Knowledge Assessment: A Modal Logic Approach

Vineet Padmanabhan[1*],Guido Governatori[2 ⋆] Subhasis Thakur[2]

[1] Department of Computer & Information Sciences, University of Hyderabad, India.
vineetcs@uohyd.ernet.in
[2] School of ITEE, The University of Queensland, Brisbane, Australia.
[guido,subhasis]@itee.uq.edu.au

Abstract. The *possible worlds semantics* is a fruitful approach used in Artificial Intelligence (AI) for both *modelling* as well as *reasoning* about knowledge in agent systems via modal logics. In this work our main idea is not to model/reason about knowledge but to provide a theoretical framework for *knowledge assessment* (KA) with the help of *Monatague-Scott* (MS) semantics of modal logic. In KA questions *asked* and answers *collected* are the central elements and knowledge notions will be defined from these (i.e., possible states of knowledge of subjects in a population with respect to a field of information).

Keywords: Modal & Epistemic Logics for Question Answering Systems, Question processing, Interpretation models.

1 Introduction

Modelling and reasoning about knowledge in agent systems is an active research area within the AI community [1,2]. It is often the case that the logical tool used to represent and reason about knowledge is that of modal logic[3] with the underlying *possible worlds* [3] model. There is also an *interpreted system* (IS) model which aims to give a computational flavour to S5 in terms of the states of computer processes [4,1] and this in turn makes it more suitable in one of the major application areas of knowledge reasoning namely Multi-Agent Systems (MAS). Recent works show that the IS Model can also be used for the specification of cognitive attitudes other than knowledge like *belief, desire* and *intention* (BDI) so that techniques like symbolic model checking can be used to verify the different agent properties inherent in the specification [5]. In this paper we deviate from the works above in the sense that our main idea is not to model/reason about knowledge but to provide a framework for *knowledge assessment* using some tools and techniques in modal logic.

To make the idea of knowledge assessment precise consider the list of questions given in Table1. It is common practice that for assessing a student's knowledge in elementary mathematics question formats as in Table1 is presented and is followed by a written examination. Thereafter the students answers are collected and finally the examiner returns an appreciation which usually boils down to a single number or percentage.

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[3] The modal logic KT45 (also called S5) is usually used to reason about knowledge.

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As pointed out in [6] such a testing procedure provides limited information because provided that a student gives correct answers to questions a, c and e it only shows a numerical appreciation (60 percent) of his/her work. What it hides is the information related to the student's knowledge/mastery in performing multiplication and deficiency in division operation. Moreover, the responses (answers) also indicate that there is some dependency among the questions. For instance, question e (a multidigit multiplication) in table 1 relies on elementary multiplication tested in question a. Consequently from a correct answer to question e we should infer a correct answer to question a. Obtaining and exploiting the most precise information from an assesment procedure is particulary needed in programmed courses as it reveals the weakness as well as strong points of the student's preparation and hence advices for further study can be inferred. Similarly any computer assisted instruction system should entail a module for uncovering the user's knowledge. We take motivation for this work from the *knowledge structure* (KS) theory

Table 1. An Excerpt of a test in Mathematics

a	$2 \times 378 =$	????
b	$322 \div 7 =$	????
c	$14.7 \times 100 =$	????
d	$6442 \div 16 =$	????
e	$58.7 \times 0.94 =$	????

as outlined in [6,7]. Knowledge structure theory presupposes that the *knowledge* of an individual in a particular domain of knowledge can be operationalised as the solving behaviour of that individual on a domain specific set X of *problems*. If the solution result for each problem is binarily coded by true/false, then the *knowledge state* of an individual in the given field of knowledge can be formally described as the subset of problems from X he/she is capable of solving. To tackle the problem of solution dependencies that can exist between problems of a certain field of knowledge KS theory employs the concept of a *surmise system*. The idea is to associate each problem $x \in X$ with a family of subsets of X called clauses, with the interpretation that, if a person is capable of solving x then he/she is capable of solving all problems in at-least one of these elements.

