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KINETIC PROCESSES IN PLASMA HEATING BY RESONANT MODE CONVERSION OF ALFVÉN WAVE

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ABSTRACT

An externally applied oscillating magnetic field (at a frequency near one MHz for typical Tokamak parameters) resonantly mode converts to the kinetic Alfvén wave, the Alfvén wave with the perpendicular wavelength comparable to the ion gyroradius. The kinetic Alfvén wave, while it propagates into the higher density side of the plasma after the mode conversion, dissipates due to both linear and non-linear processes and heats the plasma.

In a collisional regime both electrons and ions are heated almost at an equate rate, while in a collisionless regime, but yet if $\beta \leq 0.1$, electrons are heated linearly by the Landau damping while ions are heated nonlinearly by decay to the ion acoustic wave. When β exceeds 0.1, ion Landau damping becomes comparable to the electron Landau damping and both species can be heated linearly again at approximately the same rate.

Nonlinear heating occurs when the applied field exceeds a threshold of typically several tens of gauss. If $T_e \gtrsim 5T_i$ ($T_e \lesssim 5T_i$), sesonant decay to the ion acoustic wave (nonlinear ion Landau damping) heats ions. The nonlinear heating can be shown to occur only after the mode conversion, hence undesirable surface heating is inhibited.

In any case, if a magnetic field of 50 gauss effective amplitude is applies, approximately 10 mega joule per cubic meter of energy can be deposited in one second into the plasma. The heating rate here is faster than that in the transit time magnetic pumping by a factor of β^{-1}.

I. INTRODUCTION

Resonant absorption of an electromagnetic wave propagating into a nonuniform plasma has been known for quite some time since Budden's early paper in 1955.[1] The absorption appears due to a singular behavior of the wave equation for a cold and nonuniform plasma at the point where the frequency of the incident wave matches with that of the local plasma resonance [or upper (or lower) hybrid resonance]. Later, it was identified by Nickel et al.,[2] that the absorption is a manifestation of a mode conversion, (often called the resonant mode conversion) of the incident wave into the Langmuir wave which propagates away from the resonant region.

Extensive studies have been made on the resonant mode conversion for various waves[3] which have revealed that the singular behavior of the wave equations in all cases can be attributed to a mode conversion to a kinetic wave.

A similarly singular behavior of waves in a magnetohydrodynamic frequency range has been relatively unknown compared with those in the higher frequency range quoted above. To our knowledge it was first noticed by Gajewsky and Winterberg[4] in 1965 and slightly later by Pridmore-Brown.[5] It was also mentioned by Grad[6] in his review article in 1969. A singularity appears at the location where the externally applied frequency ω matches with the local Alfvén (shear Alfvén) frequency, $k_{\parallel}(x)v_A(x)$, where $k_{\parallel}$ and v_A are the wave number locally parallel to the ambient magnetic field and the Alfvén speed.

In view of the nature of high rate of the resonant absorption, Grossmann and Tataronis[7] as well as Hasegawa and Chen[8] have independently proposed the use of the Alfvén wave resonant absorption to heat a plasma. More recently Hasegawa and Chen[9] have pointed out that the resonant absorption of this case too can be identified as that due to the convective dissipation of the resonantly mode converted kinetic Alfvén wave and elucidated the detail of the heating process in a linear regime.

The primary purpose of the present paper is to give details of the mode conversion process and the heating process including nonlinear absorption due to decay of the converted wave (the kinetic Alfvén wave) into the ion acoustic wave.

For typical Tokamak parameters, the heating by the linear process occurs through the ion viscosity and the electron Ohmic dissipation for a temperature below a few hundred eV. For a higher temperature, but yet if $\beta \lesssim 0.1$, the linear heating is dominated by the electron Landau damping. The Landau damping appears because of existence of a parallel (with respect to the ambient magnetic field) electric field in the kinetic Alfvén wave. The ions may be heated nonlinearly in this regime depending on the wave amplitude. If β exceeds 0.1, the ion Landau damping can become comparable to the electron Landau damping and the both species are heated equally.

Because the collisionless heating in the present scheme depends on the existence of a parallel electric field (which is accompanied by the kinetic Alfvén wave due to its small perpendicular wavelength), magnetic compression is unncessary as in the case of the transit time magnetic pumping.[10] Consequently the present heating rate is faster than that in the transit time magnetic pumping by a factor of β^{-1}. In addition, because the perpendicular wavelength in the present scheme ($\sim\rho_i$) is much smaller, plasma loss associated with the heating is expected much smaller.

The nonlinear heating occurs due to parametric decays of the converted kinetic Alfvén wave. In the present work, we consider the decay processes where the pump kinetic Alfvén wave decays into an another kinetic Alfvén wave plus an ion acoustic wave, which, depending on the temperature ratio T_e/T_i, can be either weakly or heavily damped. Both resonant decay ($T_e \gg T_i$) and decay through nonlinear ion Landau damping ($T_e \lesssim 5T_i$) are considered. It is found that, owing to its short perpendicular wave-length ($\bar{\lambda} = k_\perp^2 c_s^2/\omega_{ci}^2 \sim O(1)$), the parametric coupling coefficients are larger than the classic values obtained under ideal magnetohydrodynamic approximation by a factor $\bar{\lambda}\omega_{ci}/\omega_A$. Here, ω_{ci} and ω_A are, respectively, ion cyclotron and kinetic Alfvén wave frequencies. For typical Tokamak plasmas, the externally applied field only needs to be of several tens of gauss to overcome the threshold condition. Both electrons and ions are heated in the nonlinear processes.

We further point out that the threshold amplitude of the external field (before the mode conversion) for a parametric decay is much higher than that of the kinetic Alfvén wave, hence undesirable surface heating by a direct nonlinear coupling of the external field is inhibited.

We present the mode conversion process in Section II, the linear absorption mechanism in Section III, the nonlinear absorption mechanism in Section IV and concluding remarks in Section V.

II. RESONANT MODE CONVERSION OF ALFVÉN WAVE

If an ideal set of magnetohydrodynamic equations are used, the wave equation in a nonuniform plasma for the x component of the plasma displacement vector $\underset{\sim}{\xi}$ can be written[8]

$$\frac{d}{dx}\left(\frac{\varepsilon\,\alpha\,B_0^2}{\alpha k_y^2\,B_0^2-\varepsilon}\,\frac{d\xi_x}{dx}\right)-\varepsilon\xi_x=0. \tag{1}$$

In this expression x axis is taken in the direction of the nonuniformity, z axis that of the ambient magnetic field $\underset{\sim}{B}_0$ and a perturbation of a form $\xi_x(x)\exp i(k_y y+k_z z-\omega t)$ is considered. ε and α are functions of the background plasma parameters,

$$\begin{aligned}\varepsilon(x) &= \omega^2 m_i n_0 \mu_0 - k_z^2 B_0^2\\ &= B_0^2\,(\omega^2/v_A^2 - k_z^2)\,,\end{aligned} \tag{2}$$

and

$$\alpha(x) = 1+\frac{\gamma\beta\omega^2}{2}\left(\omega^2-\frac{\gamma\beta k_z^2\,v_A^2}{2}\right)^{-1}, \tag{3}$$

where m_i, n_0, μ_0, γ, β, v_A are respectively, the ion mass, the plasma number density, the space permeability, ratio of the specific heats, pressure ratio of the plasma to the magnetic field $(= 2\mu_0 P/B_0^2)$ and the Alfvén speed $[= B_0(\mu_0 m_i n_0)^{-1/2}]$.

If we assume that the frequency ω and the parallel wave number k_z are fixed by the structure of the external source, $\varepsilon(x)$ can in general become zero at a certain value of $x(= x_0)$. At the point x_0, the applied frequency resonates with the local Alfvén wave. The solution of the wave equation (1) near the resonant point x_0 can then be written,[8]

$$\xi_x = c\,\ell n(x-x_0), \quad x > x_0$$

$$= c(\ell n\,|x-x_0| + i\pi), \quad x < x_0. \qquad (4)$$

The imaginary part in this expression appears through the analytic continuation of the solutions in the ranges $x > x_0$ and $x < x_0$ and represents the resonant absorption of the external perturbation by the plasma. The absorption rate can be shown[8] to be almost a hundred percent if k_y is chosen to be one over the scale size of the plasma nonuniformity, κ $(= \partial\,\ell n n_0/\partial x)$.

The logarithmic singularity appears because of the fact that the ideal magnetohydrodynamic approximation does not allow a propagation of the (shear) Alfvén wave across the magnetic field. Consequently it is expected that if a finite Larmor radius correction is introduced, the fact that ions can no longer be tied to a magnetic line of force will enable a propagation across the magnetic field and eliminate the singularity. That is, the solution (4) is no longer valid within the range $|x-x_0| \lesssim \rho_i$, where

ρ_i is the ion gyroradius. Effect of finite electron inertia can also contribute in the same way and eliminates the singularity in the range $|x-x_0| \lesssim \rho_s$, where $\rho_s = \rho_i(T_e/T_i)^{1/2}$, and T_e and T_i are electron and ion temperatures. To take these effects into accout, we have to use kinetic equations. We consider only the mode conversion process here, hence we ignore dissipative effects. We use the Vlasov equation for ions and the drift kinetic equation for electrons.

The linearized Vlasov equation for the perturbed distribution function of ions reads

$$\frac{\partial f_i}{\partial t} + \underset{\sim}{v} \cdot \frac{\partial f_i}{\partial \underset{\sim}{x}} + \frac{e}{m_i} (\underset{\sim}{E} + \underset{\sim}{v} \times \underset{\sim}{B}) \cdot \frac{\partial f_i^{(0)}}{\partial \underset{\sim}{v}}$$

$$+ \frac{e}{m_i} (\underset{\sim}{v} \times \underset{\sim}{B}_0) \cdot \frac{\partial f_i}{\partial \underset{\sim}{v}} = 0 \ , \qquad (5)$$

where $\partial \underset{\sim}{B}/\partial t = - \underset{\sim}{\nabla} \times \underset{\sim}{E}$ and $\underset{\sim}{E}$ is the wave electric field.

The unperturbed distribution function is given by

$$f_i^{(0)} = g(x+v_y/\omega_{ci}) f^{(0)}(v) \ , \qquad (6)$$

with $f_0(v)$ being the Maxwell distribution with the ion temperature T_i and g being a function representing the plasma nonuniformity in the x direction.

