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Kinetostatic and Dynamic Modeling of Flexure-Based Compliant Mechanisms: A Survey

Flexure-based compliant mechanisms are becoming increasingly promising in precision engineering, robotics, and other applications due to the excellent advantages of no friction, no backlash, no wear, and minimal requirement of assembly. Because compliant mechanisms have inherent coupling of kinematic-mechanical behaviors with large deflections and/or complex serial-parallel configurations, the kinetostatic and dynamic analyses are challenging in comparison to their rigid-body counterparts. To address these challenges, a variety of techniques have been reported in a growing stream of publications. This paper surveys and compares the conceptual ideas, key advances, and applicable scopes, and open problems of the state-of-the-art kinetostatic and dynamic modeling methods for compliant mechanisms in terms of small and large deflections. Future challenges are discussed and new opportunities for extended study are highlighted as well. The presented review provides a guide on how to select suitable modeling approaches for those engaged in the field of compliant mechanisms. [DOI: 10.1115/1.4045679]

1 Introduction

Flexure-based compliant mechanisms generally refer to mechanical devices gaining some or all of their mobility through elastic deformation of flexible members [1], which by nature possess the benefits of monolithic structure and variable stiffness over their rigid-body counterparts, thereby reducing friction, backlash, wear, no need for lubrication and assembly. Owing to these uniqueness, compliant mechanisms have been widely applied, including constant-force generation [2], multistable switches [3], micro-electro mechanical systems (MEMS) [4], precision positioning stages and grippers [5–7], micro/nanomanipulations [8], fast servotools in precision machining [9], servovalves [10], energy harvesting [11], microvibration suppression [12], alignment of optics, robotic actuation [13], and so forth.

During the last three decades, a variety of techniques and methodologies have been developed for kinetostatic and dynamic modeling of compliant mechanisms, such as the pseudo-rigid-body model (PRBM) [14], Castigliano's second theorem [15], compliance matrix method [16], elastic beam theory [17], two-port dynamic stiffness model [18], Ryu's method [19], and beam constraint model [20]. These methodologies have enabled significant advances in designing compliant mechanisms. However, the kinetostatic and dynamic modeling of compliant mechanisms remain challenging owing to their intrinsic coupling of kinematic and elastomechanical behaviors. In addition, large-deflection analysis and complex serial-parallel configurations often encountered in compliant mechanisms lead to intractable modeling procedures. Although many solutions are now available for kinetostatic and dynamic modeling of compliant mechanisms [14–20], there has been less effort on clarifying and comparing the conceptual ideas, advantages, disadvantages, discrepancies, and applicable scopes among these modeling methods. It would be difficult for designers with little experience in the field of compliant mechanisms to find a starting point from where they can be guided toward the modeling issues [21] and to identify which method is most suitable for their specific applications. As stated in Ref. [21], this problem will be enlarged by the amount of knowledge that a designer should possess, such as mechanics of materials, mechanical dynamics, kinematics of mechanisms, and nonlinear mechanics.

A review has already focused on synthesis approaches to compliant mechanisms including topology optimization and rigid-body-replacement methods [21]. In this paper, key concepts and recent advances on the state-of-the-art kinetostatic and dynamic modeling approaches for compliant mechanisms involving small and large deflections are surveyed, compared, and summarized. Details of synthesis methods for compliant mechanisms, such as topology optimization [22–30], building block approach [31,32], freedom and constraint topology (FACT) method [33,34], are out of the scope of this paper and will not be discussed in detail. Extensive studies can be found in Refs. [21–34] and related references [35,36].

The remainder of this paper is organized as follows: Progress on parametric modeling of compliant mechanisms is briefly overviewed in Sec. 2. Details of various modeling approaches, classified as (i) flexure hinges, (ii) kinetostatics of small deflection, (iii) kinetostatics of large deflection and (iv) issues on the dynamics, are, respectively, described from Secs. 3–6. Future challenges and open research topics are discussed in Sec. 7 and followed by conclusions in Sec. 8.

2 Brief Overview: Modeling Advances in Compliant Mechanisms

In this section, key advances on compliant mechanisms in terms of kinetostatic and dynamic modeling during the past three

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decades are briefly described in chronological order. A detailed survey of different types of modeling approaches involving small and large deflections will be carried out from Secs. 3–6.

2.1 Overview of Key Advances. Since the proposal of Her and Midha in 1980s [37], compliant mechanisms, in only the last three decades, have drawn ever-increasing research interests due to the inherent characteristics of precision motion without friction, backlash and wear over their rigid-body counterparts. In such a monolithic mechanism, flexible members such as flexure hinges transmit force and movement by elastic deformation with a similar function to gears and joints in traditional rigid-link mechanisms.

Compliant mechanisms can be classified into partially compliant mechanisms and fully compliant mechanisms [1], and the latter can be further subdivided into lumped, distributed, and hybrid compliant mechanisms (see Fig. 1). Partially compliant mechanisms often bear large deflection widely applied in areas such as statically balanced mechanisms [38–41], constant-force mechanisms [42–44], and joint of robots [45]. Fully compliant mechanisms are usually manufactured in monolithic structures, which are widely applied in precision engineering with static and dynamic applications. A majority of the present fully compliant mechanisms are designed with small deflection but serial-parallel configurations increase the kinetostatic and dynamic modeling complexity. On the other hand, nonlinearities in modeling compliant mechanisms with intermediate or large deflections pose design challenges for some newly emerging dynamic applications with large workspaces [46–50].

As an important element in compliant mechanisms, research on the displacement–force relationship of flexure hinges and flexible beams has been a popular topic [53]. The pioneer work can be traced back to the investigations of Paros and Weisbord [54], wherein the theoretical equations of compliance for circular flexure hinges were derived. On the other hand, elliptic integral solutions [55,56] and the chain algorithm [57] were two useful techniques for the kinetostatic analysis of flexible beams with large deflection prior to the ground-breaking PRBM proposed in the 1990s [58–60]. Although the accuracy of elliptic integral solutions is high, the modeling procedures of this method along with the chain algorithm are relatively complicated. The disadvantage was overcome by the PRBM wherein each flexure member is treated as rigid links connected by a revolute joint attached with a torsional spring. Consequently, analyzing compliant mechanisms can be simplified as an issue of rigid-body mechanisms. The pseudo-rigid-body model has been proven over time to be an effective tool for analysis and synthesis of compliant mechanisms involving both small and large deflections [61–65]. In 2001, the first book in the context of compliant mechanisms appeared, in which the pseudo-rigid-body model was systematically introduced [1].

Before and after 2000, many efforts were devoted to the compliance modeling of all kinds of notch flexure hinges with small deflection, such as circular, hyperbolic, and elliptic flexure pivots [66–69]. Afterward, many investigations focused on modeling the kinetostatic force–deflection relationship of flexure hinges for their design. A comparative review on the accuracy of compliance equations for circular flexure hinges within a wide range of geometric parameters was provided in 2008 [70]. These investigations on the kinetostatics of flexure hinges provided powerful tools and guidelines on designing multitudinous flexure hinges [71–75].

As to the modeling of compliant mechanisms with small deformation, Castigliano's second theorem was utilized to model the kinetostatics of bridge-type flexure amplifiers in 2003 [76], while elastic beam theory was employed for the kinetostatics of this type of amplifier in 2006 [77]. These two methods are now widely used to design compliant mechanisms with small deflections [78–81]. However, inner-force analysis is required in these two methods, resulting in great complexity in serial-parallel compliant mechanisms. As a result, the compliance matrix method was developed for complex configurations [82–87]. Some theorized investigations on the compliance matrix method for compliant mechanisms were also conducted [87–89], in which the matrix operation enabled easy analysis of complex compliant mechanisms.

Another aspect on the linear kinetostatic modeling of compliant mechanisms was the finite element method. The theoretical compliance matrix of circular flexure hinges was converted into the elemental stiffness matrix in the framework of the finite element method in 2008 [90]. With similar conversion formulas, the kinetostatic and dynamic modeling of all kinds of flexure-hinge-based compliant mechanisms were carried out based on the finite element method without dealing with the complicated issue of variable cross section in flexure hinges [91,92].

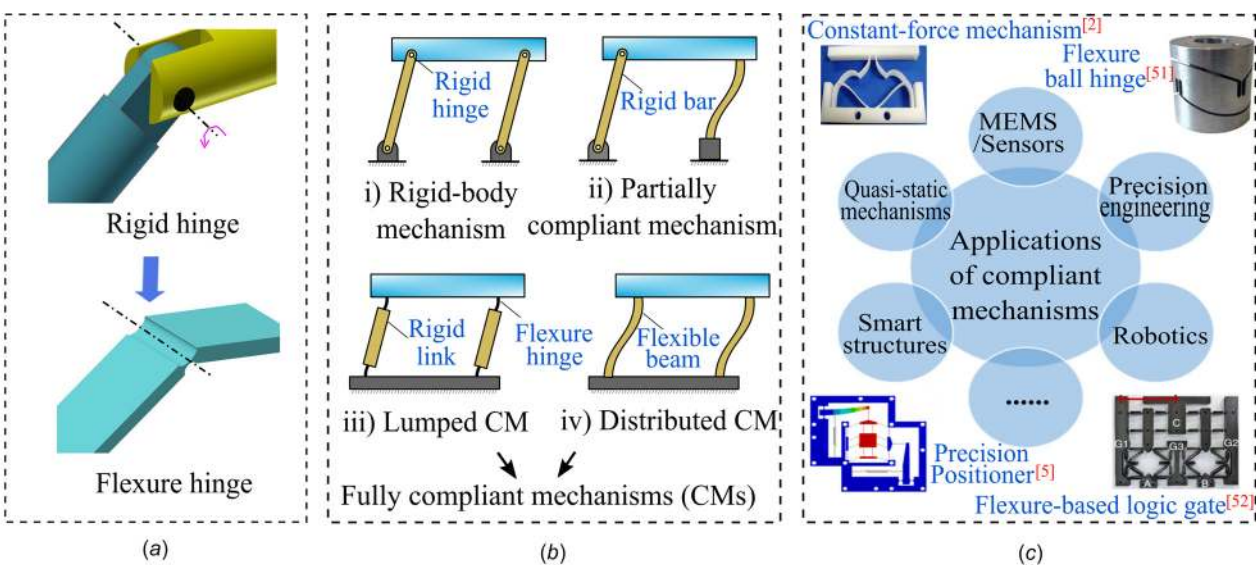

Fig. 1 Conceptual illustration of flexure-based compliant mechanisms and rigid-body mechanisms as well as application examples of compliant mechanisms [2,5,51,52] (Reprinted with permission from Wang and Xu [2]. Copyright 2018 by Elsevier; Reprinted with permission from Qin et al. [5]. Copyright 2014 by IEEE; Reprinted with permission from Teo et al. [51]. Copyright 2014 by Springer; Reprinted with permission from Song et al. [52] (Open Access)): (*a*) conceptual comparison between rigid hinges and flexure hinges, (*b*) conceptual comparison between rigid-body mechanisms and complaint mechanisms, and (*c*) exemplary engineering applications of complaint mechanisms.