In this work we describe a theoretical framework based on the possible worlds model to capture the main ingredients of KS theory as mentioned above which in turn can be used for Knowledge Assessment. Since our main aim is with respect to the *assessment* of knowledge we need to have a definition of knowledge that can fit in with this intuition. Hence, instead of defining knowledge as truth in all possible worlds, which is the common interpretation given for knowledge models based on possible worlds semantics, an agent's knowledge is *explicitly* described at a state/(in our case with respect to a question q) by a set of sets of states (set of prerequisites for the question q). In other words, our possible worlds framework for knowledge assessment is based on *Montague-Scott (MS)* semantics rather than the usual Kripke semantics.

In the comming sections we briefly discuss the knowledge structure theory along with surmise systems and outline the technical apparatus of MS-structures. Then we

show how MS-strucutres can be used as a tool for Knowledge Assessment and conclude the paper with a discussion.

2 Knowledge Structures, Surmise Systems and MS-models

As mentioned in the previous section a knowledge structure consists of a finite set $\mathbb{Q}$ together with a collection $\mathscr{K}$ of subsets of $\mathbb{Q}$ wherein the elements of $\mathbb{Q}$ are the *questions* and the members of $\mathscr{K}$ are the *knowledge states*. For example, assume that the set of questions in Table (1) is given for a test. Now, any student who took the test is characterised by the subset of questions he/she correctly answered and this subset constitutes his/her knowledge state. So for instance, we can have $\mathscr{K}_1 = \{a,b,c\}$, $\mathscr{K}_2 = \{d\}$, $\mathscr{K}_3 = \emptyset$ representing respectively the knowledge states corresponding to three students. What we can infer from the knowledge states is that the first student gave correct responses to questions a, b and c whereas the last student to none at all. Similarly one can come up with a collection $\mathscr{K}$ of knowledge states representing all possible knowledge states by observing a population of students as given in (1).

$$\mathscr{K} = \{\emptyset, \{\mathtt{a}\}, \{\mathtt{d}\}, \{\mathtt{a},\mathtt{b},\mathtt{c}\}, \{\mathtt{a},\mathtt{d},\mathtt{e}\}, \{\mathtt{b},\mathtt{c},\mathtt{d},\mathtt{e}\}, \{\mathtt{a},\mathtt{b},\mathtt{c},\mathtt{d},\mathtt{e}\}\} \tag{1}$$

It should be noted that not any subset of $\mathbb{Q}$ needs to be a knowledge state as solution dependencies could exist among the members of the set $\mathbb{Q}$. Therefore $\mathscr{K}$ comprises of all those subsets of $\mathbb{Q}$ which constitutes the set of all *empirically expectable* solution patterns. Also, from (1) it can be seen that questions b and c belong exactly to the same knowledge states, i.e., $\{\mathtt{a},\mathtt{b},\mathtt{c}\}$ and $\{\mathtt{b},\mathtt{c},\mathtt{d},\mathtt{e}\}$. Hence as mentioned in [7] one can say that b and c *define* the same notion. But this is not the case for questions b and e because they are disinguished by the knowledge state $\{\mathtt{a},\mathtt{b},\mathtt{c}\}$ and this means b and e test different skills. As pointed out above solution dependencies can exist between problems of a certain field of knowledge. In our case question e in Table1. relies on question a and hence from a correct response to question e we should infer a correct response to a, i.e., we say that we surmise mastery of question a from mastery of question e. In general *we want to infer from the knowledge of one question the complete knowledge of at least one set of questions among some list of sets*. We call these sets the clauses for the original question q. For example let $\mathbb{Q}$ denote the set of questions in table 1 and ν a mapping that associates to any element q in $\mathbb{Q}$ a non-empty collection $\nu(\mathtt{q})$ of subsets of $\mathbb{Q}$ as given in Figure1. Here question a has only one clause which

1. $\nu(\mathtt{a}) = \{\{\mathtt{c}\}\}$	2. $\nu(\mathtt{b}) = \emptyset$	3. $\nu(\mathtt{c}) = \emptyset$
4. $\nu(\mathtt{d}) = \{\{\mathtt{a},\mathtt{c}\},\{\mathtt{b}\}\}$	5. $\nu(\mathtt{e}) = \{\{\mathtt{a},\mathtt{b},\mathtt{c}\}\}$	

Fig. 1. Solution dependencies for the questions in Table1.

is that of c and question b has the empty set as its only clause. What this means is that

there is only one way to know question a which is through the acquisition of question c while there is no prerequisite for b.

