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Kinetic Scheme for Computational Aeroelastic Analysis of 2-D Airfoils in Transonic Flows

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Abstract

A new formulation based on a Kinetic scheme namely the Kinetic Flux Vector Splitting scheme on Moving Grids(KFMG) for solving Euler equations has been developed for unsteady flow computations around oscillating bodies. The necessary grid movement has been achieved using spring analogy method on structured grids. An integrated Aeroelastic code has been developed by coupling the KFMG Euler solver with a 2-DOF structural dynamics model of an airfoil. Results obtained for steady and unsteady flows problems are compared with literature data and are discussed in detail. The aeroelastic response characteristics for NACA 64A006 airfoil undergoing pitch and plunge motions have been obtained using the present aeroelastic code. The well known 'Transonic Dip' phenomenon has been captured for NACA 64A006 airfoil using the present Computational Aeroelastic code.

Nomenclature

U_∞	=	free stream velocity
h	=	plunge distance
α	=	pitch angle
x_α	=	distance between mass center and elastic axis
r_α	=	radius of gyration of airfoil mass about the elastic axis
ω_h	=	uncoupled plunging frequency of airfoil
ω_α	=	uncoupled pitching frequency of airfoil
U^*	=	non-dimensional flutter speed
k_c	=	reduced frequency based on chord length
μ	=	airfoil-to-air mass ratio
ζ_h	=	uncoupled mechanical damping parameter corresponding to plunging
ζ_α	=	uncoupled mechanical damping parameter corresponding to pitching

I. Introduction

SLENDER aerospace vehicles in general undergo oscillations in flight due to rapid manoeuvres, control surface deflections, jet pulses leading to changes in aerodynamic forces. These dynamic oscillations play a major role in moving the center of pressure and generating shock oscillations on wings, fins and core body by making the flow unsteady. Hence, the unsteady flow needs to be investigated even at the initial design stage. Setting up of experimental facility for simulating unsteady aerodynamic and aeroelastic behaviour poses additional problems of introducing complex mechanism to induce vibration or oscillation of the configuration. Also, the experiments can give only limited data and the accuracy of the data dependent on the type of instrumentation used. The progress in the numerical solution of steady flows has been closely correlated with the development of computing platforms having lower price-to-performance ratio as well as to the progress of very efficient numerical schemes. Therefore, it seems logical to make use of some of these improved techniques that are found successful in the solution of steady flows for analyzing time dependent flows. Study of unsteady aerodynamic characteristics of oscillating bodies involves development of more advanced CFD solvers coupled with the dynamics of the oscillating or deforming bodies termed as fluid-structure interaction problems. Therefore, the prediction of unsteady flows around oscillating or flexing bodies and coupled problems like fluid-structure interaction problems can be more effectively addressed by modern CFD tools.

Guruswamy et al[1] have initially developed a Potential code LTRAN2 for analyzing unsteady flow past oscillating bodies and then they have developed Euler and Navier-Stokes solvers with an appropriate structural model to deal with unsteady and aeroelastic analysis of complex 3-D geometries. Batina[2] has developed an Euler code based on van Leer scheme to predict the unsteady flow past 2-D and 3-D bodies. Ambrosi et al[3] have developed an Euler code using Roe's scheme to predict unsteady flow past oscillating airfoils.

In the present work, a new scheme developed by the authors called Kinetic Flux vector splitting scheme on Moving Grids (KFMG) which has been successfully verified against standard 2-D/3-D steady and unsteady flows has been used along with a 2-D structural dynamics model for computing aeroelastic behaviour of NACA 64A006 airfoil. This numerical experiment is for further demonstration of the ability of the new FKMG method in predicting flutter. In this case, a two-degree of freedom structural model of an airfoil undergoing pitch and plunge oscillations has been considered. An integrated CFD Euler code using KFMG scheme along with the structural dynamic model leading to a computational aeroelastic code has been developed. This aeroelastic code has been tested for a particular test case to obtain different stability modes namely, stable, neutrally stable and unstable conditions. Further, the code has been applied to predict the well known 'Transonic Dip' phenomenon which occurs when the flutter speed U^* reduces with the Mach number and reaches a minimum value at certain transonic Mach number resulting in a dip of the flutter boundary with a noticeable decrease in the flutter speed. The computational aeroelastic code has been applied to capture the Transonic Dip phenomenon of NACA 64A006 airfoil. Numerical results are presented for predicting force and moment coefficients for steady, unsteady flows on NACA 64A006 airfoil in transonic flows. Also, the aeroelastic behaviour of 2-DOF model with unsteady aerodynamic forces computed by using KFMG solver is presented for Mach numbers 0.7 to 0.86 in several increments.

