Knowledge Representation in Fuzzy Logic

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(Invited Paper)

***Abstract*—The conventional approaches to knowledge representation, e.g., semantic networks, frames, predicate calculus, and Prolog, are based on bivalent logic. A serious shortcoming of such approaches is their inability to come to grips with the issue of uncertainty and imprecision. As a consequence, the conventional approaches do not provide an adequate model for modes of reasoning which are approximate rather than exact. Most modes of human reasoning and all of common sense reasoning fall into this category.**

Fuzzy logic, which may be viewed as an extension of classical logical systems, provides an effective conceptual framework for dealing with the problem of knowledge representation in an environment of uncertainty and imprecision. Meaning representation in fuzzy logic is based on test-score semantics. In this semantics, a proposition is interpreted as a system of elastic constraints, and reasoning is viewed as elastic constraint propagation. Our paper presents a summary of the basic concepts and techniques underlying the application of fuzzy logic to knowledge representation and describes a number of examples relating to its use as a computational system for dealing with uncertainty and imprecision in the context of knowledge, meaning, and inference.

***Index Terms*—Approximate reasoning, fuzzy logic, knowledge representation.**

I. Introduction

KNOWLEDGE representation is one of the most basic and actively researched areas of AI [4], [5], [30], [31], [36], [37], [39], [46], [47]. And yet, there are many important issues underlying knowledge representation which have not been adequately addressed. One such issue is that of the representation of knowledge which is lexically imprecise and/or uncertain.

As a case in point, the conventional knowledge representation techniques do not provide effective tools for representing the meaning of or inferring from the kind of everyday type facts exemplified by the following.

1) *Usually* it takes *about an hour* to drive from Berkeley to Stanford in *light* traffic.

2) Unemployment is *not likely* to undergo a *sharp* decline during the next *few* months.

3) *Most* experts believe that the likelihood of a *severe* earthquake in the *near* future is *very low.*

The italicized words in these assertions are the labels of fuzzy predicates, fuzzy quantifiers, and fuzzy probabilities. The conventional approaches to knowledge representation lack the means for representing the meaning of fuzzy concepts. As a consequence, the approaches based on first-order logic and classical probability theory do not provide an appropriate conceptual framework for dealing with the representation of common sense knowledge, since such knowledge is by its nature both lexically imprecise and noncategorical [36], [37], [63].

The development of fuzzy logic was motivated in large measure by the need for a conceptual framework which can address the issues of uncertainty and lexical imprecision. The principal objective of this paper is to present a summary of some of the basic ideas underlying fuzzy logic and to describe their application to the problem of knowledge representation in an environment of uncertainty and imprecision. A more detailed discussion of these ideas may be found in Zadeh [59], [60], [65], [67] and other references.

II. Essentials of Fuzzy Logic

Fuzzy logic, as its name suggests, is the logic underlying modes of reasoning which are approximate rather than exact. The importance of fuzzy logic derives from the fact that most modes of human reasoning—and especially common sense reasoning—are approximate in nature. It is of interest to note that, despite its pervasiveness, approximate reasoning falls outside the purview of classical logic largely because it is a deeply entrenched tradition in logic to be concerned with those and only those modes of reasoning which lend themselves to precise formulation and analysis.

Some of the essential characteristics of fuzzy logic relate to the following.

In fuzzy logic, exact reasoning is viewed as a limiting case of approximate reasoning.

In fuzzy logic, everything is a matter of degree.

Any logical system can be fuzzified.

In fuzzy logic, knowledge is interpreted as a collection of elastic or, equivalently, fuzzy constraint on a collection of variables.

Inference is viewed as a process of propagation of elastic constraints.

Fuzzy logic differs from traditional logical systems both in spirit and in detail. Some of the principal differences are summarized in the following [62].

Truth: In bivalent logical systems, truth can have only two values: true or false. In multivalued systems, the truth value of a proposition may be an element of: a) a finite set; b) an interval such as [0, 1]; or c) a boolean algebra. In fuzzy logic, the truth value of a proposition may be a

Manuscript received March 1, 1989. This work was supported in part by NASA under Grant NCC-2-275 and by the Air Force Office of Scientific Research under Grant 89-0084.