Definition 1 *A surmise system on a finite set $\mathbb{Q}$ is a mapping ν that associates to any element q in $\mathbb{Q}$ a nonempty collection $\nu(q)$ of subsets of $\mathbb{Q}$ and satisfies the following conditions;*

1. *Any clause for question q contains q*
2. *If $q' \in C$, with C a clause for question q, there exists some clause C' for q' satisfying $C' \subseteq C$.*
3. *Any two clauses for question q are incomparable in the sense that neither is included in the other.*

We denote a surmise system by $(\mathbb{Q}, \nu)$.

2.1 MS/Neighbourhood Semantics

Montague-Scott semantics, also known as Neighbourhood semantics is considered the most general kind of possible worlds semantics in the sense that it is compatible with retaining the classical truth-table semantics for the truth-functional operators. In this section we outline the main ingredients of neighbourhood semantics needed to develop a framework for knowledge assessment.

Definition 2 *A neighbourhood model is a structure*

$$\mathscr{M} = \langle \mathbb{W}, \pi, \nu \rangle$$

where $\mathbb{W}$ is a set of worlds and $\pi(w)$ is a truth assignment to the primitive propositions for each state $w \in \mathbb{W}$. Intuitively $\pi(p) = \{w_1, w_2\}$ represents the fact that p is true at w_1, w_2 and false at $\mathbb{W} \backslash \{w_1, w_2\}$. $\nu(w)$ is a mapping from $\mathbb{W}$ to sets of subsets of $\mathbb{W}$, i.e., $\nu : \mathbb{W} \to \wp(\wp(\mathbb{W}))$. $\langle \mathbb{W}, \nu \rangle$ is called a neighbourhood frame.

The basic idea of this definition is that each world w of $\mathbb{W}$ has associated with it a set $\nu(w)$ of propositions that are necessary at w. Since a proposition in possible worlds semantics is a subset of $\mathbb{W}$[4] the set of propositions necessary at w, $\nu(w)$, is a set of subsets of w. There are no assumptions about ν except that it is a function from $\mathbb{W} \to \wp(\wp(\mathbb{W}))$ and $\nu(w)$ may be any set of propositions including the empty set. When interpreted in terms of knowledge in agent-systems the members of $\nu(w)$ can be considered as the propositions an agent knows. We will talk more about this knowledge interpretation in the next section. Inorder to state the truth conditions of a neighbourhood model we need to take care of the definition of a truth set.

Definition 3 *The truth set, $|| A ||^{\mathscr{M}}$, of the formula A in the model $\mathscr{M}$ is the set of worlds in $\mathscr{M}$ at which A is true; formally*

$$|| A ||^{\mathscr{M}} = \{w \text{ in } \mathscr{M} : \mathscr{M}, w \models A\}$$

[4] In possible worlds semantics (any kind) a proposition is identified with a set of possible worlds.

Definition 4 *(Truth Conditions) Let w be a world in a model* $\mathscr{M} = \langle \mathbb{W}, \pi, \nu \rangle$.

- $\mathscr{M}, w \models \Box A \Leftrightarrow || A ||^{\mathscr{M}} \in \nu(w)$
- $\mathscr{M}, w \models \Diamond A \Leftrightarrow (\mathbb{W} - || A ||^{\mathscr{M}}) \notin \nu(w)$

Example 1 *Let* $\mathbb{W} = \{a,b,c\}$, $\pi(p) = \{a,b\}, \pi(q) = \{b,c\}$ *and* $\nu(a) = \{\{b\},\{a,c\}\}$, $\nu(b) = \{\{a,c\},\{a\},\{a,b\}\}$ *and* $\nu(c) = \{\emptyset,\{a\},\{b,c\}\}$ *be a neighbourhood model* $\mathscr{M}$ *according to Definition 2. Then some of the formulae that are satisfied by* $\mathscr{M}$ *are*