The perturbed distribution function of electrons obeys the drift kinetic equation,[11]

$$\frac{\partial f_e}{\partial t} + v_z \frac{\partial f_e}{\partial z} + \left(\frac{E_y}{B_0} + v_z \frac{B_x}{B_0} \right) \cdot \frac{\partial f_e^{(0)}}{\partial x} - \frac{e}{m_e} E_z \frac{\partial f_e^{(0)}}{\partial v_z} = 0. \tag{7}$$

For the field equations, we assume a low but finite β plasma with $\beta \sim (m_e/m_i)^{1/2}$. In this regime, it is convenient to use the fact that magnetic compression by the wave is negligible, which enables one to use a scaler potential φ to express the perpendicular components (x, y components) of the wave electric field,

$$\underset{\sim}{E}_\perp = - \underset{\sim}{\nabla}_\perp \varphi , \tag{8}$$

since this choice makes $(\underset{\sim}{\nabla} \times \underset{\sim}{E})_z = 0$. Because the wave is electromagnetic, however, we have to use E_z which is not equal to $- \partial\varphi/\partial z$. For E_z, we hence use a different potential, ψ;

$$E_z = - \frac{\partial \psi}{\partial z} , \tag{9}$$

and $\varphi \neq \psi$.[12]

The potentials φ and ψ must satisfy suitable field equations. Following Kadomtsev,[12] we take Poisson's equation

$$\nabla_\perp^2\phi + \frac{\partial^2\psi}{\partial z^2} = \frac{e}{\varepsilon_0}(n_e - n_i), \tag{10}$$

and Ampére's law in the z direction

$$(\underset{\sim}{\nabla}\times\underset{\sim}{H})_z = J_{iz} + J_{ez} \quad , \tag{11}$$

which after substituting $\partial\underset{\sim}{B}/\partial t = -\underset{\sim}{\nabla}\times\underset{\sim}{E}$ reduces to

$$\frac{\partial}{\partial z}\nabla_\perp^2(\varphi-\psi) = \mu_0\frac{\partial}{\partial t}\left(J_{iz} + J_{ez}\right) \quad . \tag{12}$$

In these expressions, the number density n and the current density J are given by the distribution function f

$$n_j = n_0 \int f_j d\underset{\sim}{v} , \tag{13}$$

and

$$\underset{\sim}{J}_j = q_j n_0 \int \underset{\sim}{v} f_j d\underset{\sim}{v} \quad . \tag{14}$$

The set of equations from (5) to (14) describes the electromagnetic waves in a magnetohydrodynamic frequency range without restrictions to the size of the perpendicular wavelength (but with a restriction of no compressional magnetic field perturbation).

To study the resonant mode conversion of the Alfvén wave, we make some further simplications. First we assume a frequency range which is smaller than the ion cyclotron frequency. This allows us to use the quasi-neutrality condition and Eq. (10) becomes

$$n_i = n_e \quad . \tag{10'}$$

Next, we assume that k_y, the wave number perpendicular to the magnetic field as well as to the density gradient, is much smaller than ρ_i^{-1} and that the scale size of the density gradient is much larger than the ion gyroradius, so that

$$\frac{\omega}{\omega_{ci}} \gg k_y \kappa \, \rho_i^2 \quad . \tag{15}$$

This assumption allows us to ignore the term v_y/ω_{ci} in g in Eq. (6) which is equivalent to the neglection of a drift wave.

Finally, we assume that the wavelength in the x direction near the mode conversion point is small but larger than the ion gyroradius so that we can expand the wave equation in the power of $\rho_i d/dx$.[13]

With these assumptions Fourier amplitude of the number density and the current density perturbations are given by

$$\frac{en_i}{\varepsilon_0} = \frac{\omega_{pi}^2}{\omega_{ci}^2} \left\{ \frac{d}{dx} \left[1 + \frac{3}{4} \rho_i^2 \frac{d^2}{dx^2} \right] \left(g \frac{d\varphi}{dx} \right) - g k_y^2 \varphi \right\} + \frac{\omega_{pi}^2}{\omega^2} k_z^2 \, g \, \psi , \tag{16}$$

$$\frac{en_e}{\varepsilon_0} = \frac{\omega_{pe}^2}{v_{Te}^2} g \psi, \tag{17}$$

$$\mu_0 J_{iz} = \frac{\omega_{pi}^2}{c^2} \frac{k_z}{\omega} g \psi, \tag{18}$$

and

$$\mu_0 J_{ez} = - \frac{\omega_{pe}^2}{c^2 v_{Te}^2} \frac{\omega}{k_z} g \psi . \tag{19}$$

In these expressions, the ion and the electron plasma frequencies, ω_{pi} and ω_{pe} are constants and refer to the values at the maximum plasma density where $g(x)$ is normalized to unity, c and v_{Te} are the speed of light and the electron thermal speed, respectively.

To study the resonant mode conversion to the kinetic Alfvén wave, we consider only near the resonant point $x=x_0$ where $\omega^2 = k_z^2 v_A^2/g(x=x_0)$. We can then further simplify the above expressions using the assumption of a low β plasma so that $v_A^2 \gg c_s^2 = T_e/m_i$. This assumption eliminates a possibility of a simultaneous coupling to the ion acoustic wave. The wave equation can then be derived by eliminating ψ from Eqs. (10'), (12) and (16) to (19),

$$\left(\frac{\omega^2}{k_z^2\, v_A^2} \frac{3}{4} \rho_i^2 \frac{d^3}{dx^3} + \frac{d^2}{dx^2} \frac{1}{g} \frac{T_e}{T_i} \rho_i^2 \frac{d}{dx} \right) \left(g \frac{d\varphi}{dx} \right)$$

$$+ \left[\frac{d}{dx} \left(\frac{\omega^2}{k_z^2\, v_A^2} g - 1 \right) \frac{d}{dx} - k_y^2 \left(\frac{\omega^2}{k_z^2\, v_A^2} - 1 \right) \right] \varphi = 0, \tag{20}$$

where the Alfvén speed v_A^2 is that of the maximum density and g is normalized by unity.

We can immediately notice that if we put $\rho_i \to 0$ in this expression, the wave equation reduces to

$$\frac{d}{dx} \left[\varepsilon(x) \frac{d\varphi}{dx} \right] - k_y^2 \, \varepsilon(x) \, \varphi = 0, \tag{21}$$

which has an identical structure to the magnetohydrodynamic wave equation (1) near $\varepsilon = 0$. It is also noted that in a uniform plasma, $g = 1$, Eq. (20) gives two decoupled wave equations

$$\nabla_\perp^2 \varphi = 0, \tag{22}$$

and

$$\left[\bar{\rho}^{\,2} \frac{d^2}{dx^2} + \left(\frac{\omega^2}{k_z^2\, v_A^2} - 1 \right) \right] \varphi = 0, \tag{23}$$

where $\bar{\rho}^2 = (\frac{3}{4} + \frac{T_e}{T_i}) \rho_i^2$. $k_y \ll d/dx$ is used in Eq. (23).

Equation (22) represents a quasistatic electromagnetic perturbation (cut off mode) associated with an external source. In the absence of a source, this wave equation represents a surface wave. Equation (23) is the wave equation for a body or bulk wave, the Alfvén wave with finite ion gyroradius and finite electron inertia correction. We call this wave the kinetic Alfvén wave.

We can thus identify that Eq. (20) represents a coupling between a surface magnetohydrodynamic mode or an externally applied electromagnetic perturbation and the kinetic Alfvén wave. From Eq. (23), we can see that the kinetic Alfvén wave propagates, after the mode conversion, to a higher density side where $k_z^2\ v_A^2\ (x) < k_z^2\ v_A^2\ (x=x_0)$, as analogous to the case of the Bernstein wave.[14]

To study the mode conversion, we must specify the actual density profile. Because the wave after the mode conversion is expected to propagate in the higher density side, the solution depends either the converted wave can propagate all the way across the plasma column or dissipates significantly before it reaches the other side. As will be seen in the next section, damping of the kinetic Alfvén wave per wavelength is shown to be on the order of 10^{-2}, hence if the plasma size in the x direction (or r direction in a cylindrical plasma) measured by the ion gyroradius is at least 10^2, we can assume the plasma to be semi-infinite in the x direction.

As a simple example we take a linear profile for the plasma density such that

$$g(x) = \kappa x + a, \tag{24}$$

where x is a normalized distance whose origin is located at the resonant point where $g(x=0)\ \omega^2/k_z^2\ v_A^2 = 1$, or $a\,\omega^2/k_z^2\ v_A^2 = 1$, with $0 < a < 1$. Equation (20) is then reduced near $x \sim 0$ to a simple form

$$\rho^2 \frac{d^2E_x}{dx^2} + \kappa x E_x = E_0, \tag{25}$$

where

$$\rho^2 = \left(\frac{3}{4} + \frac{k_z^2\ v_A^2}{\omega^2} \frac{T_e}{T_i} \right) \rho_i^2 \ , \tag{26}$$

and

$$E_x = - \frac{\partial\varphi}{\partial x} . \tag{27}$$

E_0 is an integration constant representing a nominal value of E_x at a large negative x (E_x associated with the external source field).

In the derivation we also assumed that $|dg/dx/g| << |d\varphi/dx/\varphi|$, that is, the variation of the wave amplitude is much faster than the variation of the density (WKB approximation). The wave equation of the type Eq. (25) has been studied extensively in relation to the mode conversion of the electromagnetic wave to the Langmuir wave. It is well known that the solution can be expressed in terms of the Airy functions.[15]

If we introduce a scale length

$$\Delta = (\rho^2/\kappa)^{1/3}, \tag{28}$$

and use a normalized electric field intensity

$$\bar{E}_x = -\frac{E_x}{E_0}\frac{(\kappa\rho)^{2/3}}{\pi}, \tag{29}$$

the general solution is given by

$$\bar{E}_x = c_1 A_i(-x/\Delta) + c_2 B_i(-x/\Delta) + G_i(-x/\Delta), \tag{30}$$

where A_i, B_i are Airy functions and G_i is a function involving integrals of A_i and B_i.[15]

Because we consider here a plasma which is extended to a semi-infinite domain in the $x > 0$ direction, the suitable boundary condition for the kinetic Alfvén wave is to accept only the right going wave (no reflection at $x \to \infty$) and that which has no divergence at $x \to -\infty$. We can find then $c_2 = 0$ and $c_1 = i$. The asymptotic solution for $|x/\Delta| \gg 1$ can then be written as

$$E_x = - \frac{\pi^{1/2} E_0}{(\kappa \rho)^{2/3}} \left(\frac{\Delta}{x} \right)^{1/4} \exp \left\{ i \left[\frac{2}{3} \left(\frac{x}{\Delta} \right)^{3/2} + \frac{\pi}{4} \right] \right\} + \frac{E_0}{\kappa x} , \tag{31}$$

for $x > 0$,

$$E_x = \frac{E_0}{\kappa x} , \tag{31'}$$

for $x < 0$.

