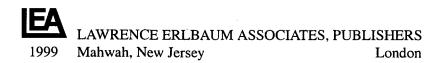
Knowing and Teaching Elementary Mathematics

Teachers' Understanding of Fundamental Mathematics in China and the United States

Liping Ma University of California, Berkeley



Contents

••

۰.

Foreword Acknowledgments Introduction		ix xiii xvii
1.	Subtraction With Regrouping: Approaches To Teaching A Topic	1
	 The U.S. Teachers' Approach: Borrowing Versus Regrouping 2 The Chinese Teachers' Approach: "Decomposing a Higher Value Unit" 7 Discussion 21 Summary 26 	
2.	Multidigit Number Multiplication: Dealing With Students' Mistakes	28
-	 The U.S. Teachers' Approach: Lining Up Versus Separating Into Three Problems 29 The Chinese Teachers' Approach: Elaborating the Concept of Place Value 38 Discussion 52 Summary 54 	
3.	Generating Representations: Division By Fractions	55
	 The U.S. Teachers' Performance on Calculation 56 The Chinese Teachers' Performance on Calculation 58 The U.S. Teachers' Representations of Division by Fractions 64 The Chinese Teachers' Approach to the Meaning of Division by Fractions 72 Discussion 80 Summary 82 	

-

CONTENTS

4.	Exploring New Knowledge: The Relationship Between Perimeter And Area	84
	How the U.S. Teachers Explored the New Idea 85 How the Chinese Teachers Explored the New Idea 90 Discussion 103 Summary 106	
5.	Teachers' Subject Matter Knowledge: Profound Understanding Of Fundamental Mathematics	107
·	 A Cross-Topic Picture of the Chinese Teachers' Knowledge: What Is Its Mathematical Substance? 108 Knowledge Packages and Their Key Pieces: Understanding Longitudinal Coherence in Learning 113 Elementary Mathematics as Fundamental Mathematics 116 Profound Understanding of Fundamental Mathematics 118 Summary 123 	
6.	Profound Understanding Of Fundamental Mathematics: When And How Is It Attained	125
	 When Is Profound Understanding of Fundamental Mathematics Attained?: What the Preteaching Groups Knew About the Four Topics 126 Profound Understanding of Fundamental Mathematics: How It Is Attained? 129 Summary 142 	
7.	Conclusion	144
ą;	 Address Teacher Knowledge and Student Learning at the Same Time 146 Enhance the Interaction Between Teachers' Study of School Mathematics and How to Teach It 147 Refocus Teacher Preparation 149 Understand the Role That Curricular Materials, Including Textbooks, Might Play in Reform 150 Understand the Key to Reform: Whatever the Form of Classroom Interactions Might Be, They Must Focus on Substantive Mathematics 151 	
		154 156 161 163

۲

viii

)