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IEEE Log Number 8928831.

1041-4347/89/0300-0089$01.00 © 1989 IEEE

fuzzy subset of any partially ordered set, but usually it is assumed to be a fuzzy subset of the interval [0, 1] or, more simply, a point in this interval. The so-called *linguistic* truth values expressed as *true, very true, not quite true*, etc., are interpreted as labels of fuzzy subsets of the unit interval.

Predicates: In bivalent systems, the predicates are crisp, e.g., *mortal, even, larger than*. In fuzzy logic, the predicates are fuzzy, e.g., *tall, ill, soon, swift, much larger than*. It should be noted that most of the predicates in a natural language are fuzzy rather than crisp.

Predicate Modifiers: In classical systems, the only widely used predicate modifier is the negation, *not*. In fuzzy logic, there is a variety of predicate modifiers which act as hedges, e.g., *very, more or less, quite, rather, extremely*. Such predicate modifiers play an essential role in the generation of the values of a linguistic variable, e.g., *very young, not very young, more or less young*, etc. [57].

Quantifiers: In classical logical systems there are just two quantifiers: *universal* and *existential*. Fuzzy logic admits, in addition, a wide variety of fuzzy quantifiers exemplified by *few, several, usually, most, almost always, frequently, about five*, etc. In fuzzy logic, a fuzzy quantifier is interpreted as a fuzzy number or a fuzzy proportion [61].

Probabilities: In classical logical systems, probability is numerical or interval-valued. In fuzzy logic, one has the additional option of employing linguistic or, more generally, fuzzy probabilities exemplified by *likely, unlikely, very likely, around 0.8, high*, etc. [65]. Such probabilities may be interpreted as fuzzy numbers which may be manipulated through the use of fuzzy arithmetic [24].

In addition to fuzzy probabilities, fuzzy logic makes it possible to deal with fuzzy events. An example of a fuzzy event is: *tomorrow will be a warm day*, where *warm* is a fuzzy predicate. The probability of a fuzzy event may be a crisp or fuzzy number [56].

It is important to note that from the frequentist point of view there is an interchangeability between fuzzy probabilities and fuzzy quantifiers or, more generally, fuzzy measures. In this perspective, any proposition which contains labels of fuzzy probabilities may be expressed in an equivalent form which contains fuzzy quantifiers rather than fuzzy probabilities.

Possibilities: In contrast to classical modal logic, the concept of possibility in fuzzy logic is graded rather than bivalent. Furthermore, as in the case of probabilities, possibilities may be treated as linguistic variables with values such as *possible, quite possible, almost impossible*, etc. Such values may be interpreted as labels of fuzzy subsets of the real line.

A concept which plays a central role in fuzzy logic is that of a possibility distribution [59], [8], [28]. Briefly, if X is a variable taking values in a universe of discourse U, then the *possibility distribution* of X, Π_X, is the fuzzy set of all possible values of X. More specifically, let $\pi_X(u)$ denote the possibility that X can take the value u, $u \in U$. Then the membership function of X is numerically equal to the *possibility distribution function* $\pi_X(u)$: $U \rightarrow [0, 1]$, which associates with each element $u \in U$ the possibility that X may take u as its value. More about possibilities and possibility distributions will be said at a later point in this paper.

It is important to observe that in every instance fuzzy logic adds to the options which are available in classical logical systems. In this sense, fuzzy logic may be viewed as an extension of such systems rather than as a system of reasoning which is in conflict with the classical systems.

Before taking up the issue of knowledge representation in fuzzy logic, it will be helpful to take a brief look at some of the principal modes of reasoning in fuzzy logic. These are the following, with the understanding that the modes in question are not necessarily disjoint.