In the above expression, the first term in Eq. (31) represents the kinetic Alfvén wave and the second term as well as the expression in Eq. (31') the source field.

As is expected, the kinetic Alfvén wave propagates in the higher density side ($x \geq 0$) of the resonant point as can be seen in Eq. (30). In Eq. (30) it is worth noting that the first few peak amplitudes of the kinetic Alfvén wave after the mode conversion are given by $E_0(\kappa\rho)^{-2/3}$ with effective wave number of $(\kappa/\rho^2)^{1/3}$, while away from the resonant point, say at $x \sim \kappa^{-1}$, the amplitude and the wave-number of the kinetic Alfvén wave become $E_0(\kappa\rho)^{-1/2}$ and ρ^{-1}, respectively. The qualitative feature of the mode converted kinetic Alfvén wave is shown in Fig. 1.

We can see that the solution for $x < 0$ is identical to the one obtained under the ideal magnetohydrodynamic approximation.[8] Hence the plasma impedance, consequently the absorption rate, seen from the external circuit remains to be unchanged from the previous magnetohydrodynamic calculation.[8]

On the other hand when the converted wave is reflected at the other side of the plasma the solution at $x < 0$ is modified from the magnetohydrodynamic result and the external plasma impedance changes accordingly. However, it can be shown again that if the e-folding distance of the converted kinetic Alfvén wave is smaller than the plasma size, the plasma impedance becomes the same as the magnetohydrodynamic result.[9]

In conclusion, if the mode converted wave dissipates well inside the plasma the absorption rate is independent of the dissipation mechanism and remains same as the result of the magnetohydrodynamic calculation; the absorbed energy is approximately $B_y^2/2\mu_0 \big|_{x \sim 0}$ per each cycle of the oscillation.

Detailed physical dissipation mechanism of the kinetic Alfvén wave which will be presented in the next two sections give processes in which the absorbed energy is eventually thermalized by each species.

III. THE LINEAR DAMPING OF THE KINETIC ALFVÉN WAVE

We have seen in the previous section that the kinetic Alfvén wave is excited by the resonant mode conversion of an externally applied oscillating electromagnetic source. A natural step to be followed is to identify the physical dissipation mechanism of the kinetic Alfvén wave. For this purpose, we now take a uniform plasma but introduce dissipative mechanism to the kinetic equations for both electrons and ions. It is convenient to divide into collisional and collisionless regimes depending on whether the collisional damping dominates over the Landau damping or vice versa.

In the collisional regime, the damping due to ions is the ion viscous damping and that due to electrons is the electron Ohmic dissipation. The former dissipates the perpendicular electric field energy while the latter the parallel electric field energy. The collisionless dissipation is the Landau damping of the parallel electric field both by electrons and ions. This aspect differs from the transit time magnetic pumping where the collisionless damping is due to the magnetic transit time damping which dissipates the compressional component (z component) of the magnetic field.

To study these dissipative processes, it is convenient to study the detailed properties of the kinetic Alfvén wave without restricting to the size of the perpendicular wavelength as well as without neglecting the coupling to the ion acoustic wave. This can be done by including the Fokker-Planck collision term in the right hand side of the Vlasov equation[16] and recalculating the number density and the current density perturbations,

$$\frac{en_i}{\varepsilon_0} = -\frac{\omega_{pi}^2}{v_{Ti}^2}\left[1 - I_0(\lambda_i)e^{-\lambda_i}\right]\varphi$$

$$- i\,\frac{\omega_{pi}^2}{v_{Ti}^2}\,\frac{\nu_{ii}}{\omega}\,\varphi \begin{Bmatrix} \frac{7}{10}\lambda_i^2 & \lambda_i \ll 1 \\ \frac{3(\pi+1)}{8\sqrt{\pi}}\lambda_i^{1/2} & \lambda_i \gg 1 \end{Bmatrix}$$

$$+ \frac{\omega_{pi}^2 k_z^2}{\omega^2} I_0(\lambda_i)e^{-\lambda_i}(1-i\delta_i)\,\psi, \qquad (16')$$

$$\frac{en_e}{\varepsilon_0} = \frac{\omega_{pe}^2}{v_{Te}^2}(1+i\delta_e)\,\psi, \qquad (17')$$

$$\mu_0 J_{iz} = \frac{\omega_{pi}^2}{c^2}\frac{k_z}{\omega} I_0(\lambda_i)e^{-\lambda_i}(1-i\delta_i)\psi, \qquad (18')$$

$$\mu_0 J_{ez} = -\frac{\omega_{pe}^2}{c^2 v_{Te}^2}\frac{\omega}{k_z}(1+i\delta_e)\,\psi, \qquad (19')$$

where $\lambda_i = k_x^2\,\rho_i^2$. Here, $I_0(\lambda_i)$ is the modified Bessel function of the first kind and ν_{ii} is the ion-ion collision rate. δ_i and δ_e represent fractional damping rates by ions and electrons. In a collisional regime δ_e is given by

$$\delta_e = \frac{\omega\nu_{ei}}{(k_z v_{Te})^2}, \qquad (32)$$

where ν_{ei} is the electron ion collision rate. The ion viscous damping is shown in the term propertional to ν_{ii} in Eq. (16'). In a collisionless regime,

$$\delta_i = 2\sqrt{\pi}\ \beta_i^{-3/2} \exp\left(-\beta_i^{-1}\right), \tag{33}$$

$$\delta_e = \sqrt{\pi}\ \beta_i^{-1/2} (T_i/T_e)^{1/2} (m_e/m_i)^{1/2}, \tag{32'}$$

where β_i is defined for the ion pressure, $\beta_i = 2v_{Ti}^2/v_A^{\ 2}$ and that at the resonant point.

The Landau damping may be reduced in a toroidal plasma due to the trapped particles in the local mirror field.

δ's obtained above do not by themselves represent the fractional heating rate of the species because different components of the fields are dissipated. To obtain the phase and amplitude relations among different components of the field and also to study the mode structure in this frequency range including the ion acoustic wave, we rederive here the dispersion relation using the charge and the current densities obtained in Eqs. (16') to (19').

We again ignore δ for these purposes. From the quasineutrality condition, we have

$$\left(\frac{\omega_{pi}^2\ k_z^2}{\omega^2} I_0 e^{-\lambda_i} - \frac{\omega_{pe}^2}{v_{Te}^2}\right)\psi - \frac{\omega_{pi}^2}{v_{Ti}^2}\left(1 - I_0 e^{-\lambda_i}\right)\varphi = 0, \tag{34}$$

and from the Ampere's law,

$$\left(\frac{\omega_{pi}^2}{c^2 k_x^2} I_0 e^{-\lambda i} - \frac{\omega_{pe}^2}{c^2 k_x^2} \frac{\omega^2}{k_z^2 \ v_{Te}^2} + 1 \right) \psi - \varphi = 0. \tag{35}$$

The argument of the Bessel function is $\lambda_i (= k_x^2 \ \rho_i^2)$. The dispersion relation is obtained by eliminating φ and ψ,

$$\left(I_0 e^{-\lambda_i} - \frac{\omega^2}{k_z^2 \ c_s^2} \right) \left[1 - \frac{\omega^2}{k_z^2 \ v_A^2} \frac{1}{\lambda_i} \left(1 - I_0 e^{-\lambda_i} \right) \right] = \frac{\omega^2}{k_z^2 \ v_{Ti}^2} \left(1 - I_0 e^{-\lambda_i} \right), \tag{36}$$

where c_s is the ion sound speed, $(T_e/m_i)^{1/2}$. Equation (36) shows clearly the linear coupling of the ion acoustic wave (zero of the first parenthesis) and the Alfvén wave (zero of the second parenthesis). For a low β plasma the coupling is weak and the dispersion relation for the each mode becomes approximately,

$$\frac{\omega^2}{k_z^2 \ v_A^2} = \frac{\lambda_i}{1 - I_0 e^{-\lambda_i}} + \frac{T_e}{T_i} \lambda_i, \tag{37}$$

for the kinetic Alfvén wave and

$$\frac{\omega^2}{k_z^2 \ c_s^2} = \frac{I_0 e^{-\lambda_i}}{1 + (1 - I_0 e^{-\lambda_i}) T_e/T_i} , \tag{38}$$

for the ion acoustic wave. It is interesting to note that the frequency deviation due to the finite ion gyroradius effect, λ_i, is positive for the Alfvén wave and negative for the ion acoustic wave. The dispersion relations for small and large λ_i become for the kinetic Alfvén wave

$$\omega^2 = k_z^2 v_A^2 \left[1 + k_x^2 \rho_i^2 (3/4 + T_e/T_i)\right] \quad \text{for} \quad \lambda_i \ll 1 , \qquad (37')$$

$$= k_z^2 v_A^2 k_x^2 \rho_i^2 (1+T_e/T_i) \quad \text{for} \quad \lambda_i \gg 1 ,$$

and for the ion acoustic wave,

$$\omega^2 = k_z^2 c_s^2 \left(1 - k_x^2 \rho_i^2\right) \left(1 + k_x^2 \rho_i^2 T_e/T_i\right)^{-1} \quad \text{for} \quad \lambda_i \ll 1 , \qquad (38')$$

$$= \frac{k_z^2 c_s^2}{\sqrt{2\pi}\, k_x \rho_i} (1+T_e/T_i)^{-1} \quad \text{for} \quad \lambda_i \gg 1 .$$

Using these relations, we can obtain the phase and amplitude relations among different components of the wave fields. We list below this relation for the kinetic Alfvén wave

$$
\begin{aligned}
E_x &= -\, i k_x \varphi \\
E_y &= -\, i k_y \varphi \\
E_z &= i k_z \frac{T_e}{T_i} (1 - I_0 e^{-\lambda_i}) \varphi \\
B_x &= i \frac{k_y k_z \varphi}{\omega} \left[1 + \frac{T_e}{T_i} (1 - I_0 e^{-\lambda_i}) \right] \\
B_y &= -i \frac{k_x k_z \varphi}{\omega} \left[1 + \frac{T_e}{T_i} (1 - I_0 e^{-\lambda_i}) \right] .
\end{aligned}
\tag{39}
$$

Because $k_x \sim \rho_i^{-1} \gg k_y \gg k_z$, we see that the dominant field components are E_x and B_y. In a toroidal plasma they correspond to radial electric field and poloidal magnetic field. It is also important to note that the wave accompanies an electric field in the direction of the ambient magnetic field. Another important point to note is that the field amplitude is enhanced compared with that of the external source. For example, if we take the y component of the wave magnetic field B_y, from Eqs. (30) and (39) we see

$$
| B_y | \sim \frac{1}{(\kappa \rho_i)^{2/3}} \left(1 + \frac{T_e}{T_i} k_x^2 \rho_i^2 \right) | B_{sy} | \tag{40}
$$

near the mode conversion region,

$$
| B_y | \sim \frac{1}{(\kappa \rho_i)^{1/2}} \left(1 + \frac{T_e}{T_i} k_x^2 \rho_i^2 \right) | B_{sy} | \tag{41}
$$

away from the mode conversion region,

where B_{sy} is the amplitude of the source field.

