KINEMATIC AND DYNAMIC REFERENCE FRAMES

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ABSTRACT

The fundamental properties of kinematic and dynamic reference frames are defined, first as an abstract concept and second in a practical (although idealized) thought experiment. A four dimensional space-time description in coordinate free notation illustrates the properties, limitations, and relationships between kinematic and dynamic reference frames. Kinematic reference frames can be defined quite rigorously. Dynamic reference frames cannot be defined so well, but are nonetheless very useful. In practice a combination has been generally adopted. Presently we can materialize purely kinematic terrestrial and celestial reference frames.

INTRODUCTION

Ultimately our observations are applied to the study of the physical world. We identify or label observations using the term "coordinates". This labeling is an orderly way to classify events and objects. We can also extend this function in discussing interactions among "labels" (coordinates), and tend to give the coordinates physical significance. As long as we completely understand what we are doing there is nothing wrong with this practice. However, we must be very careful that we correctly interpret the physical significance of coordinates, and not study some phenomenon of a reference frame that we have invented.

For example, in studying earth satellites, one can use the earth's equator as a reference. The equations of motion will take a form that depends on the theoretical definition used to describe the equator's motion in space. Depending on the choice of definition, satellite perturbations will arise. In the Kozai and Kinoshita (1973) theory, for example, no change in semimajor axis (a) is predicted. In another approach, Balmino (1974), there will be perturbations in a. Both are correct. At present there is an anomaly in the Lageos satellite orbit with the existence of an unexplained change in a. Rather thorough analysis has failed to provide a good physical explanation. We do not

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suggest that the anomaly is related to the Kozai or Balmino reference frame. We do question if it may be due to reference frame related phenomena. There are also examples of phenomena in the solar system that may have the same origin: for example, the excess secular change of the obliquity of the ecliptic ($0''.31$/century). Considerable effort has been given to seeking a geophysical explanation. However, the answer may lie in the reference frame itself.

The purpose of this article is twofold: to describe the principles and to present observational accuracies. The inexorable improvement in accuracy will certainly continue and we should be designing a reference system that will be durable, in definition, yet able to undergo successive improvements in materialization.

It is important to draw a careful distinction between a reference frame and a reference system. The former is simply a mathemtical description. For example, the use of three orthogonal unit vectors in three-space to define the coordinates of a rotating frame is a reference frame. A reference system, however, is much more. It is often based on a reference frame, and includes a prescribed procedure to materialize the system, including methods of observation and reduction of data. Therefore, once a reference frame has been defined, it is unambiguous and immutable ever after. However, a reference system can change considerably, depending upon the prescribed procedure. The Conventional International Origin (CIO), defined by the adopted latitudes of the five ILS stations, is a reference frame. It has become rather impractical to express observations with respect to this frame. On the other hand, the BIH zero meridian comes closer to being a reference system. Though it lacks the conceptual simplicity and elegance of the CIO, the BIH zero meridian is a convenient system for expressing observations of UT. We hope to maintain the distinction between frame and system in this paper.

One last point: since we are studying the physical world, we must choose the correct description, given our limited understanding of the universe. This immediately leads us to a discussion of General Relativity (GR), though not because there are relativistic effects that must now be taken into account (though there are). The development of the concept of GR for reference frames has been covered by Moritz (1979, 1981) and the relevant effects in the solar system have been discussed by Brumberg (1981). There is a growing number of books on GR, which can provide any amount of needed detail (e.g. Misner et al. 1973). We prefer to view GR as the necessary vehicle for approaching the subject because the concept is essentially involved with the fundamental nature of reference systems. In essence, GR is a denial of absolute motion, and therefore absolute reference frames, and can be used as an analysis of reference frames. In a recent review of experimental tests of GR (Will 1979), no competing theory has received any observational confirmation. The very measurements discussed here will be used as tests of GR and competing theories. Therefore, although we can presently treat the departures of GR from Newtonian mechanics as small corrections (as we do with other phenomena, such as refraction), increased measurement accuracy and new measurements will require more complete treatment. We might as well take the correct view now, and create a reference system that will remain correct in principle as the measurement accuracy increases.