In addition to investigations on the linear modeling of compliant mechanisms, much effort was also devoted to the large-deflection issue. In 2006, the beam constraint model was proposed to provide a kinetostatic solution for flexure mechanisms when the deflection is within 10% of the beam length based on the continuum beam theory and Taylor series expansion [93–96]. The characteristic of this method lies in the captured load-stiffening effects. The beam constraint model was classified as an intermediate-range kinetostatic method of compliant mechanisms in "Handbook of Compliant Mechanisms" [97]. Afterward, this method was further enhanced by including shear effects [98] and extended for large-deflection analysis of flexure beams [99,100], namely, the chained beam constraint model.

On the other hand, since the prediction accuracy of the pseudo-rigid-body model is limited in estimating larger end slope of flexible beams, efforts have been focused on improving the performance of the pseudo-rigid-body model by including more parameters, such as the 2*R* (*R* denotes a revolute pair), 3*R*, 5*R*, and *RPR* (*P* denotes a prismatic pair) pseudo-rigid-body models [101–104]. Moreover, some multi-axis flexure hinges with complex configurations and large deflection, such as the Cartwheel flexure hinge, were analyzed based on the pseudo-rigid-body model [105–107]. Recent progress on the pseudo-rigid-body model was to include mass factors for the dynamic analysis of flexure mechanisms [108]. Moreover, the kinetostatics of large-deflection compliant mechanisms has emerged as an optimizing problem instead of the usual way of formulating the load equilibrium equations [109–112]. In these energy-based methods, the kinetostatic analysis of compliant mechanisms was resolved in the framework of the principle of elastic energy minimization.

During the past three decades, the dynamic issue has been also an important aspect of designing compliant mechanisms. However, research on the dynamics of compliant mechanisms has not been extensively studied. Two books in the field of compliant mechanisms [1,97] mainly involve the kinetostatic issue. To sum up, the previous dynamic modeling of compliant mechanisms was mainly based on Lagrange's method [113–130]. In recent years, some newly dynamic modeling approaches, such as the pseudo-rigid-body model with mass factors [108] and the dynamic stiffness modeling method based on d'Alembert's principle [18,131,132], were developed for the dynamic analysis of compliant mechanisms.

The existing Lagrange-based dynamic modeling methods for compliant mechanisms can be further generalized into three categories. When flexure hinges/flexible beams are modeled as elastic joints connected to rigid links and the dynamic model is built by simplifying compliant mechanisms as the rigid-link mechanisms, this version is known as the PRBM-based method [113–118]. For example, a PRBM-based dynamic model was built for parallel-guided compliant mechanisms [114] based on Lagrange's equation and the dynamic equivalence. In the second category, the equivalent input/output stiffness of compliant mechanisms is first modeled by using a variety of kinetostatic methods, the dynamic model can be then formulated by calculating elastic and kinetic energies based on Lagrange's equation with motion degrees-of-freedom (DOFs) as the variables, all these solutions can be classified as a lumped-parameter dynamic model [119–123]. The third category is termed the distributed-parameter model with finite number of DOFs [124–132], in which compliant mechanisms are usually discretized into several submembers and the dynamic model is established by calculating total elastic and kinetic energies. For example, a method similar to the rigid-multibody dynamics [19,126] was used to design flexure manipulators by several groups [127–130]; moreover, various finite element dynamic models have been developed in the last decades [92,124,125].

2.2 Distinction Between Two Sets of Terminologies.

The kinetostatic and dynamic modeling of compliant mechanisms should be distinguished from the following two relevant concepts:

(1) The first distinction is synthesis design of compliant mechanisms by employing the rigid-body-replacement method [21,133], building block approach [31,32], screw theory [134,135], FACT method [33,34] or topological optimization technique [22–30]. In the case of the rigid-body-replacement method, compliant mechanisms are designed through creating the kinematic model of a basic rigid-link mechanism and then replacing rigid joints with flexure hinges, while the conceptual configuration is searched without a priori knowledge in the topological optimization. The challenge for synthesis design lies in identifying the best configurations, while the target of kinetostatic and dynamic modeling of compliant mechanisms mainly focuses on their performance prediction, influence analysis, and parametric optimization by virtue of mathematical formulating. It is noted that the pseudo-rigid-body model was not only utilized for the synthesis of compliant mechanisms [21] but also widely applied to their kinetostatic and dynamic analyses. Readers should distinguish these subtle differences for clear understanding.

(2) The second concept is the term "kineto-elastodynamics" and the term "flexible multibody dynamics" in the context of mechanical dynamics and in the field of aerospace [136,137]. The objectives of these two disciplines are both on the dynamic analysis of rigid-body mechanisms or mechanical systems considering negative elastic deformation and on how to eliminate the vibration at high speeds. In contrast, compliant mechanisms make use of elastic deformation to transmit motions and forces. Classical methods of relative, absolute, and hybrid coordinates in the flexible multibody dynamics and kineto-elastodynamic model [136,137] would not be directly applicable to the kinetostatic and dynamic modeling of compliant mechanisms due to their distinguishing configurations and different characteristics of deformation.

2.3 Modeling Challenges for Compliant Mechanisms.

Challenges associated with the kinetostatic and dynamic modeling of flexure-based compliant mechanisms mainly arise from the following three inherent factors:

2.3.1 *Coupling of Kinematic and Elastomechanical Behaviors.*

In rigid-link mechanisms, kinematics, statics, and dynamics are usually analyzed in sequence and the kinematics is only dependent on geometric parameters decoupling from the kinetic behaviors. However, compliant mechanisms rely on the elastic deformation to transmit forces and create desired motion DOFs. Modeling of compliant mechanisms requires simultaneous solution of kinematic and elastomechanical behaviors.

2.3.2 *Serial-Parallel Configurations.*

Serial-, parallel- and their hybrid-kinematic configurations are frequently designed in flexure-guided manipulators due to their own advantages and disadvantages in terms of stroke range, mechanical bandwidth and output stiffness [5–9]. The over-constraint configuration in many compliant mechanisms poses some complexity for their kinetostatic and dynamic formulating.

2.3.3 *Nonlinear Large Deflections.*

In addition to hard and time-consuming modeling procedures at the level of large-deflection flexure beams, greater difficulty comes from modeling the whole compliant mechanisms with large deflection. Although several modeling methods such as the pseudo-rigid-body-model and energy-based methods [109–112] are available for modeling large-deflection compliant mechanisms, more concise and accurate methods for dynamic issues of complex compliant mechanisms are still intractable.

2.4 Classification of Modeling Methods for Compliant Mechanisms.

Although versatile finite element packages such as ANSYS are widely available, these packages give less insight into

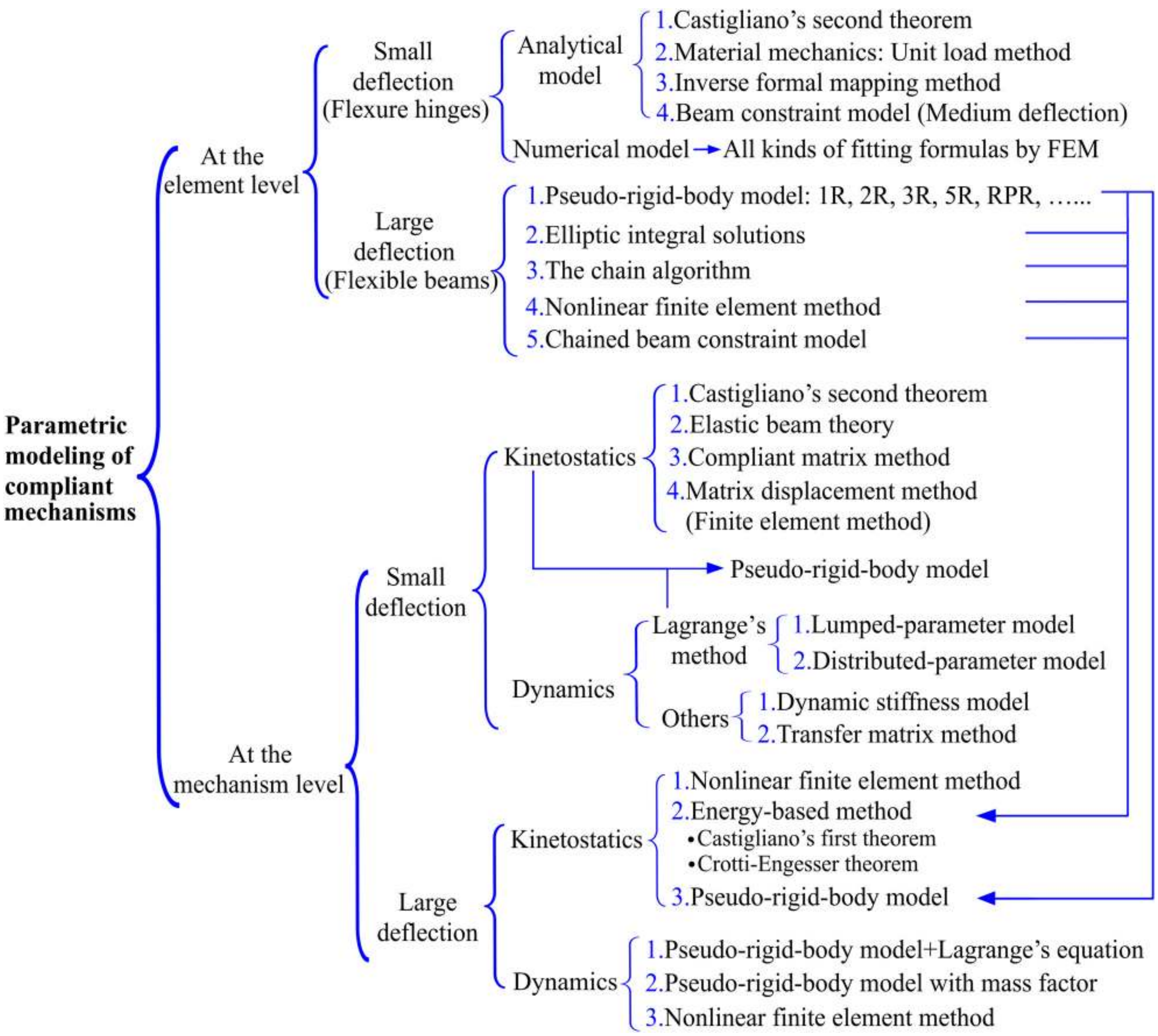

Fig. 2 Category of the kinetostatic and dynamic modeling approaches for compliant mechanisms. (It is noticed that synthesis methods for compliant mechanisms, such as the building block approach, topology optimization, FACT method, and screw theory, are not included here and can refer to Refs. [21–35,133–135]).

the intrinsic deformation of compliant mechanisms from the perspective of designers. Moreover, their modeling step is somewhat time-consuming and is not appropriate for the early stage of design where conceptual ideas should be analyzed and evaluated in an intuitive sense and in a short period of time. Therefore, continuous efforts have been devoted to developing parametric modeling methods for the kinetostatic and dynamic analysis of compliant mechanisms, as summarized in Fig. 2. Detailed surveys for each category will be discussed in the following Secs. 3–6.