If we take a Tokamak plasma $\kappa \sim r^{-1} \sim 10^{-3}\,\rho_i^{-1}$, hence the y component of the wave magnetic field can be enhanced by a factor of a hundred near the mode conversion point.

Using these results, let us now calculate the actual heating rate for each species. In a collisional regime, the ion heating is dominated by the viscous damping of the transverse component of the wave field. The heating rate is given by

$$n_0 \frac{dT_i}{dt} = \frac{1}{2}\ \mathrm{Re}(\underset{\sim}{J}\cdot\underset{\sim}{E}^*)_{ion}$$

$$= \begin{cases} 0.7\ \nu_{ii}(k_x\rho_i)^2\ \dfrac{\varepsilon_0|E_x|^2}{2}\ \dfrac{\omega_{pi}^2}{\omega_{ci}^2} & \text{for} \quad k_x\rho_i < 1 \\[2ex] 0.9\ \nu_{ii}(k_x\rho_i)^{-1}\ \dfrac{\varepsilon_0|E_x|^2}{2}\ \dfrac{\omega_{pi}^2}{\omega_{ci}^2} & \text{for} \quad k_x\rho_i > 1\ . \end{cases} \tag{42}$$

Because k_x, $|E_x|^2$ and $\omega_{pi}^2/\omega_{ci}^2$ are all functions of x, the heating rate varies as function of the distance away from the mode conversion point, $x = 0$. However, the variations tend to cancel among each other and the heating rate remains roughly constant and is given approximately by

$$n_0 \frac{dT_i}{dt} \sim \nu_{ii} \frac{|B_y(\kappa^{-1})|^2}{2\mu_0}\ , \tag{42'}$$

where $B_y(\kappa^{-1})$ is the value of the wave magnetic field at $x = \kappa^{-1}$, given by Eq.(41).

The electron heating in a collisional regime is governed by the Ohmic dissipation of the field aligned current,

$$n_0 \frac{dT_e}{dt} = \frac{1}{2} \operatorname{Re}(J_z E_z^*)$$

$$= \nu_{ei} \frac{T_e}{T_i} \frac{1}{\lambda_i} (1 - I_0 e^{-\lambda_i})^2 \frac{\varepsilon_0 |E_x|^2}{2} \frac{\omega_{pi}^2}{\omega_{ci}^2} \frac{\omega^2}{k_z^2 v_{Te}^2} . \qquad (43)$$

If we compare this heating rate with that of the ions in Eq. (42), we see that

$$\frac{dT_i/dt}{dT_e/dt} \sim \frac{\nu_{ei}(T_e/T_i)(\omega^2/k_z^2 v_{Te}^2)}{\nu_{ii}}$$

$$\sim \left(\frac{m_i}{m_e}\right)^{1/2} \left(\frac{T_i}{T_e}\right)^{3/2} \frac{1}{\beta_i} . \qquad (44)$$

For most Tokamak parameters, this ratio remains order unity. Hence, in a collisional regime electrons and ions are heated approximately at an equal rate. However, electrons are heated in the parallel direction while ions in the perpendicular direction.

In a collisionless regime, the linear heating occurs due to the Landau damping. Hence, particles are heated in the parallel direction; $n_0 dT/dt \sim \operatorname{Re}(J_z E_z^*/2)$. The Landau damping rate for trapped electrons by a local mirror is reduced by the bounce motion. We assume here for simplicity a case of a straight magnetic field. The heating rate for ions in a collisionless regime is then given by

$$n_0 \frac{dT_i}{dt} = \omega \frac{\delta_i}{2} \beta_i \left(\frac{T_e}{T_i} \right)^2 \frac{I_0 e^{-\lambda_i}}{\lambda_i} (1-I_0 e^{-\lambda_i})^2 \frac{\varepsilon_0 |E_x|^2}{2} \frac{\omega_{pi}^2}{\omega_{ci}^2} , \tag{45}$$

where as before β_i (= const) is defined for the plasma parameter at the resonant point and δ_i is given by Eq. (33). Other quantities except ω is a function of position. A maximum heating is achieved at $\lambda_i \sim 1$.

The heating rate for electrons in a collisonless regime is given by

$$n_0 \frac{dT_e}{dt} = \omega \ \delta_e \frac{T_e}{T_i} \frac{1}{\lambda_i} (1-I_0 e^{-\lambda_i})^2 \frac{\varepsilon_0 |E_x|^2}{2} \frac{\omega_{pi}^2}{\omega_{ci}^2} , \tag{46}$$

where δ_e is given by Eq. (32'). In Eqs. (45) and (46), $\varepsilon_0 |E_x|^2 \ \omega_{pi}^2/\omega_{ci}^2/2$ can be approximately be identified as the wave magnetic field energy $B_y^2/2\mu_0$.

The ratio of the heating rate for ions and electrons in the collisionless regime becomes

$$\frac{dT_i/dt}{dT_e/dt} \sim \frac{(1/2)\beta_i I_0 e^{-\lambda_i} T_e/T_i \, \delta_i}{\delta_e} \tag{47}$$

$$\sim \left(\frac{2m_i}{m_e} \right)^{1/2} \left(\frac{T_e}{T_i} \right)^{3/2} e^{-\beta_i^{-1}}$$

for $\lambda_i \ll 1$.

The factor β_i on the ion heating rate appears because the ion Landau damping is possible only through the coupling to the ion acoustic wave. This ratio is negligibly small for a low β plasma, that is only electrons can be heated in a low β plasma. However, when β_i approaches unity, the ratio becomes order unity. For example, if $\beta_i = 0.2$, $T_e/T_i = 2$ makes this ratio unity and the corresponding value of δ_e becomes 0.13. Because of the trapped particle effect this value of δ_e is an overestimate. The true damping rate will probably be an order magnitude less.

IV. NONLINEAR DAMPING OF THE KINETIC ALFVÉN WAVE

In this section, we consider nonlinear heating processes due to parametric decay instabilities excited by the converted kinetic Alfvén wave. Here, we are interested in the decay processes where the pump kinetic Alfvén wave decays into an another kinetic Alfvén wave and an ion acoustic wave. The ion acoustic wave can be either weakly or heavily damped depending on the temperature ratio T_e/T_i. In the former case $T_e \gg T_i$, resonant decay occurs. In the latter case $T_e \lesssim 5T_i$, the decay instability is made through nonlinear ion Landau damping; i.e., it is an induced scattering process. Both these two cases are considered here.

In order to illustrate the physics more clearly, we adopt a simple model of a uniform plasma to which a self-consistent pump wave $\varphi_0(x,t)$ (the kinetic Alfvén wave) of the following form

$$\varphi_0(\underset{\sim}{x},t) = \frac{1}{2}\{\varphi_0 \exp[-i(\omega_0 t + \underset{\sim}{k}_0\cdot\underset{\sim}{x})] + c.c\}, \qquad (48)$$

is applied. Here, $(\omega_0, \underset{\sim}{k}_0)$ satisfies the linear dispersion relation for kinetic Alfvén wave given by Eq. (37). Effects due to nonuniformities and spatial localization of the pump wave will not be discussed in the present work. The pump field φ_0 is assumed to be sufficiently weak so that only interactions up to $O(|\varphi_0|^2)$ need to be kept. Furthermore, since decay instabilities are considered here, we ignore the upper sideband as being off-resonant and only discuss the couplings among the pump wave $(\pm\omega_0, \pm\underset{\sim}{k}_0)$, the lower-sideband $(\omega_-, \underset{\sim}{k}_-) = (\omega_s - \omega_0, \underset{\sim}{k}_s - \underset{\sim}{k}_0)$ and the low frequency wave $(\omega_s, \underset{\sim}{k}_s)$. Note, again, $(\pm\omega_0, \pm\underset{\sim}{k}_0)$ and $(\omega_-, \underset{\sim}{k}_-)$ are kinetic Alfvén waves, and $(\omega_s, \underset{\sim}{k}_s)$ is the ion acoustic mode. For low-β plasmas, $|\omega_0| \sim |k_z v_A| > |\omega_s| \sim |k_z c_s|$. To further simplify the analyses, we make the additional assumptions that $|k_\perp \rho_i| < 1$ and $T_e/T_i > 1$, so that we need only keep the effects of finite electron inertia. The dynamics of both species are then described by the following drift kinetic equation

$$\frac{\partial f_j}{\partial t} + v_z \frac{\partial f_j}{\partial z} + \underset{\sim}{\nabla}_\perp \cdot (\underset{\sim}{v}_\perp f)_j + \left(\frac{q}{m}\right)_j [E_z + (\underset{\sim}{v}_\perp \times \underset{\sim}{B}_\perp)\cdot \underset{\sim}{e}_z]_j \frac{\partial f_j}{\partial v_z} = 0, \qquad (49)$$

where

$$\underset{\sim}{v}_{\perp j} = (\underset{\sim}{v}_E + \underset{\sim}{v}_P + \underset{\sim}{v}_B)_j, \quad j = e, i, \qquad (50)$$

with $\underset{\sim}{v}_E = \underset{\sim}{E}_\perp \times \underset{\sim}{B}_0 / B_0^2$, $\underset{\sim}{v}_{Pj} = (m/qB_0^2)_j \, (d\underset{\sim}{E}_\perp/dt)$ and $\underset{\sim}{v}_B = v_z \underset{\sim}{B}_\perp / B_0$. $f_j(\underset{\sim}{x}, v_z, t)$ is the drift distribution function and other notations are standard. Note here, $\frac{d}{dt}$ contains a convective term; i.e., $\frac{d}{dt} = \frac{\partial}{\partial t} + (\underset{\sim}{v} \cdot \underset{\sim}{\nabla})$.

