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KINEMATICAL FORM FACTORS IN THE PERIPHERAL MODEL

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ABSTRACT

In potential theory, the "vertex functions" for resonance formation contain kinematical form factors, which depend on the spin of the resonance and are simply related to penetration factors. Analogous form factors are introduced for the relativistic vertex functions of the one-pion exchange model. The resulting vertex functions behave like the Born term functions near the pion pole, but go to constants as the momentum transfer goes to infinity, for arbitrary resonance spin. For elastic nucleon-nucleon scattering, the total form factor reduces to a simple pole.

1. INTRODUCTION

In this paper we are concerned with peripheral interactions in which one spinless particle (π , η or K or any scalar particle) is exchanged. To state it clearly, we shall neglect such improvements of the peripheral model as vector meson exchange, Reggeization and unitarization, although we are convinced that each of these improvements is necessary.

For a reaction

$$a + b \rightarrow c + d \quad (1)$$

e.g., the one-pion exchange matrix element has the form

$$T_{\pi} = V_{a\pi c}(s_c, t) \frac{1}{m_{\pi}^2 - t} V_{b\pi d}(s_d, t). \quad (2)$$

The factor $(m_{\pi}^2 - t)^{-1}$ is the propagator of the virtual pion (we neglect a possible form factor of the propagator), and the remaining two factors are the vertex factors, each depending on the square of the momentum transfer $\Delta^2 = -t$ and on the square of the effective mass (s_c or s_d) of the group of particles (c or d) created at that vertex. To simplify our discussion we shall focus our attention on reactions where the effective mass distributions are dominated by resonances, although the more general case can be treated in a rather similar way. For our purpose we also are not interested in effects which depend on the widths of the resonances. Thus we effectively put $s_c = m_c^2$ and $s_d = m_d^2$ and treat reaction (1) like a two-particle reaction (quasi-two-particle reaction, see Fig. 1).

The simplest peripheral model is the pole approximation of one-pion exchange, which consists of replacing t by m_{π}^2 in the vertex factors, i.e., putting the vertex factors on the mass shell. Knowing

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the spin and parity of the resonances one usually calculates the vertex factors in lowest order perturbation theory according to Feynman's rules. This is the Born term model, hereafter abbreviated as BTM. In the case that the particle a is spinless, the BTM gives

$$V_{a\pi c}^B(t) = \left(\frac{a_c(t)}{a_c(m_\pi^2)} \right)^{j_c} V_{a\pi c}(m_\pi^2), \quad (3)$$

where j_c is the spin of the resonance, and a_c is the momentum of a in the c rest frame

$$a_c(t) = \frac{1}{2m_c} \left\{ [(m_c - m_a)^2 - t][(m_c + m_a)^2 - t] \right\}^{\frac{1}{2}} \quad (4)$$

and $V_{a\pi c}(m_\pi^2)$ is the product of a coupling constant and a spin function. The more complicated baryon vertex will be discussed in Section 3.

Since a_c diverges like $(-t)$ as $-t \rightarrow \infty$, it thus compensates (for $j_c \neq 0$) to a certain extent the decrease of the cross-section with t due to the t in the propagator, in gross contradiction to experiment. In order to save the BTM it appears necessary to introduce strong form factors ¹⁾. In perturbation theory such form factors would arise from the higher order corrections of the vertex functions. They may be physically interpreted as a result of a finite extension of the interaction region at the vertices. In the absorption model where one tries to avoid such form factors by taking into account a strong absorption in channels with small impact parameters ²⁾, one invariably gets into troubles with reactions in which both c and d have spins ≥ 1 (double resonance production) due to the occurrence of the factor $(a_c)^{j_c}$ in $V_{a\pi c}^B$ and a corresponding factor in $V_{b\pi d}^B$ ^{2), 3), 4)}.

Empirically, for large values of $-t$, the simple pole approximation ⁵⁾

$$V_{a\pi c}^{\text{pole}}(t) = \text{const. } V_{a\pi c}(m_\pi^2) \quad (5)$$

(with const. = some constant of the order ^{of} 1) appears to be a much better approximation than the BTM, as represented by (3). The aim of the present paper is to derive a model in which the vertex functions do go to constants as $-t \rightarrow \infty$, and which reduces to the BTM near the pion pole. Such a model arises very naturally from considerations well-known in potential theory. Each vertex is given a spatial extension of radius R . The factor in Eq. (3) then results from a change in the penetration probability of the angular momentum barrier; in fact, it should be interpreted as the ratio of the square roots of the penetration factors off and on the mass shell. For $a_c R \ll \ell$, i.e., if classically the particles stay outside of their interaction region, the penetration factor is proportional to $(a_c R)^{2\ell}$ as seen in Eq. (17) below. In this case the familiar factor in Eq. (3) arises. For arbitrary values of $a_c R$, however, the penetration factor cannot be greater than 1, since it is directly related to the probability to enter the interaction volume. Hence, for small impact parameters $\ell/a_c \ll R$ the penetration factor goes to one, the angular momentum barrier becomes negligible, and as a consequence the vertex factor will never exceed a certain constant value.

In addition to this essentially kinematical factor, there could well exist a proper "dynamical form factor", which would appear even for s wave resonances where the angular momentum barrier is absent. To simplify matters in the following no dynamical form factor is introduced. In Section 2 we shall shortly discuss the penetration factors in non-relativistic potential scattering. The relevant formulae can be found in any standard text book of nuclear physics [e.g., Blatt and Weisskopf ⁶⁾].