1) Categorical Reasoning: In this mode of reasoning, the premises contain no fuzzy quantifiers and no fuzzy probabilities. A simple example of categorical reasoning is:

Carol is slim
Carol is very intelligent

Carol is slim and very intelligent

In the premises, *slim* and *very intelligent* are assumed to be fuzzy predicates. The fuzzy predicate in the conclusion, *slim and very intelligent*, is the conjunction of *slim* and *very intelligent.*

Another example of categorical reasoning is:

Mary is young
John is much older than Mary

John is (much older ∘ *young).*

where (*much_older* ∘ *young*) represents the composition of the binary fuzzy predicate *much_older* with the unary fuzzy predicate *young*. More specifically, let π_{much_older} and π_{young} denote the possibility distribution functions associated with the fuzzy predicates *much_older* and *young*, respectively. Then, the possibility distribution function of John's age may be expressed as [59]

$$\pi_{Age(John)}(u) = \vee_v(\pi_{much\_older}(u, v) \wedge \pi_{young}(v)$$

where $\vee$ and $\wedge$ stand for max and min, respectively.

2) Syllogistic Reasoning: In contrast to categorical reasoning, syllogistic reasoning relates to inference from premises containing fuzzy quantifiers [64], [11]. A simple example of syllogistic reasoning is the following:

most Swedes are blond
most blond Swedes are tall

$most^2$ *Swedes are blond and tall*

where the fuzzy quantifier *most* is interpreted as a fuzzy proportion and $most^2$ is the square of *most* in fuzzy arithmetic [24].

3) Dispositional Reasoning: In dispositional reasoning the premises are dispositions, that is, propositions which are preponderantly but necessarily always true [66]. An example of dispositional reasoning is:

heavy smoking is a leading cause of lung cancer

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to avoid lung cancer avoid heavy smoking

Note that in this example the conclusion is a maxim which may be interpreted as a dispositional command. Another example of dispositional reasoning is:

usually the probability of failure is not very low
usually the probability of failure is not very high

(2 *usually* $\ominus$ 1) *the probability of failure is not very low and not very high*

In this example, *usually* is a fuzzy quantifier which is interpreted as a fuzzy proportion and 2 *usually* $\ominus$ 1 is a fuzzy arithmetic expression whose value may be computed through the use of fuzzy arithmetic. ($\ominus$ denotes the operation of subtraction in fuzzy arithmetic.) It should be noted that the concept of *usuality* plays a key role in dispositional reasoning [64], [66], and is the concept that links together the dispositional and syllogistic modes of reasoning. Furthermore, it underlies the theories of non-monotonic and default reasoning [33], [34], [35], [45].

4) Qualitative Reasoning: In fuzzy logic, the term *qualitative reasoning* refers to a mode of reasoning in which the input-output relation of a system is expressed as a collection of fuzzy if-then rules in which the antecedents and consequents involve linguistic variables [58], [69]. In this sense, qualitative reasoning in fuzzy logic bears some similarity to—but is not coextensive with—qualitative reasoning in AI [6], [14], [29].

A very simple example of qualitative reasoning is:

volume is small if pressure is high
volume is large if pressure is low

---

volume is ($w1 \wedge$ *high* + $w2 \wedge$ *large*) *if pressure is medium*

where + should be interpreted as infix max; and

$$w1 = sup(high \wedge medium)$$

and

$$w2 = sup(low \wedge medium)$$

are weighting coefficients which represent, respectively, the degrees to which the antecedents *high* and *low* match the input *medium*. In $w1$, the conjunction *high* $\wedge$ *medium* represents the intersection of the possibility distributions of *high* and *low*, and the supremum is taken over the domain of *high* and *medium*. The same applies to $w2$.

Qualitative reasoning underlies many of the applications of fuzzy logic in the realms of control and systems analysis [48], [42], [50]. In this connection, it should be noted that fuzzy Prolog provides an effective knowledge representation language for qualitative reasoning [1], [2], [38], [69].

III. Meaning and Knowledge Representation

In a general setting, knowledge may be viewed as a collection of propositions, e.g.,

Mary is young
Pat is much taller than Mary
overeating causes obesity
most Swedes are blond
tomatoes are red unless they are unripe
usually high quality goes with high price
if pressure is high then volume is low

To constitute knowledge, a proposition must be understood. In this sense, meaning and knowledge are closely interrelated. In fuzzy logic, meaning representation—and thus knowledge representation—is based on test-score semantics [60], [65].