Let $f_j = f_j^{(0)} + f_j^{(1)} + f_j^{NL}$ and using the two potentials φ and ψ defined in Eqs. (8) and (9), we obtain the linear response $f_j^{(1)}$ as

$$f_e^{(1)} = f_e^{(0)} \bar{\psi} \ , \tag{51}$$

and

$$f_i^{(1)} = - \bar{\lambda} \ f_i^{(0)} \varphi + c_s^2 \frac{\partial f_i^{(0)} / \partial v_z}{v_z - \omega / k_z} \bar{\psi} \quad , \tag{52}$$

where $\bar{\psi} = e\psi/T_c$, $\bar{\varphi} = e\varphi/T_c$ and $\bar{\lambda} = k_\perp^2 \, c_s^2/\omega_{ci}^2$. Equations (51) and (52) are valid for both kinetic Alfvén and ion acoustic waves. Note, however, since $(\omega_s, \underset{\sim}{k}_s)$ is an electrostatic mode, $\varphi_s = \psi_s$; while for the kinetic Alfvén modes, $(\omega_-, \underset{\sim}{k}_-)$ and $(\omega_0, \underset{\sim}{k}_0)$, $\varphi_{-,0} \neq \psi_{-,0}$.

In dealing with the nonlinear analyses, let us note that because the kinetic Alfvén waves have $\bar{\lambda} \sim O(1)$, our results are, therefore, valid in the regime $\bar{\lambda} \gg \omega_0/\omega_{ci}$; while the classic magnetohydrodynamic results of Sagdeev and Galeev[17] are valid in the opposite limit. The details of nonlinear interactions are different for the different modes as well as species and will be treated separately.

IV. A. Ion Acoustic $(\omega_s, \underset{\sim}{k}_s)$ Mode

Since we are interested in either the resonant decay to the ion acoustic mode or the induced scattering decay when this mode is heavily damped, only nonlinear perturbations up to $O(|\varphi_0|)$ need to be kept here; i.e., we only have to calculate $f_j^{(2)}$. For the electrons, $|\omega_s|$, $|\omega_0| \ll \omega_{ce}$ and $|k_\perp \rho_e| \ll |k_\perp \rho_i| < 1$, $\underset{\sim}{v}_{pe}$, thus, can be neglected. The dominant nonlinear contributions to $f_e^{(2)}$ is then found to originate from, referring to Eq. (49), the $\underset{\sim}{\nabla}_\perp \cdot [\underset{\sim}{v}_B(\underset{\sim}{\Omega}_0) f_e^{(1)}(\underset{\sim}{\Omega}_-) + (0 \rightleftarrows -)]$ term as well as the $(q/m)_e \{[\underset{\sim}{v}_E(\underset{\sim}{\Omega}_0) \times \underset{\sim}{B}_\perp(\underset{\sim}{\Omega}_-)] \cdot \underset{\sim}{e}_z + (0 \rightleftarrows -)\} (\partial f_e^{(0)}/\partial v_z)$ term. Here, we have denoted $\underset{\sim}{\Omega}_0 \equiv (\omega_0, \underset{\sim}{k}_0)$, $\underset{\sim}{\Omega}_0 \equiv (\omega_-, \underset{\sim}{k}_-)$ and $\underset{\sim}{\Omega}_s \equiv (\omega_s, \underset{\sim}{k}_s)$. With $|\omega_s| \ll |k_{zs} v_{Te}|$, $f_e^{(2)}(\underset{\sim}{\Omega}_s)$ is given by

$$f_e^{(2)}(\underset{\sim}{\Omega}_s) \simeq f_e^{(0)}[\bar{\psi}_B(\underset{\sim}{\Omega}_s) + \bar{\psi}_E(\underset{\sim}{\Omega}_s)], \qquad (53)$$

where $\bar{\psi}_B$ corresponds to the $\underset{\sim}{v}_B f_e^{(1)}$ term;

$$\bar{\psi}_B(\underset{\sim}{\Omega}_s) = \frac{ic_s^2}{2\omega_{ci} k_{\parallel s}} (\underset{\sim}{k}_s \times \underset{\sim}{k}_0) \cdot \underset{\sim}{e}_z \left[\frac{k_{\parallel 0} \bar{\lambda}_- (1+\lambda_0)}{\omega_0} - \frac{k_{\parallel -} \bar{\lambda}_0 (1+\bar{\lambda}_-)}{\omega_-} \right] \bar{\varphi}_- \bar{\varphi}_0 , \qquad (54)$$

and $\bar{\psi}_E$ corresponds to the $\underset{\sim}{v}_E \times \underset{\sim}{B}_\perp$ term;

$$\bar{\psi}_E(\underset{\sim}{\Omega}_s) = \frac{ic_s^2}{2\omega_{ci}k_{\parallel s}} (\underset{\sim}{k}_s \times \underset{\sim}{k}_0)\cdot \underset{\sim}{e}_z \left[\frac{k_{\parallel 0}(1+\bar{\lambda}_0)}{\omega_0} - \frac{k_{\parallel -}(1+\bar{\lambda}_-)}{\omega_-} \right] \bar{\varphi}_- \bar{\varphi}_0 . \tag{55}$$

In deriving Eqs. (54) and (55), we have made use of the fact that $\underset{\sim}{\Omega}_0$ and $\underset{\sim}{\Omega}_-$ are kinetic Alfvén waves and, hence, ψ (or E_z) and $\underset{\sim}{B}_\perp$ are related to φ by Eq. (39).

In treating the ions, we note first that because $|\omega_-|$, $|\omega_0| >> | k_{\parallel 0}v_{Ti}|$, $|k_{\parallel -}v_{Ti}|$, $\underset{\sim}{v}_B$ has negligible contribution and, from Eq. (52), $f_i^{(1)}$ of the kinetic Alfvén waves is reduced to

$$f_i^{(1)}(\underset{\sim}{\Omega}) = - \bar{\lambda} f_i^{(0)} \bar{\varphi}(\underset{\sim}{\Omega}) + \frac{k_z v_z}{\omega} f_i^{(0)} \frac{T_e}{T_i} \bar{\psi}(\underset{\sim}{\Omega})$$
$$\equiv f_{i\perp}^{(1)} + f_{iz}^{(1)} , \quad \text{for } \underset{\sim}{\Omega} = \underset{\sim}{\Omega}_0,\ \underset{\sim}{\Omega}_- . \tag{56}$$

Secondly, we note that $\underset{\sim}{v}_{pi}$ contains a nonlinear term from its convective part; i.e., $\underset{\sim}{v}_{pi}^{(2)} = (m/qB_0^2)_i (\underset{\sim}{v}\cdot\underset{\sim}{\nabla})_i \underset{\sim}{E}_\perp$. However, it can be shown that the contribution to the $\underset{\sim}{\nabla}_\perp \cdot [\underset{\sim}{v}_\perp f]_i$ term from $\underset{\sim}{v}_{pi}^{(2)} f_{i\perp}^{(0)}$ cancels to the order ω_0/ω_{ci} with that from $\underset{\sim}{v}_E f_{i\perp}^{(1)}$. Thus, the only net contribution comes from the $\underset{\sim}{v}_E f_{iz}^{(1)}$ term; which corresponds to the usual $(\underset{\sim}{v}_E \cdot \underset{\sim}{\nabla}) v_z$

convective nonlinear (ponderomotive) force term in the parallel (to $\underset{\sim}{B}_0$) equation of motion. Another important nonlinear contribution comes from the $(\underset{\sim}{v}_E \times \underset{\sim}{B}_\perp)\cdot\underset{\sim}{e}_z$ term, similar to that of the electrons. Combining these nonlinear terms, we obtain

$$f_i^{(2)}(\underset{\sim}{\Omega}_s) \simeq c_s^2 \frac{\partial f_i^{(0)}/\partial v_z}{v_z - \omega_s/k_{zs}} [\bar{\Psi}_c(\underset{\sim}{\Omega}_s) + \bar{\Psi}_E(\underset{\sim}{\Omega}_s)], \qquad (57)$$

where $\bar{\Psi}_c$ corresponds to the $\underset{\sim}{\nabla}_\perp\cdot[\underset{\sim}{v}_E f_{iz}^{(1)}]$ nonlinearity and is given by

$$\bar{\Psi}_c(\underset{\sim}{\Omega}_s) = - i \frac{c_s^2}{2\omega_{ci}k_{zs}} (\underset{\sim}{k}_s \times \underset{\sim}{k}_0)\cdot\underset{\sim}{e}_z \left(\frac{k_{z0}\bar{\lambda}_0}{\omega_0} - \frac{k_{z-}\bar{\lambda}_-}{\omega_-} \right) \bar{\varphi}_- \bar{\varphi}_0 ,$$

and $\bar{\Psi}_E(\underset{\sim}{\Omega}_s)$ is given in Eq. (55). Substituting Eqs. (51), (52), (53) and (57) into the quasi-neutrality equation

$$\left[n_e^{(1)}(\underset{\sim}{\Omega}_s) + n_e^{(2)}(\underset{\sim}{\Omega}_s)\right] = \left[n_i^{(1)}(\underset{\sim}{\Omega}_s) + n_i^{(2)}(\underset{\sim}{\Omega}_s)\right], \qquad (59)$$

we have

$$\varepsilon_s(\underset{\sim}{\Omega}_s)\bar{\varphi}_s = \Lambda_s[F(\bar{\lambda}) + \varepsilon_s(\underset{\sim}{\Omega}_s)]\bar{\varphi}_0\bar{\varphi}_- , \qquad (60)$$

where

$$\varepsilon_s = 1 + \bar{\lambda}_s + \chi_i , \qquad (61)$$

$$\chi_i = - c_s^2 \int \frac{\partial f_i^{(0)}/\partial v_z}{v_z - \omega_s/k_{zs}} dv_z, \tag{62}$$

$$\Lambda_s = \frac{\Lambda}{k_{zs}} \left(\frac{k_{z0}}{\omega_0} - \frac{k_{z-}}{\omega_-} \right) \simeq \frac{\Lambda}{\omega_0} , \tag{63}$$

$$\Lambda = - i\, c_s^2 \frac{(\underset{\sim}{k}_s \times \underset{\sim}{k}_0) \cdot \underset{\sim}{e}_z}{2\omega_{ci}} , \tag{64}$$

and

$$F(\bar{\lambda}) = \bar{\lambda}_0 + \bar{\lambda}_- + \bar{\lambda}_0 \bar{\lambda}_- - \bar{\lambda}_s ; \tag{65}$$

Equation (60) describes the coupling of the ion acoustic mode to the lower-sideband kinetic Alfvén wave via the pump.