II. Kinetic Flux Vector Splitting Scheme on Moving Grids (KFMG)

Different upwind schemes are in use to solve Euler equations and in the present study, Kinetic Flux Vector Splitting (KFVS) scheme has been chosen. The KFVS scheme essentially addresses Boltzmann equation of kinetic theory of gases and is transformed into Euler equations through a moment method strategy. Extensive studies are carried out using 2-D and 3-D formulations of KFVS scheme in different flow regimes[4,5]. The 1-D KFMG solver based on KFVS scheme developed by Kulkarni and Deshpande[6] revealed that the KFVS scheme has inherent simplicity, especially when applied to moving grid problems. The simplicity stems from the fact that the molecular velocity term present in the governing equations at the Boltzmann level is replaced by the term involving the difference between the molecular velocity and the grid velocity. The above aspect has been exploited in deriving the fluxes for moving grids. The details are presented in Ref.[7]. It is interesting to note that, the KFMG and KFVS split fluxes can be related to one another through a matrix containing the grid velocities and the grid velocity components w_1 and w_2 at each grid point during the (n+1) time step are calculated using the location of the point in n and (n+1) time steps. Because of this close relationship between KFVS and KFMG it is very easy to develop KFMG code by introducing minor modifications in a KFVS code.

The moving wall boundary has been treated using Kinetic Moving Boundary Condition (KMBC) and the fixed outer boundary has been treated using Kinetic Outer Boundary Condition (KOBC). These boundary conditions were derived at molecular level and by using moment method strategy the expressions for Euler level have been obtained. The KOBC is so general that, it can treat any type of outer/out flow boundary which may be subsonic / supersonic.

III. Dynamic Mesh Algorithm

Between two successive time steps, the body changes its orientation in case of oscillation and hence the grid has to be moved or regenerated around the deformed body. Regeneration of grid at each time step is not only laborious but is also computationally expensive. Hence, a moving grid strategy given by J.T.Batina[2] based on 'spring analogy' is implemented on structured grids to move the grid so as to get a good quality grid around the oscillating body. The mesh is moved in conformity with the instantaneous position of the body by modeling each edge of each quadrilateral by a spring. The spring stiffness for a given edge i-j is taken to be inversely proportional to the length of the edge. Grid points on the outer boundary are held fixed and the instantaneous locations of the points on the inner boundary are prescribed. At each time step, the static equilibrium equations in the x and y directions are solved iteratively at each interior node (i,j) of the grid for the displacements. The spring analogy method is used to obtain the grid around the new orientation of the oscillating airfoil at every time level.

IV. Computational Aeroelastic Code

A typical section having two degrees of freedom (2-DOF) involving pitch and plunge oscillations of an airfoil is considered here to derive the aeroelastic equations of motion. The definitions of variables and sign conventions are shown in Fig.1. It is assumed that the airfoil is rigid and the amplitude of oscillation is small. It is also assumed that there is no coupling in mechanical damping.

