KNOWLEDGE-AIDED STAP ALGORITHM USING CONVEX COMBINATION OF INVERSE COVARIANCE MATRICES FOR HETEROGENOUS CLUTTER

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ABSTRACT

Knowledge-aided space-time adaptive processing (KA-STAP) algorithms, which incorporate *a priori* knowledge into radar signal processing methods, have the potential to substantially enhance detection performance while combating heterogeneous clutter effects. In this paper, we develop a KA-STAP algorithm to estimate the *inverse* interference covariance matrix rather than the covariance matrix itself, by combining the inverse of the covariance known *a priori*, $\mathbf{R}_0^{-1}$, and the inverse sample covariance matrix estimate $\hat{\mathbf{R}}^{-1}$. The computational load is greatly reduced due to the avoidance of the matrix inversion operation. We also develop a cost-effective algorithm based on the minimum variance (MV) criterion for computing the mixing parameter that performs a convex combination of $\mathbf{R}_0^{-1}$ and $\hat{\mathbf{R}}^{-1}$. Simulations show the potential of our proposed algorithm, which obtain substantial performance improvements over prior art.

Index Terms— Space-time adaptive processing, knowledge-aided techniques, airborne radar applications.

1. INTRODUCTION

Space-time adaptive processing (STAP) techniques have been well developed following the landmark publication by Brennan and Reed [1] since 1973. A significant increase in output signal-to-interference-plus-noise-ratio (SINR) for airborne radar applications can potentially be achieved by a joint-domain optimization of the spatial and temporal degrees-of-freedom (DOFs) [2]. The STAP must employ secondary data, generally taken from range cells adjacent to the cell under test (CUT), to estimate the covariance matrix $\mathbf{R}$ in the optimal detector [3]. Prior work ([4, 5] and the references therein) have focused on algorithms with the underlying assumption that the training samples are independent and identically distributed (i.i.d) with the same covariance matrix as the primary data (so called homogeneous training). However, it is widely understood that the clutter environments are often heterogeneous (or non-i.i.d) [6, 7]. For example, clutter reflectivity varies spatially and target-like signals frequently reside within the training data. Thus, merely using the sample covariance matrix estimate $\hat{\mathbf{R}}$ results in significant output SINR performance degradation.

To mitigate the delerious effects of the heterogeneity in the secondary data, knowledge-aided (KA) STAP techniques, which make use of an *a priori* knowledge of the clutter covariance matrix, have recently gained significant attention [8–12]. In KA-STAP, two questions have to be answered: the first is how to derive the prior co-

variance matrix from the terrain knowledge of the clutter and how to estimate the real interference covariance matrix with the prior knowledge [8–10]; the second is how to apply the covariance matrix estimate in the filtering [11, 12]. A number of techniques have been shown to result in superior detection performance when the limited sample support is used in highly nonstationary clutter environments. However, most of the KA-STAP techniques studied previously inevitably require matrix inversion which has complexity of $\mathcal{O}(M^3)$, where M is the dimension of the matrix.

In this paper, we propose a KA-STAP algorithm which estimates the *inverse* interference covariance matrix instead of the covariance matrix itself, by combining the inverse prior covariance matrix $\mathbf{R}_0^{-1}$ and the inverse sample covariance matrix estimate $\hat{\mathbf{R}}^{-1}$. We consider a linear combination of $\mathbf{R}_0^{-1}$ and $\hat{\mathbf{R}}^{-1}$, more precisely, $\mathbf{R}^{-1} = \alpha\mathbf{R}_0^{-1} + \beta\hat{\mathbf{R}}^{-1}$. Because the computation of $\hat{\mathbf{R}}^{-1}$ can be simplified by the matrix inversion lemma and may be obtained recursively, our proposed algorithm has considerably lower complexity compared with the scheme proposed by Stoica et al. in [10]. Futhermore, in Stoica et al.'s scheme, one must replace $\mathbf{R}$ with $\hat{\mathbf{R}}$ under the assumption of homogeneous training when estimating the mixing parameter, while our algorithm is simpler, does not require such assumption and can be represented by a convex combination of filters [13]. To this end, we develop a cost-effective algorithm based on the minimum variance (MV) criterion for computing the mixing parameter that performs the convex combination. The simulations show significant performance improvement brought by our proposed KA-STAP algorithm.