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In Section 3 we utilize the analogy to potential scattering theory to give the new formulae for the relativistic matrix elements. Arguments of this type have already been used by Clegg ⁷⁾. In potential theory, the masses of the particles forming the resonance are always real, whereas we need the penetration factor for the case that one of the masses (namely \sqrt{t}) is imaginary. This extrapolation from potential theory is formally trivial, but of course it is a model. Its value depends on the smallness of other effects (dynamical form factors). Section 4 finally contains a comparison with experiments and a few remarks about the relevance of our work with respect to other improvements of the BTM.

2. PENETRATION FACTOR IN NON-RELATIVISTIC POTENTIAL SCATTERING

We describe the scattering of two particles with relative momentum $2k$ in their c.m.s. in terms of a complex, radially symmetrical, potential which vanishes outside a radius R . The potential shall simply be characterized by the logarithmic radial derivative of the wave function for all angular momenta ℓ at $r = R$:

$$f_{\ell}(x) = R \left. \frac{dU_{\ell}(r)/dr}{U_{\ell}(r)} \right|_{r=R} = g_{\ell}(x) - i h_{\ell}(x) \quad \left. \begin{array}{l} x = k \cdot R \\ h_{\ell}(x) \geq 0 \\ g_{\ell}(x) \end{array} \right\} \text{real} \quad (6)$$

The internal (or resonance) scattering amplitude for a special partial wave is then given by

$$A_{\ell}^{\circ}(\vartheta) = \frac{2\ell+1}{k} a_{\ell}^{\circ} P_{\ell}(\cos \vartheta) \quad (7)$$

with

$$a_e^0 = \frac{S_e}{g_e - \Delta_e - i(S_e - h_e)} \quad (8)$$

The $S_e(x)$, $\Delta_e(x)$ are defined in terms of the spherical Bessel and Neumann functions j_l and n_l :

$$y \frac{d}{dy} \ln [y n_e(y) + i y j_e(y)] \Big|_{y=x} = \Delta_e(x) + i S_e(x) \quad (9)$$

and are explicitly given by

$$S_e(x) = x \cdot v_e(x) \quad (10)$$

$$\Delta_e(x) = -x \sqrt{v_e(x)(v_e'(x) - v_e(x))} \quad (11)$$

with

$$v_e(x) = \frac{1}{x^2 [n_e^2(x) + j_e^2(x)]} \quad (12)$$

$$v_e'(x) = \left[\frac{d}{dx} (x n_e(x)) \right]^2 + \left[\frac{d}{dx} (x j_e(x)) \right]^2 \quad (13)$$

For the reaction cross-section one finds

$$\sigma_{r,e} = \frac{4\pi}{k^2} (2l+1) t_e(x) \quad (14)$$

with

$$t_e(x) = \frac{s_e h_e}{(s_e - h_e)^2 + (s_e + h_e)^2} \quad (15)$$

To demonstrate the physical meaning of $v_e(x)$ we assume that all particles entering the interaction region $r \leq R$ move only towards the centre with a high momentum $K \gg k$ (independent of ℓ) without returning to the surface (condition of continuum theory). One then has $f_e \approx -ikR$ and hence

$$t_e \approx \frac{k}{K} v_e(x) \quad (16)$$

In this case t_e describes simply the probability of the particle to enter the reaction volume: the factor k/K arises from the impedance mismatch due to the difference in the momenta inside and outside (if K is very big most particles will be reflected and hence never enter!), the $v_e(x)$, however, must be interpreted as a penetration factor which describes the probability of the particle to penetrate the potential barrier, or - in three-dimensional language - the probability to hit the target particle. The particle which would classically by-pass the target according to (12) still has a finite reaction chance

$$v_e(x) \approx \left(\frac{x^\ell}{(2\ell-1)!!} \right)^2 \quad \text{for } x \ll \ell \quad (17)$$

This exhibits the well-known dependence on $x = kR$. For particles, however, which classically would enter easily the interaction region this probability should be approximately one. This also follows immediately from definition (12)

$$v_e(x) \approx \frac{1}{1 + \ell(\ell+1)/2x^2} \approx 1 \quad \text{for } x \gg \ell \quad (18)$$

which indicates that $v_e(x)$ is within 10% of its asymptotic value 1 for $x > 3 \sqrt{\ell(\ell+1)/2}$.

For s wave scattering we find

$$V_0(x) = 1 \quad \text{for all } x, \quad (19)$$

as has to be expected. The functions $v_\ell(x)$ [and also $v'_\ell(x)$] were tabulated by Feshbach and Lax⁸⁾.

In particular we have

$$v_1(x) = \frac{x^2}{1+x^2}, \quad v_2(x) = \frac{x^4}{9+3x^2+x^4} \quad (20)$$

As is well known the penetration factor $v_\ell(x)$ enters as an important factor in the partial widths of a resonance for the various channels. We can immediately deduce this from formula (8) if we remember that a resonance occurs for an energy E'_s for which the real part of the denominator of the resonance scattering amplitude vanishes

$$(g_e - \Delta_e)_{E=E'_s} = 0 \quad (21)$$

It is common to introduce a "formal resonance energy" E_s by the condition

$$g_e(x(E)) \Big|_{E=E_s} = 0 \quad (22)$$

and to expand the logarithmic derivative in the neighbourhood of this energy

$$g_e(x(E)) = \frac{d}{dE} g_e(x(E)) \Big|_{E=E_s} (E-E_s) + \dots \quad (23)$$

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Then the resonance scattering amplitude (8) can be written in the Breit-Wigner form :

$$a_e^0 = \frac{-\frac{1}{2} \Gamma_{e,i}^s}{(E - E_s') + \frac{i}{2} \Gamma_e^s} \quad (24)$$

where

$$E_s' = E_s - \gamma_s \Delta_e(x) \quad (25)$$

is the actual resonance energy, and

$$\begin{aligned} \Gamma_{e,i}^s &= \gamma_s \cdot 2 S_e(x) = 2 \gamma_s \times U_e(x) \\ \Gamma_{e,r}^s &= \gamma_s \cdot 2 h_e(x) \\ \Gamma_e^s &= \Gamma_{e,i}^s + \Gamma_{e,r}^s \end{aligned} \quad (26)$$

the width of the initial channel, the reaction width (i.e., the total width of the rest of the channels), and the total width, respectively.

