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Kinetic Theory of Collective Modes in Classical Liquids*)

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By solving the linearized Vlasov-Boltzmann equation it is shown that zero sound can exist in classical liquids. The effective interatomic potential is shown to be expressed in terms of the direct correlation function. The real and imaginary parts of the frequency are expressed analytically for both small and large values of wave-vector k, but in general are obtained numerically. In our solution of the dispersion relation, first (ordinary) sound and zero sound originate from the same pole; the former is the solution for smaller k and the latter is that for larger k. We believe that the collective modes observed in classical liquids by neutron scattering experiments should be interpreted as zero sound. The imaginary part of the diffusion pole which contributes to the line width of the quasi-elastic peak, becomes small for the wave-vector where the form factor $S(k)$ has peaks.

§ 1. Introduction

Recently, the new collective modes in classical liquids are observed in the slow neutron inelastic scattering experiments.[1] The well-known collective modes in liquids are the ordinary (first) sound, which can be described by hydrodynamical equation. These new modes observed in the neutron scattering experiments belong to the range of so large wave-vector k and frequency ω that the hydrodynamical description cannot be applied and they have an origin different from first sound. These new collective modes provide an explicit indication of the "solid-like" phenomena in liquids; one, for example, is that the line-width of the quasi-elastic peak of the neutron scattering experiments is narrower than Dk^2 (D; diffusion constant) and oscillates against wave-vector k (de Gennes narrowing[2]).

These modes have the same origin as those for plasma oscillation, lattice vibrations, zero sound in Fermi liquid, elementary excitations in liquid He^4, and sloppy spin waves; these modes are described as the oscillation in the self-consistent field formed by surrounding particles. Already, Vlasov[3] suggested that a new sound different from first sound can exist in the neutral particle system. Klimontovich and Silin[4] called the collective modes formed by a self-consistent field self-consistent sound, or zero sound after Landau's Fermi liquid theory.

*) The main result of this investigation has been reported at the conference on the inelastic scattering of neutrons at Japan Atomic Energy Research Institute, during November 20th–22nd, 1967 (Report No. JAERI-1157, in Japanese).

Takimoto and Ninham[5] examined zero sound in a gas. They considered that the condition for the existence of zero sound is as follows:

$$\text{mean free path } l \gg \text{wave length } \lambda \gg \text{mean particle distance } d, \quad (1\cdot1)$$

and concluded that the condition (1·1) is realized by rarefying a gas. Their results, however, show that the ratio of the damping factor of zero sound to the frequency becomes larger as the density of a gas is reduced and zero sound cannot exist in a rarefied gas. The mean free path was related to the interparticle collisions through the bare two-body potential, so that the gas must be rarefied if it is to be in the collisionless regime. Some part of the collision processes, however, contributes to forming the self-consistent field, and the mean free path must be related to the reduced collisions through the effective two-body potential. From this standpoint high density favours the existence of zero sound.

Recently Nelkin and Ranganathan[6] obtained the line shape of the dynamical structure factor $S(k, \omega)$ by solving the linearized Vlasov equation with a special initial condition.

The purpose of the present paper is to examine the behaviors of the poles of first sound, zero sound and diffusion by solving the dispersion equation derived from Vlasov-Krook equation with the aid of the method of fluctuations in distribution function developed by Hashitsume.[8]

In § 2, we investigate the kinetic equation for the one-body distribution function in liquids. In § 3, the method of fluctuations in the distribution function is summarized and using this method and the zero-th moment sum rule, we determine the interatomic effective potential which is contained in the kinetic equation for the one-body distribution function given in § 2. In § 4, we solve the dispersion equation obtained by the procedure prescribed in § 3, and the dispersion curve and damping of zero sound and first sound are obtained. The last section is devoted to a summary and discussion.

§ 2. Kinetic equation for the one-body distribution function

Bogoliubov and Kirkwood have shown that in a rarefied gas, the one-body distribution function obeys the Boltzmann equation. However, it is not certain what kind of kinetic equation can describe the behavior of the classical liquids in small scale of time and space as observed by the inelastic neutron scattering experiments. Assuming that the n-body distribution functions can be represented as the time-independent functionals of the one-body distribution function and can be expanded in powers of the density, Bogoliubov and his followers derived the kinetic equation for a dense gas which contains the three- and four-body collision terms. This equation is not only too complicated to use for the practical problems, but also has the difficulty that the four-body collision term diverges. This is an essential difficulty in extending Bogoliubov's method so as to meet

the requirements of the kinetic theory of liquids.