3 Modeling of Flexure Hinges

Flexure hinges undergo elastic deformation relative to adjacent stiffer regions in compliant mechanisms. Normally, the difference of compliance and stiffness is reached by geometric characteristics of deformation regions. Depending on these characteristics, flexure hinges are often designed with single or multiple axes. Notch-type flexure hinges with single axis are often profiled as rectangular section (also known as leaf spring), circular, corner-filleted, elliptic, parabolic, hyperbolic or other conic profiles and hybrid cross sections [15,138] (see Fig. 3). Each type of profiles provides unique mechanical properties to suit different requirements of design [67], such as discrepant motion accuracy, out-of-plane stiffness, or different levels of stress concentration. Generally, the target of kinetostatic modeling for flexure hinges is to obtain their force–deflection relationship, namely, the compliance matrix. A general form of the compliance matrix of flexure hinges for planar problems can be expressed as follows [53,68]:

$$\boldsymbol{C} = \begin{bmatrix} \Delta x/F_x & 0 & 0 \\ 0 & \Delta y/F_y & \Delta\theta/F_y \\ 0 & \Delta y/M_z & \Delta\theta/M_z \end{bmatrix} = \begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & c_\alpha \\ 0 & c_\alpha & c_\theta \end{bmatrix} = \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & k_\alpha \\ 0 & k_\alpha & k_\theta \end{bmatrix}^{-1} \quad (1)$$

where c_x, c_y, c_α, c_θ and k_x, k_y, k_α, k_θ are the compliance and stiffness coefficients. F_x, F_y, and M_z are the tensile, shear, and moment loads with corresponding deflections Δx, Δy, and $\Delta\theta$.

Since the pioneer works on formulating circular flexure hinges by Paros and Weisbord [54], plenty of methods have been presented to describe the kinetostatics of all kinds of flexure hinges. In the literature, there are basically two categories for the compliance modeling of flexure hinges, namely, analytical formulas and

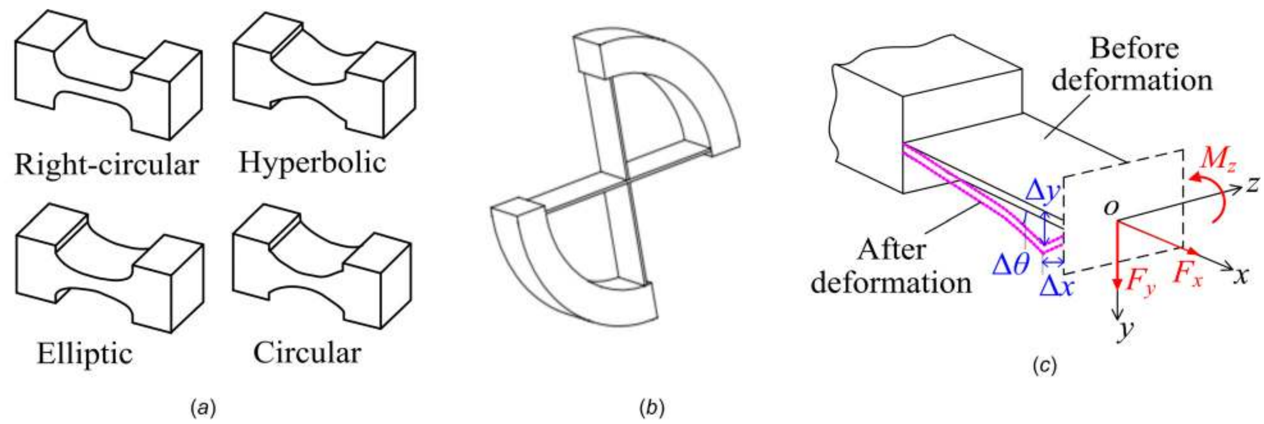

Fig. 3 Exemplary flexure hinges and the coordinate definition: (*a*) typical notch flexure hinges, (*b*) multi-axis flexure hinges, and (*c*) coordinate system of planar flexure hinges

empirical equations; the former can be subdivided again into Castigliano's second theorem [139–145], unit load method in mechanics of materials [69,146–149], and inverse conformal mapping [150]. Compliance coefficients of single-axis flexure hinges in Eq. (1) can be further solved as the following unified integral form based on analytical modeling methods [53,68]:

$$\begin{cases} c_x = \dfrac{\Delta x}{F_x} = \dfrac{1}{Ed}\displaystyle\int_0^l \dfrac{1}{h(x)}dx & c_y = \dfrac{\Delta y}{F_y} = \dfrac{12}{Ed}\displaystyle\int_0^l \dfrac{x^2}{h^3(x)}dx + \dfrac{E}{\kappa G}\cdot c_x \\ c_\theta = \dfrac{\Delta\theta}{M_z} = \dfrac{12}{Ed}\displaystyle\int_0^l \dfrac{1}{h^3(x)}dx & c_\alpha = \dfrac{\Delta\theta}{F_y} = \dfrac{12}{Ed}\displaystyle\int_0^l \dfrac{x}{h^3(x)}dx \end{cases} \tag{2}$$

where E is the Young's modulus, l and d are the length and out-of-plane thickness of a flexure hinge, G is the shear modulus, κ is the shape factor, $h(x)$ is the in-plane thickness of the profile. It is noticed that the shear effect is included and Eq. (2) would be slightly different from expressions in some literature due to the different coordinate definitions.

3.1 Analytical Formulas. Usually, design of notch-type flexure hinges is confined to small deformation but variable profiles increase the modeling complexity. Smith et al. [66] extended the compliance model in Ref. [54] for elliptic flexure hinges, but the shear effect is not a part of these two studies. Castigliano's second theorem is a popular method for the compliance modeling of flexure hinges due to its straightforward concept and easy operation. For example, closed-form compliance equations for corner-filleted, elliptic, and hyperbolic flexure hinges were derived by Lobontiu and coworkers [67,68,139,140]; compliance equation for V-shaped flexure hinges was built by Tian et al. [141]. Other similar compliance modeling include hybrid flexure hinges by Zhang et al. [142]; semicircular flexure hinges by Horacio et al. [144], L-shape hinges considering high-order shear effects by Nguyen et al. [145] and others [143]. In addition to Castigliano's second theorem, the unit load method is another useful technique for deriving the compliance of flexure hinges [69]. Chen et al. [146,147] unified the profiles of several conic flexure hinges and obtained a generalized compliance model, while profiles of several flexure hinges were fitted as the rational Bessel curve for general formulating of compliance by Vallance et al. [148] and the rational B-spline curve in Ref. [75]. Apparently, compliance models for the same flexure hinge in Refs. [68], [69], and [146] are identical and can be summarized as solving the integrals in Eq. (2) for different profiles of flexure hinges.

3.2 Empirical Equations. Empirical equations by finite element analysis are favorable considering the fact that few of the previous closed-form formulas can accurately predict the compliance characteristics of flexure hinges in a large range of geometric parameters. Smith et al. [151] developed empirical equations for the compliance of circular flexure hinges. Schotborghet al. [152] derived empirical equations for circular and corner-filleted flexure hinges with dimensionless design graphs. Yong et al. [70] compared the accuracy of several compliance models for circular flexure hinges with the finite elemental results as the benchmark and established empirical compliance models for use in a large range of geometric parameters. Other empirical compliance models for corner-filleted and multinotched flexure hinges can refer to those in Refs. [153–155].

3.3 Other Works. Complex flexure hinges are combination of simple flexures, which can act as revolute and prismatic joints [138]. For the compliance modeling of multi-axis flexure hinges such as the Cartwheel flexure hinge, some authors have suggested the pseudo-rigid-body model [105,106], compliance matrix method [156] and Ryu's method [157]. Other contributions to the compliance formulating of flexure hinges, among many, were those investigated by Lobontiu et al. [158–161], wherein a general model of compliance was established for segmentally symmetric and curve-axis flexure hinges. Recently, the dynamic-PRBM and a dynamic stiffness matrix were separately developed by Su et al. [108] and Ling et al. [162] for the purpose of dynamic analysis of large-deflection flexure hinges and notch flexure hinges with varying cross section.

3.4 Discussion on this Study. For notch-type flexure hinges, the essence of compliance modeling is to solve similar integrals in Eq. (2), while compliance formulating of complex multi-axis flexure hinges can be regarded as a modeling issue of compliant mechanisms. Castigliano's second theorem and the unit load method are two straightforward approaches for the compliance modeling of notch flexure hinges. However, depending on the geometric aspect ratio of flexure hinges, large deviation exists among different compliance models and even in components of the same model. Several compliance models for circular flexure hinges have been compared in a phenomenological sense [70]. However, the influence factors should be further investigated. For instance, even the shear effect has already been considered, the prediction accuracy in some previous compliance models still deteriorates with the increase of the minimum thickness of flexure hinges meaning other factors would still exist limiting the modeling accuracy; for another example, different components in one compliance matrix derived by the same method have discrepant prediction errors [66,69,70]. Some researchers recommended tailoring accurate components from different compliance matrices [163]. However, it relies on experience and is intractable for designing new flexure hinges. Empirical modeling is a reliable way to analyze flexure hinges with high accuracy but it is time-consuming, noninsightful and nonuniversal for a new type of flexure hinges.

4 Kinetostatic Modeling of Compliant Mechanisms With Small Deflection

There have been a considerable number of publications on the kinetostatic modeling of small-deflection compliant mechanisms [5–12,164]. The implementation complexity on this issue lies in the difficulty to deal with serial-parallel configurations often existing in compliant mechanisms [5–7]. In general, the kinetostatic modeling of compliant mechanisms within the regime of small deflection falls into four categories: Castigliano's second theorem, elastic beam theory, compliance matrix method, and finite element method (matrix displacement method).

4.1 Castigliano's Second Theorem

4.1.1 Conceptual Idea. The key procedure of Castigliano's second theorem for the kinetostatic modeling of compliant mechanisms is to solve the total strain energies of all flexure members in compliant mechanisms, like the tensile, shear, and bending strain energies. Input and output displacements of compliant mechanisms can be calculated as the first-order differential of the total strain energy with respect to their corresponding external force; this procedure can be formulated as

$$U = \sum_{i=1}^{n}\left(\int_0^{l_i}\frac{N_i^2(x)}{2EA_i(x)}dx + \int_0^{l_i}\frac{S_i^2(x)}{2\kappa GA_i(x)}dx + \int_0^{l_i}\frac{M_i^2(x)}{2EI_i(x)}dx\right) \quad (3)$$

$$u_{\text{in}} = \frac{\partial U}{\partial f_{\text{in}}} = F_1(f_{\text{in}}, f_{\text{out}}), \quad w_{\text{out}} = \frac{\partial U}{\partial f_{\text{out}}} = F_2(f_{\text{in}}, f_{\text{out}}) \quad (4)$$

where A_i and I_i are the area and moment of inertia of the cross section of the ith flexure member. n is the total number of flexure members. F_1 and F_2 are the notation of functions. $N_i(x)$, $S_i(x)$, and $M_i(x)$ are the inner tensile force, shear force, and bending moment of the ith flexure member as shown in Fig. 4, which should be solved in advance as the function of external loads based on force equilibrium equations and other necessary displacement boundary conditions. In some cases, there is no external output force excited on compliant mechanisms; thus, a dummy force is often assumed and setting it to zero later [76].