The rest of the paper is organized as follows. In section 2, we briefly describe the problem statement. Section 3 focuses on the key principle of the proposed KA-STAP and Section 4 describes the methods to estimate the mixing parameter. Section 5 presents numerical examples illustrating the performance improvement of the proposed algorithm. Finally, conclusions are drawn in Section 6.

2. PROBLEM STATEMENT

The system under consideration is a pulsed Doppler radar residing on an airborne platform. The radar antenna is a uniformly spaced linear array antenna consisting of N elements. Radar returns are collected in a coherent processing interval (CPI), which is referred to as the 3-D radar data-cube shown in Fig. 1(a), where L denotes the number of samples collected to cover the range interval. The data is then processed at one range of interest, which corresponds to a slice of the CPI data-cube. This slice is a $J \times N$ matrix which consists of $N \times 1$ spatial snapshots for J pulses at the range of interest. It is convenient to stack the matrix column-wise to form the $M \times 1$, $M = JN$ vector $\mathbf{r}(i)$, termed a space-time snapshot [1]. Given a space-time snapshot, radar detection is a binary hypothesis problem, where hypothesis
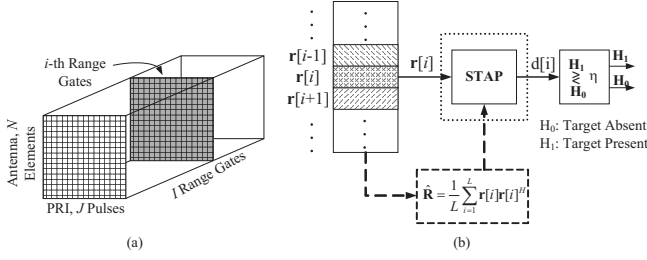
Fig. 1. (a) The Radar CPI datacube. (b) The STAP schematic.

$\mathbf{H}_0$ corresponds to target absence and hypothesis $\mathbf{H}_1$ corresponds to target presence. The radar space-time snapshot is then expressed for each of the two hypotheses in the following form,

$$\begin{aligned} \mathbf{H}_0 &: \mathbf{r} = \boldsymbol{\nu} \\ \mathbf{H}_1 &: \mathbf{r} = \alpha \mathbf{s}_t + \boldsymbol{\nu}, \end{aligned} \tag{1}$$

where α is a complex gain and $\mathbf{s}_t$ is the target space-time steering vector, which is the $M \times 1$ normalized space-time steering vector in the space-time look-direction. Vector $\boldsymbol{\nu}$ encompasses any undesired interference or noise component of the data including clutter $\mathbf{c}$, jamming $\mathbf{j}$ and thermal noise $\mathbf{n}$. These three components are assumed to be mutually uncorrelated. Thus, the $M \times M$ covariance matrix $\mathbf{R}_\nu$ of the undesired clutter-plus-jammer-plus-noise component can be modelled as

$$\mathbf{R}_\nu = E\{\boldsymbol{\nu}\boldsymbol{\nu}^H\} = \mathbf{R}_c + \mathbf{R}_j + \mathbf{R}_n, \tag{2}$$

where H represents Hermitian transpose, $\mathbf{R}_c = E\{\mathbf{c}\mathbf{c}^H\}$, $\mathbf{R}_j = E\{\mathbf{j}\mathbf{j}^H\}$ and $\mathbf{R}_n = E\{\mathbf{n}\mathbf{n}^H\}$ denote clutter, jamming and noise covariance matrix respectively. In practice, the interference-plus-noise covariance matrix $\mathbf{R}_\nu$ is typically unknown. The common approach is to estimate it from the *secondary* data set which does not contain the signal of interest ($\mathbf{r} = \boldsymbol{\nu}$). In this context, we can refer the interference-plus-noise covariance matrix $\mathbf{R}_\nu$ as $\mathbf{R}$.