The "reduced width"

$$\gamma_s = - \left(\frac{dg_e}{dE} \right)_{E=E_s}^{-1} \quad (27)$$

is a positive quantity and gives a measure for the strength with which a wave entering the interaction volume is coupled to the particular resonance state s . Because of (26) the partial widths $\Gamma_{\ell,i}^s$ of the resonance s for the initial channel i contain three factors :

- 1) the probability of the particle to get within the range of the target particle (penetration factor v_ℓ),
- 2) the probability to be not reflected on the surface ($\sim k$),
- 3) the probability to react in the resonance state ($\sim \gamma_s$).

The energy dependence arises dominantly through the first two factors.

Generally the probability for a reaction from an initial channel i through the resonance state s to a definite final channel f , is given by

$$t_\ell(i \rightarrow f) = \frac{\left(\frac{1}{2} \Gamma_{e,i}^s\right) \left(\frac{1}{2} \Gamma_{e,f}^s\right)}{(E - E_s')^2 + \left(\frac{1}{2} \Gamma_e^s\right)^2} \quad (28)$$

For the differential cross-section for this transition we then get

$$\frac{d\sigma_\ell(i \rightarrow f)}{d\Omega} = \frac{(2\ell+1)^2}{k^2} P_e^2(\omega, \vartheta) \cdot t_\ell(i \rightarrow f) = |T_{if}(\ell, \vartheta)|^2 \quad (29)$$

Here T_{if} , defined by the above equation, should be interpreted as the invariant transition amplitude corresponding to a Feynman graph which contains the resonance state as a "propagator", i.e.,

$$T_{if}(l, \vartheta) = \sum_{m=-l}^{m=+l} v_i(lm, \vartheta_i, \varphi_i) \frac{1}{(E-E'_s) + \frac{i}{2} \Gamma_e^s} v_f^*(lm, \vartheta_f, \varphi_f) \quad (30)$$

with θ the angle between the directions (θ_i, φ_i) and (θ_f, φ_f) .

By comparison we find

$$v_i(lm, \vartheta_i, \varphi_i) = \sqrt{4\pi \gamma_s} R v_e(k_i R) Y_l^m(\vartheta_i, \varphi_i) \quad (31)$$

and a similar expression for v_f . From this expression the close connection between the coupling constant of the initial particles with the resonance state and the reduced width can be immediately seen. Important for our considerations, however, is the observation that the momentum (and energy) dependence in this approximation enters dominantly through the penetration factor. For small momenta $k_i = k_f = k \ll \frac{\ell}{R}$ we get the familiar multipole expansion

$$v(lm, \vartheta, \varphi) = \sqrt{4\pi \gamma_s} R \frac{(kR)^\ell}{(2\ell-1)!!} Y_l^m(\vartheta, \varphi) \quad (32)$$

which corresponds to the approximation employed in the BTM as, for example, indicated by the vertex dependence Eq. (3).

All these formulae are perfectly well-known in nuclear physics. They have been applied to the N^* pion-nucleon resonance as early as 1954⁹⁾. After the advent of the Chew-Low model, in which v_1 is replaced by $(k \cdot R)^2$, they were commonly disregarded. It is nevertheless true that the Chew-Low model gives a poor description of the N^* resonance shape, especially for $E > E'_s$. A more recent calculation of Layson¹⁰⁾ clearly demonstrates the importance of the penetration factor.

3. RELATIVISTIC VERTEX FACTORSa) The meson vertex

The considerations of the last Section show that the Born term expression (3) for the meson vertex is appropriate for small values of $a_c R$ only. The vertex function containing the complete penetration factor (neglecting a possible "dynamical" form factor) is given by

$$V_{a\pi c}(t) = \left(\frac{v_{jc}(a_c(t) \cdot R_c)}{v_{jc}(a_c(m_\pi^2) \cdot R_c)} \right)^{\frac{1}{2}} \cdot V_{a\pi c}(m_\pi^2) \quad (33)$$

where a_c is given by Eq. (4), v_{jc} is given by Eq. (12), and R_c is the "radius" of the vertex $a\pi c$. To give some meaning to this radius, it would of course be necessary to introduce a potential for the vertex function. But instead of doing so, we merely refer to the common "range argument", stating that

$$R_c \leq \frac{1}{m_x} \quad (34)$$

where m_x is the mass of the lowest state which the incident particle a and the virtual pseudoscalar e can exchange.

From Eq. (21) we see that $V_{a\pi c} \rightarrow V_{a\pi c}^B$ for $a_c \cdot R_c \rightarrow 0$, and from Eq. (22), $V_{a\pi c} \rightarrow v_{a\pi c}^{\text{pole}}$ as given by (5), for $-t \rightarrow \infty$. This is exactly what we need for the peripheral model: the Born term near the pole $t = m_\pi^2$, and no divergence as $-t \rightarrow \infty$.