$\mathscr{M}, b \models \Box p$ *(since* $|| p ||^{\mathscr{M}} = \{a,b\} \in \nu(b)$*)*
$\mathscr{M}, b \models \Diamond p$ *(since* $\{a,b,c\} - || p ||^{\mathscr{M}} = \{a,b,c\} - \{a,b\} = c \notin \nu(b)$*)*
$\mathscr{M}, c \models \Box\Diamond p$ *(since* $|| \Diamond p ||^{\mathscr{M}} = \{b,c\} \in \nu(c)$*)*
$\mathscr{M}, a \models \Box\Box p$ *(since* $|| \Box p ||^{\mathscr{M}} = \{b\} \in \nu(a)$*)*
$\mathscr{M}, c \models \Box\bot$ *(since* $|| \bot ||^{\mathscr{M}} = \emptyset \in \nu(c)$*)*
$\mathscr{M}, a \models \Box(p \wedge q)$ *(since* $|| p \wedge q ||^{\mathscr{M}} = \{b\} \in \nu(a)$*)*
$\mathscr{M}, a \not\models \Box p$ *(since* $|| p ||^{\mathscr{M}} = \{a,b\} \notin \nu(a)$*)*

The last two items in the above list needs special mention. Note that $\mathscr{M}, a \models \Box(p \wedge q)$ but $\mathscr{M}, a \not\models \Box p$. In the case of a relational structure if we fix the valuations of p and q it is not possible to show that $\Box(p \wedge q)$ is true at a but $\Box p$ is false at a. The reason is that $\Box p$ is false at a forces a to have an accessible world in which p is false. There is only one such world (c) where p is false. However, if c is accessible from a, then $\Box(p \wedge q)$ will no longer be true at a (since if p is false at c then so is $p \wedge q$). The above example shows that the axiom $\Box(\psi \wedge \varphi) \rightarrow \Box\psi \wedge \Box\varphi$ is not valid in the case of neighbourhood frames. In the next section we will demonstrate why such axioms need to be avoided in th case of knowledge assessment.

3 Assessing Knowledge

In this section we show how to use the technical apparatus of Neighbourhood models as outlined above for knowledge assessment. We write the modal connectives as **K** to emphasise the knowledge aspect. Initially we do not want to bind **K** with any properties but just as a replica of the modal operators.

Consider a neighbourhood model $\mathscr{M} = \langle \mathbb{W}, \pi, \nu \rangle$ where $\mathbb{W} = \{\mathtt{a},\mathtt{b},\mathtt{c},\mathtt{d},\mathtt{e}\}$ be the set of questions as given in Table1 and ν be as in Figure. 1. Let π be given as follows

$$\pi(*) = \{\mathtt{a},\mathtt{c}\},\ \pi(\div) = \{\mathtt{b}\}$$

By $\pi(*) = \{\mathtt{a},\mathtt{c}\}$ we mean that multiplication is true/holds for questions a and c. For question e this need not be the case because to solve e one needs the knowledge of both multiplication and division. Similar argument holds in the case of $\pi(\div) = \{\mathtt{b}\}$. Now we can say that a model $\mathscr{M}$ and question q *satisfies* the *knowledge* of multiplication if and only if the *truth set* of multiplication is in the *list of sets* related to question q. Formally

$$\mathscr{M}, \mathtt{q} \models \mathbf{K}(*) \Leftrightarrow || * || \in \nu(\mathtt{q}) \tag{2}$$