An optimal STAP, in the maximum SINR sense, is given by $\boldsymbol{\omega}_{opt} = \gamma \mathbf{R}^{-1}\mathbf{s}_t$, where γ is an arbitrary scalar. Normally, since $\mathbf{R}$ is unknown, secondary data $\{\mathbf{r}(k)\}_{k=1}^K$ is employed to estimate the covariance matrix by means of the well-known formula

$$\hat{\mathbf{R}} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{r}(k)\mathbf{r}^H(k), \tag{3}$$

where $\mathbf{r}(k)$ denotes the received data vector at the time instant k and K denotes the number of training samples. Such estimate can be sufficiently accurate when K is at least twice as great as M and the training samples are assumed i.i.d. However, it has been widely recognized that the clutter environments are often heterogeneous and the impact of the heterogeneity on the STAP performance loss has been investigated in [6]. Thus, KA signal processing is becoming an important technique to combat the heterogeneity [9]. In [8, 10], the covariance matrix is estimated by combining an initial guess of the covariance matrix $\mathbf{R}_0$, derived from the digital terrain database or the data probed by radar in previous scans, and the sample average covariance matrix estimate in the present scan $\hat{\mathbf{R}}$, so that

$$\mathbf{R} = \alpha \mathbf{R}_0 + \beta \hat{\mathbf{R}}. \tag{4}$$

Formula (4) can be restricted to a convex combination [10] as follows

$$\mathbf{R} = \alpha \mathbf{R}_0 + (1 - \alpha)\hat{\mathbf{R}} \quad \alpha \in (0, 1). \tag{5}$$

With the estimated covariance matrix, many STAP algorithms including data prewhitening [8,9] and knowledge-aided constraints [11] improve clutter mitigation performance. However, most of these KA-STAP algorithms require a matrix inversion operation with a complexity of $\mathcal{O}(M^3)$, which motivates us to develop a novel KA-STAP with lower complexity and greater flexibility.

3. PROPOSED KA-STAP ALGORITHM

The principle of our proposed algorithm is detailed in this section. We consider the inverse covariance matrix estimate by using a linear combination of the inverse covariance matrices instead of the covariance matrix itself, more precisely, given by

$$\mathbf{R}^{-1} = \alpha \mathbf{R}_0^{-1} + \beta \hat{\mathbf{R}}^{-1}. \tag{6}$$

To reduce the number of mixing parameters to be estimated, a convex combination is considered as follows

$$\mathbf{R}^{-1} = \eta \mathbf{R}_0^{-1} + (1 - \eta)\hat{\mathbf{R}}^{-1}, \tag{7}$$

where $\eta \in (0, 1)$ [13].

The sample average inverse covariance matrix can be simplified by the matrix inversion lemma [14] such that

$$\hat{\mathbf{R}}^{-1}(n + 1) = \lambda^{-1}\hat{\mathbf{R}}^{-1}(n) - \frac{\lambda^{-2}\hat{\mathbf{R}}^{-1}(n)\mathbf{r}(n)\mathbf{r}^H(n)\hat{\mathbf{R}}^{-1}(n)}{1 + \lambda^{-1}\mathbf{r}^H(n)\hat{\mathbf{R}}^{-1}(n)\mathbf{r}(n)}, \tag{8}$$

where λ is a forgetting factor. Thus, the inverse covariance matrix estimate can be recursively calculated, which brings a significant reduction in the computational complexity. The remaining work is to effectively estimate the mixing parameter η.