IV. B. Kinetic Alfvén (ω_-, $\underset{\sim}{k}_-$) Mode

For this mode we have to calculate both the charge and parallel current density perturbations to $O(|\bar{\varphi}_0|^2)$; i.e., $f_j^{(3)}(\underset{\sim}{\Omega}_-)$ in order to take into account the induced scattering process.

Thus, nonlinear contributions due to both $f_j^{(1)}(\underset{\sim}{\Omega}_s)$ and $f_j^{(2)}(\underset{\sim}{\Omega}_s)$ must be included. Let us first consider the electrons. Again, $\underset{\sim}{v}_{pe}$ has negligible contribution and dominant nonlinear contributions come from the $\underset{\sim}{v}_B^*(\underset{\sim}{\Omega}_0)[f_e^{(1)}(\underset{\sim}{\Omega}_s) + f_{\underset{\sim}{e}}^{(2)}(\underset{\sim}{\Omega}_s)]$ term as well as the

$[\underset{\sim}{v}_E(\underset{\sim}{\Omega}_s) \times \underset{\sim}{B}^*_\perp(\underset{\sim}{\Omega}_0)]\cdot\underset{\sim}{e}_z$ term. It turns out, however, that $n_e^{(2)}$ is negligible compared to $n_i^{(2)}$ due to the canceling of dominant nonlinear terms; while $n_e^{(3)}$ is given by

$$n_e^{(3)}(\underset{\sim}{\Omega}_-) \simeq ic_s^2 \frac{(1+\bar{\lambda}_0)k_{z0}}{2\omega_{ci}k_{z-}-\omega_0} (\underset{\sim}{k}_s\times\underset{\sim}{k}_0)\cdot\underset{\sim}{e}_z\bar{\varphi}_0^* \, n_e^{(2)}(\underset{\sim}{\Omega}_s). \qquad (66)$$

Perturbations in parallel current density can then be obtained from the continuity equation;

$$J_{ze}^{(2)}(\underset{\sim}{\Omega}_-) \simeq e\underset{\sim}{k}_{\perp -}\cdot[\underset{\sim}{v}_E(\underset{\sim}{\Omega}_s)n^*(\underset{\sim}{\Omega}_0) + \underset{\sim}{v}_E^*(\underset{\sim}{\Omega}_0)n^{(1)}(\underset{\sim}{\Omega}_s)]_e/k_{z-} , \qquad (67)$$

and

$$J_{ze}^{(3)}(\underset{\sim}{\Omega}_-) \simeq e[\underset{\sim}{k}_{\perp -}\cdot\underset{\sim}{v}_E^*(\underset{\sim}{\Omega}_0)n^{(2)}(\underset{\sim}{\Omega}_s) - \omega_- n^{(3)}(\underset{\sim}{\Omega}_-)]_e/k_{z-} . \qquad (68)$$

As to the ions, because $|\omega_-| \gg |k_{z-}v_{Ti}|$, the dynamic is mainly in the perpendicular (to $\underset{\sim}{B}_0$) plane. Thus, we can neglect J_{zi} and, from the continuity equation, we find the ion density perturbations to be

$$n_i^{(2)}(\underset{\sim}{\Omega}_-) \simeq \underset{\sim}{k}_{\perp -}\cdot[N_0\underset{\sim}{v}_p^{(2)} + \underset{\sim}{v}_E(\underset{\sim}{\Omega}_s)n^*(\underset{\sim}{\Omega}_0) + \underset{\sim}{v}_E^*(\underset{\sim}{\Omega}_0)\, n^{(1)}(\underset{\sim}{\Omega}_s)]_i/\omega_- , \qquad (69)$$

and

$$n_i^{(3)}(\underset{\sim}{\Omega}_-) \simeq \underset{\sim}{k}_{\perp -} \cdot [\underset{\sim}{v}_E^*(\underset{\sim}{\Omega}_0) n^{(2)}(\underset{\sim}{\Omega}_s)]_i / \omega_- \; . \tag{70}$$

Here, as discussed in Sec. (IV. A), $v_{pi}^{(2)}(\underset{\sim}{\Omega}_-)$ is the non-linear ion polarization drift due to the convective $(\underset{\sim}{v}_\perp \cdot \underset{\sim}{\nabla}_\perp)$ term.

The two field equations, Eqs. (10') and (12), including nonlinear perturbations become at $\underset{\sim}{\Omega} = \underset{\sim}{\Omega}_-$

$$n_e^{(1)} + n_e^{(2)} + n_e^{(3)} = n_i^{(1)} + n_i^{(2)} + n_i^{(3)} \; , \tag{71}$$

$$\frac{\partial}{\partial z} \nabla_\perp^2 (\varphi - \psi) = \mu_0 \sum_{j=e,i} \frac{\partial}{\partial t} \left(J_{zj}^{(1)} + J_{zj}^{(2)} + J_{zj}^{(3)} \right) , \tag{72}$$

which, after substituting the linear responses Eqs. (16') to (19'), reduce to

$$\varepsilon_A(\underset{\sim}{\Omega}_-)\bar{\varphi}_- = \frac{1}{N_0 \omega_- \bar{\lambda}_- (1+\bar{\lambda}_-)} \Big[\omega_- (n_-^{(2)} + n_-^{(3)}) - (1+\bar{\lambda}_-) k_{z-} \left(J_{z-}^{(2)} + J_{z-}^{(3)} \right) / e \Big] , \tag{73}$$

where

$$\varepsilon_A(\underset{\sim}{\Omega}_-) = 1 - \frac{k_{z-}^2 v_A^2}{\omega_-^2} \quad (1+\bar{\lambda}_-), \tag{74}$$

$$n_-^{(2),(3)} = [n_i(\underset{\sim}{\Omega}_-) - n_e(\underset{\sim}{\Omega}_-)]^{(2),(3)} , \tag{75}$$

and

$$J_{z-}^{(2),(3)} \cong J_{ze}^{(2),(3)}(\underset{\sim}{\Omega}_-) \quad . \tag{76}$$

In deriving Eq. (73), we have noted that $\underset{\sim}{\Omega}_-$ is a resonant kinetic Alfvén mode and, hence, $|\varepsilon_A| << 1$. Substituting Eqs. (66), (67), (68), (69) and (70) for the nonlinear charge density and parallel current density perturbations into Eq. (73), it reduces to

$$\tilde{\varepsilon}_A(\underset{\sim}{\Omega}_-)\bar{\varphi}_- = \Lambda_A F(\bar{\lambda})\bar{\varphi}_0^* \; \bar{\varphi}_s \; , \tag{77}$$

where

$$\tilde{\varepsilon}_A = \varepsilon_A - \varepsilon_A^{(3)} \; , \tag{78}$$

$\varepsilon_A^{(3)}$ is due to third order perturbations

$$\varepsilon_A^{(3)} = |\bar{\varphi}_0|^2 \; |\Lambda|^2 \; \frac{(1+\bar{\lambda}_0)}{k_{zs}\omega_-} \left[1 - \frac{k_{z0}\omega_-(1+\bar{\lambda}_0)}{k_{z-}\omega_0} \right] \cdot \left(\frac{k_{z0}}{\omega_0} - \frac{k_{z-}}{\omega_-} \right), \tag{79}$$

and

$$\Lambda_A = \Lambda/[\omega_-\bar{\lambda}_-(1+\bar{\lambda}_-)] \simeq -\Lambda/[\omega_0\bar{\lambda}_-(1+\bar{\lambda}_-)]. \tag{80}$$

Combining the two coupled equations, Eqs. (60) and (77), we then derive the following dispersion relation for the parametric decay instabilities

$$\left(\varepsilon_A - \tilde{\varepsilon}_A^{(3)}\right)\varepsilon_s = \Lambda_A\Lambda_s F^2(\bar{\lambda})|\bar{\varphi}_0|^2 , \tag{81}$$

where

$$\tilde{\varepsilon}_A^{(3)} = \varepsilon_A^{(3)} + \Lambda_A\Lambda_s F(\bar{\lambda})|\bar{\varphi}_0|^2 . \tag{82}$$

With $T_e >> T_i$, the acoustic wave is weakly damped and we have the resonant decay instability. In this case, $\varepsilon_A^{(3)}$ can be neglected. Let $\omega_s = \omega_{sr} + i\gamma$ and $\omega_- = -\omega_A + i\gamma$, where ω_{sr} and $\omega_A = \omega_0 - \omega_{sr}$ satisfy, respectively, the dispersion relations for the ion acoustic and kinetic Alfvén waves, Eqs. (38) and (37). Equation (81) then reduces to

$$\frac{\partial\varepsilon_{sr}}{\partial\omega_{sr}}\frac{\partial\varepsilon_{Ar}}{\partial\omega_A}(\gamma+\Gamma_A)(\gamma+\Gamma_s) = \left|\frac{\Lambda}{\omega_0}\right|^2 \frac{F^2(\bar{\lambda})|\bar{\varphi}_0|^2}{\bar{\lambda}_-(1+\bar{\lambda}_-)} , \tag{83}$$

$$\frac{\partial\varepsilon_{sr}}{\partial\omega_{sr}} = \frac{2(1+\bar{\lambda}_s)}{\omega_{sr}} , \tag{84}$$

$$\frac{\partial \varepsilon_{Ar}}{\partial \omega_A} = \frac{2}{\omega_A} , \tag{85}$$

and Γ_A, Γ_s are the corresponding linear damping rates. From Eq. (83), we can deduce the threshold pump field by letting γ=0. Well above the threshold field, the growth rate is given by

$$\gamma_D \simeq \frac{\omega_{ci}}{4} \left(\frac{\omega_{sr}}{\omega_A} \right)^{1/2} \left|\frac{B_{\perp 0}}{B_0}\right| \beta^{-1/2} \frac{|F(\bar{\lambda})\sin\theta|}{[(1+\bar{\lambda}_0)(1+\bar{\lambda}_-)(1+\bar{\lambda}_s)]^{1/2}} . \tag{86}$$