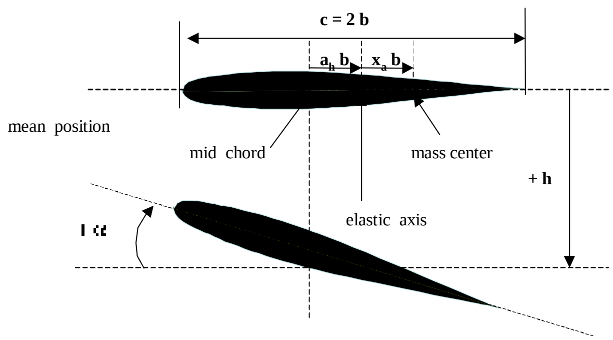

Figure 1 : Definition of parameters for a 2-DOF structural dynamics model

Considering the inertia forces, damping forces, elastic forces and aerodynamic forces, the equations of motion derived from energy considerations, for a two degrees of freedom as shown in Fig.1 are given here[8].

$$
\begin{aligned}
m_a \ddot{h} + S\,\ddot{\alpha} + C_h \dot{h} + K_h h &= Q_h \\
S\,\ddot{h} + I_\alpha \ddot{\alpha} + C_\alpha \dot{\alpha} + K_\alpha h &= Q_\alpha
\end{aligned}
\qquad \textbf{(1)}
$$

Where (.) represents the derivative with respect to time. Here, Q_h and Q_α are generalised air loads due to plunge and pitch motions respectively. The equations of motion given by Eq.(1) can be written in matrix form as,

$$[M]\{u''\} + [C]\{u'\} + [K]\{u\} = \{F\} \qquad \textbf{(2)}$$

where, u is the vector of displacements corresponding to the two degrees of freedom for the structure. Here, [M], [C] and [K] represent the mass, damping and stiffness matrices respectively and [F] is the vector of aerodynamic loads and these are given by,

$$[M]=\begin{bmatrix} 1 & x_\alpha \\ x_\alpha & r_\alpha^2 \end{bmatrix};\ [C]=\begin{bmatrix} \zeta_h & 0 \\ 0 & \zeta_\alpha \end{bmatrix};\ [K]=\left(\frac{2}{U^* k_c}\right)^2 \begin{bmatrix} \left[\dfrac{\omega_h}{\omega_\alpha}\right]^2 & 0 \\ 0 & r_\alpha^2 \end{bmatrix};$$

$$\{F\} = \frac{4}{\pi \mu k_c^2} \begin{Bmatrix} -C_L \\ 2C_m \end{Bmatrix} \ and\ \{u\} = \begin{Bmatrix} h \\ \alpha \end{Bmatrix}$$

Here, k_c is the reduced frequency, μ is the airfoil-to-air mass ratio, ξ is the non-dimensionalised plunging displacement and α is the pitch angle. The structural equations of motion are solved using the well known linear acceleration method. The KFMG based Euler solver has been coupled with this structural dynamics model to integrate both aerodynamic and structural equations simultaneously for obtaining the aeroelastic response. The solution procedure to solve the integrated fluid-structure interaction equations is described through a flow chart in Fig.2.