In the present paper we take a different approach such that the multiple scattering between liquid particles contribute mainly to forming the effective two-body potential and that the motion of the particles is essentially governed by this modified potential via the binary collisions. As a representation of this picture, we can write the following kinetic equation for the one-body distribution function:

$$\frac{\partial f}{\partial t}+\boldsymbol{v}\frac{\partial f}{\partial \boldsymbol{r}}+\frac{\boldsymbol{F}_{\text{self}}}{m}\frac{\partial f}{\partial \boldsymbol{v}}=I[f],$$

$$\boldsymbol{F}_{\text{self}}(\boldsymbol{r},t)\equiv-\int\frac{\partial V_{\text{eff}}(|\boldsymbol{r}-\boldsymbol{r}'|)}{\partial \boldsymbol{r}}f(\boldsymbol{r}',\boldsymbol{v}',t)\,d\boldsymbol{r}'d\boldsymbol{v}', \tag{2·1}$$

where $I[f]$ is the two-body collision term, $V_{\text{eff}}(r)$ an effective potential between two particles, whose functional form should be determined by the sum rule, and m is the mass of a particle.

Although Kirkwood's or Rice-Allnatt's kinetic equation[7] for liquids does not contain the Vlasov term explicitly, there is a term which is essentially transformed to the Vlasov term with the effective potential $V_{\text{eff}}(r)=-k_BT\ln\cdot g(r)$ ($g(r)$; the radial distribution function).

Landau's kinetic equation for a Fermi liquid can also be considered as the Vlasov-Boltzmann equation with the momentum-dependent effective interaction potential $V_{\text{eff}}(\boldsymbol{r}_1\boldsymbol{r}_2;\boldsymbol{p}_1\boldsymbol{p}_2)=\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)f_{\boldsymbol{p}_1\boldsymbol{p}_2}$, where $f_{\boldsymbol{p}_1\boldsymbol{p}_2}$ is the scattering amplitude function and $\boldsymbol{r}_i$, $\boldsymbol{p}_i$ are the position and momentum of the i-th particle.

§ 3. Two-body double time correlation function

In order to obtain the two-body double-time correlation function from the kinetic equation for the one-body distribution function, we use the method of fluctuations in distribution function; this method, formulated by Hashitsume,[8] has frequently been applied to a wide variety of problems by the Soviet school.

In his method a random function $\hat{f}$ is defined by

$$\hat{f}(\boldsymbol{r},\boldsymbol{v},t;\tilde{\boldsymbol{r}}^N,\tilde{\boldsymbol{v}}^N)=\sum_{i=1}^{N}\delta(\boldsymbol{r}-\tilde{\boldsymbol{r}}_i(t))\delta(\boldsymbol{v}-\tilde{\boldsymbol{v}}_i(t)), \tag{3·1}$$

where $\tilde{\boldsymbol{r}}_i(t)$, $\tilde{\boldsymbol{v}}_i(t)$ are the position and velocity of the i-th particle at time t. This random function is assumed to obey, on the average, the kinetic equation of its average function, $f(\boldsymbol{r},\boldsymbol{v},t)=\langle\hat{f}\rangle$, i.e. the one-body distribution function (this implies Onsager's assumption).

This assumption is expressed as

$$\frac{\partial \hat{f}}{\partial t}+\boldsymbol{v}\frac{\partial \hat{f}}{\partial \boldsymbol{r}}+\frac{\boldsymbol{F}_{\text{self}}}{m}\frac{\partial \hat{f}}{\partial \boldsymbol{v}}=I[\hat{f}]+y(\boldsymbol{r},\boldsymbol{v},t), \tag{3·2}$$

where $y(\boldsymbol{r}, \boldsymbol{v}, t)$ is a random function whose statistical property should be determined by the fluctuation-dissipation theorem as follows. If the generalized thermodynamical velocity $\dot{x}$ and the force $X \equiv -\partial \dot{S}/\partial \dot{x}$, where S is the entropy, are defined by and subject to the relation $\dot{x} = -\hat{\gamma} X + y$, then, according to the fluctuation-dissipation theorem,[9] in general the random quantity y must have the property $\langle |y|^2 \rangle_\omega = 2k_B \hat{\gamma}$. In our case, using the definition of entropy $S = -k_B \int \hat{f} \times \ln \hat{f} d\boldsymbol{r} d\boldsymbol{v}$ and putting $\dot{x} = I[f] + y$, the generalized thermodynamical force X is obtained in the form