To give an example if we substitute question a from Table 1 in place of q we get

$$\mathscr{M}, \mathtt{a} \not\models \mathbf{K}(*)\ \ (\textit{since}\ || * ||^{\mathscr{M}} \notin \nu(\mathtt{a})) \tag{3}$$

because $||*|| = \{\mathtt{a},\mathtt{c}\}$ and $\{\mathtt{a},\mathtt{c}\} \notin \nu(\mathtt{a})$. From a knowledge assessment perspective (3) has much to offer. For instance, suppose that we have a collection $\mathscr{K}$ of knowledge states as given in (1) in Section 2. where we have a set $\{\mathtt{a}\}$. Then (3) shows the incomplete knowledge of a student with respect to multiplication. In other words (3) helps in assessing a student's knowledge in multiplication with respect to (from the viewpoint of) the answer set provided by him/her. In this case we can assess that a correct response to question a is not enough for a student to solve (have complete knowledge of) other questions related to multiplication. In a similar manner from (3) we can also reason about a student's lack of knowledge in *division* because

$$\mathscr{M}, \mathtt{a} \not\models \mathbf{K}(\div) \;\; (\textit{since} \; || \div ||^{\mathscr{M}} \notin \nu(\mathtt{a})).$$

It should be kept in mind that it is possible to make the model $\mathscr{M}$ satisfy certain conditions so as to fit in with the notion of a surmise system as outlined in the previous section. For instance, the first item of Definition 1 generalises the reflexivity condition for a relation and we will show later on how to give such conditions for a neighbourhood model $\mathscr{M}$. Now, let us take d and repeat the same process. This time we can see that

$$\mathscr{M}, \mathtt{d} \models \mathbf{K}(*) \;\; (\textit{since} \; || * ||^{\mathscr{M}}, \textit{i.e.}, \{\mathtt{a},\mathtt{c}\} \in \nu(\mathtt{d})) \tag{4}$$

holds which tells us that a student who has provided the answer set d *knows* or have mastered multiplication. From (4) we can also infer

$$\mathscr{M}, \mathtt{d} \models \mathbf{K}(\div) \;\; (\textit{since} \; || \div ||^{\mathscr{M}}, \textit{i.e}, \{\mathtt{b}\} \in \nu(\mathtt{d})) \tag{5}$$

which shows that a student who has provided the answer set d *knows* division. At the same time from (4) and (5) we get

$$\mathscr{M}, \mathtt{d} \not\models \mathbf{K}(* \wedge \div) \; (|| * \wedge \div ||^{\mathscr{M}}, \textit{i.e}, \{\mathtt{a},\mathtt{b},\mathtt{c}\} \notin \nu(\mathtt{d})) \tag{6}$$

which tells us that from answer set d one cannot assess the knowledge of both multiplication and division. For instance, from Figure1 it can be seen that question d can be mastered along two different approaches, one implying the mastery of the sole question b, the other requiring the mastering of questions a and c. In other words, according to our model, for a student to solve question d he/she needs to know multiplication or division and not both. (6) shows exactly this and more in the sense that it avoids the problem of *logical omniscience* (LO)[5] [3,8] which plague knowledge models based on possible worlds. Now let us consider e;

$$\mathscr{M}, \mathtt{e} \models \mathbf{K}(* \wedge \div) \; (|| * \wedge \div ||^{\mathscr{M}}, \textit{i.e}, \{\mathtt{a},\mathtt{b},\mathtt{c}\} \in \nu(\mathtt{e})) \tag{7}$$

(7) shows the mastery/knowledge of a student in multiplication and division with respect to question e or in other words a student who has provided the answer set e knows both multiplication and division. There are two main reasons for having such

[5] LO usually refers to a family of related *closure* conditions. In the case of (6) we avoid *closure under conjunction*, i.e., the condition that if an agent *i* knows both φ and ψ, then agent *i* knows $\varphi \wedge \psi$.

an assessment procedure; 1) It is usually the case that in an oral examination teachers strongly reduce the number of questions by making inferences from the collected answers and 2) because of 1 they can specifically select the next question. These two features also show the superior efficiency of oral testing over written testing. Any good automated procedure should encompass these features and exploit them to minimise the test duration. Our aim in this paper is to give a theoretical model based on modal logic to account for the above mentioned features. Of course there are other models (probabilistic models) that can account for such an assessment proceudre but the main idea here is to show the usability of modal logic as a tool for knowledge assessment.