In deriving Eq. (86), we have let $(\underset{\sim}{k}_s \times \underset{\sim}{k}_0)\cdot \underset{\sim}{e}_z = (\underset{\sim}{k}_- \times \underset{\sim}{k}_0)\cdot \underset{\sim}{e}_z$ $= k_{\perp -} k_{\perp 0} \sin\theta$ and used the relation between $B_{\perp 0}$ and $\bar{\varphi}_0 (= e\varphi_0/T_e)$ expressed in Eq. (39). The growth rate obtained here is larger than that of the ideal magnetohydrodynamic results due to Sagdeev and Galeev[17] by a factor of $\bar{\lambda}\omega_{ci}/\omega_A$. This enhancement is expected because, due to the finite $\bar{\lambda}$'s, nonlinear effects induced by the $\underset{\sim}{E} \times \underset{\sim}{B}_0$ drifts of electrons and ions do not cancel each other to the order of $\bar{\lambda}$ in the case considered here; while only ion polarization drift (which is smaller than the $\underset{\sim}{E} \times \underset{\sim}{B}_0$ drift by a factor $\frac{\omega_A}{\omega_{ci}}$) contributes in the ideal magnetohydrodynamic limit. Furthermore, unlike the magnetohydrodynamic case in which only the backscattering is allowed, three different types of decay are possible here as illustrated in the ω-k_z diagram in Fig. 2. Note also since $F(\bar{\lambda}) \to 0$ as $\bar{\lambda}_0 \to 0$, this decay process is pertinent to the pump wave being a kinetic Alfvén wave.

Let us now consider the case with $T_e \lesssim 5T_i$ such that the ion acoustic wave is heavily Landau damped by ions. In this case, the decay instability is made through non-linear ion Landau damping. i.e., it is an ion-induced scattering process or, sometimes also called, a quasimode decay instability. From Eq. (81), the growth rate γ_N is obtained to be

$$\gamma_N = -\frac{\omega_A}{2} \,\mathrm{Im}\left[\frac{\Lambda_A\Lambda_s F^2(\bar{\lambda})\,|\bar{\varphi}_0|^2}{\varepsilon_s} + \tilde{\varepsilon}^{(3)}_A\right] - \Gamma_A \,. \quad (87)$$

Note, from Eq. (82), $\tilde{\varepsilon}^{(3)}_A$ does not contribute to the growth rate and γ_N further reduces to

$$\gamma_N = \frac{\omega_{ci}}{8}\,\beta^{-1}\left|\frac{B_{\perp 0}}{B_0}\right|^2\left(\frac{\omega_{ci}}{\omega_A}\right)\frac{F^2(\bar{\lambda})\sin^2\theta}{(1+\bar{\lambda}_0)(1+\bar{\lambda}_-)}\,\frac{\mathrm{Im}\chi_i}{|\varepsilon_s|^2} - \Gamma_A \,, \quad (88)$$

which has its maximum value at $|\omega_s| \simeq |k_{zs}v_{Ti}|$ and

$$(\gamma_N)_{max} \simeq \frac{\omega_{ci}}{8} \beta^{-1} \left| \frac{B_{\perp 0}}{B_0} \right|^2 \left(\frac{\omega_{ci}}{\omega_A} \right) \frac{F^2(\bar{\lambda}) \sin^2\theta}{(1+\bar{\lambda}_0)(1+\bar{\lambda}_-)} \frac{T_e/T_i}{(1+\bar{\lambda}_s)^2 + (T_e/T_i)^2} - \Gamma_A . \tag{89}$$

Again, we remark that (i) the growth rate obtained here is larger than the classic magnetohydrodynamic value by a factor $(\bar{\lambda}\omega_{ci}/\omega_A)^2$, (ii) our results are pertinent to the pump wave being a kinetic Alfvén wave and (iii), similar to the resonant decay instability, three types of decay are possible here (c.f. Fig. 2). Note that the threshold pump field depends on Γ_A which, in the collisionless regime, is mainly due to electron Landau damping and, typically, $\Gamma_A/\omega_{ci} \sim O(10^{-2})$. Then for a reasonable choice of other parameters; such as $\beta \sim 10^{-2}$, $\omega_{ci}/\omega_A \sim 10$ and $T_e \sim T_i$, the threshold amplitude of $B_{\perp 0}$, $(B_{\perp 0})_{th}/B_0 \sim 10^{-2}$. Now, as discussed in Section II and shown in Eq. (30), with the pump wave being excited by the resonant mode conversion, its amplitude is enhanced by an Airy factor $(\kappa\rho)^{-2/3}$ compared with the externally applied field. Hence, even with an applied field of several tens of gauss, the kinetic Alfvén wave will have an amplitude of several hundred gauss which can exceed the threshold value with B_0 in the order of 10 K gauss.

Based on these findings, we can make some important remarks on nonlinear surface heating problems. For a realistic application of a RF heating concept to a reactor plasma, one of the basic question is the power penetration into the core of the plasma. When the externally applied RF field has a large amplitude, nonlinear interactions at the plasma surface tends to heat only the surface area and inhibit the power penetration. This problem will not arise in the present heating scheme because (1) the amplitude of the externally applied RF field is much smaller [by a factor of $(\kappa\rho_i)^{1/2}$] than that of the kinetic Alfvén wave, and (2) the nonlinear coupling coefficient of the external field to the ion acoustic wave is also much smaller (by a factor of ω_0/ω_{ci}) than that of the kinetic Alfvén wave.

Finally, we discuss the effects of finite ion Larmor radius; which so far have been neglected to simplify the analyses. As is well known, finite ion Larmor radius has the effect of reducing the field experienced by the ions by a factor $I_0(\lambda_i)e^{-\lambda_i} \simeq 1 - \lambda_i$ for $\lambda_i = k_\perp^2 \rho_i^2 < 1$. Thus, the $\underset{\sim}{E}\times\underset{\sim}{B}_0$ currents of electrons and ions do not cancel each other to the order of λ_i. It then can be shown that the only modification of finite λ_i to the results obtained here is that there are additional terms of $O(\lambda_i)$ in the expression of $F(\overline{\lambda})$ and the main features of our results remain unchanged.

Effects of orbit diffusion become important when the spectrum developes into a wide range. They contribute to an additional dissipation mechanism. These effects will be discussed in a separate paper.

V. CONCLUDING REMARKS

We have shown that an externally applied oscillating magnetic field can be absorbed by a non-uniform plasma at the Alfvén resonant surface. The absorption is due to a linear mode conversion of the applied oscillating magnetic field to the kinetic Alfvén wave. In a toroidal plasma a kinetic Alfvén wave propagates almost parallel to the magnetic field and into a higher density side spiraling along the toroidal direction. Plasma is heated when this kinetic Alfvén wave is dissipated by various physical processes. We have shown that the wave can be dissipated both linearly and nonlinearly by plasma particles. Both collisional and collisionless damping can be considered as the dissipation mechanism. When the plasma temperature is low, typically below a few hundred electron volts, collisional damping takes place. Collisional damping by ions originates from the ion viscosity and perpendicular field energy is dissipated. This will lead to an increase of perpendicular ion temperature. The viscous damping is maximized here because the perpendicular wavelength of the kinetic Alfvén wave is comparable to the ion gyroradius. Collisional damping by electrons is due to

Ohmic dissipation of parallel component of the wave electric field. Electrons are heated in the parallel direction by this mechanism. We have found that, in the collisional regime, the heating rate of electrons and ion are approximately the same.

In a collisionless regime the linear heating occurs due to Landau damping of the parallel component of the electric field by both electrons and ions. When β is smaller than 0.1, electron Landau damping dominates. However, when β is larger than 0.1, ion Landau damping dominates. Electron Landau damping is reduced due to the trapped electron effects, however, the damping rate due to the circulating electron is large enough for the wave to be dissipated in the plasma.

If the applied oscillating magnetic field has an intensity larger than a few tens of gauss, nonlinear dissipation of the kinetic Alfvén wave is expected. Because of the spatial resonance effect, the externally applied magnetic field is enhanced near the resonant point and the kinetic Alfvén wave, which is excited by the mode conversion at the resonant point, carries this enhanced amplitude and propagates into the plasma with slowly decreasing amplitude. Away from the resonant point, the magnetic field amplitude of the kinetic Alfvén wave is given approximately by $(\kappa\rho_i)^{-1/2}$ times the externally applied magnetic field amplitude. This enhancement makes the nonlinear process to occur more easily. The nonlinear process we have considered is a parametric decay of the kinetic Alfvén wave into the ion acoustic wave. It was shown that when the electron temperature is much larger than the ion temperature, the resonant three wave decay can take place, while if the electron temperature is comparable to the ion temperature, nonlinear ion Landau damping can occur. In either cases it is expected that both electrons and ions are heated approximately at an equal rate in this nonlinear process. Here, electrons are heated by the linear Landau damping of the kinetic Alfvén wave and ions are heated by linear and nonlinear Landau damping of the excited ion acoustic wave and the kinetic Alfvén wave respectively. In the derivation of the nonlinear coupling coefficient, we have found that the parametric decay is possible only with a pump with a finite perpendicular wavelength for a reasonable range of plasma parameters and amplitude of the pump. This finding is very important in the application to plasma heating because it indicates that undesirable surface heating by a direct nonlinear coupling between the applied field and plasma is inhibited.

In reality if the damping rate of the kinetic Alfvén wave is large enough so that the kinetic Alfvén wave completely dissipates before it reaches to another resonant point at the other side of the plasma, the absorption rate which is obtained from the magnetohydrodynamic calculation gives the correct energy disposition rate. Because the wavelength of the kinetic Alfvén wave is on the order of the ion gyroradius, this means if the fractional damping rate per cycle of the wave is larger than the ratio of the ion gyroradius to the minor radius of the plasma, ρ_i/r, a total absorption of the kinetic Alfvén wave takes place. For a reactor plasma ρ_i is approximately 10^{-3} m while r is on the order of 1 m hence the fractional damping rate needs to be larger than 10^{-3}. This condition is easily satisfied by taking any of the dissipation mechanism considered here. However, to obtain a uniform heating it is desirable to have the fractional damping rate not to be too large and most desirably on the order of 10^{-2}.

