association. We intend to investigate this in the near future.

Another important aspect that we left out of these simulations is that of membership selection. Before one can derive kinematic ages one has to select stars that are members of the association. The selection process will have to be against a large population of background stars. Errors in this selection process will lead to an additional error source for the kinematic ages. On the other hand, membership selection will increase the number of stars that one can use to derive kinematic ages. Whether this will lead to a substantial improvement remains to be investigated. Work along these lines is in progress.

Finally, observations so far show that velocity dispersions in clusters and associations are small. They are of the order of $1\,\mathrm{km\,s^{-1}}$ for open clusters. For associations they are ~ 2–$8\,\mathrm{km\,s^{-1}}$ and are probably inflated by the presence of undetected spectroscopic binaries. Even if the velocity dispersions are at the high end of the range, one is still in the regime where the observational errors are large compared with the internal velocities. This is even worse for associations further away than Upper Scorpius, for which moreover the distance errors are generally also larger.

In Section 1 we discussed the discrepancy between nuclear and kinematic ages, which was illustrated by the ages listed in Table 1. Based on the above discussion of the ages in Table 1 and on the considerations regarding the simulations, we conclude that this discrepancy is most likely caused by underestimates of the kinematic age.

The *HIPPARCOS* data will not allow us to derive accurate kinematic ages using the methods outlined in Section 3. What is at least needed in the tracing back of stellar orbits in time is a correction for the effects of Galactic tides. Part of the observational problem (distances and proper motions) will be solved by the planned astrometric space experiment *GAIA* (see e.g. Lindegren & Perryman 1995). This experiment aims at astrometry at the 20-micro-arcsecond accuracy level. This would lead to accuracies in the velocities of $15\,\mathrm{m\,s^{-1}}$ at a distance of 160 pc. The distances would then be accurate to 0.3 per cent. However, tracing back space motions would require radial velocities of comparable accuracy.

5 CONCLUSIONS AND FUTURE WORK

We studied the problem of deriving kinematic ages and initial configurations of OB associations using simple *N*-body simulations. We used a slightly modified version of the NBODY2 code of Aarseth (1995). The synthetic associations were generated subject to the condition that they are unbound stellar groups. We used simple initial conditions in order to assess whether, based on the linear expansion model, accurate kinematic ages and initial configurations can be derived from astrometric data.

For an association located at 160 pc, the distance of the Upper Scorpius subgroup of Sco OB2, the results show that the accuracy of kinematic ages is limited by the errors on proper motions and the effects of *N*-body interactions for the low velocity dispersion associations. For the associations with higher velocity dispersions the limitations are the errors in individual distances of member stars and the effects of the Galactic tidal field.

In the case of the tracing back of stellar motions on the sky, the kinematic ages are always underestimated. The derived ages eventually converge towards the same value (4 Myr) for associations older than ~4–6 Myr. The ages derived from the expansion coefficient can be underestimated as well as overestimated, depending on the coordinate direction used as well as the magnitude of the effects of virtual expansion. The derived initial sizes are overestimates of the real initial size and should not be trusted.

Forthcoming astrometric space missions, such as *GAIA*, will be able to solve the observational problem for the nearby associations. However, at a more fundamental level the accuracy of kinematic ages is limited because of the use of the linear expansion model. The motions of the association members should actually be traced back, taking at least the Galactic tidal field into account. This requires knowledge of the space motions of the stars.

To assess what can really be learned about the evolution of associations by studying their kinematics, improvements in the modelling are needed. In the discussion we pointed out that our simulations are not very realistic. That can be improved by better initial conditions for the stars, and by including stellar evolution and the membership selection procedure. More importantly, a binary population and the effects of the parental molecular cloud should be included. The binaries will introduce the observational problem of disentangling orbital motion and space motion, and the molecular cloud will affect the dynamics of the evolving association.

Straightforward tracing back of the motions of the stars always leads to underestimated ages. The ages derived from the expansion coefficient can only be trusted if the velocity dispersion is more than about twice as large as the observational errors. Therefore we conclude that the current discrepancy between kinematic and nuclear ages is most likely caused by underestimates of the kinematic age.

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APPENDIX A: TRANSFORMATION OF POSITIONS AND VELOCITIES AND THE CORRECTION FOR VIRTUAL EXPANSION

The simulations are done in the rotating coordinate system shown in Fig. 1 (Section 2.2). Hence it is necessary to perform a coordinate transformation to calculate the observed positions, proper motions and radial velocities as seen from the Sun. Fig. A1 shows the transformation for the case where the centre of the association is in the fourth quadrant of the Galactic plane. The results described below are similar for other quadrants.