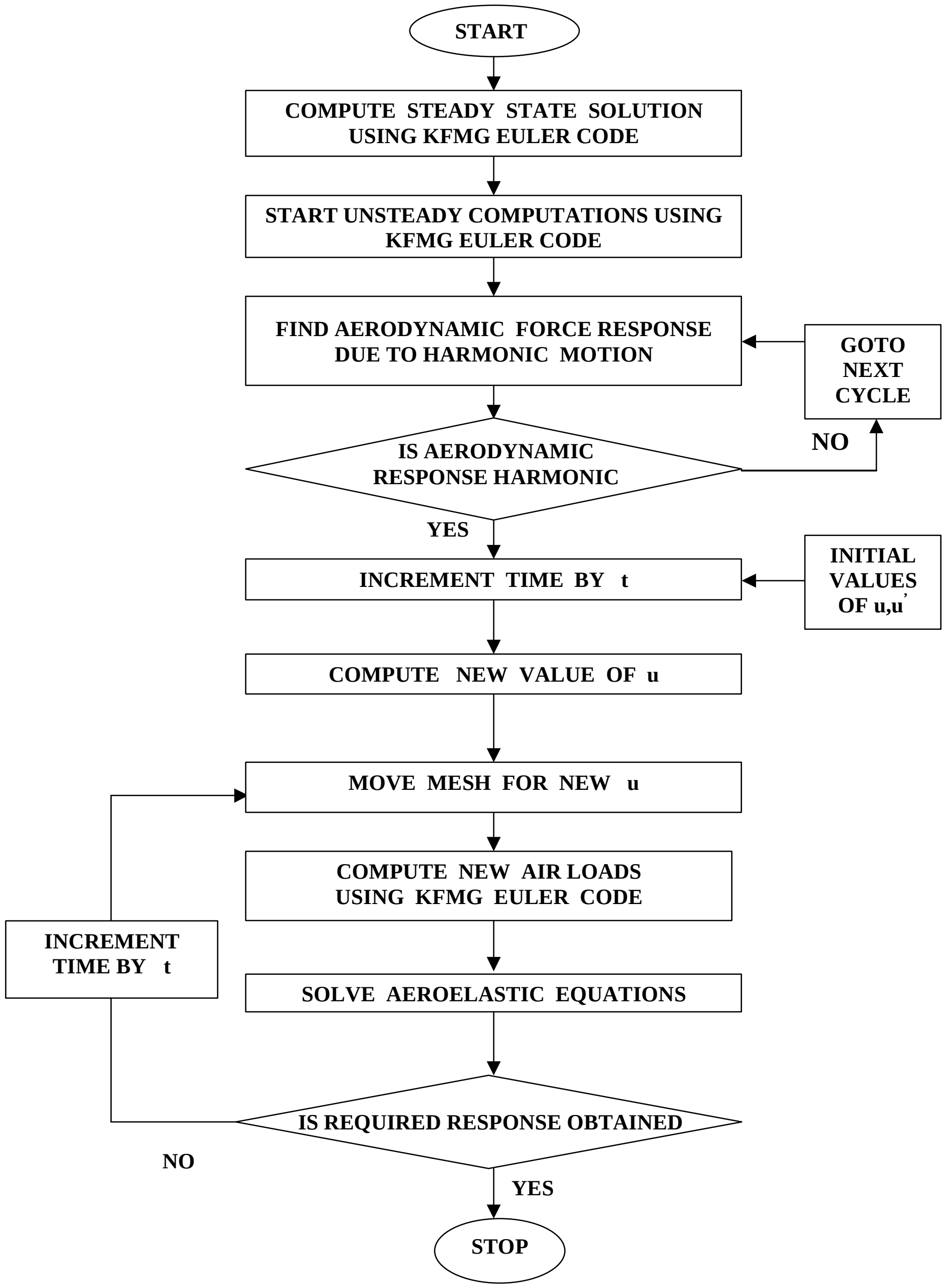

Figure 2 : Procedure for aeroelastic analysis

V. Results and Discussions

A. Dynamic mesh

The capabilities of the dynamic mesh algorithm are demonstrated by considering the grid around NACA 64A006 airfoil. The grid is generated by an elliptic grid generator ELGRID-TM based on Thomas-Middelcoff procedure. In this example, the airfoil is pitched about the quarter chord with amplitude of 5 degrees and the mesh at pitch angles of +5 deg. and -5 deg. have been obtained and are shown in Fig.3. The mesh moves smoothly as the airfoil pitches,

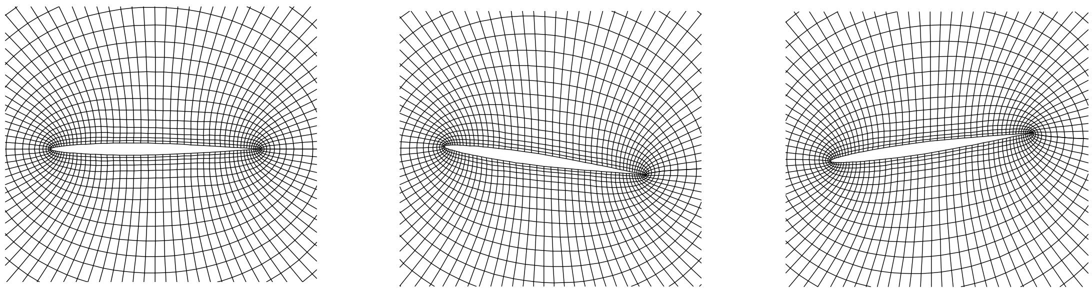

Figure 3 : 2-D Dynamic mesh algorithm showing the initial grid and the grids at different orientations of the 64A006 airfoil

and the procedure is completely general in the sense that it can treat any realistic oscillating airfoil motions.