$$X \equiv -\frac{\partial \dot{S}}{\partial \dot{x}} \doteqdot \mathcal{F}/f_0 \quad (f_0 \equiv \langle \hat{f} \rangle, \ \mathcal{F} \equiv \hat{f} - f_0)$$

and the relation between $\dot{x}$ and X leads to

$$\dot{x} = I[\hat{f}] + y = \int \left. \frac{\delta I}{\delta f(\boldsymbol{r}, \boldsymbol{v}', t)} \right|_{f_0} \mathcal{F}(\boldsymbol{r}, \boldsymbol{v}', t) d\boldsymbol{v}' + y$$

$$= \int \left. \frac{\delta I}{\delta f(\boldsymbol{r}, \boldsymbol{v}', t)} \right|_{f_0} f_0(\boldsymbol{v}') X(\boldsymbol{r}, \boldsymbol{v}', t) d\boldsymbol{v}' + y ,$$

so that the random function y must have the following property:

$$\langle y(\boldsymbol{v}) y^*(\boldsymbol{v}') \rangle_{k\omega} = -2k_B \gamma(\boldsymbol{v}, \boldsymbol{v}'),$$

$$\gamma(\boldsymbol{v}, \boldsymbol{v}') \equiv \left. \frac{\delta I(\boldsymbol{v}, [f])}{\delta f(\boldsymbol{r}, \boldsymbol{v}', t)} \right|_{f_0} f_0(\boldsymbol{v}'). \tag{3·3}$$

We adopt here the Krook model[10] as an approximation for the collision term to perform the practical calculation, i.e.

$$I(\boldsymbol{v}, [\hat{f}]) = \nu \{ n\Phi - \hat{f} \}, \tag{3·4}$$

where

$$\Phi \equiv \left(\frac{m}{2\pi k_B T} \right)^{3/2} \exp\left[-\frac{m(\boldsymbol{v} - \boldsymbol{u})^2}{2k_B T} \right],$$

$$n \equiv \int \hat{f} d\boldsymbol{v}, \quad \boldsymbol{u} \equiv \frac{1}{n} \int \boldsymbol{v} \hat{f} d\boldsymbol{v} ,$$

$$T \equiv \frac{m}{3k_B n} \int (\boldsymbol{v} - \boldsymbol{u})^2 \hat{f} d\boldsymbol{v} ,$$

ν: collision frequency (parameter).

Carrying out the functional differentiation $I(\boldsymbol{v}, [f])$ with respect to f, we obtain

$$\gamma(\boldsymbol{v}, \boldsymbol{v}') = \nu \left\{ f_0(\boldsymbol{v}) \delta(\boldsymbol{v} - \boldsymbol{v}') - \frac{1}{n_0} \left[1 + \frac{m}{T_0} \boldsymbol{v}\boldsymbol{v}' \right. \right.$$

$$\left. \left. + \frac{3}{2} \left(1 - \frac{mv^2}{3k_B T_0} \right) \left(1 - \frac{mv'^2}{3k_B T_0} \right) \right] f_0(\boldsymbol{v}) f_0(\boldsymbol{v}') \right\}. \tag{3·5}$$

Linearizing Eq. (3·2) and taking the Fourier transform of $\mathcal{F}$ in space and time

$$F_{k\omega}(\boldsymbol{v}) = \int \mathcal{F}(\boldsymbol{r}, \boldsymbol{v}, t) e^{i\omega t - i\boldsymbol{k}\boldsymbol{r}} d\boldsymbol{r} d\boldsymbol{v}\,, \tag{3·6}$$

and with the aid of the equation of continuity, we obtain the following simultaneous equations*) (3·9) and (3·10) for the unknown $\delta n_{k\omega}$, $\delta T_{k\omega}$ defined by

$$\delta n_{k\omega} = \int F_{k\omega} d\boldsymbol{v}\,, \tag{3·7}$$

$$\delta T_{k\omega} = \frac{1}{n_0}\int \left\{\frac{mv^2}{3k_B} - T_0\right\} F_{k\omega} d\boldsymbol{v}\,, \tag{3·8}$$