A basic idea underlying test-score semantics is that a proposition in a natural language may be viewed as a collection of elastic or, equivalently, fuzzy constraints. For example, the proposition *Mary is tall* represents an elastic constraint on the height of Mary. Similarly, the proposition *Jean is blonde* represents an elastic constraint on the color of Jean's hair. And, the proposition *most tall men are not very agile* represents an elastic constraint on the proportion of men who are not very agile among tall men.

In more concrete terms, representing the meaning of a proposition p through the use of test-score semantics involves the following steps.

1) Identification of the variables $X_1, \cdots, X_n$ whose values are constrained by the proposition. Usually, these variables are implicit rather than explicit in p.

2) Identification of the constraints $C_1, \cdots, C_m$ which are induced by p.

3) Characterization of each constraint C_i by describing a testing procedure which associates with C_i a test score τ_i representing the degree to which C_i is satisfied. Usually τ_i is expressed as a number in the interval [0, 1]. More generally, however, a test score may be a probability/possibility distribution over the unit interval.

4) Aggregation of the partial test scores $\tau_1, \cdots, \tau_m$ into a smaller number of test scores $\bar{\tau}_1, \cdots, \bar{\tau}_k$, which are represented as an *overall vector test score* $\tau = (\bar{\tau}_1, \cdots, \bar{\tau}_k)$. In most cases $k = 1$, so that the overall test score is a scalar. We shall assume that this is the case unless an explicit statement to the contrary is made.

It is important to note that, in test-score semantics, the meaning of p is represented not by the overall test score τ but by the procedure which leads to it. Viewed in this perspective, test-score semantics may be regarded as a generalization of truth-conditional, possible-world, and model-theoretic semantics. However, by providing a computational framework for dealing with uncertainty and dispositionality—which the conventional semantical systems disregard—test-score semantics achieves a much higher level of expressive power and thus provides a basis for representing the meaning of a much wider variety of propositions in a natural language.

In test-score semantics, the testing of the constraints induced by p is performed on a collection of fuzzy relations which constitute an *explanatory database*, or *ED* for short. A basic assumption which is made about the explanatory database is that it is comprised of relations whose meaning is known to the addressee of the meaning-representation process. In an indirect way, then, the testing and aggregation procedures in test-score semantics may be viewed as a description of a process by which the meaning of p is composed from the meanings of the constituent relations in the explanatory database. It is this explanatory role of the relations in *ED* that motivates its description as an *explanatory database*.

As will be seen in the sequel, in describing the testing procedures we need not concern ourselves with the actual entries in the constituent relations. Thus, in general, the description of a test involves only the frames of the constituent relations, that is, their names, their variables (or attributes), and the domain of each variable.

As a simple illustration of the concept of a test procedure, consider the proposition $p \triangleq$ *Maria is young and attractive*. The *ED* in this case will be assumed to consist of the following relations:

$$ED \triangleq POPULATION\,[Name;\ Age;\ \mu Attractive] + YOUNG\,[Age;\ \mu] \qquad (3.1)$$

in which + should be read as "and," and $\triangleq$ stands for "denotes."

The relation labeled *POPULATION* consists of a collection of triples whose first element is the name of an individual, whose second element is the age of that individual, and whose third element is the degree to which the individual in question is attractive. The relation *YOUNG* is a collection of pairs whose first element is a value of the variable *Age* and whose second element is the degree to which that value of *Age* satisfies the elastic constraint characterized by the fuzzy predicate *young*. In effect, this relation serves to calibrate the meaning of the fuzzy predicate *young* in a particular context by representing its denotation as a fuzzy subset, *YOUNG*, of the interval [0, 100].

With this *ED*, the test procedure which computes the overall test score may be described as follows.

1) Determine the age of Maria by reading the value of *Age* in *POPULATION*, with the variable *Name* bound to Maria. In symbols, this may be expressed as

$$Age(Maria) = {}_{Age}POPULATION\,[Name = Maria].$$

In this expression, we use the notation ${}_{Y}R[X = a]$ to signify that X is bound to a in R and the resulting relation is projected on Y, yielding the values of Y in the tuples in which $X = a$.