4.1.2 Key Advances. Castigliano's second theorem was employed to formulate the kinetostatics of bridge-type compliant amplifying mechanisms by Lobontiu and Garcia [76] and was later widely used in the field of compliant mechanisms [165,166]. From Ref. [76], one can observe that modeling procedures and the resulting model are somewhat complicated even for such a simple configuration. Even so, Castigliano's second theorem is very useful for some applications with the characteristics of guiding flexure beams. For example, input/output stiffness of several XYZ flexure-based manipulators were analyzed based on Castigliano's second theorem, respectively, by Yong and coworkers [167–169], Kenton and Leang et al. [170] and Gu and coworkers [171]. Moreover, as stated in Sec. 3, Castigliano's second theorem was frequently employed for the compliance modeling of all kinds of notch-type flexure hinges [139–145]. In the case of more complicated design, Ueda et al. [172,173] derived the force–deflection relationship of a flexure gripper to measure its tip force based on Castigliano's second theorem; they also employed this method to build the two-port force–displacement model of a nested multistage flexure amplifier [174,175]. Other applications of this method can be found in the literature, such as Yeom et al. [176] for an elliptic-type amplifier; Du et al. [177] for a 6DOFs flexure mechanism as well as a vertical nanopositioner in Ref. [120].

4.1.3 Discussion on this Study. The key feature of Castigliano's second theorem is the concise form of strain energy, making it particularly suitable for flexure-beam-guided compliant mechanisms with simple configurations, such as plenty of XYZ flexure-based manipulators in Refs. [167–171]. However, it becomes somewhat complicated to implement inner-force analysis for compliant mechanisms with complex configurations.

4.2 Elastic Beam Theory

4.2.1 Conceptual Idea. The key procedure of elastic beam theory for the kinetostatic modeling of compliant mechanisms is similar to Castigliano's second theorem in terms of energy form. The first step is normally to express the inner force of each flexible beam as the function of external loads. Constrained reactions should be also calculated in advance as the function of external loads by virtue of boundary conditions. Then, the deflection formula and the principle of energy conservation or the principle of virtual work [178] in mechanics of materials can be employed to formulate the kinetostatics of compliant mechanisms [80]

$$M_i(x) = \text{EI}\cdot y_i'' \quad (5)$$

$$\frac{1}{2}f_{\text{in}}\cdot x_{\text{in}} - \frac{1}{2}f_{\text{out}}\cdot x_{\text{out}} = \sum_{i=1}^{n}\left(\int_0^{l_i}\frac{N_i^2(x)}{2EA_i(x)}dx + \int_0^{l_i}\frac{S_i^2(x)}{2\kappa GA_i(x)}dx + \int_0^{l_i}\frac{M_i^2(x)}{2EI_i(x)}dx\right) \quad (6)$$

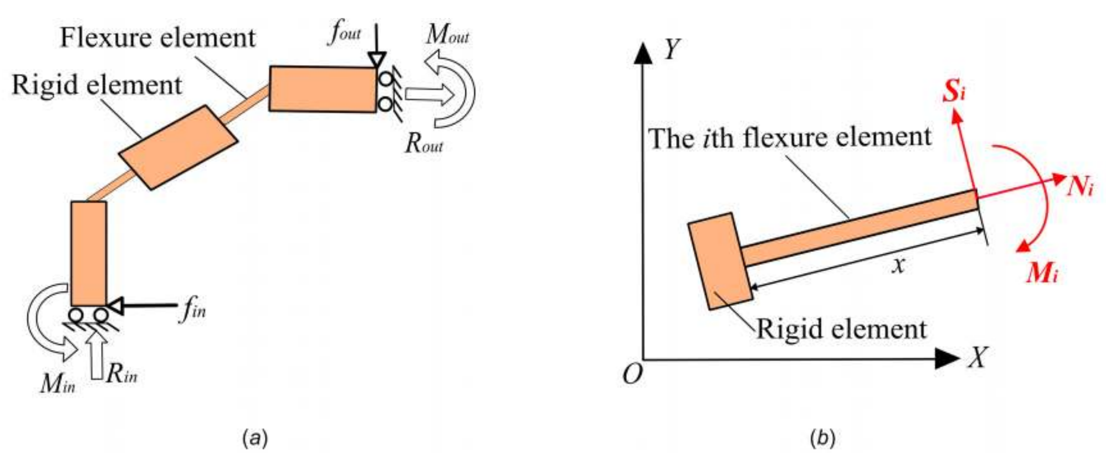

Fig. 4 Kinetostatic modeling based on Castigliano's second theorem: (*a*) exemplary complaint mechanism and (*b*) notation of force in local flexure element

where y_i is the deflection of the ith flexible beam. The prime denotes the derivative with respect to spatial coordinate x. Other variables in Eqs. (5) and (6) are denoted in Fig. 4. In Eq. (6), the key idea of the principle of energy conservation is that work done by the input force along its corresponding input displacement is equal to the tensile, shear, and bending strain energies as well as the opposite work by the output force.

4.2.2 Key Advances. Elastic beam theory was widely used for bridge-type compliant mechanisms [179–182]. Ma et al. [77] derived the analytical formula of the displacement amplification ratio for bridge-type compliant mechanisms based on elastic beam theory. Subsequently, plenty of similar analytical formulas sprung up, such as those by Mottard and St-Amant [183], Hwang et al. [184], Xu and Li [185], Chen et al. [79], Shao et al. [186], Liu and Yan [187], and Higuchi and Higuchi [188]. In view of the low prediction accuracy of some previous models, Qi et al. [78] and Ling et al. [80] separately proposed enhanced analytical formulas based on elastic beam theory. Moreover, other investigations were dedicated to this problem by accounting for the compliance of all members [81,189,190]. Wei and Shirinzadeh [191] proposed a general model for kinetostatic analyzing of three kinds of bridge-type flexure amplifiers. Ling [192] extended this general model for simultaneously analyzing their kinetostatics and dynamics with a concept of two-port dynamic stiffness model. The existing kinetostatic formulas for bridge-type compliant mechanisms are listed in Table 1. Detailed comparison on their prediction accuracy can be found in Ref. [192]. As another typical application, the kinetostatics of parallel four-link flexure mechanisms was analyzed with elastic beam theory in Refs. [193] and [194]. Other studies such as those in Refs. [195] and [196] can be also regarded as a variation of elastic beam theory.

4.2.3 Discussion on this Study. Elegant analytical formulas can be obtained with elastic beam theory that is insightful for revealing the deformation characteristics of compliant mechanisms. The major disadvantage of this method, however, lies in its complicated inner-force analysis for complex configurations. Hence, some members in compliant mechanisms, such as the input port, are often assumed to be rigid for the conciseness of modeling but with some loss of accuracy [77–81,192].

4.3 Compliance Matrix Method

4.3.1 Conceptual Idea. The basic idea of this method is to transfer the compliance of each flexure member from the local coordinate frame into an assigned reference coordinate system. Kinetostatics of compliant mechanisms can be modeled based on the principle of compliance summation in the serial chain and stiffness summation in a parallel structure. As shown in Fig. 5, the output compliance matrix with respect to the fixed end for serial, parallel and hybrid chains can be formulized as [82,83]

$$\boldsymbol{C}_j^o = \sum_{i=1}^{n} \boldsymbol{T}_i^j \boldsymbol{C}_i (\boldsymbol{T}_i^j)^T \tag{7}$$

$$\boldsymbol{C}_j^o = \left(\sum_{i=1}^{n} (\boldsymbol{T}_i^j \boldsymbol{C}_i (\boldsymbol{T}_i^j)^T)^{-1} \right)^{-1} \tag{8}$$

$$\boldsymbol{C}_A^o = \boldsymbol{T}_1^A \boldsymbol{C}_1 (\boldsymbol{T}_1^A)^T + \left[\left(\sum_{i=2}^{4} \boldsymbol{T}_i^A \boldsymbol{C}_i (\boldsymbol{T}_i^A)^T \right)^{-1} + \left(\sum_{i=5}^{8} \boldsymbol{T}_i^A \boldsymbol{C}_i (\boldsymbol{T}_i^A)^T \right)^{-1} \right]^{-1} \tag{9}$$

where C_i is the compliance matrix of the ith flexure member in the local coordinate. The coordinate transformation matrix $\boldsymbol{T}_i^j$ can be found in the literature such as Refs. [82] and [83].

4.3.2 Key Advances. The compliance matrix method can be recognized as an efficient toolkit for a wide range of compliant mechanisms with complex configurations [198–203]. Pham and Chen [198] employed the compliance matrix method for parallel compliant mechanisms, while this method was extensively used to analyze flexure-based precision positioning stages by the groups of Li, Xu [82–84,199] and other researchers [204,205]. However, it is difficult to obtain the detailed displacements in compliant mechanisms. To overcome this limitation, the effort has gone into combining the compliance matrix method with the inverse-kinematic model [202]. It is noted that the input/output

Table 1 Theoretical models of the displacement amplification ratio for bridge-type compliant mechanisms

Methods	Refs.	Analytical models of displacement amplification ratio	Configurations
Simplified model	[77]	$R_{\text{amp}} = \cot\theta$	
	[76]	$R_{\text{amp}} = (\sqrt{L^2 \sin^2\theta - 2L\Delta x \cos\theta - \Delta x^2} + L\sin\theta)/\Delta x$	
	[197]	$R_{\text{amp}} = \left\lvert \dfrac{\sin\theta - \sin(\theta - \Delta\theta)}{\cos\theta - \cos(\theta - \Delta\theta)} \right\rvert$	
	[78]	$R_{\text{amp}} = \dfrac{\int_{\theta-\Delta\theta}^{\theta} \cot u du}{\Delta\theta} = \ln(\dfrac{\sin\theta}{\sin(\theta - \Delta\theta)}) \Big/ \Delta\theta$	
Elastic beam theory	[48]	$R_{\text{amp}} = \dfrac{K_l L^2 \sin\theta \cos\theta}{2K_\theta \cos^2\theta + K_l L^2 \sin^2\theta}$	Parallel
	[77]		Parallel
	[188]		Rhombic
	[185]	$R_{\text{amp}} = \dfrac{K_l L^2 \sin\theta \cos^3\theta}{2K_\theta + K_l L^2 \cos^2\theta \sin^2\theta}$	Parallel
	[78]	$R_{\text{amp}} = \dfrac{K_l LH}{4K_\theta + K_l H^2}$ $(H = L\tan\theta)$	Parallel
	[80]	$R_{\text{amp}} = \dfrac{K_l L^2 \sin\theta \cos\theta}{12K_\theta \cos^2\theta + K_l L^2 \sin^2\theta}$	Rhombic
	[80]	$R_{\text{amp}} = \dfrac{K_l LH + K_l K_\theta H L_2 / 2K'_\theta}{(4K_\theta + K_l H^2) + 2K_l K_\theta (1/K'_l + H^2/4K'_\theta)}$	Parallel
	[186]	$R_{\text{amp}} = \dfrac{L}{2K_\theta \cot\theta / K_l L + L\tan\theta}$	Rhombic
Others		[81,189–192]	