MEASUREMENTS

All measurements are carried out in two steps: establishing a standard, and specifying a procedure for comparison fo the standard with the object system. Measurements are essentially four-tuple quantities, each consisting, in principle, of four numbers. Generally one accepts the standard and analyses the comparison object. However, one can reverse the process. For example, the meridian passage of a star or the return of a laser pulse from a satellite can be used to set a clock. Satellite-determined datum scale have necessitated reinterpretation of the surveyor's scale. An algebra for discussing these four-tuple quantities has been developed (Eddington 1946).

The usual fundamental standards are the centimeter, the second, and the gram, on which all our measurements are based. In fact, the speed of light (c) has become the operational definition of our length scale. This is most easily understood by introducing some formal ideas. Having taken GR as our description of the universe, we consider the metric ($g_{\mu\nu}$) using the summation convention:

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu = (\eta_{\mu\nu} + h_{\mu\nu})\, dx^\mu\, dx^\nu$$

$$\mu,\nu = 0,\ 1,\ 2,\ 3 \tag{1}$$

$$x^0 = ct, \quad x^1 = x, \quad x^2 = y, \quad x^3 = z \tag{2}$$

$$\eta_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{3}$$

$g_{\mu\nu}$, $\eta_{\mu\nu}$, $h_{\mu\nu}$ are metric tensors.
$\eta_{\mu\nu}$ is the Minkowski tensor of flat space time or special relativity.
$h_{\mu\nu}$ is the contribution of the gravitational field or curvature of space time. A particularly important solution of the field equations for a spherically symmetric static (i.e., stationary) field is due to Schwarzschild. It is normally written in polar coordinates:

$$ds^2 = (1 - \frac{2GM}{r})dt^2 - \frac{dr^2}{(1 - \frac{2GM}{r})} - r^2 d\theta^2 - r^2 \sin^2\theta\, d\lambda^2 \tag{4}$$

or in its isotropic form:

$$r = r'\left(1 + \frac{2GM}{4r'}\right)^2 \qquad (5)$$

$$ds^2 = \left(\frac{1 - \frac{GM}{2r}}{1 + \frac{GM}{2r}}\right)^2 dt^2 - \left(1 + \frac{GM}{2r}\right)^4 \left(dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\lambda^2\right) \qquad (6)$$

In this case we can write approximately

$$g_{\mu\nu} = \begin{bmatrix} (1 - \frac{2GM}{r}) & 0 & 0 & 0 \\ 0 & -(1 + \frac{2GM}{r}) & 0 & 0 \\ 0 & 0 & -(1 + \frac{2GM}{r}) & 0 \\ 0 & 0 & 0 & -(1 + \frac{2GM}{r}) \end{bmatrix}$$

When the units of GM are geometrodynamic centimeters (i.e., GM/c^2), we have $GM_\odot$ = 1.476 km for the sun, and $GM_\oplus$ = 0.4438 cm for the earth. One must realize that the Schwarzschild solution is only one possible solution to the field equations. It is not correct in principle, since it represents a universe with only one body, a point mass. However, it is a very good approximation on the scale of the solar system, even though it leaves out all nonlinear and rotation terms (e.g., The Lense Thirring Effect). See the textbooks for metrics containing these terms (e.g., Ohanian, 1976).