If we multiply the target space-time steering vector $\mathbf{s}$ at both sides of (7), the formula will lead to a combination of two filters given by

$$\begin{aligned} \mathbf{R}^{-1}(n)\mathbf{s} &= \eta \mathbf{R}_0^{-1}\mathbf{s} + (1 - \eta)\hat{\mathbf{R}}^{-1}(n)\mathbf{s}, \\ \Rightarrow \boldsymbol{\omega}(n) &= \eta(n)\boldsymbol{\omega}_0(n) + (1 - \eta(n))\hat{\boldsymbol{\omega}}(n), \end{aligned} \tag{9}$$

where we define $\boldsymbol{\omega}(n) = \mathbf{R}^{-1}(n)\mathbf{s}$, $\boldsymbol{\omega}_0(n) = \mathbf{R}_0^{-1}(n)\mathbf{s}$ and $\hat{\boldsymbol{\omega}}(n) = \hat{\mathbf{R}}^{-1}(n)\mathbf{s}$. The systematic diagram of the proposed KA-STAP algorithm is shown in Fig. 2. Thus, the mixing parameter η can be optimized in the sense of minimizing output power. The optimum mixing parameter which minimizes the cost function $\mathcal{L}(\eta)$ is given by

$$\eta_o = \arg\min_\eta \mathcal{L}(\eta) = \arg\min_\eta \mathbf{E}\left\{|\boldsymbol{\omega}^H(n)\mathbf{r}(n)|^2\right\}. \tag{10}$$

By equating the gradient of the cost function $\mathcal{L}(\eta)$ with respect to η to zero, we get

$$\eta_o = \frac{\Re\left\{\mathbf{s}^H(\hat{\mathbf{R}}^{-1} - \mathbf{R}_0^{-1})\mathbf{R}\hat{\mathbf{R}}^{-1}\mathbf{s}\right\}}{\mathbf{s}^H(\mathbf{R}_0^{-1} - \hat{\mathbf{R}}^{-1})\mathbf{R}(\mathbf{R}_0^{-1} - \hat{\mathbf{R}}^{-1})\mathbf{s}}. \tag{11}$$

where $\Re\{\cdot\}$ denotes the real part of a complex value. However, because $\mathbf{R}$ in (11) is unknown, we have to come up with an adaptive algorithm to estimate η in real time using the MV criterion, which will be presented in the following section.
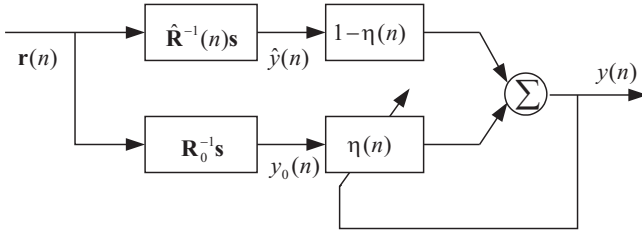
Fig. 2. The systematic diagram of the proposed KA-STAP algorithm.

4. MIXING PARAMETER ESTIMATION

In this section, an adaptive algorithm is developed to estimate the mixing parameter $\eta(n)$. Here, we borrow the ideas from [13] which deal with the adaptation of $\eta(n)$ for the convex combination of two adaptive filters. It should be remarked that our adaptation of η is derived based on the minimum variance (MV) criterion for complex systems, which is an extension of previous results that dealt only with real variables and the mean square error (MSE) criterion.

For the adaptation of the mixing parameter $\eta(n)$, we use a gradient descent method to minimize the output power of the overall filter, say $p(n) = |y(n)|^2$, where $y(n)$ is the output of the overall filter which is a function of $\eta(n)$, given by

$$y(n) = \eta(n)y_0(n) + [1 - \eta(n)]\hat{y}(n), \qquad (12)$$

where $y_0(n) = \boldsymbol{\omega}_0^H \mathbf{r}(n)$ and $\hat{y}(n) = \hat{\boldsymbol{\omega}}^H(n)\mathbf{r}(n)$ are the outputs of the two filters at time n. Instead of directly modifying $\eta(n)$, an auxiliary variable $\varepsilon(n)$ is adapted to restrict $\eta(n)$ to an interval [0,1] via a sigmoidal function such that [13]

$$\eta(n) = \text{sgm}[\varepsilon(n)] = \frac{1}{1 + e^{-\varepsilon(n)}}. \qquad (13)$$