$$\alpha_{11}\delta n_{k\omega}/n_0 + \alpha_{12}\delta T_{k\omega}/T_0 = \frac{i}{n_0}\int \frac{y_{k\omega} d\boldsymbol{v}}{\omega + i\nu - \boldsymbol{k}\boldsymbol{v}}\,, \tag{3·9}$$

$$\alpha_{21}\delta n_{k\omega}/n_0 + \alpha_{22}\delta T_{k\omega}/T_0 = \frac{im}{n_0 T_0}\int \frac{v^2 y_{k\omega}}{\omega + i\nu - \boldsymbol{k}\boldsymbol{v}} d\boldsymbol{v}\,, \tag{3·10}$$

where

$$\alpha_{11} = 1 - (iy/z)J - (p + 2ixy)(J-1),\ \alpha_{12} = (iy/2z)[J - 2z^2(J-1)],$$
$$\alpha_{21} = 3 - (2iy/z)[z^2(J-1) + J] - (p + 2ixy)[2z^2(J-1) + 2J - 3],$$
$$\alpha_{22} = 3 - (i\nu/\omega + i\nu)[(2z^4 + z^2 + 1)J - 2z^4 - 2z^2],$$
$$z \equiv x + iy = (\omega + i\nu)/2kv_T,\quad v_T \equiv \sqrt{k_B T_0/m},\quad p \equiv n_0 V_{\text{eff}}(k)/k_B T_0,$$
$$J(z) \equiv \frac{z}{\sqrt{\pi}}\int_{-\infty}^{\infty} \frac{e^{-t^2}}{z - t} dt\,.$$

The above equations are solved for $\delta n_{k\omega}$ to derive the dynamical form factor $S(k, \omega) = \langle|\delta n_{k\omega}|^2\rangle$. The functional form of $V_{\text{eff}}(k)$ is then determined so as to satisfy the zero-th moment sum rule,

$$n_0 S(k) = \langle|\delta n|^2\rangle_k = \frac{1}{2\pi}\int S(k, \omega) d\omega\,. \tag{3·11}$$

Because of the computational difficulty in this procedure, we use an approximate $S(k, \omega)$ determined from the collisionless equation in the calculation. Since the Vlasov term has an important contribution principally in the collisionless regime, this approximation is to be a sufficiently good one. In this collisionless approximation Eq. (3·2) is written as

$$\frac{\partial \mathcal{F}}{\partial t} + \boldsymbol{v}\frac{\partial \mathcal{F}}{\partial \boldsymbol{r}} + \frac{\boldsymbol{F}_{\text{self}}}{m}\frac{\partial f_0}{\partial \boldsymbol{v}} = -\nu\mathcal{F} + y \tag{3·12}$$

*) This calculation is essentially similar to that in the plasma problems studied by Sitenko and Gurin.[11]

(we take the limit $\nu \to 0$ later)

and Eq. (3·5) reduces to

$$\langle y(\boldsymbol{v}) y^*(\boldsymbol{v}') \rangle = 2k_B \nu f_0(\boldsymbol{v}) \delta(\boldsymbol{v}-\boldsymbol{v}'). \tag{3·13}$$

From Eq. (3·12) $\delta n_{k\omega}$ is given by

$$\delta n_{k\omega} = \int \frac{i y_{k\omega}(\boldsymbol{v}')}{s_k(\boldsymbol{v}')} d\boldsymbol{v}' \Big/ \left\{ 1 + \int \frac{H_k(\boldsymbol{v}')}{s_k(\boldsymbol{v}')} d\boldsymbol{v}' \right\}, \tag{3·14}$$

where

$$H_k(\boldsymbol{v}) \equiv \frac{V_{\text{eff}}(\boldsymbol{k})}{m} \boldsymbol{k} \frac{\partial f_0}{\partial \boldsymbol{v}},$$

$$s_k(\boldsymbol{v}) \equiv \omega + i\nu - \boldsymbol{k}\boldsymbol{v}.$$

From Eqs. (3·13) and (3·14) we obtain

$$S(k, \omega) \equiv \langle |\delta n|^2 \rangle_{k\omega} = -\frac{2k_B T_0}{V_{\text{eff}}(k)} \frac{1}{\omega} \cdot \text{Im}\left(\frac{1}{\varepsilon_{k\omega}} \right),$$

where

$$\varepsilon_{k\omega} \equiv 1 - \frac{V_{\text{eff}}(k)}{k_B T_0} n_0 \{ J(z) - 1 \}. \tag{3·15}$$