In Fig. A1 R is the distance from the Galactic Centre to the Sun and R_C and D_C are the distances of the OB association to the Galactic Centre and the Sun, respectively. The positions and velocities in the (x, y, z) coordinate system have to be transformed to the (ξ, η, ζ) system. This is achieved by first rotating the (x, y, z) coordinates over an angle θ around the Z-axis to find the (x', y', z') coordinates (where θ can be calculated from ℓ_C, D_C and R_C), and then performing a translation from the latter system over a distance D_C to find the (ξ, η, ζ) coordinates. The coordinates are then in a heliocentric system. The Galactic coordinates (ℓ, b) are given by

$$\begin{aligned}
\ell &= \pi - \arctan\left(\left|\frac{\eta}{\xi}\right|\right) && (\xi > 0, \eta > 0),\\
\ell &= \pi + \arctan\left(\left|\frac{\eta}{\xi}\right|\right) && (\xi > 0, \eta < 0),\\
\ell &= \arctan\left(\left|\frac{\eta}{\xi}\right|\right) && (\xi < 0, \eta > 0),\\
\ell &= 2\pi - \arctan\left(\left|\frac{\eta}{\xi}\right|\right) && (\xi < 0, \eta < 0),\\
b &= \arcsin\left(\frac{\zeta}{\sqrt{\xi^2 + \eta^2 + \zeta^2}}\right).
\end{aligned} \tag{A1}$$

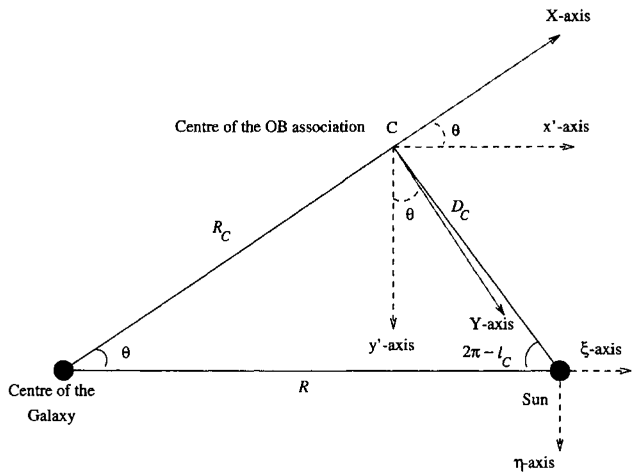

Figure A1. Transformation from the rotating coordinate system centred on the OB association to a coordinate system centred on the Sun. The (x, y, z) coordinates are first rotated around the Z-axis over an angle θ to transform them to the (x', y', z') system. After that the coordinates are transformed by a translation to the (ξ, η, ζ) system (the ζ- and z'-axes point upwards). The positions and velocities in the latter system are transformed to observables on the sky.

The velocities (v_x, v_y, v_z) can be transformed to the (ξ, η, ζ) system by a rotation over the angle θ. The transformation to radial velocities and proper motions can be done by defining the unit vectors

$$\hat{\boldsymbol{r}} = (-\cos\ell\cos b, \sin\ell\cos b, \sin b)\,,$$
$$\hat{\boldsymbol{\ell}} = (\sin\ell, \cos\ell, 0)\,,$$
$$\hat{\boldsymbol{b}} = (\cos\ell\sin b, -\sin\ell\sin b, \cos b)\,, \qquad \text{(A2)}$$

in the (ξ, η, ζ) system and calculating the inner product of these with (v_ξ, v_η, v_ζ) to find $(v_{\rm rad}, v_\ell, v_b)$. The proper motions follow from $\mu_\ell \cos b = v_\ell/4.74d$ and $\mu_b = v_b/4.74d$, where v is the velocity in km s^{-1}, d the distance in pc and μ the proper motion in arcsec yr^{-1}.

A1 Correction for virtual expansion

A non-zero radial motion of the association with respect to the line of sight through its centre causes an apparent expansion or contraction. This is referred to as virtual expansion (see Section 3). If this radial component of the association's motion is $\boldsymbol{V}_{\rm sys}$, its direction is given by the unit vector $\hat{\boldsymbol{r}}_{\rm C}$ for the coordinates $(\ell_{\rm C}, b_{\rm C})$ of the centre of the association. For a star located at (ℓ, b) the radial velocity and proper motions due to $\boldsymbol{V}_{\rm sys}$ are given by the inner product of $\boldsymbol{V}_{\rm sys}$ and the unit vectors $\hat{\boldsymbol{r}}$, $\hat{\boldsymbol{\ell}}$ and $\hat{\boldsymbol{b}}$ for the position of the star. The observed motions are

$$v_{\rm rad} = V_{\rm sys}\left[\cos(\ell_{\rm C} - \ell)\cos b_{\rm C}\cos b + \sin b_{\rm C}\sin b\right],$$
$$\mu_\ell \cos b = \frac{V_{\rm sys}}{4.74d}\cos b_{\rm C}\sin(\ell_{\rm C} - \ell)\,, \qquad \text{(A3)}$$
$$\mu_b = \frac{V_{\rm sys}}{4.74d}\left[\sin b_{\rm C}\cos b - \sin b\cos b_{\rm C}\cos(\ell_{\rm C} - \ell)\right],$$

where $V_{\rm sys} = |\boldsymbol{V}_{\rm sys}|$.

The correction for virtual expansion is done by assuming that the observed radial velocity of the star is due to $\boldsymbol{V}_{\rm sys}$ and calculating the latter from the first of equations (A3). The proper motions due to the radial motion of the association as a whole can then be calculated and subtracted from the observed proper motions to get the corrected proper motions. The radial velocity as well as the distance to the star are required to calculate this correction. The correction for the effects of the solar motion proceeds in the same way but then $\boldsymbol{V}_{\rm sys}$ is known and given by the reflex of the Sun's motion with respect to the local standard of rest.

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