The normalized update equation for $\varepsilon(n)$ is given by [13]

$$\varepsilon(n+1) = \varepsilon(n) - \frac{\mu_\varepsilon}{2[\sigma_\varepsilon + q(n)]}\frac{\partial p(n)}{\partial \varepsilon(n)}. \qquad (14)$$

where μ_ε is a step size, σ_ε is a small positive constant and $q(n)$ can be expressed by

$$q(n+1) = (1 - \lambda_q)|y_0(n) - y_1(n)|^2 + \lambda_q q(n), \qquad (15)$$

where λ_q is a forgetting factor. It was shown that better behavior is obtained by the normalized adaptation of the mixing parameter and the selection of λ_q is rather easy [13]. Because ε should be a real number, we have to modify the recursion of [13]. Expanding the cost function, we obtain

$$
\begin{aligned}
|y(n)|^2 &= \left|\eta(n)[y_0(n) - \hat{y}(n)] + \hat{y}(n)\right|^2 \\
&= \eta(n)^2 |y_0(n) - \hat{y}(n)|^2 \\
&\quad + 2\eta(n)\Re\left\{[y_0(n) - \hat{y}(n)]\hat{y}(n)^*\right\} + |\hat{y}(n)|^2.
\end{aligned} \qquad (16)
$$

To derive the recursion in (14), we firstly simplify $\frac{\partial p(n)}{\partial \varepsilon(n)}$ as follows

$$\frac{\partial p(n)}{\partial \varepsilon(n)} = \frac{\partial p(n)}{\partial \eta(n)}\frac{\partial \eta(n)}{\partial \varepsilon(n)}, \qquad (17)$$
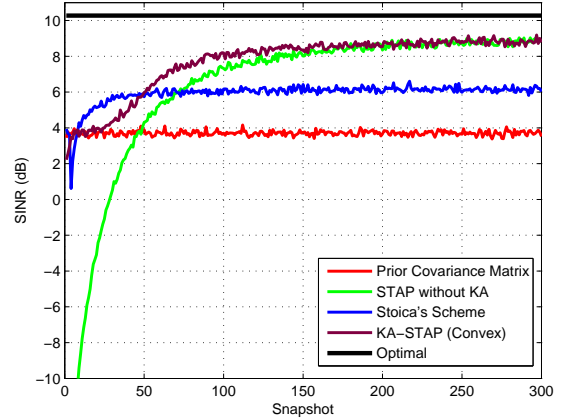

Fig. 3. The SINR performance against the number of snapshots for the proposed algorithm with $\lambda = 0.998$, $\lambda_q = 0.7$, $\sigma_\varepsilon = 0.001$, $\mu_\varepsilon = 5$ and $\varepsilon \in [-4, +4]$.

where

$$
\begin{aligned}
\frac{\partial p(n)}{\partial \eta(n)} &= \frac{\partial |y(n)|^2}{\partial \eta(n)} \\
&= 2\eta(n)|y_0(n) - \hat{y}(n)|^2 \\
&\quad + 2\Re\left\{[y_0(n) - \hat{y}(n)]\hat{y}(n)^*\right\},
\end{aligned} \qquad (18)
$$

and

$$\frac{\partial \eta(n)}{\partial \varepsilon(n)} = -\frac{-e^{-\varepsilon(n)}}{[1 + e^{-\varepsilon(n)}]^2} = \eta(n)[1 - \eta(n)]. \qquad (19)$$

Thus, the adaptation of $\varepsilon(n)$ in (14) can be rewritten as

$$
\begin{aligned}
\varepsilon(n+1) = \varepsilon(n) &- \frac{\mu_\varepsilon}{\sigma_\varepsilon + q(n)}\Big\{\eta(n)|y_0(n) - \hat{y}(n)|^2 \\
&+ \Re\big\{[y_0(n) - \hat{y}(n)]\hat{y}(n)^*\big\}\Big\}\eta(n)[1 - \eta(n)].
\end{aligned} \qquad (20)
$$

To further improve the behavior of the adaptation, we need to constrain ε to an interval $[\varepsilon^-, \varepsilon^+]$. Such constraint restricts η in the interval $(0, 1)$, otherwise, the adaptation will stop when η is equal to 0 or 1.