B. Steady and Unsteady Flows

The formulations of KFVS based Euler solver on dynamically moving meshes, are demonstrated by performing calculations on NACA 64A006 airfoil with grid size 101 X 41. The outer boundary has been chosen as a circle with

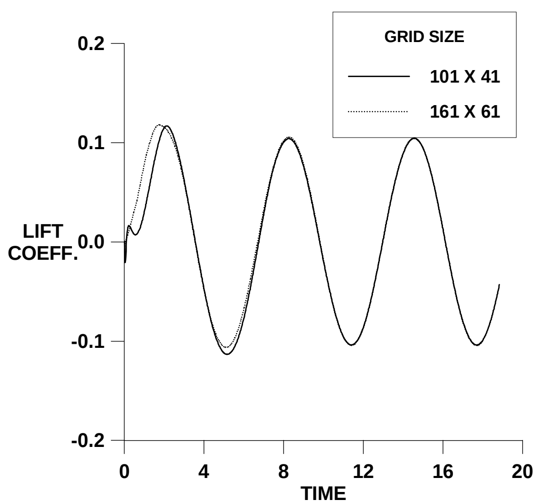

Figure 4 : Variation of unsteady lift coefficient with time - Grid refinement studies

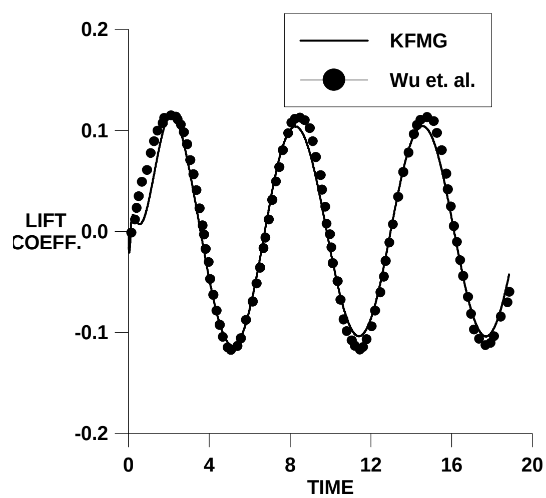

Figure 5 : Variation of unsteady lift coefficient with time – Comparison of results

radius equal to 20 times the chord of the airfoil. Initially, steady state calculations were performed on the airfoil at a free stream Mach number of $M_\infty = 0.85$ with zero angle of attack. Reconstruction of fluxes for obtaining space accuracy and fourth order Runge-Kutta method for time accuracy have been implemented in the solver. The steady flow results have been taken as the input data to perform unsteady calculations for the airfoil, which oscillates harmonically about the quarter chord point with amplitude of 0.01 rad. and a reduced frequency based on chord of 0.1 at Mach 0.85 and the mean angle of attack of 0^o. Two grid sizes namely, 101X41 and 161X61 have been considered for performing unsteady flow computations. The results obtained on these grids are plotted in Fig. 4 and it can be seen that the solution remains same on both the grids after first cycle of oscillation. The time evolution of the lift coefficient is plotted with non-dimensional time along with comparison[8] in Fig. 5 and it is found that the unsteady aerodynamic coefficients compare well with corresponding results available in literature.

C. Aeroelastic Analysis

At the end of the third cycle of forced sinusoidal oscillations, the airfoil was released and allowed to follow pitch and plunging motions dictated by the structural dynamic equations. The structural parameters used in the present study are : $\omega_h / \omega_\alpha = 0.2$, $a_h = -0.5$, $x_\alpha = 0.25$, $U^* = 5.5$, $k_c = 0.1$ and the same parameters have been used by Wu et al[8] in a similar study. The value of airfoil-to-air mass ratio (μ) has been parametrically varied from 110 to 150 during this phase of calculations to study the effect of the airfoil-to-air mass ratio on the flutter characteristics[9]. As shown in Figures 6 and 7, it was found that variations in the airfoil-to-air mass ratios can lead to stable damped oscillations, neutrally stable oscillations, or divergent (flutter) oscillations.

