

Knowing and Teaching Elementary Mathematics

*Teachers' Understanding of Fundamental
Mathematics in China and the United States*

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