5. NUMERICAL EXAMPLES

In this section we assess the proposed KA-STAP algorithm in an airborne radar application under certain heterogenous clutter condition.
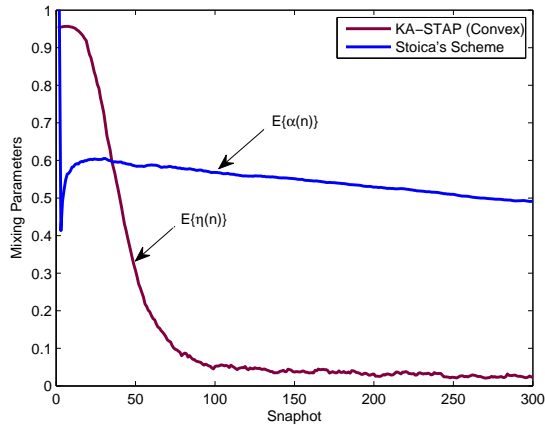
Fig. 4. Ensemble-average of $\eta(n)$ and $\alpha(n)$ for our proposed scheme and Stoica et al.'s scheme, respectively.

The parameters of the radar platform are shown in Table 1. We assume that the clutter-to-noise-ratio (CNR) is fixed at 40 dB and there is no jammer. Assuming that the calibration-on-clutter is known, the prior clutter covariance matrix can be calculated using these radar parameters. To model the heterogenous clutter, the spectral variation is introduced and target-like signals are Poisson-seeded over 300 training snapshots [6, 9]. We investigate the SINR performance against the number of snapshots for our proposed algorithm and the behavior of the mixing parameter η. In the simulation, the filter $\hat{\boldsymbol{\omega}}$ is implemented by using the recursive least squares (RLS) algorithm with $\lambda = 0.998$. The parameters for the adaptive algorithm adapting $\eta(n)$ are set as $\lambda_q = 0.7$, $\sigma_\varepsilon = 0.001$ and $\mu_\varepsilon = 5$. Furthmore, we constrain ε to the interval [-4,+4]. All presented results are averages over 1000 independent Monte-Carlo runs.

Fig. 3 shows the SINR performance against the number of snapshots for our proposed algorithm. We compare it with two component filters and Stoica et al.'s scheme. The curves show that our proposed KA-STAP with convex combination algorithm outperforms Stoica et al.'s scheme almost 3dB at the steady-state, although Stoica et al.'s scheme converges faster in the first 50 snapshots. We also note that Our proposed algorithm performs, most of time, better than two component filters. The component filter using the prior covariance matrix has fixed performance and after 50 snapshots its performance is exceeded by another component filter using the estimated covariance matrix. In Fig. 4, the mixing parameters $\eta(n)$ and $\alpha(n)$ for our proposed scheme and Stoica et al.'s scheme are shown, respectively. The experimental curves were estimated from the ensemble-average of 1000 independent runs. Note that they have different meanings that $\eta(n)$ is the mixing parameter for the convex combination of two inverse matrices and $\alpha(n)$ is the the mixing parameter for the convex combination of two matrices. Our mixing parameter η is adapted with the normalized algorithm according to the output power and converges to a steady-state value close to 0.

6. CONCLUSIONS

In this paper, motivated by the fact that knowledge-aided space-time adaptive processing (KA-STAP) algorithms have the potential to substantially enhance detection performance combating heteroge-

neous clutter effects, we developed a KA-STAP algorithm to estimate the inverse interference covariance matrix rather than the covariance matrix itself, by combining the prior covariance matrix inverse, $\mathbf{R}_0^{-1}$, and the inverse sample covariance matrix estimate $\hat{\mathbf{R}}^{-1}$. The computational load was greatly reduced due to the avoidance of the matrix inversion operation. Furthermore, an adaptive algorithm based on the MV criterion for the mixing parameter was developed for performing the convex combination with complex signals. The results showed the potential of our proposed algorithm and a substantial performance improvement over prior art.

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