Most modern procedures for measuring length involve transmitting a photon, or other electromagnetic radiation, and measuring a time interval. This is possible due to the fact that the path of a photon is governed by:

$$ds = 0 \qquad (8)$$

We send a photon from A to B, reflect it from B back to A, and measure the transit (Δt) time with a clock at A. Since dt is the coordinate time of the observer, integrating 1, 4, or 6 along the path gives the relationship between the time interval and the distance in the observer's frame. In flat space time we have simply $\rho_{AB} = \Delta t/2$. In curved space time we have a more complicated line integral. The effects of space curvature on range distance measurement are illustrated in Table 1 for a simple geometry, where the photon's path is along the radius to the mass. For doppler measurements, an oscillator on a satellite or spacecraft is used to generate a radio signal. The rate of this oscillator depends upon the potential it experiences, and is also known as the "effect of time dilation". For close earth satellites,

Table 1. Space-time curvature effect on range measurement.

Mass	Range Measurement to:	
Earth	Lageos	0.62 cm
Earth	Synchronous Satellite	1.59 cm
Earth	Moon	3.63 cm
Earth	Sun	8.93 cm
Sun	Lageos	-12.5 cm
Sun	Synchronous Satellite	-62.8 cm
Sun	Moon	-741. cm
Sun	Sun ($1R_\odot$)	-16.9 km

this effect has been important for many years (Gaposchkin and Wright 1969). The time dilation effect has been used as one of the fundamental tests of GR (Vessot and Levine 1979). We know, of course, that the effect of atmospheric refraction at zenith on range measurements is 2.1 m.

By using property (8), we are essentially using a clock to measure a distance in terms of the speed of light. It is, therefore, more correct to consider the metric scale imposed by a standard clock and an adopted value of the speed of light. In doing so we have as standards the second and the gram. However, if we use an atomic clock to measure Δt, then the time will be atomic time and the length scale will be tied to the atomic second. There is no a priori guarantee that the time in 2, 4, or 5 is atomic time. Atomic clocks are based on physical processes, which occur at the atomic level. We would like to have a clock that is governed by macroscopic (dynamic) processes. Such a clock is called a geometrodynamic clock. There is a body of theory, largely unverified both theoretically and observationally, that would predict a difference between atomic time and geometrodynamic time. This theory is based on the Large Number Hypothesis (LNH) of Dirac. Wesson (1980) gives the most recent summary of the status of the idea. If valid, it predicts a difference in time scale. This is also known as a the Variable G (or $\dot{G}$) Hypothesis, but that is patently a misnomer. The LNH predicts a quadratic depature of the two time scales (a linear difference would be only a question of definition) and an increase in mass of the universe. The present upper limit, observationally, is $< 2 \times 10^{-10}$/year. Using this limit, the systematic error in range measurement across the earth's orbit is only 0.03 cm. For the present this error can be ignored in establishing the length scale. Since the dynamics of the solar system run on geometrodynamical time, departures of the two time systems would show up in establishing the dynamic reference frame. It is in that discussion that possible differences must be reckoned with.

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IDEALIZED KINEMATIC SYSTEM

Consider n points in a four-dimensional Minkowsky Space. Consider that each point can transmit to any other point and receive a reflected photon, or electromagnetic radiation. Each point also has a clock, and all clocks have been synchronized. Therefore each point (i) can measure Δt to each other point (j) and compute $\rho_{ij}(t)$. Each point can then show on a sign or broadcast by radio, which is the same thing as both signals going at the speed of light, the ρ_{ij} and t. Another point can observe each of the n-1 points and similarly broadcast the result. An external observer can make a tabulation of the ρ_{ij}. The only problem in constructing a time history is to identify the times. If we simply require that point i's observation of point j be the same as point j's observation of point i, then an ordering by coordinate time can be used (the time each clock shows for its particle). Of course this will generally not be the same reading shown by the central observer's clock. By identifying the coordinate time with each configuration, the observer can construct a polyhedron and its temporal evolution in three-space, or its motion through four-space. Notice that this polyhedron is defined, even in the presence of potential fields, accelerations, and rotations. It rigorously defines a kinematic reference system.