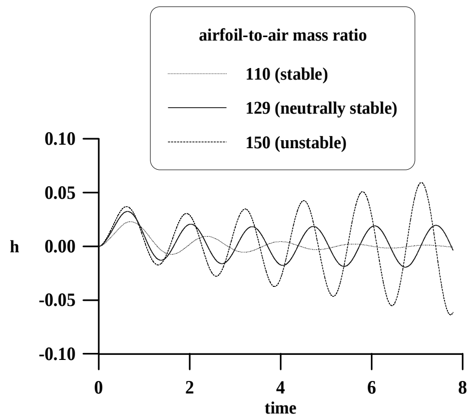

Figure 6 : Aeroelastic response characteristics – Variation of plunge distance with time

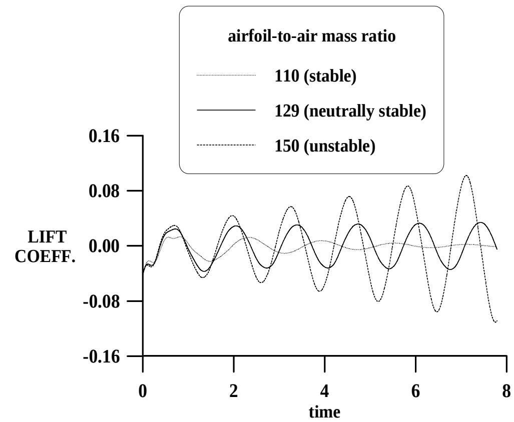

Figure 7 : Aeroelastic response characteristics – Variation of lift coefficient with time

D. Transonic Dip phenomenon

A very interesting phenomenon called 'Transonic dip' is very important in aeroelastic study at transonic flow. This phenomenon occurs when the flutter speed U^* reduces with Mach number and reaches a minimum at a certain value of M_∞ and this minimum value is a function of μ. Therefore, a dip of the flutter boundary with a noticeable decrease in the flutter speed occurs when the flow is transonic. This is called 'Transonic Dip'. The shock waves play a dominant role in the mechanism of transonic dip phenomenon. This phenomenon has been studied by Isogai[10] for the NACA 64A010 airfoil using an integrated aeroelastic code LTRAN2 wherein flow solver based on Transonic small perturbation theory is coupled with a 2-DOF structural dynamics model. In the present work, the transonic dip phenomenon for a NACA 64A006 airfoil undergoing pitch and plunge motions are captured using KFMG Euler code. The grid required for this analysis has been chosen to be the same as that mentioned in previous sub sections (i.e.,101 x 41).

In order to obtain flutter boundary (U^* versus Mach number - curve for neutral stability) of an airfoil undergoing pitch and plunge motions, the airfoil-to-air mass ratio (μ) and reduced frequency k_c are held fixed and U^* is varied for a given M_∞. It has been already shown that for the structural parameters mentioned in the previous sub section and free stream Mach number of 0.85, the neutrally stable response is obtained for $\mu = 129$ and flutter speed U^*=5.5. Now, the value of μ (=129) has been kept constant and U^* corresponding to neutrally stable response is found for different M_∞. The method used for obtaining neutrally stable mode holding μ fixed is as follows:

- For every Mach number, both steady and unsteady calculations are performed. Then, aeroelastic computations are performed to find the neutrally stable conditions and the corresponding value of U^*. In Fig.8 the variation of pitch angle, plunge displacement, lift and moment coefficients with time are plotted corresponding to M_∞=0.8 and μ=129. The three modes of oscillations namely, stable, neutrally stable and unstable conditions are