These are the same formulae as Mermin's[12] obtained by the Green's function method, and also Aronson's[13] by the diagram method, but their interatomic potential is bare one, instead of effective. Recently Nelkin et al.[6] obtained their formula for $S(k, \omega)$ by solving the Vlasov equation with a special initial condition. Only if the effective potential is proportional to the direct correlation function, Eq. (3·15) is the same as Nelkin's as will be shown below. Using Eq. (3·15) and with the aid of the Kramers-Krönig relation, Eq. (3·11) reduces to

$$n_0 S(k) = \frac{n_0}{1 + n_0 \beta V_{\text{eff}}(k)},$$

$$(\beta \equiv 1/k_B T_0)$$

from which we obtain the following effective potential:

$$n_0 V_{\text{eff}}(k) = -k_B T_0 n_0 C(k), \tag{3·16}$$

where $n_0 C(k) \equiv 1 - 1/S(k)$ is the direct correlation function. This is identical with the result obtained by Zwanzig[14] by the method of the variational principle.

§ 4. The dispersion curve

Proceeding in the manner outlined in the previous section, we may obtain $S(k, \omega)$ where collision terms are also taken into account. However, we in-

vestigate here only the behaviors of the poles of $S(k, \omega)$ associated with the sound and the diffusion. The dispersion equation is obtained from the denominator of $S(k, \omega)$, that is,

$$\alpha_{11}\alpha_{22}-\alpha_{12}\alpha_{21}=0$$

which is written in the form

$$\left(3-2i\frac{y}{z}J\right)\left[1-i\frac{y}{z}J-(p+2ixy)(J-1)\right]$$
$$+\frac{i}{2}\frac{y}{z}(1+p-2x^2)\left[2z^2(J-1)-J\right]=0\,, \qquad (4\cdot1)$$

where

$$p\equiv-n_0C(k) \quad \text{and} \quad z\equiv x+iy\equiv\frac{\omega+i\nu}{\sqrt{2}v_Tk}\,.$$

We now solve this dispersion equation in the two limiting cases: (i) in the collisonless regime ($\omega\tau\gg1$, $\tau\equiv1/\nu$) and (ii) in the hydrodynamical regime ($\omega\tau\ll1$) as follows.

(i) collisionless regime ($\omega\tau\gg1$)

We find the dispersion curve from Eq. (4·1) as the real part of the root ω,

$$\frac{\mathrm{Re}\,\omega}{v_Tk}\fallingdotseq\sqrt{3+p}\,, \qquad (4\cdot2)$$

and the damping,

$$\frac{\mathrm{Im}\,\omega}{v_Tk}\fallingdotseq-\frac{\sqrt{2\pi}x_0e^{-x_0^2}}{2d(x_0)}+\frac{1}{2d(x_0)}\left\{\frac{8}{3}\frac{x_0}{p^2}+\frac{2}{3x_0p}\left(1+\frac{1}{p}\right)\right\}\frac{\nu}{v_Tk}-\frac{\nu}{v_Tk}\,, \qquad (4\cdot3)$$

where

$$d(x)\equiv(2x^2-1)F(x)-x\fallingdotseq\frac{1}{2x^3}+\frac{1}{2x^5}+\cdots\,,$$

$$F(x)\equiv e^{-x^2}\int_0^x e^{t^2}dt\,,$$

$$x_0\equiv\sqrt{p/2(1-p\alpha)+3/2}\,,$$

$$\alpha\equiv2\sqrt{\frac{p}{2}+\frac{3}{2}}F\left(\sqrt{\frac{p}{2}+\frac{3}{2}}\right)-\left(1+\frac{1}{p}\right).$$

When $p\gg3$, the dispersion curve, Eq. (4·2), is written as

$$(\mathrm{Re}\,\omega)^2\fallingdotseq\frac{v_Tk^2}{S(k)}\,,$$

which is identical with $\langle\omega^2\rangle_k^{\mathrm{coh}}\equiv\int\omega^2S(k,\omega)\,d\omega$.