For each time there are $N = n(n - 1)/2$ measured distances. Ignoring for the moment how or why we should establish a Cartesian Coordinate System and certain degeneracies, there are 3η coordinates. Five parameters would need to be imposed, (an origin and orientation). There are many ways to select these five parameters. For example, one might prescribe three coordinates for one point and one coordinate for two other points. Or one might choose the origin so that the weighted mean of the x, y, and z coordinates is zero and there is not net rotation. In either case, one can then produce a time-ordered history of the coordinates' changes.

This type of network could, for example, be realized with a system of satellites such as the Global Positioning System (GPS). At each instant of time, this polyhedron would define a system quite independently of origin or orientation, motion or rotation. The relative motion of points would be obvious, because it would be directly reflected in the changes in ρ_{ij}. This polyhedron is the ultimate pure kinematic system and serve as the definition of a reference system. In general, it is not too useful since the ρ_{ij} can vary with time, and the measurements of other coordinates referred to it will also vary, even if their scalar separations are invariant. Nevertheless, this is the kinematic reference frame we will set up, a terrestrial polyhedron of interstation baselines. Indeed, we expect the motion to be small and hope they will be regular. We will set up such a system using baseline determinations drawn from the laser ranging of Lageos (Latimer, Gaposchkin 1977), GPS (Goad 1981), or Very Long Baseline Interferometry VLBI (Robertson 1981). Bender (1981) gives a prescription for realizing such a terrestrial system, which Mueller (1981) would call a Conventional Terrestrial System (CTS). In both cases the attempt is to model the temporal changes in ρ_{ij}, using plate tectonic models, and to refer the origin of the equivalent Cartesian Coordinates to the center of mass, using the dynamic constraints from the laser tracking of Lageos.

If we take a general polyhedron and make it very large with the observer within it, then the analysis is reduced to measuring angles τ_{ij} either optically or with VLBI. However, the time now becomes the proper time of the observer. Of course, the τ_{ij} must be corrected for the "aberration" that is due to the motion of the observer. This aberration is classically known and easily derivable from the special relativity Lorentz transformation in the neighborhood of the telescope. There is also the deflection of the earth's gravity field (0".0003), and the sun's gravity field (from 0".0041 to 1".75), and even for photons passing near Jupiter (0".0163).

There is an interesting and important difference between the polyhedron and the unit sphere kinematic system. Establishment of the polyhedron does not depend upon the relative motion of the observer in any way. If the system were rigid and non-rotating, the observer could reconstruct it even if he was moving at a high velocity. With the unit sphere, even if the emitters are "fixed," the observer will see variations due to his motion and to the presence of a potential field. Of course, with our knowledge of the earth's rotation, orbital motion, and the potential fields in the solar system--which will cause the geodesic precession of 1".9/century (Eddington 1924), one can reduce the observations to a non-rotating heliocentric system. This system would then produce a reference frame with no motion, if the sources were apparently fixed.

A kinematic celestial system can be realized optically with telescopes, whether ground-based or satellite-based (Fricke 1981, Kovalevsky 1981, or by means of VLBI observations of quasars. Quasars have considerable attraction; it is believed that among the expected 40,000 observable extra-galactic objects, we can find a sufficient number to establish a kinematic unit sphere polyhedron (i.e., a celestial reference system). Such a system, related to the distant objects in the universe, would constitute an inertial reference system that could be used as a Conventional Inertial System (CIS); Mueller (1981). If our present ideas of the universe are correct, the quasars should also be geometrically fixed, i.e. the τ_{ij} are constants. Of course this needs verification (Robertson (1981).

Finally, if the baselines monitored as the terrestrial polyhedron are also the baselines of a VLBI observatory, we will be able to obtain a direct observation of the relationship between these two purely kinematic reference frames. This then defines the link between the two systems (the CTS and the CIS), avoiding the complication of the complex theory of the earth's motion in space: precession, nutation, polar motion, and earth rotation. Of course, one can choose to present this link in terms of a conventional theory, with small observed

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corrections. This is in fact exactly what happens with an "adopted" precession and nutation series and published polar motion. Such a procedure has the added feature of being independent of the ecliptic.