It is common, although somewhat misleading for the analogy to the potential case, to write

$$V_{a\pi c}(t) = F(t) \cdot V_{a\pi c}^B(t) \quad (35)$$

where F is a "form factor", normalized to unity at $t = m_\pi^2$. Insertion of (3) into (33) gives

$$F(t) = \frac{a_c^{j_c}(m_\pi^2)}{a_c^{j_c}(t)} \left(\frac{v_{j_c}(a_c(t) \cdot R_c)}{v_{j_c}(a_c(m_\pi^2) \cdot R_c)} \right)^{\frac{1}{2}} \quad (36)$$

This looks complicated, but with the explicit expressions (19) and (20) for v_{j_c} it gives $F \equiv 1$ for $j_c = 0$ (s wave resonance),

$$F(t) = \left(\frac{1 + R_c^2 a_c^2(m_\pi^2)}{1 + R_c^2 a_c^2(t)} \right)^{\frac{1}{2}} \quad \text{for } j_c = 1 \quad (37)$$

$$F(t) = \left(\frac{9 + 3 R_c^2 a_c^2(m_\pi^2) + R_c^4 a_c^4(m_\pi^2)}{9 + 3 R_c^2 a_c^2(t) + R_c^4 a_c^4(t)} \right)^{\frac{1}{2}} \quad \text{for } j_c = 2. \quad (38)$$

As we have stressed already in the Introduction, we do not pretend that we have found the general vertex function. What we do claim is that the penetration factor model (35) makes more sense than the BTM (3). In the region $0 < t < m_\pi^2$, this is demonstrated by potential theory. Since we do not expect abrupt changes of the matrix elements near $t = 0$, the penetration factor has to be continued to small negative values of t as well. As $t \rightarrow -\infty$ finally, (33) is still meaningful whereas (3) is meaningless: one cannot use a Born approximation when the matrix element becomes infinite.

b) The baryon vertex

The model for the baryon vertex is complicated by the fact that the incident nucleon carries spin $\frac{1}{2}$. Instead of Eq. (3) we have

$$V_{b\pi d}^B(t) = \left(\frac{(m_b + m_d)^2 - t}{(m_b + m_d)^2 - m_\pi^2} \right)^{\pm \frac{1}{2}} \left(\frac{b_d(t)}{b_d(m_\pi^2)} \right)^{\ell_d} \cdot V_{b\pi d}(m_\pi^2) \quad (39)$$

where ℓ_d is the orbital angular momentum of the vertex, and b_d is defined in the same way as a_c

$$b_d(t) = \frac{1}{2m_d} \left\{ [(m_b + m_d)^2 - t][(m_b - m_d)^2 - t] \right\}^{\frac{1}{2}} \quad (40)$$

The plus sign in the first exponent of (39) applies for scalar coupling and for $N\pi N^*$ coupling, whereas the minus sign applies for pseudoscalar coupling. Whereas Eq. (3) went without derivation, we have to say a few words about the derivation of Eq. (39). To this end we define the momenta

$$k_\pm = \sqrt{(m_b \pm m_d)^2 - t}, \quad b_d = \frac{1}{2m_d} k_+ k_- \quad (41)$$

The t dependence of the Born terms is then given by k_+ for scalar coupling, k_- for pseudoscalar coupling and by $k_+^2 k_- = 2m_d k_+ b_d$ for $N\pi N^*$ coupling¹⁾. The t dependence of the general spin $\frac{1}{2}$, spin 0, spin j vertex is given by¹¹⁾

$$k_\pm \cdot b_d^{j - \frac{1}{2}} \quad (39')$$

where $k_+(k_-)$ appears for even (odd) intrinsic parity. By means of (41), (39') leads to (39).

There are two points in Eq. (39) which need discussion. Firstly, the separation of the total spin j into an orbital part and an intrinsic part s is ambiguous. In order to illustrate this, consider the simple case of pseudoscalar $N\pi N$ coupling. This gives rise to a pole in the πN elastic scattering amplitude. When the nucleon is described in terms of 2-component spinors, the pole arises in the $p_{\frac{1}{2}}$ state, and thus we have the orbital angular momentum $\ell = 1$. If we use 4-component spinors on the other hand, we see that the relevant matrix element connects the large components of one nucleon with the small components of the other nucleon (this comes from the factor γ_5 for pseudoscalar coupling). In this case we would interpret the momentum dependence of the matrix element as a purely relativistic effect which has nothing to do with penetration of a centrifugal barrier. The origin of k_{\perp} is then simply the weight of the small components. Since our model relies on analogy to the non-relativistic potential theory, we adopt the common interpretation of a pure p wave interaction.

The rest goes in analogy with the meson vertex. We write

$$V_{b\pi d}(t) = F(t) \cdot V_{b\pi d}^R(t) \quad (42)$$

In the case of a p wave, in particular for pseudoscalar $N\pi N$ coupling and vector $N\pi N^*$ coupling, we have

$$F(t) = \left(\frac{1 + R_d^2 b_d^2(m_\pi^2)}{1 + R_d^2 b_d^2(t)} \right)^{\frac{1}{2}} \quad (43)$$

analogous to Eq. (37).