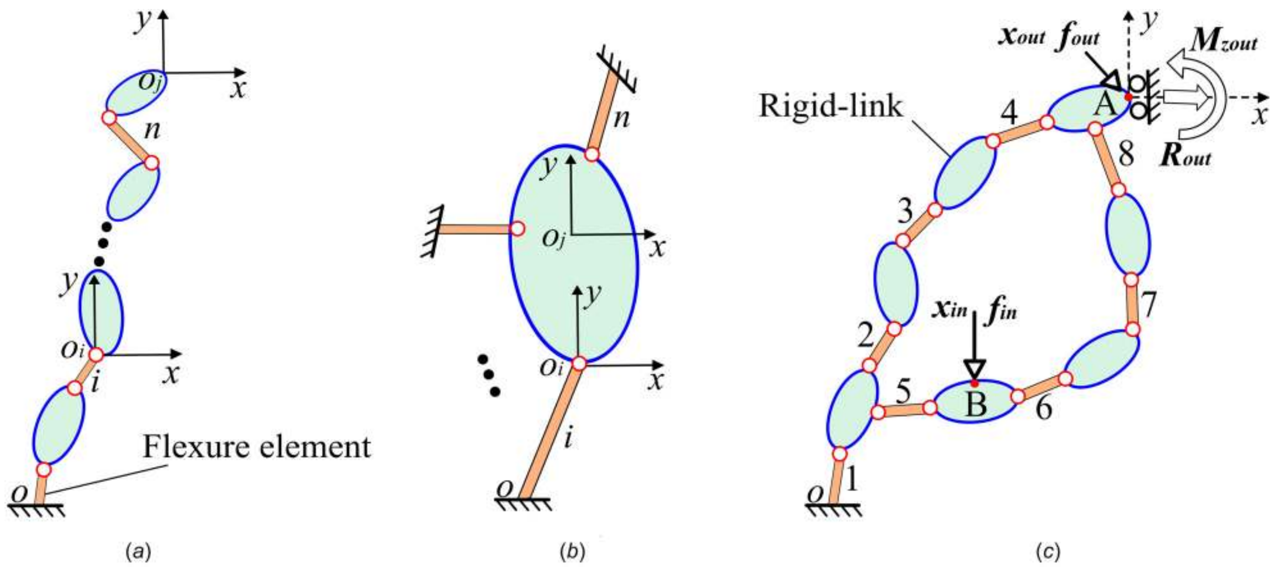

Fig. 5 Exemplary kinetostatic modeling with the compliance matrix method: (*a*) serial, (*b*) parallel, and (*c*) serial-parallel

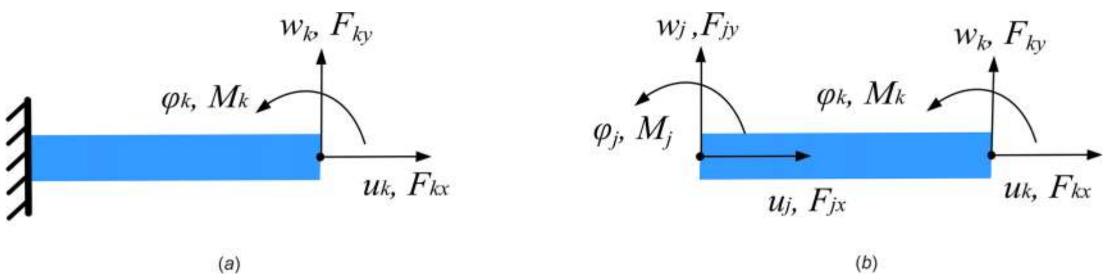

Fig. 6 Comparison of coordinate frame between the compliance matrix method and the finite element method (matrix displacement method): (*a*) compliance matrix method and (*b*) matrix displacement method

displacements coincide with the points of actuation force in the above investigations [198–205]. The modeling procedure will become more complicated with multiple actuation forces, and when the solved displacement is not coincident with the points of actuation force. Lobontiu and coworkers [87,88] provided investigations on the kinetostatic modeling of serial-parallel compliant mechanisms with multiple forces and the case of displacements at different points of actuation force. A general two-port kinetostatic model of compliant mechanisms was also established based on the compliance matrix method in Ref. [205].

4.3.3 Discussion on this Study. The compliance matrix method has emerged as a versatile kinetostatic modeling technique for compliant mechanisms with complex configurations. Compared to Castigliano's second theorem and elastic beam theory, there is no need for inner-force analysis, thus the compliance matrix method by nature possesses some inherent advantages in terms of conciseness. This merit is particularly useful for serial-parallel compliant mechanisms. On the other hand, coupled compliance between the shear force and bending moment is considered in the compliance matrix method. One challenge of this method, however, lies in the fact that the output and input stiffness are separately modeled with reduplicative procedures. In addition, the modeling procedure becomes complicated with multiple actuation forces and when the solved displacements are not coincident with the points of actuation force [87,88].

4.4 Finite Element Method (Matrix Displacement Method)

4.4.1 Conceptual Idea. The finite element method has been employed in the field of compliant mechanisms for a long time [206–208]. The underlying idea of this method is also based on matrix transformation but is different from the compliance matrix method (Fig. 6). It can be viewed from Fig. 6 that the number of degrees-of-freedom for planar flexure beams is six in the finite element method, while the flexure beam is usually considered to be fixed at the end of the preceding rigid block in the compliance matrix method.

4.4.2 Key Advances. Considering the varying cross section in all kinds of notch flexure hinges, a mathematic formula transferring the theoretical compliance matrix into the elemental stiffness matrix of flexure hinges was proposed by Wang and Zhang [90]. This formula is advantageous in the framework of the finite element method and was later applied to all kinds of flexure-hinge-based compliant mechanisms [91,209]. Li and Hao [210] derived the elemental stiffness matrix of flexible beams based on screw theory. Ling et al. [209] presented a straightforward kinetostatic modeling approach for serial-parallel compliant mechanisms by directly building the nodal force equilibrium equation without extra procedures of assembling the global stiffness matrix. A tree-structure method similar to the finite element method was proposed in Ref. [211] to transfer the loads into the local flexure element but not the common way of stiffness matrix assembling in the finite element method. Some other modeling methods discussed in Ref. [212] can be also regarded as variants of the finite element method.

4.4.3 Discussion on this Study. The obvious benefit of the finite element method is that it can be applied to a wide class of compliant mechanisms and has higher prediction accuracy over other kinetostatic modeling approaches. Moreover, the inner displacement information can be obtained with single or multiple

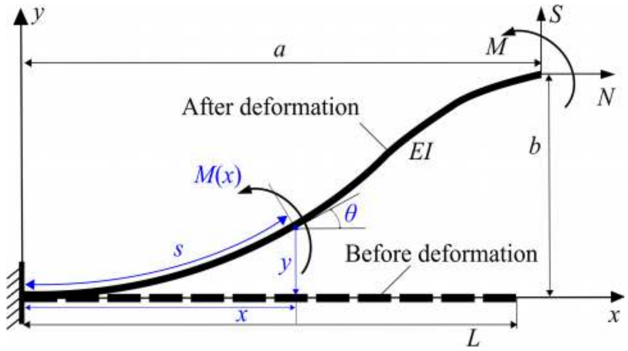

Fig. 7 Conception of the pseudo-rigid-body model: (*a*) continuum model of large-deflection beams and (*b*) corresponding pseudo-rigid-body model

actuation loads. However, degrees-of-freedom in the finite element model are usually huge.

5 Kinetostatic Modeling of Compliant Mechanisms With Large Deflection

Compliant mechanisms with large deflection have been popular research topics [1,37,58,94]. In general, the present kinetostatic modeling methods for large-deflection analysis of compliant mechanisms can be summarized as six categories: (1) pseudo-rigid-body model, (2) beam constraint model, (3) elliptic integral solutions, (4) nonlinear finite element method, (5) chain algorithm, and (6) energy-minimization-based solutions. Some of these methods are aimed at single flexure beams while others can deal with compliant mechanisms. It should be noticed that the pseudo-rigid-body model, initially proposed for large-deflection problems, has been also widely used for modeling small-deflection compliant mechanisms.

Figure 7 provides the kinetostatic model of a large-deflection beam subjected to the tip loads of tensile force N, shear force S and bending moment M. The bending equation can be generally formulated as

$$M(x) = \mathrm{EI}\frac{\mathrm{d}\theta}{\mathrm{d}s} = \mathrm{EI}\frac{\mathrm{d}^2y/\mathrm{d}x^2}{\left[1+(\mathrm{d}y/\mathrm{d}x)^2\right]^{3/2}} \tag{10}$$

Equation (10) can be simplified as a linear model if the term "dy/dx" is ignored for small-deflection analysis. In addition, the force balance equation of large-deflection flexure beams should be described after deformation, i.e.,

$$\begin{cases} M(x) = M + S\cdot(L-x) & \text{(small deflection)} \\ M(x) = M + S\cdot(L-x) - N\cdot(b-y) & \text{(second-order theory)} \\ M(x) = M + S\cdot(a-x) - N\cdot(b-y) & \text{(large deflection)} \end{cases} \tag{11}$$

Equation (10) together with Eq. (11) describes the kinetostatic model of flexure beams under different levels of deflection. The key problem associated with the kinetostatic analysis of large-deflection beams is to solve the deflections $\{a, b\}$ and the slope θ under the tip loads $\{N, S, M\}$ by solving Eqs. (10) and (11) as well as necessary axial-deformation equations which are not shown here. It is noticed that the constitutive relationship of materials in Eq. (10) are usually isotropic and homogeneous where the conventional elastic beam theory is applicable. Nonconventional materials such as plastic may be used in compliant mechanisms and their constitutive relationship is different from Eq. (10).

5.1 Pseudo-Rigid-Body Model

5.1.1 Pseudo-Rigid-Body Model of Flexure Beams at the Element Level. Previous research on the pseudo-rigid-body model was mainly for three types of flexure members: Fixed-free beams, short flexure pivots, and fixed-guided beams [58–60]. Figure 8 shows a fixed-free flexure beam with its pseudo-rigid-body model. The beam is decoupled into a joint with a torsional spring and two rigid-link bars. Its deflection path is emulated by kinematic trajectory of rigid-link mechanisms, while the force–deflection relationship is approximated by the spring that represents the pivot's stiffness. The key issue is to find the position of the characteristic pivot and the characteristic stiffness of spring. The detailed procedure is to approximate the tip trajectory $\{a, b\}$ and slope θ of the pseudo-rigid-body model subjected to external loads $\{F, M\}$ with respect to the exact continuum model by means of optimization strategies [1,58].