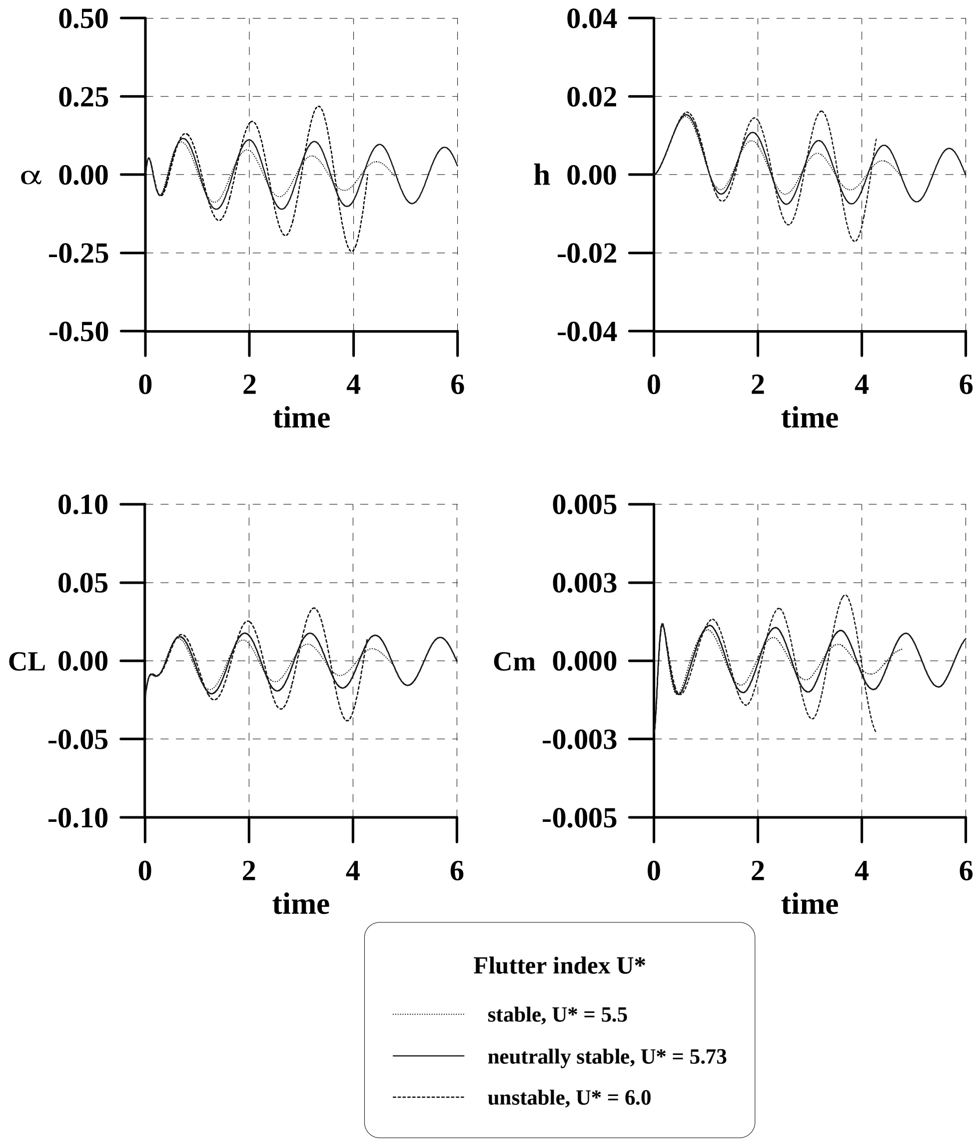

Figure 8 : Aeroelastic response characteristics of NACA 64A006 airfoil with free stream Mach number of 0.7

shown for this case corresponding to different value of U^*. From this, a value of U^* corresponding to neutrally stable condition for this Mach number (say 0.8) has been arrived at and the value U^* is found to be 5.73.