IDEALIZED DYNAMIC SYSTEMS

Until now we have discussed motion, but only in terms of the change of relative position in time, i.e., the path of a particle in Minkowski event space. We have not needed to relate the path of a point in this space. We have needed only a clock, a transmitter, and a receiver of photons or electromagnetic radiation to construct and maintain a well-defined reference system. One consequence of a metric theory of gravity is equations of motion, which take many forms. The motion of a free particle along a geodesic is derivable from the variational principle that

$$\delta \int_{s_1}^{s_2} ds$$

is stationary. One form is the Equation of Geodesic Deviation:

$$\frac{D^2\xi}{Ds^2} + R^{\alpha}_{\beta\gamma\delta} \frac{dx^{\beta}}{ds} \xi^{\gamma} \frac{dx^{\delta}}{ds} = 0 \qquad (9)$$

This gives the separation vector $\bar{\xi}$ between two nearby geodesics in terms of the path of a test particle and the Riemiann curvatures tensor $R^{\alpha}_{\beta\gamma\gamma}(g)$. Such an equation can be used to study, for example, the motion in an orbiting spaceship by writing g in a rotating system. In this case

$$\frac{dt}{ds} = 1 \quad , \quad \frac{dx^i}{ds} = 0 \quad , \quad i = 1, 2, 3$$

and

$$\frac{D^2\xi^{\alpha}}{Ds^2} + R^{\alpha}_{0\gamma 0} \xi^{\gamma} = 0 \qquad (10)$$

This leads to the tidal force tensor

$$R^{k}_{0\ell 0} = \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial x^k \partial x^{\ell}}$$

which has the simple form for a point mass:

$$R_{0\ell 0} = \begin{bmatrix} \frac{GM}{c^2 r^3} & 0 & 0 \\ 0 & \frac{GM}{c^2 r^3} & 0 \\ 0 & 0 & -\frac{2GM}{c^2 r^3} \end{bmatrix} \tag{11}$$

The equation (9) can also be used to define the motion of an earth satellite. One determines the geodesic motion ($\overline{\xi}$) of the satellite with respect to the earth's center of mass ($\overline{x}$), which moves on a geodesic.

The importance of equation (9) is that it displays the fact that no inertial system has been defined. Any particle can serve as a reference so long as the metric or the Riemann curvature tensor is known. However, this just begs the question and substitutes knowledge of the metric for knowledge of the inertial frame. For example, to write (11) we used the fact that the particles' (earth's) motion around the earth (sun) was due to the space curvature caused by the mass of the earth (sun). But we do not know the masses a priori. Not even Einstein succeeded in formulating a theory that predicted masses. Then we go to the more usual form of the equations of motion

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0 \tag{12}$$

or

$$\frac{du^\mu}{ds} + \Gamma^\mu_{\alpha\beta}\, u^\alpha\, u^\beta = 0 \tag{13}$$

where the Christophel Symbol is

$$\Gamma^\mu_{\alpha\beta} = \frac{1}{2} g^{\mu\nu}\,(g_{\nu\alpha,\beta} + g_{\beta\nu,\alpha} - g_{\alpha\beta,\nu}) \tag{14}$$

and where the comma (,) denotes differentiation. Equation (14) is completely equivalent, but the $\overline{x}$ now refer to an "inertial reference frame." In a sense this is tautology, in that an inertial reference frame is being defined as the frame in which these equations are true. There is a final form of the equations of motion known as the Parameterized Post Newtonian Approximation (PPN). First derived by Eddington and Clark (1938), the PPN or its derivative is used for all analyses of orbits in the solar system. The complete treatment of the restricted two-body problem is given by Hagihara (1930-31).