Howell and coworkers [58–60] developed the pseudo-rigid-body model and provided the characteristic parameters for several kinds of flexure beams subject to different end-force loadings based on the elliptic integral solutions [1]. Since one rotational pivot is used in the original pseudo-rigid-body model, it is called the 1R model. The pseudo-rigid-body model with variable parameters and the case of different types of beams were studied by Dado [213], Lyon [214], and Kimball and Tsai [215]. In an attempt to overcome the limitation of dependence of the 1R model on the types of end-force loading, several variations of the pseudo-rigid-body model were developed. For example, Su [101] presented a 3R pseudo-rigid-body model with high accuracy for a larger range of deflection and with load-independence coefficients, wherein the flexure beam was divided into four rigid segments with three elastic joints. A set of new characteristic parameters of the 3R pseudo-rigid-body model was re-optimized by Chen et al. [216] based on a new multi-objective optimization strategy.

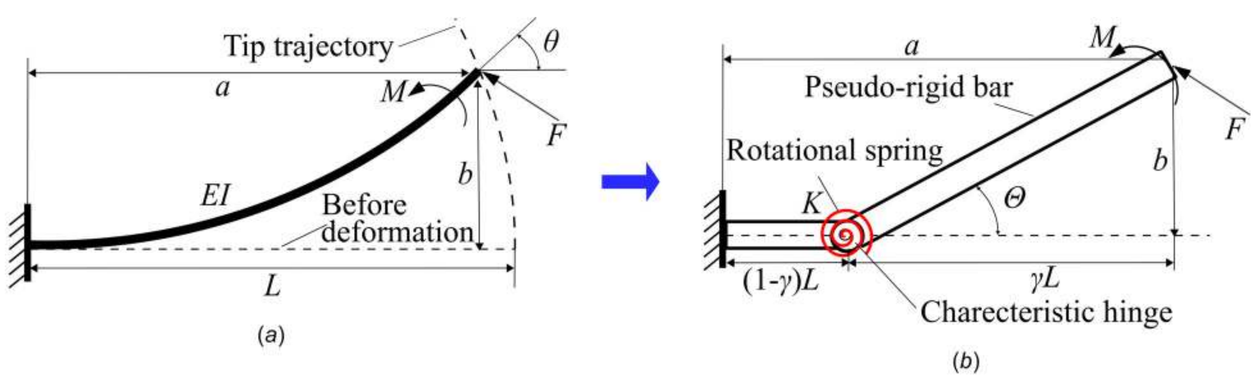

Fig. 8 Mechanical model of large-deflection flexure beams: (*a*) continuum model of large-deflection beams and (*b*) corresponding pseudo-rigid-body model

As reported, the 3*R* model [101,216] can predict the kinematic behaviors of large-deflection beams subjected to multiple end forces. However, the 3*R* model becomes more complicated owing to the introduction of three pivots. Yu et al. [102] reduced the joints and proposed a 2*R* pseudo-rigid-body model. In addition to the 2*R* and 3*R* models, a RPR model considering tensile effects and a 5*R* model for offsetting inflection points were also developed [103,104,217]. Besides, the pseudo-rigid-body model with shear effects in short pivots for robotic joints [218], circular-shape flexure beams [219] and a general matrix for all kinds of pseudo-rigid-body models [220] were studied by Su and Venkiteswaran. Saggere and Kota [221] introduced a finite element type of model in which the beam was divided into more than three segments joined by torsional springs. Other developments on the pseudo-rigid-body model can be found in the works by Šalinić and Nikolić [222] and Valentini and Pennestrì [223].

5.1.2 Applying the Pseudo-Rigid-Body Model to Compliant Mechanisms. The key idea of utilizing the pseudo-rigid-body model for the analysis of compliant mechanisms is to substitute the flexure pivots and flexible beams in compliant mechanisms with the PRBM parameters, while other parts of compliant mechanisms are assumed as rigid bodies. As a consequence, the pseudo-rigid-body model can be considered as a bridge connecting compliant mechanisms with rigid-link mechanisms. Modeling of compliant mechanisms can thus be solved in the framework of the traditional rigid-body mechanics.

Howell and Midha [224] presented in detail the pseudo-rigid-body model for the kinetostatic analysis of compliant mechanisms based on the loop closure theory and the principle of virtual work. Lyon et al. [225] established the kinetostatic model of parallel-guiding and slider-crank compliant mechanisms based on the pseudo-rigid-body model, while Yu and coworkers [114,226] built the PRBM-based dynamic model of large-deflection parallel-guided compliant mechanisms. Other considerable investigations include those such as partially compliant mechanisms by Tanik et al. [227], bistable compliant mechanisms by Pucheta and Cardona [228], constant-force mechanisms by Aten et al. [229], Cartwheel flexure hinges by Pei and coworkers [105,106], to mention a few. A computational design tool of compliant mechanisms for personalized animatronics was also developed by Disney Research with the pseudo-rigid-body model [230]. In addition to compliant mechanisms, the pseudo-rigid-body model was also applied to analyze the large deflection of carbon nanotubes [231,232] and human spines to predict implant-induced changes on motion [61].

Experiences in the last decades have also shown the pseudo-rigid-body model can be a potentially efficient tool for some types of small-deflection compliant mechanisms. For example, the pseudo-rigid-body model has been widely used for the kinetostatic and dynamic analyses of flexure grippers by modeling flexure hinges as equivalent joints with a spring, such as several types of flexure-based grippers designed by Chen and coworkers [233,234], Wang et al. [235], Tian and coworkers [117,236,237], and so on. Other pioneering works were applying the pseudo-rigid-body model to the design and analysis of precision positioning stages by Li and coworkers [238,239], Wan and Xu [240], Tian and coworkers [241–243], Liu et al. [244], and so forth.

5.1.3 Discussion on this Study. The benefit in the use of the pseudo-rigid-body model comes from transmitting compliant mechanisms into equivalent rigid-body mechanisms. This facilitates the use of the wealth of existing rigid-body mechanics knowledge for the solution of compliant mechanisms. The use of pseudo-rigid-body model provides a quick way to test concepts and thus reduces the effort to obtain final concepts. Although the 2*R*, 3*R*, RPR and 5*R* pseudo-rigid-body models are more versatile, the 1*R* model was widely used for compliant mechanisms owing to its simplicity. However, the load dependency in the 1*R* model makes it not well-suited for complex loads and complicated configurations as well as for free vibration analysis of compliant mechanisms. On the other hand, procedures of the pseudo-rigid-body model for modeling compliant mechanisms is usually performing the kinematic solution with loop closure theory or kinematic approximation; then, carrying out the static analysis by the virtual work principle and at last establishing the dynamic model based on Lagrange's equation. It shows some complicacy for complex configurations. Therefore, kinematic approximations and mass lumping are usually adopted with limited accuracy. Moreover, characteristic parameters in the pseudo-rigid-body model were optimized under kinematic conditions which may be inaccurate for the dynamic analysis of compliant mechanisms, especially for high-frequency solutions.

5.2 Other Methods

5.2.1 Beam Constraint Model. The beam constraint model proposed by Awtar and coworkers [93–96] provides a closed-form model based on the continuum beam theory and Taylor series expansion. Since a linear form of Eq. (10) was used in the beam constraint model, it is mainly suitable for flexure beams within an intermediate deformation range (10% of the beam length). The characteristics of this model lies in the captured load-stiffening effects [94]. Furthermore, the constraint behavior of flexure beams in terms of their stiffness and error motion is specified with this method [245]. Zhao et al. [246] developed the analytical static model for the Cartwheel flexure hinge while Malaeke and Moeenfard [245] investigated the mixed flexure-rigid-link mechanisms with the beam constraint model. Recently, Chen et al. included the shear effect [98] and developed the beam constraint model for large-deflection analysis of planar and spatial flexure beams, namely, the chained beam constraint model [99,100], in which a flexible beam was divided into a few elements and each element was modeled by the beam constraint model. The second-order Taylor series in the beam constraint model was expanded to the third order [247]. Moreover, the beam constraint model was used to analyze grippers and accelerometers [248,249]. The (chained) beam constraint model formulates medium/large-deflections by capturing load-stiffening effects. Previous investigations on this method were mainly focused on single beams or simple compliant mechanisms. Further applying it for more complex compliant mechanisms with intermediate/large deflections, such as the increasingly used large-stroke flexure-based manipulators, is still an open problem.

5.2.2 Nonlinear Finite Element Method and the Chain Algorithm. The finite element method can deal with complex geometric shapes by discretizing the structure into small elements. Kinetostatic and dynamic modeling of small-deflection compliant mechanisms based on the finite element method is easy to be implemented [92,125]. However, formulizing the global stiffness matrix will become complicated with time-consuming iterative computations for nonlinear large-deflection analysis. Thus, there were limited parametric modeling cases of compliant mechanisms by employing the nonlinear finite element method. In general, the nonlinear finite element method is implanted into commercial software packages such as ANSYS and ABAQUS. On the other hand, the chain algorithm [57,250] also discretizes a structure into small elements, but unlike the finite element method, elements in the chain algorithm are treated in succession with no requirement of the inversion of assembled stiffness matrix [251]. The shooting method was often used to satisfy boundary conditions in the chain algorithm [251]. However, accuracy of the chain algorithm still depends on the resolution of discretization and the inserting interval of loads. Salamon [252] utilized a graphical, user-driven iterative technique for better convergence of the chain algorithm, while Lan and Coulter [251,253] introduced an increment-loading method into the chain algorithm to improve its accuracy. In addition, a chain algorithm element was created from pseudo-rigid-body segments and used in a chain calculation to accurately predict the force–deflection relationship of flexure beams with large deflection by Chase et al. [254]. It is noticed that investigations and applications on the chain algorithm were mainly focused on single beams but less for compliant mechanisms [255,256].

5.2.3 Elliptic Integral Solutions. Owing to its high accuracy and analytical form for the kinetostatic analysis of flexure beams, the elliptic integral method is a robust solution in the field of compliant mechanisms. The key idea is to express the solutions of large-deflection bending equations, namely, Eqs. (10) and (11), as the integral of trigonometric functions and the integral solutions can be obtained by table look up [55,56]. The elliptic integral solutions of fixed-fixed beams was provided by Lyon and Howell [257], while Kimball and Tsai deduced the elliptic integral for flexure beams with one inflection point [55]. Zhang and Chen et al. [258] established a complete elliptic integral solution for flexure beams with any number of inflection points. More recently, kinetostatics of an XY micro positioning stage with the negative stiffness mechanism was analyzed by Wang and coworkers [44,259] based on the elliptic integral solutions. From the previous advances, elliptic integral solutions were mainly limited to single beams. Research described in Refs. [44] and [259] are examples for application in compliant mechanisms. In addition, elliptic integral solutions were frequently applied to optimize the characteristic parameters in the pseudo-rigid-body model due to its high accuracy [58,59,101,102,214].