The second point concerns the question of a possible modification of the first factor in (39). Since this factor has a purely relativistic origin, it should not be modified on the basis of our present argument. We could, for example, interpret the t dependence

of k_+ as a t dependence of our radius R (Lorentz contraction). Fortunately, k_+ is almost independent of t in the region where the peripheral graph is needed (see the figures in the following Section). For the pseudoscalar vertex we may therefore replace b_d in the form factor (43) by $k_- = b_d \cdot (2m_d/k_+)$, and obtain the more convenient expression

$$F_{ps}(t) = \left(\frac{1 + R_d^2 k_-^2(m_\pi^2)}{1 + R_d^2 k_-^2(t)} \right)^{\frac{1}{2}} = \left(\frac{R_d^{-2} + (m_b - m_d)^2 - m_\pi^2}{R_d^{-2} + (m_b - m_d)^2 - t} \right)^{\frac{1}{2}} \quad (44)$$

which leads to the pole approximation (5) for $-t \rightarrow \infty$. This is also useful in peripheral interactions with identical vertices ($pp \rightarrow pp$ by π exchange, or $\bar{p}p \rightarrow \bar{\Lambda}\Lambda$ by K exchange), where the total form factor is simply the square of (44), i.e., a simple pole. It leads to closed expressions for partial wave amplitudes, which is important for application of the absorption model.

A similar modification may be introduced for the $N\bar{\pi}N^*$ vertex. Here it would be more appropriate to replace b_d in the form factor (39) by the combination $k_+ b_d / (m_b + m_d)$, which again leads to the pole approximation (5) for $-t \rightarrow \infty$. For the reaction $\bar{p}p \rightarrow \bar{N}^*N^*$ for example, the use of (39) would result in a matrix element which goes to a very small constant instead of going to zero (in the Born term model, the matrix element for this process diverges like t^2 !).

An alternative approach would be to rewrite

$$k_+^2 = 2m_b m_d \left(1 + \sqrt{1 + b_d^2 / m_d^2} \right) \quad (45)$$

and then modify b_d according to our rule for p waves. This prescription would lead to the pole approximation for $-t \rightarrow \infty$ even in the case of scalar coupling.

One final word about vertices in which the outgoing particle d is stable ($m_d < m_b + m_\pi$). In this case $k_-(m_\pi^2)$ and $b_d(m_\pi^2)$ are imaginary, a situation which is not treated in potential theory. Our form factors (43) and (44) are then the analytic continuation of the potential theory expressions. Such continuation should be reliable as long as

$$R_d^2 b_d^2(m_\pi^2) \gg -1 \quad (46)$$

which is the old requirement that the singularities of the form factors should be further away from the physical region than the pole of the propagator ¹⁾.

4. COMPARISON WITH EXPERIMENT AND REMARKS ABOUT FUTURE IMPROVEMENTS

In Fig. 2 we have drawn the squares of a few form factors for pion exchange, taking $R^2 = 1/10 m_\pi^2$, in agreement with requirement (34). The notation is rather shorthanded, one should for example write $F_{K(\pi)K^*}$ instead of F_{K^*} , in order to indicate that it is the pion which is taken off the mass shell. The spin of the f^0 resonance has been assumed to be 2. The form factor of f^0 is so flat because a_c is very big already at $t = 0$, and therefore the relative increase of a_c from $t = 0$ to $t = -0.7$ is small. One should, however, keep in mind that for high values of $-t$, $F_{f^0}^2$ will go to zero like t^{-4} [see Eq. (38)]. It would be more instructive in this case to consider instead of $F(t)$, directly the ratio of the penetration factors which is approximately constant and one for big a_c .

The baryon vertex form factors have been computed according to Eq. (43) for $N\pi N^*$ and Eq. (44) for $N\pi N$. The value of the uncertain factor $k_+(m_\pi^2)/k_+(t)$ is 0.91 at $t = -0.7$ for the $N\pi N$ vertex, and even closer to unity for the $N\pi N^*$ vertex. The big difference between $F_{N^*}^2$ and F_N^2 in the region $0.1 < -t < 0.3$ is thus simply due to the mass difference between N and N^* .

For comparison with experiment we have to take reactions where contributions from vector meson exchange are known to be small. Figures 3-5 show the differential cross-sections for $\pi^+p \rightarrow \rho p$ at 2.75 GeV/c, $\pi^+p \rightarrow \rho^0 N^*$ at 4 GeV/c, and $\bar{p}p \rightarrow \bar{N}^* N^*$ at 3.6 GeV/c, respectively. For the nucleon and the ρ meson, $R^2 = 1/10 m_\pi^2$ has been chosen, whereas for the N^* , curves are given both for $R_{N^*}^2 = 1/10 m_\pi^2$ and for $R_{N^*}^2 = 1/4 m_\pi^2$. The difference between these two curves is much bigger in Fig. 5 than in Fig. 4, since in the reaction $\bar{p}p \rightarrow \bar{N}^* N^*$ the N^* form factor occurs at both vertices. Notice that the cross-section in Fig. 5 is given on a logarithmic scale. The Born term model is wrong by two orders of magnitude, whereas our model is wrong by one order of magnitude. Figure 5 also shows the prediction of the absorption model ⁴⁾, which again is wrong by one order of magnitude. This indicates that the application of the absorption model to *our model* will give agreement with experiment.

In the π^+p reactions, where our predictions are only slightly larger than the experimental distributions, the effects of absorption will be less pronounced. This will allow one to take the absorption in the final state equal to that in the initial state. This removes one of the difficult points of the absorption model.

We have confined our model to spin zero exchange, since we do not know at present how to extend it to vector exchange. For vector exchange, the matrix element is a sum of products instead of just one product like in (2). The difficulty arises from those terms in the sum

which depend on s , the square of the c.m.s. energy, and which at high energy are known to be wrong anyway. It appears that one has to make a model for the s dependence in the first place, for example the Regge pole model.

Of course, we can also Reggeize our spin zero exchange amplitudes, but here the effects are commonly assumed to be small.