2) Test the elastic constraint induced by the fuzzy predicate *young*:

$$\tau_1 = {}_{\mu}YOUNG[Age = Age(Maria)].$$

3) Determine the degree to which Maria is attractive:

$$\tau_2 = {}_{\mu Attractive}POPULATION[Name = Maria].$$

4) Compute the overall test score by aggregating the partial test scores τ_1 and τ_2. For this purpose, we shall use the min operator $\wedge$ as the aggregation operator, yielding

$$\tau = \tau_1 \wedge \tau_1 \qquad (3.2)$$

which signifies that the overall test score is taken to be the smaller of the operands of $\wedge$. The overall test score, as expressed by (3.2), represents the compatibility of $p \triangleq$ *Maria is young and attractive* with the data resident in the explanatory database.

In testing the constituent relations in *ED*, it is helpful to have a collection of standardized translation rules for computing the test score of a combination of elastic constraints $C_1, \cdots, C_k$ from the knowledge of the test scores of each constraint considered in isolation. For the most part, such rules are *default* rules in the sense that they are intended to be used in the absence of alternative rules supplied by the user.

For purposes of knowledge representation, the principal rules of this type are the following.

1) Rules Pertaining to Modification: If the test score for an elastic constraint C in a specified context is τ, then in the same context the test score for

(*a*) *not C* is $1 - \tau$ (*negation*)
(*b*) *very C* is τ^2 (*concentration*)
(*c*) *more or less C* is $\tau^{1/2}$ (*diffusion*).

2) Rules Pertaining to Composition: If the test scores for elastic constraints C_1 and C_2 in a specified context are τ_1 and τ_2, respectively, then in the same context the test score for

(*a*) C_1 *and* C_2 is $\tau_1 \wedge \tau_2$ (*conjunction*), where $\wedge \triangleq$ *min*.
(*b*) C_1 *or* C_2 is $\tau_1 \vee \tau_2$ (*disjunction*), where $\vee \triangleq$ *max*.
(*c*) *If* C_1 *then* C_2 is $1 \wedge (1 - \tau_1 + \tau_2)$ (*implication*).

3) Rules Pertaining to Quantification: The rules in question apply to propositions of the general form *Q A's are B's*, where Q is a fuzzy quantifier, e.g., *most, many, several, few*, etc., and A and B are fuzzy sets, e.g., *tall men, intelligent men*, etc. As was stated earlier, when the fuzzy quantifiers in a proposition are implied rather than explicit, their suppression may be placed in evidence by referring to the proposition as a *disposition*. In this sense, the proposition *overeating causes obesity* is a disposition which results from the suppression of the fuzzy quantifier *most* in the proposition *most of those who overeat are obese*.

To make the concept of a fuzzy quantifier meaningful, it is necessary to define a way of counting the number of elements in a fuzzy set or, equivalently, to determine its cardinality.

There are several ways in which this can be done [60], [9], [52]. For our purposes, it will suffice to employ the concept of a *sigma-count*, which is defined as follows.

Let F be a fuzzy subset of $U = \{u_1, \cdots, u_n\}$ expressed symbolically as

$$F = \mu_1/u_1 + \cdots + \mu_n/u_n = \Sigma_i \mu_i/u_i$$

or, more simply, as

$$F = \mu_1 u_1 + \cdots + \mu_n u_n$$

in which the term μ_i/u_i, $i = 1, \cdots, n$ signifies that μ_i is the grade of membership of u_i in F, and the plus sign represents the union.

The sigma-count of F is defined as the arithmetic sum of the μ_i, i.e.,

$$\Sigma Count(F) \triangleq \Sigma_i \mu_i, \qquad i = 1, \cdots, n$$

with the understanding that the sum may be rounded, if need be, to the nearest integer. Furthermore, one may stipulate that the terms whose grade of membership falls below a specified threshold be excluded from the summation. The purpose of such an exclusion is to avoid a situation in which a large number of terms with low grades of membership become count-equivalent to a small number of terms with high membership.