5.2.4 Energy-Minimization-Based Kinetostatic Solutions. In view of the difficulties associated with the kinetic solutions in previous mechanics-model-guided methods for large-deflection compliant mechanisms, attention was devoted to the kinetostatic modeling of compliant mechanisms in the presence of large deflections by using energy-minimization-based approaches. For example, an investigation by the group of Su was devoted to the kinetostatic analysis of large-deflection compliant mechanisms based on the principle of minimum potential energy and using optimization strategies [109–111]. Chen and Ma [112] also provided a framework for the kinetostatic analysis of large-deflection compliant mechanisms based on the principle of minimum potential energy. Other similar solutions can be found in Refs. [260] and [261]. Energy-minimization-based methods are generally kinetostatic solutions for large-deflection compliant mechanisms without solving mechanical equations, but how to extend it to the dynamic issues would be difficult and is still an interesting open problem.

6 Dynamic Modeling of Compliant Mechanisms

Increasing applications of compliant mechanisms are extended to high speeds and high frequencies [5–12,262], thus determining their dynamic behavior is necessary and interesting. This can be crucial for evaluating/optimizing natural frequencies and for designing controllers. To sum up, many of the previous dynamic modeling of compliant mechanisms was mainly based on Lagrange's method [113–130]. Some improved modeling approaches [108,131,132] were recently developed for the dynamic analysis of compliant mechanisms. In the following, recent advances on the Lagrange-based modeling methods and these newly emerging approaches will be discussed in detail.

6.1 Lagrange-Based Methods. Over the past three decades, Lagrange-based dynamic modeling approaches have been developed for compliant mechanisms [113–130]. Generally speaking, the reported approaches can be roughly classified into three categories, as shown in Fig. 9.

6.1.1 PRBM-Based Dynamic Model. In the case of pseudo-rigid-body model, compliant mechanisms are first transmitted into rigid-link mechanisms by substituting flexure pivots and flexible beams with the pseudo-rigid-body model and its corresponding characteristic parameters. The dynamic model can thus be established by calculating the kinematics with the loop closure theory, then performing the static analysis with the principle of virtual work and at last calculating the elastic/kinetic energies in sequence. The detailed procedure can be found in literatures such as Refs. [113–118]. It is noticed that approximate kinematic relationships between inner members and the input/output motion DOFs are often utilized to avoid the complicated solution of kinematics with the loop closure theory [263,264]. Since the input/output motion DOFs of compliant mechanisms are usually taken as the variables of the PRBM-based dynamic model, it can be generally considered as a lumped-parameter model in such a sense.

6.1.2 Lumped-Parameter Dynamic Model. A popular and elegant methodology for the dynamic modeling of small-deflection compliant mechanisms is the so-called lumped-parameter dynamic model with the input or output motion DOFs as the variables [119–123]. Indeed, the analytical formula of fundamental frequency can be obtained with this method. As shown in Fig. 9, the input or output stiffness of compliant mechanisms is first modeled by a sort of kinetostatic methods, such as the compliance matrix method or Castigliano's second theorem. Afterward, elastic and kinetic energies are calculated in the form of motion DOFs as shown in Eq. (12). At last, the dynamic model of compliant mechanisms can be further derived by employing Lagrange's equation

$$\begin{cases} U = \frac{1}{2}K_{\text{in}} \cdot X_{\text{in}}^2 \text{ or } U = \frac{1}{2}K_{\text{out}} \cdot X_{\text{out}}^2 \\ T = \frac{1}{2}\sum_{i=1}^{n} M_i \cdot \dot{X}_i^2 + \frac{1}{2}\sum_{i=1}^{n} J_i \cdot \dot{\theta}_i^2 = F\left(\dot{X}_{\text{in}}^2\right) \text{ or } T = F\left(\dot{X}_{\text{out}}^2\right) \end{cases} \tag{12}$$

where K_{in} and K_{out} are the input and output stiffness of compliant mechanisms calculated by a sort of kinetostatic methods, respectively. X_{in} and X_{out} are the input and output motion DOFs. The superimposed dot indicates differentiation with respect to the time. M_i is the lumped mass of the ith rigid link or flexure member whose kinetics is prominent. J_i is the rotational moment of the ith rigid link. F is the sign of a function.

In Eq. (12), the elastic energy can be easily calculated with the prepared input or output stiffness at the stage of kinetostatic analysis, differing from the pseudo-rigid-body model wherein the elastic energy is the summation of all characteristic springs [233–244] and kinematic solutions are required for calculating the total elastic energy. On the other hand, the kinetic energy in Eq. (12) is dependent on the kinematics of compliant mechanisms and would be intractable to obtain for complex serial-parallel configurations. Therefore, some of the previous investigations employed approximately kinematic relationships to calculate the kinetic energy [119–123,265].

Polit et al. [122] directly derived the dynamic model of a flexure-based precision positioning stage based on Lagrange' equation, while Ferrira and coworkers [263,264] derived the dynamic model of a parallel flexure mechanism based on Jacobian matrix and Lagrange' equation. The dynamic model and analytical fundamental frequency of several flexure-beam-guided XY and XYZ nanopositioners were directly established by using Lagrange's equation by Yong and coworkers [167–169]. Other similar investigations can refer to the works by Tian and coworkers [265–268] for several kinds of flexure mechanisms. For some compliant mechanisms with complex configurations, exact solutions of kinematics are difficult; thus, approximate kinematics was employed. For example, the dynamic model of bridge-type amplifiers was built with elastic beam theory and approximate kinematic solutions by Nabae et al. [192]. In addition, the kinetostatics and dynamics of several XY precision manipulators were analyzed by combining the compliance matrix method and Lagrange's

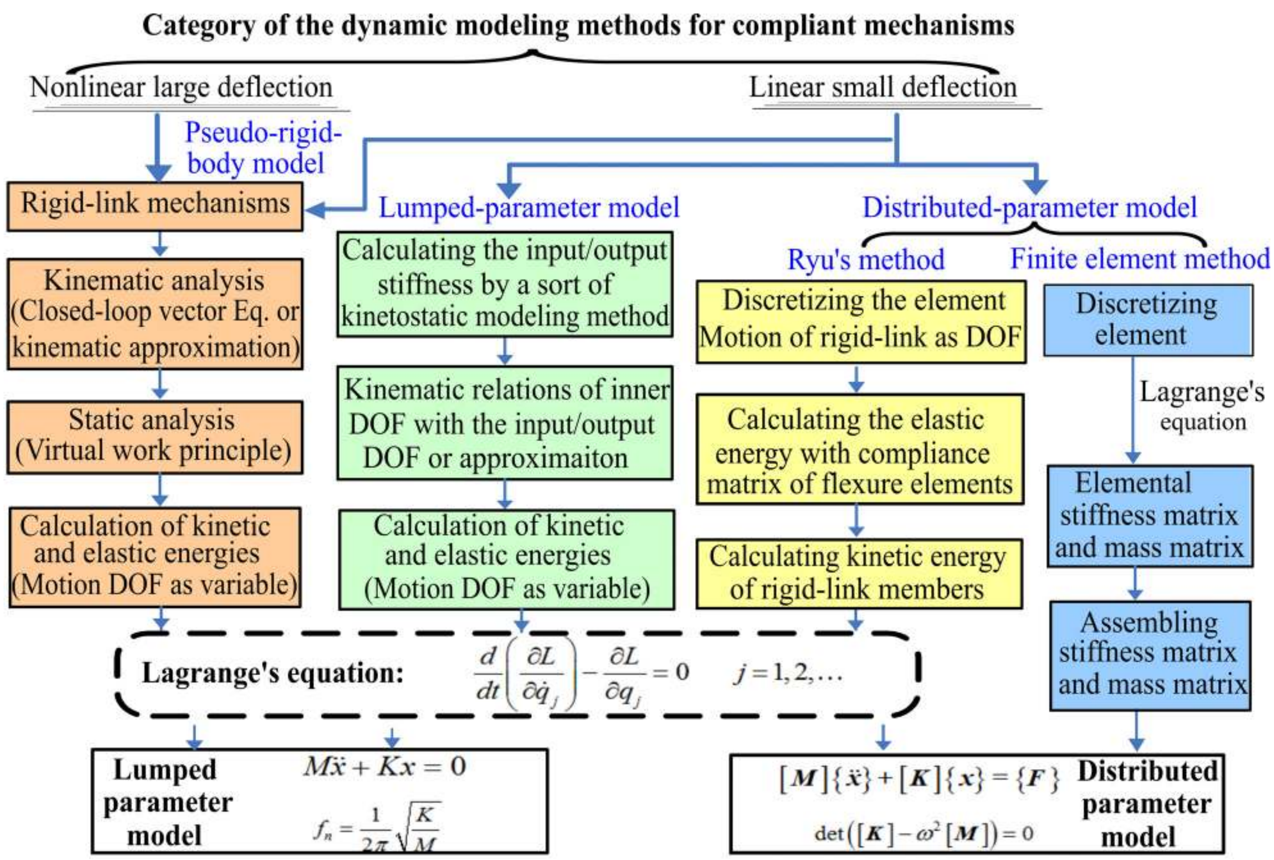

Fig. 9 Dynamic modeling of compliant mechanisms based on Lagrange's method

equation with approximate kinematics by the groups of Li and coworkers [82–84,199,269] as well as other researchers [270–272].

6.1.3 Distributed-Parameter Dynamic Model. In the distributed-parameter model, the detailed DOFs of each flexure member or rigid-body member are taken as the variables, in which compliant mechanisms are usually discretized into several subelements and the dynamic model is established by formulating the total elastic and kinetic energies and combining them with Lagrange's equation [124–130]. In the literature, two different approaches can be found for the distributed-parameter dynamic formulation of compliant mechanisms, namely, the finite element method and a rigid-multibody-similar dynamic model introduced by Ryu et al. [126].

The finite element method is versatile in handling complex configurations, leading itself well to model the kinetostatics and dynamics of compliant mechanisms. Lobontiu et al. [161] and Zhang and Hou [273] derived three-node mass matrix of circular flexure hinges for dynamic analysis of compliant mechanisms, while a force-interpolation-based mass matrix of circular flexure hinges was proposed in Ref. [125]. Rösner et al. [124] proposed an improved finite element method with Krylov model reduction scheme for the purpose of real-time control simulation of compliant mechanisms, while the finite element method based on an analytical stiffness matrix formula was developed for the dynamic modeling of flexure-hinge-based compliant mechanisms by Ling et al. [92]. Other examples of using the finite element method for the dynamic modeling of flexure manipulators can be found in Refs. [274–276].