(ii) hydrodynamical regime ($\omega\tau \ll 1$)

Expanding ω/ν in powers of $\mu \equiv 1/y$ and determining coefficients to satisfy the dispersion equation, we obtain the dispersion curve for first sound

$$\frac{\mathrm{Re}\,\omega}{v_T k} \fallingdotseq \sqrt{\frac{5}{3}+p}\,, \tag{4·4}$$

and the damping

$$\frac{\mathrm{Im}\,\omega}{(v_T k)^2} \fallingdotseq -\frac{10+4p}{10+6p}\frac{1}{\nu}\,. \tag{4·5}$$

The diffusion pole is given by $\mathrm{Re}\,\omega = 0$ and

$$\frac{\mathrm{Im}\,\omega}{(v_T k)^2} = -\frac{10(1+p)}{10+6p}\frac{1}{\nu}\,. \tag{4·6}$$

The collective modes observed in the inelastic neutron scattering experiments should be interpreted as zero sound in the collisionless regime and its dispersion curve is given by Eq. (4·2).

The dispersion curve of first sound, Eq. (4·4), is observed in the light scattering experiments. The damping of zero sound obtaind as in Eq. (4·3) consists of the Landau damping and the collision damping; the latter is independent of wave-vector k and proportional to the collision frequency ν, while the damping of first sound, Eq. (4·5), is inversely proportional to ν.

In order to obtain the dispersion curve for general wave-vector, we solved the dispersion equation numerically. The dispersion curve and damping are plotted against the wave-vector k in Fig. 1(a). In the computation we take, $\nu = 10^{11}$ sec^{-1} which corresponds to argon with density 1.37 g/cc at temperature 90°K and used the Ashcroft model[15] for $S(k)$ (in which parameters are taken as $\eta = 0.45$ and $\sigma = 3.44$Å). Since this model cannot represent properly the behavior of $S(k)$ for small k, these results should be interpreted with due reserve.

In Figs. 1(a) and 2(a) the curves denoted by "gas" mean the solution of the dispersion equation (4·1) without the Vlasov term. As shown in Fig. 1(a), the damping for a liquid is small for small k, but increases with k and approaches the damping for a gas. With increasing k the dispersion curve for a liquid also approaches asymptotically that for a gas, $\omega_k = \sqrt{2k_B T_0/m}\cdot k$, as it should.

Figure 1(b) shows the behavior of the imaginary part of the diffusion pole against wave-vector k, which becomes Dk^2 for small k. This imaginary part for liquids in Fig. 1(b) has a dip at wave-vector at which $S(k)$ has the peaks. This phenomena of the diffusion pole for a liquid may be associated with de Gennes narrowing,[2),16] that is, the line width of the quasi-elastic peak in the neutron scattering experiments becomes narrow in the vicinity of k where $S(k)$ has the peaks.

We also investigated the relation between zero sound and first sound. We solved the dispersion equation of the sound waves, varing the wave-vector k from