The *relative sigma-count*, denoted by $\Sigma Count(F/G)$, may be interpreted as the proportion of elements of F which are in G. More explicitly,

$$\Sigma Count(F/G) = \frac{\Sigma Count(F \cap G)}{\Sigma Count(G)}$$

where $F \cap G$, the intersection of F and G, is defined by

$$\mu_{F \cap G}(u) = \mu_F(u) \wedge \mu_G(u), \qquad u \in U.$$

Thus, in terms of the membership functions of F and G, the relative sigma-count of F in G is given by

$$\Sigma Count(F/G) = \frac{\Sigma_i \mu_F(u_i) \wedge \mu_G(u_i)}{\Sigma_i \mu_G(u_i)}.$$

The concept of a relative sigma-count provides a basis for interpreting the meaning of propositions of the form *Q A's are B's*, e.g., *most young men are healthy*. More specifically, if the focal variable (i.e., the constrained variable) in the proposition in question is taken to be the proportion of *B's* in *A's*, then the corresponding translation rule may be expressed as

$$Q\ A\text{'s are } B\text{'s} \rightarrow \Sigma Count(B/A) \text{ is } Q.$$

As an illustration, consider the proposition $p \triangleq$ *over the past few years Naomi earned far more than most of her close friends*. In this case, we shall assume that the constituent relations in the explanatory database are:

$$\begin{aligned} ED \triangleq\ & INCOME\ [Name;\ Amount;\ Year] + \\ & FRIEND\ [Name;\ \mu] + \\ & FEW\ [Number;\ \mu] + \\ & FAR.MORE\ [Income1;\ Income2;\ \mu] + \\ & MOST\ [Proportion;\ \mu]. \end{aligned}$$

Note that some of these relations are explicit in p; some are not; and that most of the constituent words in p do not appear in ED.

In what follows, we shall describe the process by which the meaning of p may be composed from the meaning of the constituent relations in ED. Basically, this process is a test procedure which tests, scores, and aggregates the elastic constraints which are induced by p.

1) Find Naomi's income, IN_i, in $Year_i$, $i = 1, 2, 3, \cdots$, counting backward from present. In symbols,

$$IN_i \triangleq {}_{Amount}INCOME[Name = Naomi;\ Year = Year_i]$$

which signifies that *Name* is bound to Naomi, *Year* to $Year_i$, and the resulting relation is projected on the domain of the attribute *Amount*, yielding the value of *Amount* corresponding to the values assigned to the attributes *Name* and *Year*.

2) Test the constraint induced by *FEW*:

$$\mu_i \triangleq {}_{\mu}FEW\ [Year = Year_i]$$

which signifies that the variable *Year* is bound to $Year_i$ and the corresponding value of μ is read by projecting on the domain of μ.

3) Compute Naomi's total income during the past few years:

$$TIN \triangleq \Sigma_i \mu_i IN_i$$

in which the μ_i play the role of weighting coefficients. Thus, we are tacitly assuming that the total income earned by Naomi during a fuzzily specified interval of time is obtained by weighting Naomi's income in year $Year_i$ by the degree to which $Year_i$ satisfies the constraint induced by *FEW* and summing the weighted incomes.

4) Compute the total income of each $Name_j$ (other than Naomi) during the past few years:

$$TIName_j = \Sigma_i \mu_i IName_{ji}$$

where $IName_{ji}$ is the income of $Name_j$ in $Year_i$.

5) Find the fuzzy set of individuals in relation to whom Naomi earned far more. The grade of membership of $Name_j$ in this set is given by

$$\begin{aligned} \mu_{FM}(Name_j) = {}& {}_{\mu}FAR.MORE[Income1 \\ & = TIN;\ Income2 = TIName_j]. \end{aligned}$$

6) Find the fuzzy set of close friends of Naomi by intensifying [59] the relation *FRIEND*:

$$CF \triangleq CLOSE.FRIEND \triangleq {}^2FRIEND$$

which implies that

$$\mu_{CF}(Name_j) = ({}_{\mu}FRIEND[Name = Name_j])^2$$

where the expression

$${}_{\mu}FRIEND[Name = Name_j]$$

represents $\mu_F(Name_j)$, that is, the grade of membership of $Name_j$ in the set of Naomi's friends.