Table 1 : Flutter speed index at different Mach numbers

Mach number		**0.7**	**0.75**	**0.8**	**0.85**	**0.8525**	**0.855**	**0.86**
Flight speed U^*	**μ = 129**	**6.2**	**6.0**	**5.73**	**5.5**	**5.54**	**5.65**	**6.4**
	μ = 300	**9.34**	**9.0**	**8.5**	**8.0**	**7.97**	**8.0**	**9.1**

- Similarly, the response characteristics are obtained corresponding to Mach numbers from 0.7 to 0.86. The U^* values obtained for neutral stability are also shown in Table-1 for the corresponding Mach numbers.

- In a similar way, the response characteristics for yet another μ (=300) are also obtained. Here, another set of U^* values representing neutrally stable condition is obtained corresponding to different Mach numbers and are tabulated in Table-1.

- The flutter speed U^* versus Mach number is plotted in Fig.9 and this curve represents the flutter boundary for the NACA 64A006 airfoil undergoing pitch and plunge motions. The value of μ corresponding to this boundary is 129. The change in U^* is not very significant when we compare the values of U^* at 'a' and 'b' respectively corresponding to Mach 0.7 and 0.85. Hence, a less predominant dip is seen at Mach 0.85

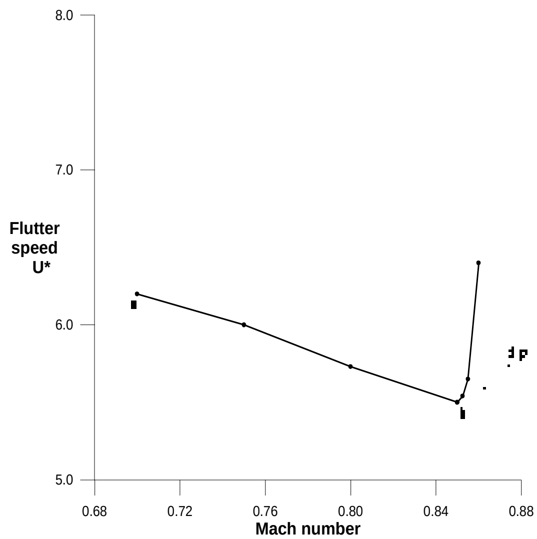

Figure 9 : Flutter boundary for μ = 129

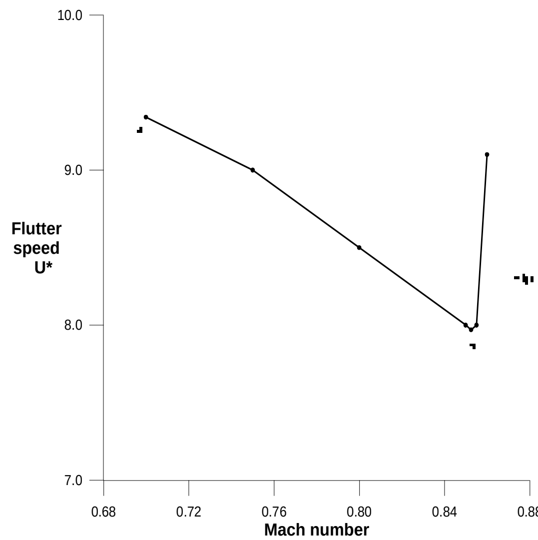

Figure 10 : Flutter boundary for μ = 300

- Considerable decrease in flutter speed U^* (also called flutter index) is seen when we move along the curve from points 'a' to 'b' (Fig.10) corresponding to Mach 0.7 to 0.8525. Here, the value of μ= 300. The percentage reduction in flutter index has been found to be approximately 30 % from Mach number 0.7 to 0.8525. This curve clearly shows the dip of the flutter boundary in transonic regime.

VI. Conclusions

The capability of computational aeroelastic code (consisting of 2-D KFMG Euler code coupled with structural dynamics model) in obtaining the aeroelastic response characteristics of NACA 64A006 airfoil has been satisfactorily demonstrated through response curves for h, α, C_L and C_m. Also, the very interesting transonic dip phenomenon is captured. This new formulation using Kinetic Flux Vector Splitting scheme on Moving Grids (KFMG) for Euler equations has been established to obtain unsteady flow characteristics. Numerical results in general compare well with available results.

Acknowledgments

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