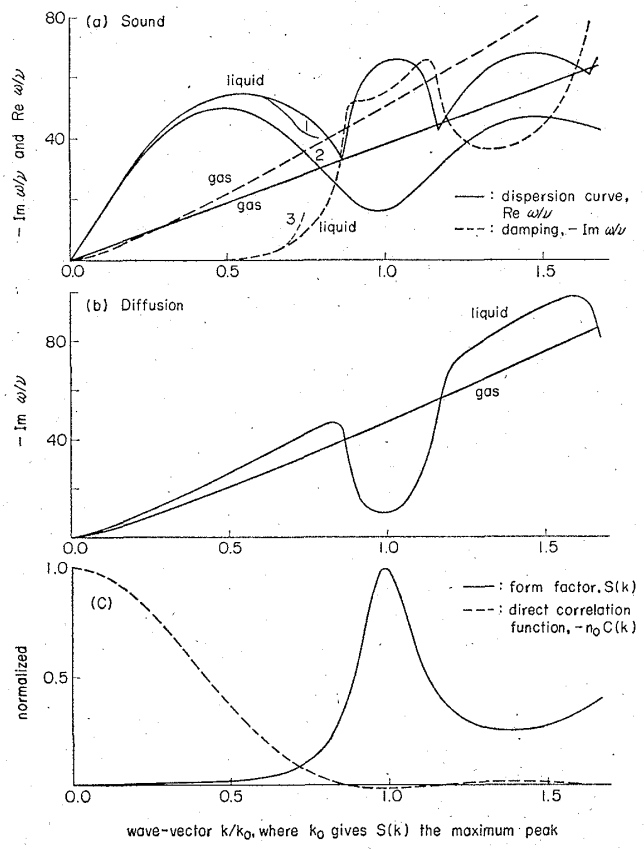

Fig. 1. (a) The dispersion curve and the damping of zero sound. Curves 1 and 3 are calculated from Eqs. (4·2) and (4·3). Curve 2: $\sqrt{k_B T_0/mS(k)}\cdot k$. The collective modes observed in the inelastic neutron scattering experiments should be interpreted as zero sound. (b) The imaginary part of the diffusion pole vs. wave-vector k. (c) The form factor $S(k)$ and the direct correlation function. The dip in the vicinity of $k/k_0=1$ may be associated with de Gennes narrowing.

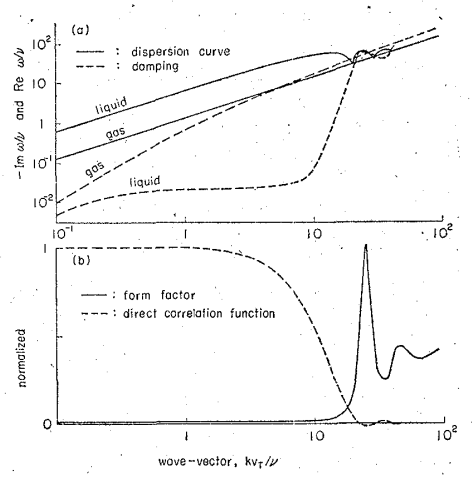

Fig. 2. The relation between zero sound and first sound. In Fig. 2(a) the curves denoted by "liquid" represent the dispersion curve and the damping of first sound for small wave-vector k, and those of zero sound for large k. First sound and zero sound originate from the same pole of the dispersion equation.

the hydrodynamical to the collisionless regime. The results are shown in Fig. 2. For small k (in hydrodynamical regime) the dispersion curve and damping represent those of first sound and for large k (in collisionless regime) those of zero sound in liquids. The ratio of the damping to frequency inceases as k increases and when k further increases into the collisionless regime, the ratio decreases. From this we find that first sound and zero sound originate from the same pole of the dispersion equation. The same behavior as this is observed experimentally in the sound absorption in liquid He^3.[17]

§ 5. Concluding remarks

The dispersion curve of zero sound in Fig. 1(a) is qualitatively in good agreement with experiment, except the odd behavior at the peak of the structure factor. The collective excitation observed in the neutron scattering experiments should be interpreted as zero sound. From Fig. 2 we conclude that the same pole of the dispersion equation gives zero sound and first sound; the former corresponds to the pole for smaller wave-vector k, the latter to the pole for larger k. The

ratio of the velocity of first sound to that of zero sound, denoted by v_1, v_0 respectively, is

$$\frac{v_1}{v_0}=\frac{\sqrt{5/3+p}}{\sqrt{3+p}}<1\,.$$

Since zero sound originates in the Vlasov term, we can consider that the existence of zero sound, which belongs to the same pole as first sound, is independent of statistics (Fermi, Bose or Boltzmann). The formula of the damping factor (4·3) shows that zero sound can be the more easily observed as $S(0)$ or $k_BT\chi_T$(χ_T; isothermal compressibility) is smaller.

As Fig. 1(b) indicates, the diffusion pole shows the phenomena corresponding to de Gennes narrowing. However, it is not clear how this pole affects the behavior of the quasi-elastic peak.

Addition of the Vlasov term to the kinetic equation in liquids explains the "solid-like" behaviors of liquids considerably well. In our calculation, the damping factor of zero sound becomes large near the wave-vectors at which $S(k)$ has peaks, due to the Landau damping, whereas Egelstaff[18] considers that the damping is small there in analogy to the lattice vibrations. As wave-vector increases, liquids gradually show gas-like behaviors rather than solid-like ones, and the Landau damping bridges the gap between them.

Although the Krook approximation for the collision term contains only the hydrodynamic poles, it is sufficient to investigate the behaviors of the hydrodynamic poles for large wave-vectors as observed in the neutron scattering experiments.

We have used Hashitsume's method in order to relate the one-body distribution function to the dynamical structure factor $S(k, \omega)$. This method has the advantage that it is founded on the principles which have clear meanings in the statistical mechanics; these are Onsager's assumption and the fluctuation-dissipation theorem. The neutron scattering experiments offer important information on the study of the kinetic properties of liquids.

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