Because the resonant absorption can absorb the oscillating magnetic field totally per each cycle of oscillations, Q value of the plasma can be on the order of unity. Therefore power input to the plasma per unit volume can be simply given approximately by $\omega B_s^2/2\mu_0$. This means, to provide with 200 MW power at $f = \omega/2\pi = 1$ MHz to a plasma with a volume of 2×10^3 m^3, one only needs an oscillating magnetic field with amplitude of a few gauss.

This magnetic field, when it is mode converted to the kinetic Alfvén wave, will have an amplitude of a few hundred gauss. However, because it has a very short wavelength and because the magnitude is still much lower than any of the dc magnetic field applied to a reactor plasma, it is considered that the plasma is not significantly disturbed by this field.

This heating method has a basic merit of using a low frequency RF field of on the order or lower than 1 MHz for which a high power source is currently available. This also presents a merit that it does not heat impurity ions by the cyclotron resonance. Most of the cyclotron resonant frequency of impurities lies above this frequency range.

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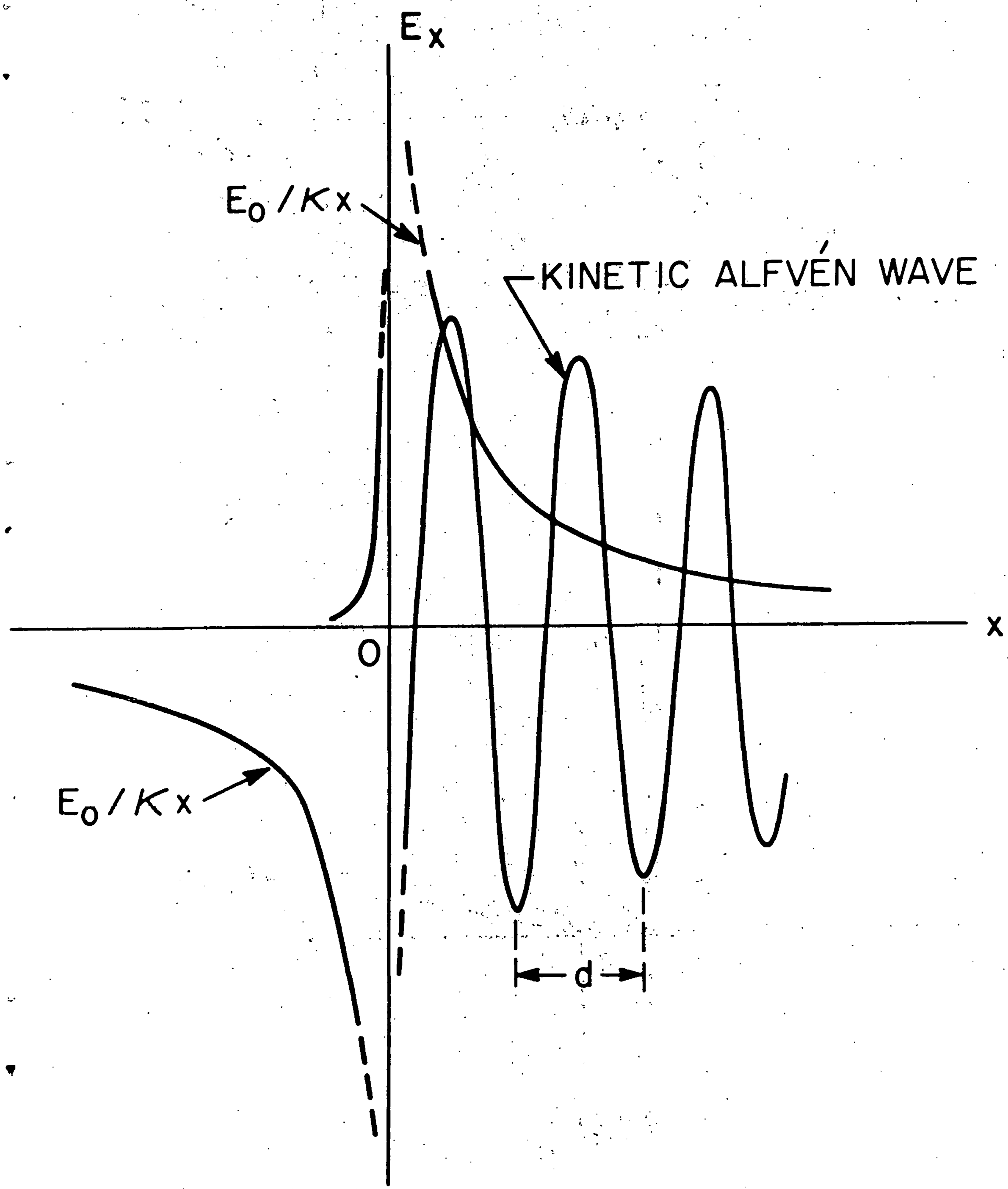
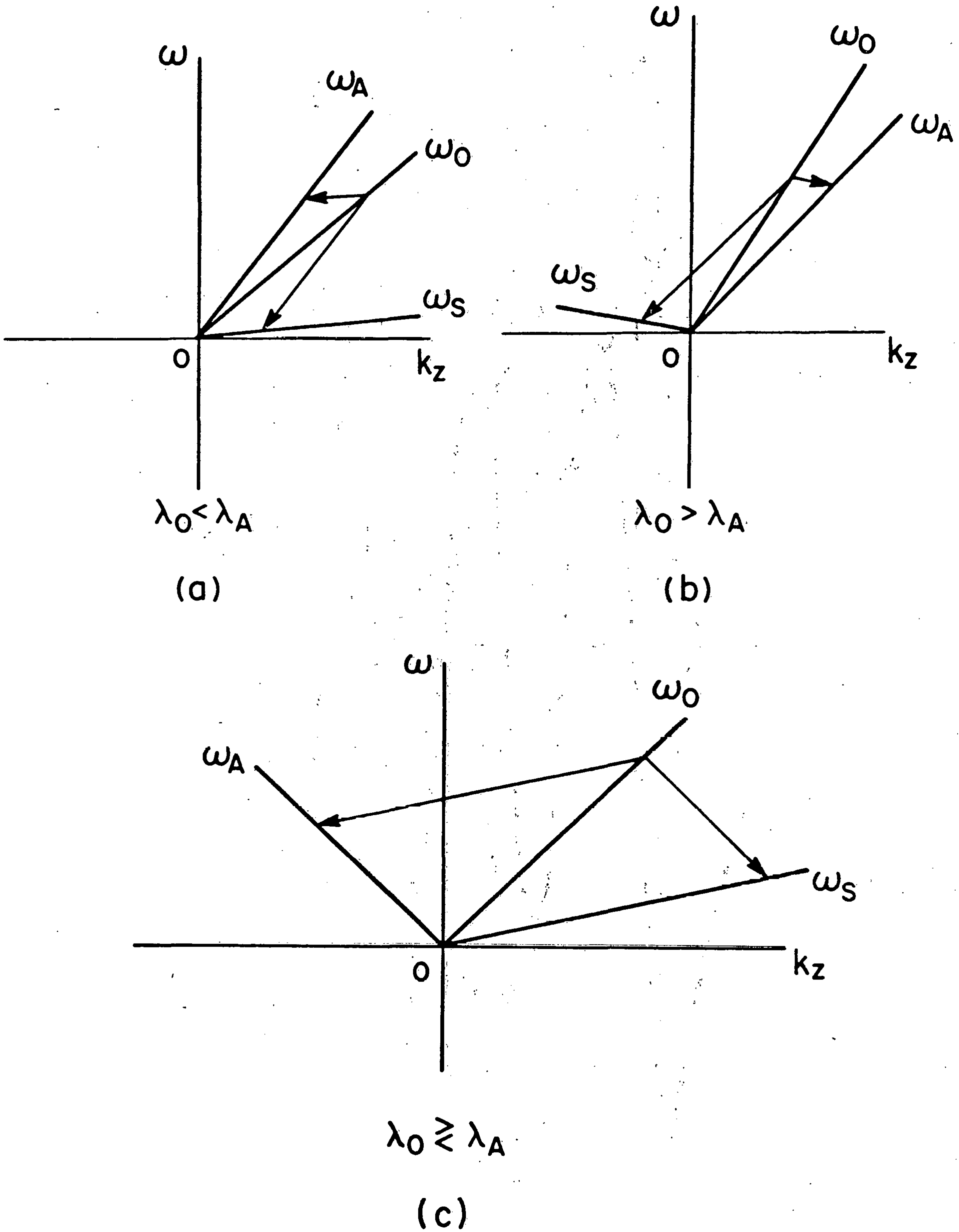
FIGURE CAPTIONS

Fig. 1 Schematic diagram of the x component (radial component) of the wave electric field near the spatial resonant point, $x = 0$. $x > 0$ region corresponds to the higher density side where the kinetic Alfvén wave is excited. The plasma heating occurs when the kinetic Alfvén wave is dissipated by wave particle interactions. The wavelength d near the resonant point is approximately given by $(3\pi)^{2/3}(\rho^2/\kappa)^{1/3} \sim 50\rho_i$.

Fig. 2 Three types of decay are possible from frequency ω_0 to ω_a depending on the sign of k_{zs} and k_{zA} as well as the size of $\lambda_0 (= k_{\perp 0}^2 c_s^2/\omega_{ci}^2)$ with respect to $\lambda_A (= k_{\perp A}^2 \; c_s^2/\omega_{ci}^z)$. In the magnetohydrodynamic limit where $\lambda_0 = 0$, only case (c) is acceptable. In case (b), $k_\perp$ is decreased while in case (b) $k_\perp$ is increased in consequence of the decay.



E_x
E_0 / Kx
KINETIC ALFVÉN WAVE
x
O
E_0 / Kx
d


ω
ω_A
ω_0
ω_S
0
k_z
$\lambda_0 < \lambda_A$
(a)
ω
ω_0
ω_A
ω_S
0
k_z
$\lambda_0 > \lambda_A$
(b)
ω
ω_0
ω_A
ω_S
0
k_z
$\lambda_0 \gtrless \lambda_A$
(c)