KINETICALLY CONSTRAINED SPIN MODELS

ABSTRACT. Kinetically constrained spin models (KSCM) are interacting particle systems with Glauber-like dynamics, no static interactions beyond hard core and reversible w.r.t. simple i.i.d product measure. The essential feature is that the creation/destruction of a particle on x can occur only if the configuration around x satisfies certain constraints (which completely define each specific model). KCSM have been introduced and studied in physical literature as examples of models showing a slow relaxation and in the attempt to model liquid/glass transition. Because of the constraints KCSM may show ergodicity breakdown (dynamical phase transition) without any equilibrium counterpart and it is this phenomenon that has been the main object of investigation in the physical community. In these lectures, starting from the basics of reversible continuous time Markov chains, we will develop rigorous tools to analyze the density and volume dependence of the relaxation time for a fairly general class of KCSM.

1. TENTATIVE PLAN

Lecture 1: Continuous time reversible Markov chains with finite state space. Mixing and relaxation times, spectral gap, logarithmic Sobolev constant, geometric tools and examples [10, 4, 5]

Lecture 2: KCSM [9, 2, 3, 13]. Definition, graphical construction of the generator and semigroup, stationary and reversible measures, ergodicity and mixing [8, 7]. Boostrap percolation models [1, 11, 6, 12] and KCSM.

Lecture 3: KCSM: analysis of the relaxation time and of the persistence function [3].

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