R E F E R E N C E S

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FIGURE CAPTIONS

- Figure 1 : Peripheral graph for a quasi-two-particle reaction. Particles a and c are mesons, while b and d are baryons.
- Figure 2 : Kinematical form factors for the vertices $\pi\pi f^0$, $\pi\pi\rho$, $K\pi K^*$, $N\pi N^*$ and $N\pi N$, for $R^2 = 1/10 m_\pi^2$.
- Figure 3 : Differential cross-section for $\pi^+p \rightarrow \rho^+p$ at 2.75 GeV/c. BTM = Born Term Model, corresponding to $R_\rho^2 = R_N^2 = 0$. Experimental values from Ref. 12), figure from Ref. 2).
- Figure 4 : Differential cross-section for $\pi^+p \rightarrow \rho^0 N^*$ at 4 GeV/c, for $R_\rho^2 = 1/10 m_\pi^2$ and two different values of $R_{N^*}^2$. Experimental values from Ref. 13), figure from Ref. 2).
- Figure 5 : Differential cross-section for $\bar{p}p \rightarrow \overline{N^*}N^*$ at 3.6 GeV/c. The broken line is the prediction of the absorption model 4). Experimental values from Ref. 14).

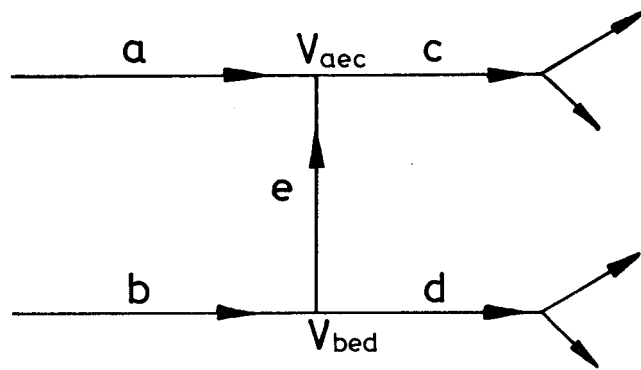


FIG. 1

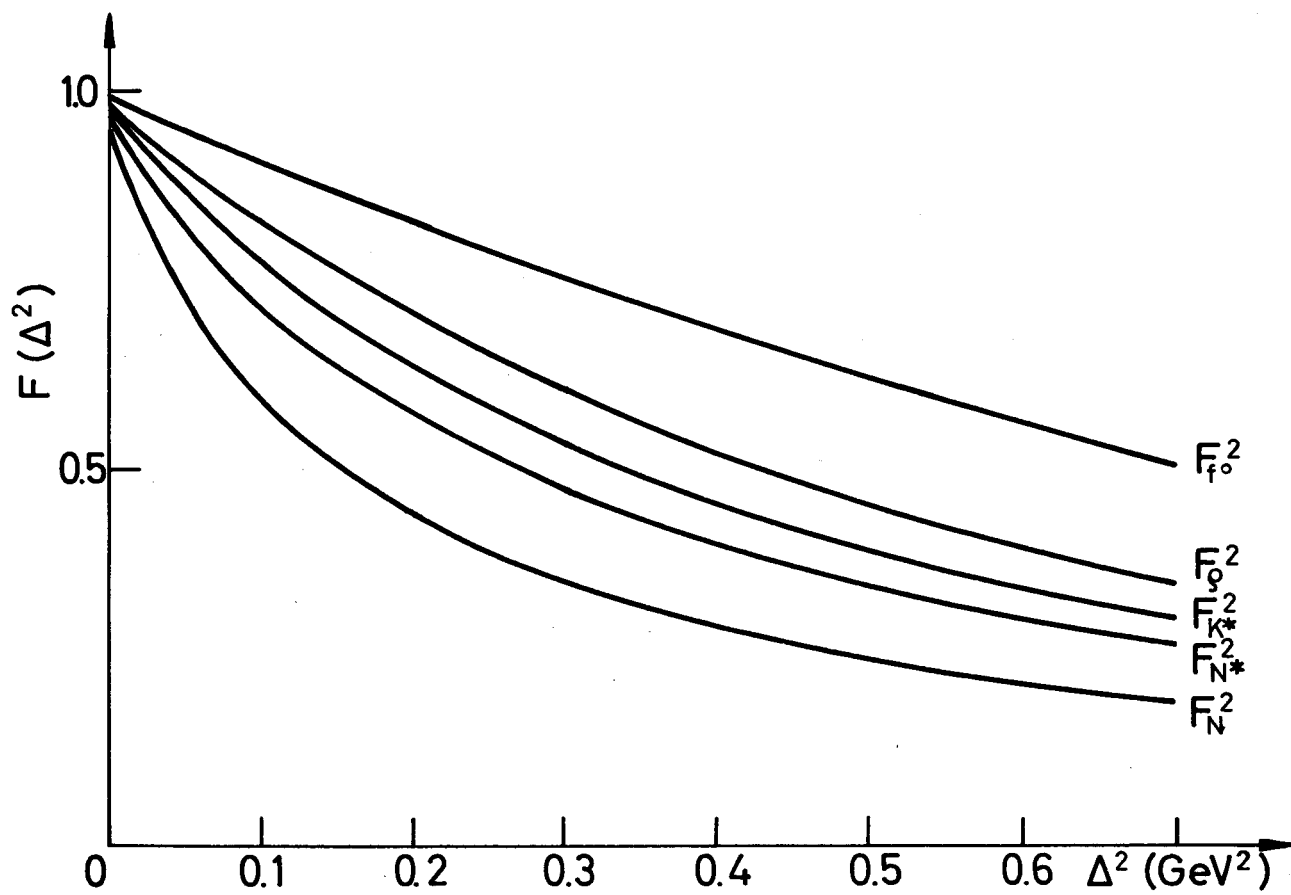


FIG. 2

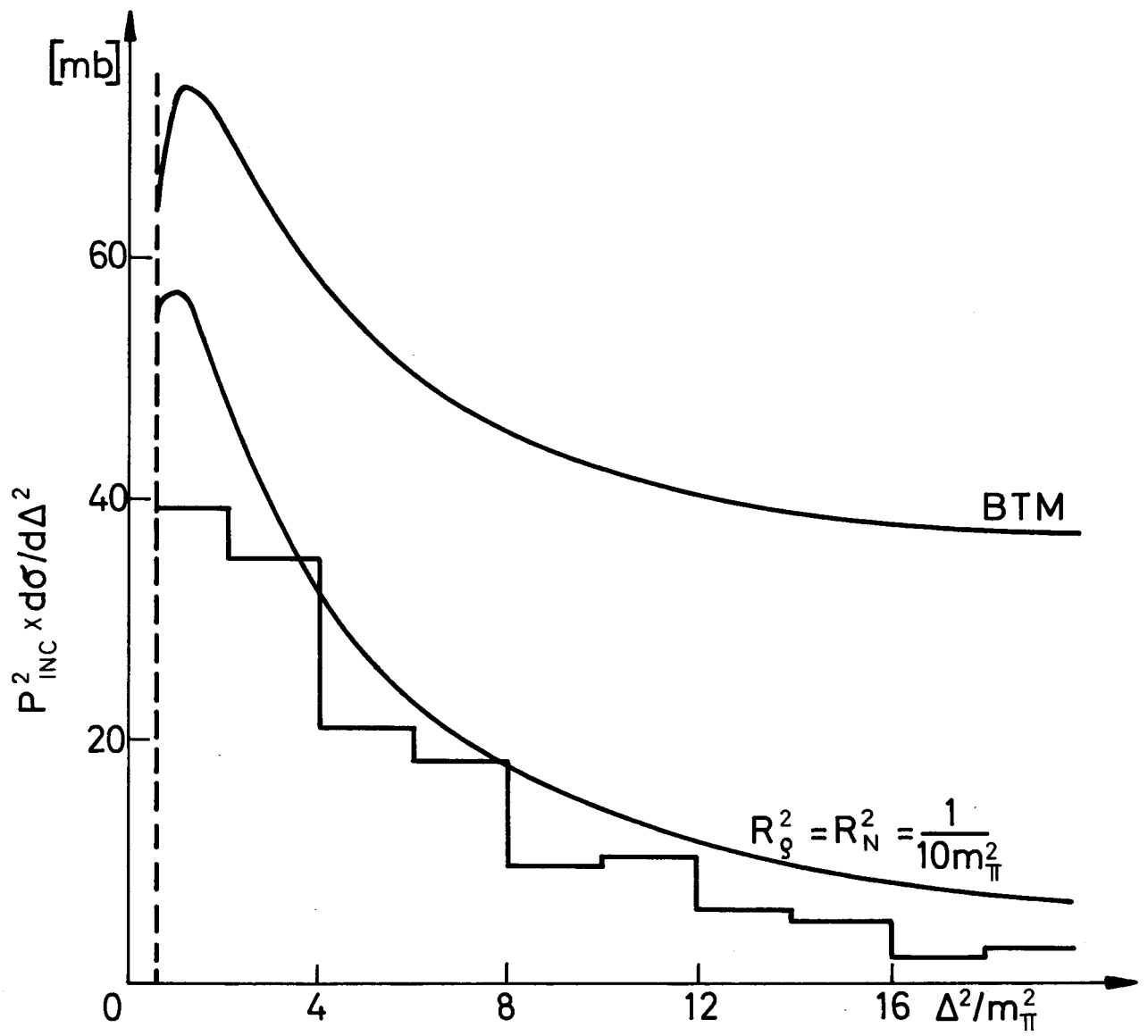


FIG. 3

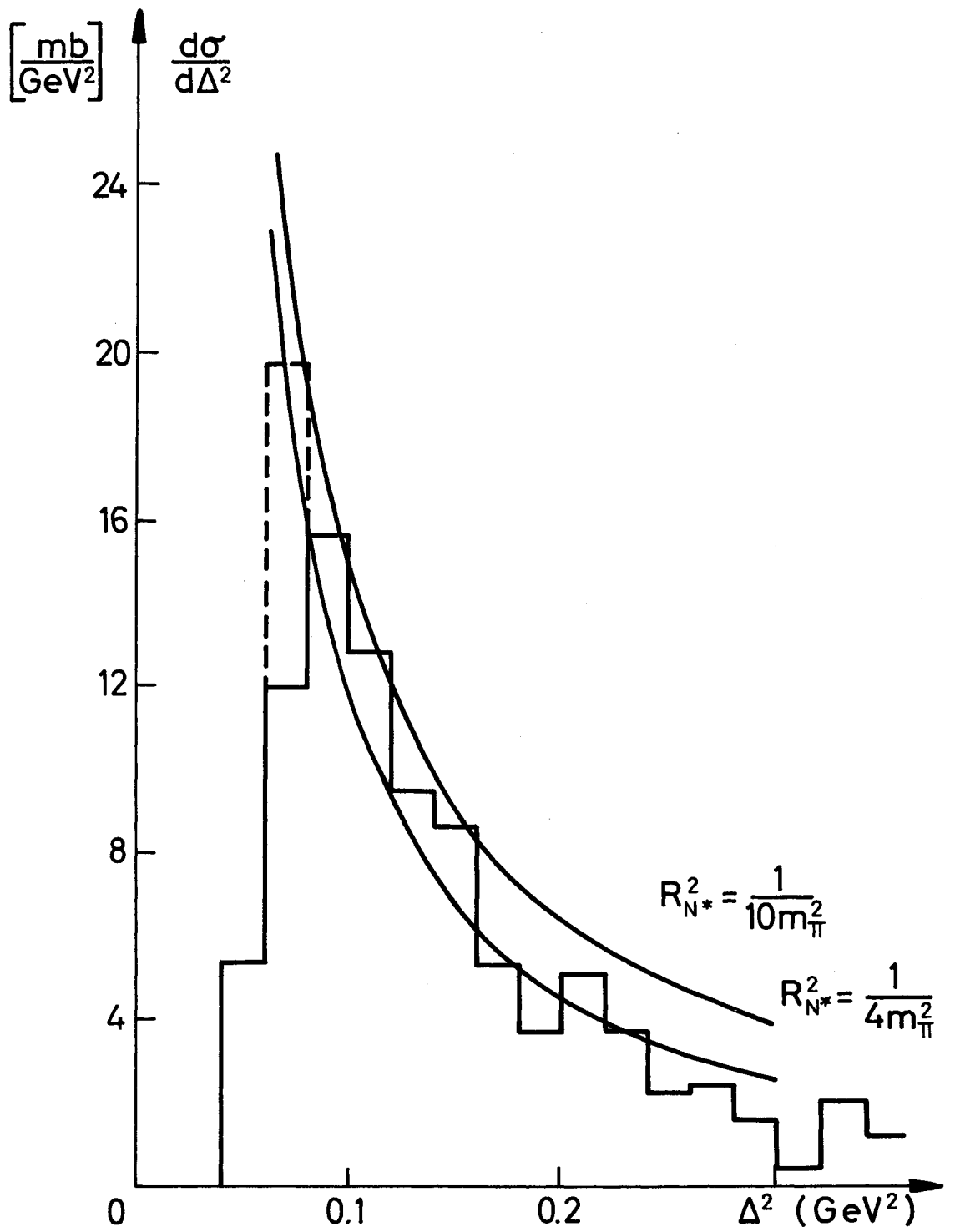


FIG. 4

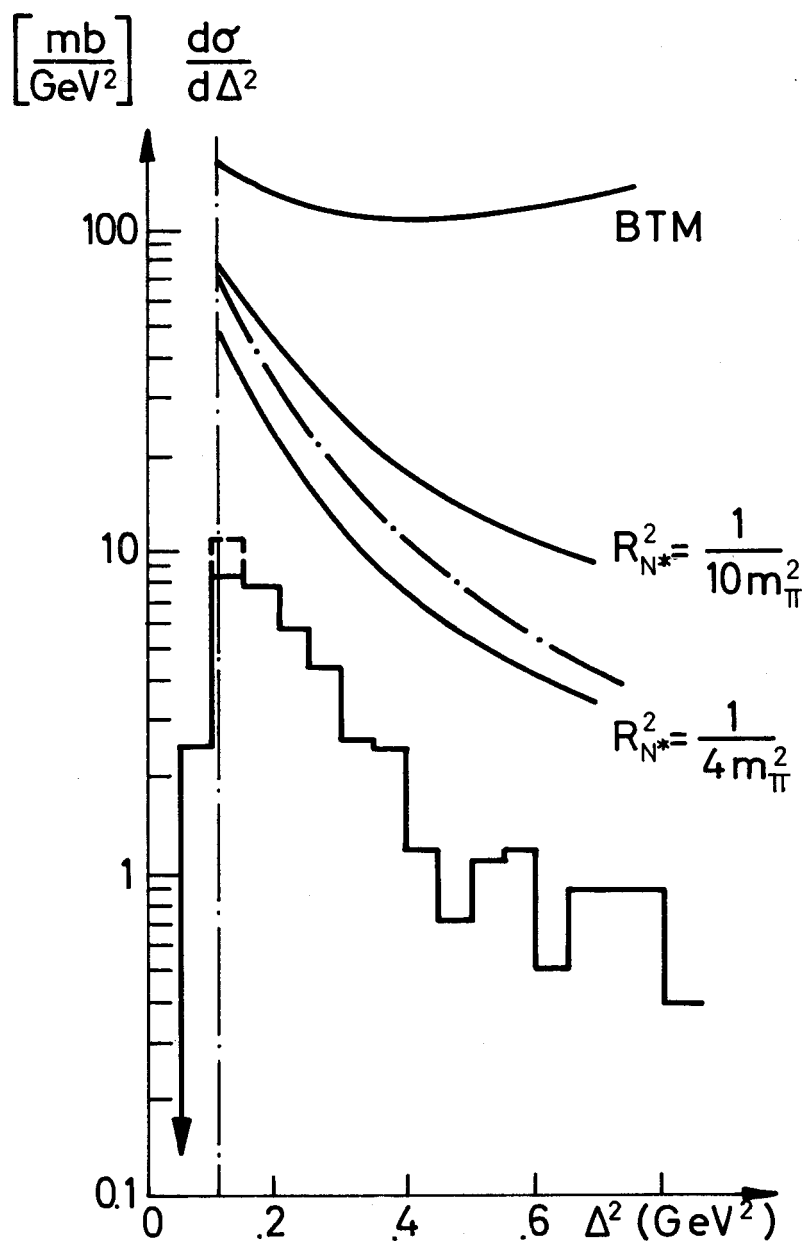


FIG 5