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We have equations of motion that allow us to connect points on a geodesic, provided we know the metric. We must also know the non-gravitational forces, which add terms on the right-hand side of 12 and 13. In principle we have a data analysis problem to recover the necessary parameters: the metric, forces, and initial conditions. If some forces are stochastic, then the modeling problem becomes correspondingly more difficult. Aside from random forces, one could hope to define, in an inertial reference frame, the motion of a particle (a satellite or planet). As more data become available, the parameters and initial conditions can be improved. It will be axiomatic, however, that the dynamic system will be constantly "improved." Its accuracy will depend on the accuracy of the data for the interval of time observed, and will deteriorate with extrapolation.

One can inquire what use will be made of this dynamic frame. It will be excellent for relating points in its own system. However, that will not depend upon its being inertial, only that it provides a consistent theory for tracing a point in four-space. The satellite's position in this frame can easily be realized (by definition). For example, using laser ranging to Lageos one will be able to calculate station coordinate differences that are nearly as accurate as the data (Bender 1981). Such baselines will be excellent, since the major uncertainty in this system is orientation (i.e., rotation).

If one seeks a dynamic frame as a reference for dynamic studies, the situation is less favorable. It is true that the satellite trajectory will define an inertial reference system with some accuracy (Kozai 1981). The exact frame depends, of course, upon the details of the force models, data reduction, and so forth. Such frames are getting better; the main uncertainty concerns rotation, as there is no clear way to separate the gravitational effects from the general rotation without observing celestial frame or Fermi-Walker transport with dyroscopes. Even so, a Fermi-Walker transported gyroscope in earth's orbit around the sun will show the $1\overset{''}{.}9$/century geodesic precession with respect to the star background (Eddington 1924). Another possibility would be to observe synchronous satellites with respect to star background with astrometric cameras or VLBI systems.

These uncertainties are long-term effects. A number of dynamic phenomena appear to be short periodic from the point of view of a satellite. The ability to treat short periodic perturbation is therefore the critical issue. In this case, the situation is reasonably good. The short-period effects of GR are generally quite small here: 0.012 cm for Lageos, 0.146 cm for the moon, and 1.8 km for Mercury (Gaposchkin 1981). The residuals due to a shift in the center of mass of the coordinates of the observing stations will appear to the satellite as a once-per-revolution effect. The accuracy of recovering the center of mass is therefore the same as the accuracy of the data and the perturbations, depending on the true anomaly. From the point of view of a satellite, the earth's polar motion with respect to an angular momentum axis appears to be strictly diurnal. Satellite doppler and laser ranging are presently giving excellent measurements of polar motion. One must be careful not to mask this with the once-per-day perturbations that are due to errors in the tesseral harmonics of order one. The determination of UT1 is not so promising, for the reasons given above.

CONCLUSIONS

In summary, one can draw the following calculations from these arguments.

1. A reference systems should be simple. That is, invariant phenomena should remain invariant when expressed in the system. Apparent motion should not reflect motion of the reference system.

2. A terrestrial reference system, one fixed to the earth, can be defined and established in a completely kinematic fashion as a polyhedron. Satellite dynamics can be used to obtain the polyhedron baselines, and the relationship of the polyhedron to the center of mass.

3. A celestial reference system can also be established kinematically. The most promising approach is to use VLBI observations of extra-galactic radio sources. These VLBI stations can also contribute to the definition of the terrestrial reference frame polyhedron baselines.

4. The motion of the terrestrial reference system, with respect to the celestial reference frame, will be monitored with VLBI.

5. Satellite or dynamic reference frames can be realized. They can permit monitoring of the terrestrial reference system with respect to the earth's angular momentum axis: nearly the rotation axis.

6. The scale of the terrestrial reference frame should be defined from a standard clock and the speed of light.

7. We should seek to define a reference system that is, in principle, a factor of 10 better than we expect to need: i.e., 0.1 cm and 0"0001. We may not immediately realize system of such accuracy.

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