3.1 Models Satisfying certain conditions

So far we have been trying to build a framework based on modal logic for knowledge assessment so as to decide what formulas should be valid for the *knowledge* reading of $\Box$ (i.e. when we interpret $\Box$ to be a modality representing knowledge). We did not impose any constraints on the model $\mathscr{M}$. Since we want to relate our knowledge assessment model with that of a surmise system we need to make sure that our model satisfies conditions given in Definition1. In this section we show how to achieve this. The following conditions can be given for items 1, 2 and 3 of Definition1.

1. $X \in \nu(w) \Rightarrow w \in X$ (*reflexivity* condition)
2. $X \in \nu(w) \Rightarrow \{w' \in W : X \in \nu(w')\} \in \nu(w)$ (*transitivity* condition)
3. $\forall X,Y \in \nu(w), X \neq Y \Rightarrow \exists x,y : x \in X, x \notin Y, y \in Y, y \notin X$ (*Any two clauses are incomparable*)

It should be kept in mind that given a function $\nu : \mathbb{W} \rightarrow \wp(\mathbb{W})$ it is always possible to define a function $f : \wp(\mathbb{W}) \rightarrow \wp(\mathbb{W})$ such that $f(X) = \{w : X \in \nu(w)\}$. In this manner we can define every function ν of Definition 2 in terms of a function like f as follows;

$$w \in f(X) \Leftrightarrow X \in \nu(w) \tag{8}$$

Hence truth conditions for $\Box A$ in terms of f can be given as

$$\mathscr{M}, w \models \Box A \Leftrightarrow || A ||^{\mathscr{M}} \in \nu(w) \Leftrightarrow w \in f(|| A ||^{\mathscr{M}}), \text{ i.e.,}$$
$$|| \Box A ||^{\mathscr{M}} = f(|| A ||^{\mathscr{M}})$$

The corresponding model conditions using (8) for reflexivity $(f(X) \subseteq X)$ and transitivity $(f(X) \subseteq f(f(X))$ is much more concise and easy to use. This alternate characterisation of ν-models is nothing more than a notational variant and should not be seen as a new model. A question which naturally comes to mind then is why not define conditions like reflexivity, transitivity etc. before hand on the set of questions so as to have a relational model (A *binary relation* on the set of questions so as to formalise the surmise idea). rather than constructing a surmise system as discussed in the previous sections. One reason for not adopting a relational model as pointed out in [6] is that the knowledge structure associated to a surmise relation is closed under intersection and union whereas that of a surmise system is closed under union alone. Put in other words, if two students characterised by their knowledge states K and K' meet and share what

they know they will both end with the union $K \cup K'$ as their common knowledge state. In the case of intersection similar motivation doesn't exist and the only argument that could be given is that the two students would decide to retain their common knowledge, i.e., $K \cap K'$ which according to [6] is weak because cognitive development is considered to be cumulative over time. And from a modal logic point of view we can avoid the problem of LO which as pointed out earlier is not a good property to have as far as knowledge assessment is concerned. Also, in the relational model the accessibility relation must be given before defining satisfiability in a world because the satisfiability of a formula containing a modal operator is defined in terms of the accessibility relation. We can avoid this using the MS-models.

4 Discussion

We have outlined a modal logic based approach for knowledge assessment where questions asked and answers collected form the main ingredients and knowledge notions are defined from these. Our approach is different when compared to other modal logic theories of knowledge in Artificial Intelligence where modelling/reasoning about knowledge is the main focal area. The current work is in the preliminary stages and lot needs to be done. We have only outlined the syntax and semantics of our framework and have completely neglected the multi-agent aspect. What we would like to have ideally is to efficiently uncover, given a student in the population, which member of $\mathscr{K}$ represents his/her knowledge state. From a multi-agent perspective we can think of modifying ν to ν_i where i represents an agent and assign the propositions he/she knows. But in the case of knowledge assessment it is not that simple because we cannot assign randomly the questions a particular agent/student knows as the assessment is done based on the questions asked and answers collected. An earlier version of this paper appeared in [9].

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