On the other hand, the use of rigid-multibody-similar dynamic approach proposed by Ryu et al. [126] seems to be a useful way because of its concise modeling procedure. This method was applied to the dynamic modeling of lumped compliant mechanisms or their composed flexure manipulators by several research groups [127–130,277–279]. As shown in Fig. 9, the expression of dynamic model in Ryu's method is closely similar to that of the finite element model at the first glance. However, they are different especially in terms of the notation of variables (DOFs). The variables in Ryu's method are the motion DOFs of rigid links while the variables in the finite element method are nodal displacements between flexure members. This discrepancy leads to distinguishing modeling procedures for these two approaches.

6.1.4 Discussion on this Study. It is pointed out that Lagrange-based dynamic modeling method is a commonly used technique in the field of compliant mechanisms with the concise form of energy. However, the challenge of this method lies in the required kinematic solutions. The coupling of kinematic and elastomechanic behaviors in compliant mechanisms has led to complicated kinetostatic modeling procedures, the whole modeling complexity will be further enlarged when involving the dynamic issues. In addition, the usual practice of kinematic approximation, mass lumping, or even mass neglecting for flexure hinges and flexible beams in some of the previous Lagrange-based dynamic models often led to limited prediction accuracy.

6.2 Other Methods. In addition to the aforementioned Lagrange-based dynamic models, some efforts have been contributed to the dynamic modeling of compliant mechanisms. For example, the transfer matrix method was employed for the frequency analysis of small-deflection compliant mechanisms in Ref. [280], which has the characteristics of easy programming and low matrix order. However, the transfer matrix method is advantageous for serial configurations and the existing solutions are difficult for plenty of the serial-parallel substructures in compliant mechanisms. Moreover, the dynamic modeling procedure in the present transfer matrix method is mutually exclusive for the simultaneous kinetostatic analysis and extra procedures are required.

For the large-deflection analysis of compliant mechanisms, the mass property was included into the pseudo-rigid-body model in

Ref. [108], which can be considered as a dynamic pseudo-rigid-body model. The characteristic parameters of mass for large deflection were optimized based on the linear continuum vibration model in the framework of small deformation in Ref. [108]. Further investigations would be interesting to compare the difference between the linear continuum vibration model and nonlinear large-deflection vibration model.

More recently, a dynamic stiffness modeling methodology based on d'Alembert's principle was proposed by Ling et al. [18,131,281,282] to model the simultaneous kinetostatics and dynamics of compliant mechanisms with small deflection in a static manner. As shown in Fig. 10, two parallel distributed- and lumped-parameter models were established based on the matrix displacement method and an improved transfer matrix method. Indeed, it is a frequency-domain modeling method by employing some concepts in the spectral method [283]. The advantage of this approach lies in the fact that the kinetostatics and dynamics of compliant mechanisms can be simultaneously modeled where the dynamic modeling is simplified as a static-similar problem without the requirements of inner-force analysis and kinematic calculation. Moreover, kinematic approximation and mass lumping are avoided. However, the resulting transcendental or polynomial eigen-problem for the solution of natural frequencies and the difficulty to formulate the dynamic stiffness matrix of some irregular members in compliant mechanisms with lumped compliance are main disadvantages of this method [131,132].

7 Discussion

Flexure-based compliant mechanisms have emerged as an increasingly used technique in modern precision manipulation, robotics, and other engineering applications. Despite the advantageous properties of compliant mechanisms, it is still challenging for researchers to perform accurate and concise modeling of compliant mechanisms owing to their coupling of kinematic and elastomechanical behaviors with large deflections and/or complex serial-parallel configurations. Therefore, more innovative solutions are still required to achieve this goal:

7.1 Accurate Modeling of Flexure Hinges With Variable Cross Section. There have been numerous kinetostatic models for all kinds of flexure hinges. However, large error exists among different analytical kinetostatic models and even in the components by the same modeling method depending on the geometric aspect ratio of flexure hinges [70]. Some other factors in addition to shear effects would influence the modeling accuracy. Empirical modeling is a reliable way to analyze flexure hinges with high accuracy but it is time-consuming and non-insightful for a new type of flexure hinges. On the other hand, only the stiffness of flexure hinges but not their mass was included in some previous dynamic modeling of flexure-hinge-based compliant mechanisms, which would be inaccurate for compliant mechanisms with distributed/hybrid compliance as well as for calculating high-order dynamic responses. Further studies are still pending for clarifying the influence factors on the modeling accuracy of flexure hinges with a wide range of geometric aspect ratio. Moreover, tailoring these influence factors to accurately and concisely formulate the kinetostatics and dynamics for all kinds of flexure hinges is of great importance.

7.2 Efficient Kinetostatic/Dynamic Modeling of Complex Serial-Parallel Compliant Mechanisms. The kinetostatic and dynamic modeling of compliant mechanisms with small deflection have been popular research topics for performance prediction and geometric parameter optimization. Castigliano's second theorem and elastic beam theory are advantageous for relatively simple configurations due to their requirement of inner-force analysis. The compliance matrix method was widely used for the kinetostatic modeling of compliant mechanisms with complex configurations. However, input and output stiffness are usually separately modeled and it will become complicated for the solution of inner displacement with multiple actuation forces [87,88]. As to the

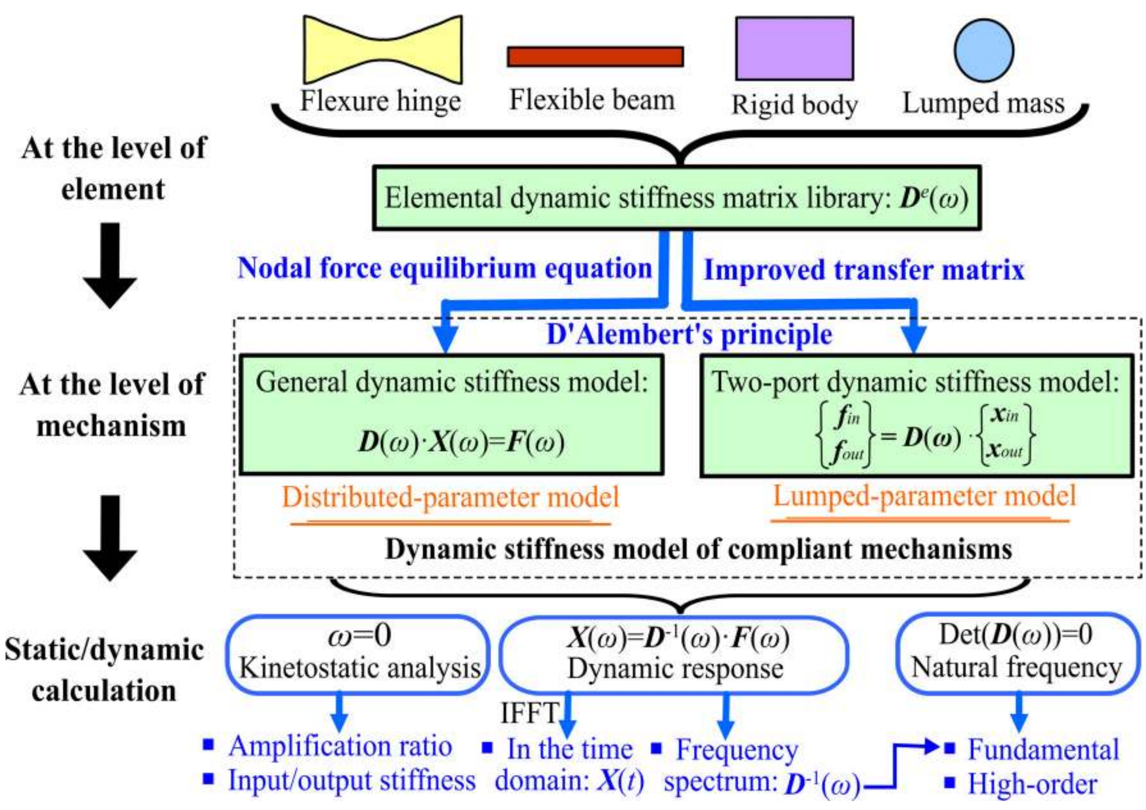

Fig. 10 Dynamic modeling procedures for compliant mechanisms based on d'Alembert's principle

dynamic modeling of compliant mechanisms by combining the aforementioned kinetostatic modeling methods with Lagrange's equation, kinematic solutions are usually necessary and it would become intractable for complex configurations. The finite element method is general and powerful but often with multiple DOFs. The matrix displacement method together with a dynamic stiffness model based on d'Alembert's principle was developed in Refs. [131] and [209] for simultaneously modeling the kinetostatics and dynamics of compliant mechanisms. However, the elemental dynamic stiffness matrix for irregular members in compliant mechanisms is difficult to formulate. Therefore, continuing to explore concise and accurate approaches for modeling kinetostatics and dynamics of compliant mechanisms with serial-parallel configurations is still an ongoing problem.

7.3 Dynamic Modeling of Intermediate-Range and Large-Deflection Compliant Mechanisms. There have been a considerable number of studies on the kinetostatic analysis of large-deflection compliant mechanisms in the literature. However, these studies are mainly focused on single flexure beams or mechanisms with relatively simple configurations, e.g., parallel four-bar compliant mechanisms. For the kinetostatics of large-deflection compliant mechanisms, several solutions are available, such as the pseudo-rigid-body model [1], (chained) beam constraint model [96], and energy-based techniques [109–112]. However, the present investigations on the dynamic modeling of compliant mechanisms with large deflection are relatively rare except the techniques by combining the pseudo-rigid-body model with Lagrange's equation [108,113–117]. In these PRBM-based dynamic models, kinematic solutions in the elastic/kinetic energies are required and it will become complicated for complex configurations. Moreover, the modeling accuracy is also limited with some of the previous studies. In recent years, there are increasingly dynamic applications of flexure-based manipulators with intermediate or even large deflection ranges, such as large-workspace precision grippers [47–50,284]. To satisfy these emerging requirements, more effort should be devoted to developing dynamic modeling methods for compliant mechanisms with intermediate and large deflections as well as revealing dynamic characteristics and new behaviors. The implementation of such an issue still remains challenging.

7.4 New Techniques and Advanced Modeling Methodologies. Recently, dynamic modeling of complex systems was powerfully implemented by using data-driven techniques such as deep learning algorithm [285,286] for the purpose of control. Improvements in the performance of data-driven techniques compensate some disadvantages in physics-based models with fundamental laws in the discipline of mechanics. Two issues would be interesting: (a) The first is to predict the kinetostatic and dynamic performance of flexure-based mechanical systems including large deflection and other nonlinearities based on data-driven techniques or hybrid modeling principles; (b) The second possible topic would be employing the powerful data-mining capability of artificial intelligence for the optimization and synthesis of compliant mechanisms with potentially new configurations and better performance.

8 Conclusions

This paper presents an overview of technologies and approaches on the kinetostatic and dynamic modeling of compliant mechanisms involving small- and large-deflections. The conceptual ideas, key modeling procedures, advantages, disadvantages, recent advances, and scope of different modeling methods in the related literature are surveyed and summarized.

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