7) Count the number of close friends of Naomi. On denoting the count in question by $\Sigma Count(CF)$, we have:

$$\Sigma Count(CF) = \Sigma_j \mu^2_{FRIEND}(Name_j).$$

8) Find the intersection of *FM* with *CF*. The grade of membership of $Name_j$ in the intersection is given by

$$\mu_{FM \cap CF}(Name_j) = \mu_{FM}(Name_j) \wedge \mu_{CF}(Name_j)$$

where the min operator $\wedge$ signifies that the intersection is defined as the conjunction of its operands.

9) Compute the sigma-count of $FM \cap CF$:

$$\Sigma Count(FM \cap CF) = \Sigma_j \mu_{FM}(Name_j) \wedge \mu_{CF}(Name_j).$$

10) Compute the relative sigma-count of *FM* in *CF*, i.e., the proportion of individuals in $FM \cap CF$ who are in *CF*:

$$\rho \triangleq \frac{\Sigma Count(FM \cap CF)}{\Sigma Count(CF)}.$$

11) Test the constraint induced by *MOST*:

$$\tau \triangleq {}_{\mu}MOST[Proportion = \rho]$$

which expresses the overall test score and thus represents the compatibility of *p* with the explanatory database.

In application to the representation of dispositional knowledge, the first step in the representation of the meaning of a disposition involves the process of *explicitation*, that is, making explicit the implicit quantifiers. As a simple example, consider the disposition

$$d \triangleq \textit{young men like young women}$$

which may be interpreted as the proposition

$$p \triangleq \textit{most young men like mostly young women}.$$

The candidate *ED* for *p* is assumed to consist of the following relations:

$$ED \triangleq POPULATION[Name; Sex; Age] + LIKE[Name1; Name\ 2; \mu] + MOST[Proportion; \mu],$$

in which μ in *LIKE* is the degree to which *Name1* likes *Name2*.

To represent the meaning of *p*, it is expedient to replace *p* with the semantically equivalent proposition

$$q \triangleq \textit{most young men are P}$$

where *P* is the fuzzy *dispositional* predicate

$$P \triangleq \textit{likes mostly young women}.$$

In this way, the representation of the meaning of *p* is decomposed into two simpler problems, namely, the representation of the meaning of *P*, and the representation of the meaning of *q* knowing the meaning of *P*.

The meaning of *P* is represented by the following test procedure.

1) Divide *POPULATION* into the population of males, *M.POPULATION*, and population of females, *F.POPULATION*:

$$M.POPULATION \triangleq {}_{Name,Age}POPULATION[Sex = Male]$$

$$F.POPULATION \triangleq {}_{Name,Age}POPULATION[Sex = Female]$$

where ${}_{Name,Age}POPULATION$ denotes the projection of *POPULATION* on the attributes *Name* and *Age*.

2) For each $Name_j$, $j = 1, \cdots, K$, in *F.POPULATION*, find the age of $Name_j$:

$$A_j \triangleq {}_{Age}\, F.POPULATION\ [Name = Name_j].$$

3) For each $Name_j$, find the degree to which $Name_j$ is young:

$$\alpha_j \triangleq {}_{\mu}YOUNG[Age = A_j]$$

where α_j may be interpreted as the grade of membership of $Name_j$ in the fuzzy set *YW* of young women.

4) For each $Name_i$, $i = 1, \cdots, k$, in *M.POPULATION*, find the age of $Name_i$:

$$B_i \triangleq {}_{Age}M.POPULATION[Name = Name_i].$$

5) For each $Name_i$, find the degree to which $Name_i$ is young:

$$\delta_i \triangleq {}_{\mu}YOUNG[Age = B_i]$$

where δ_i may be interpreted as the grade of membership of $Name_i$ in the fuzzy set *YM* of young men.

6) For each $Name_j$, find the degree to which $Name_i$ likes $Name_j$:

$$\beta_{ij} \triangleq {}_{\mu}LIKE[Name1 = Name_i;\ Name2 = Name_j]$$

with the understanding that β_{ij} may be interpreted as the grade of membership of $Name_j$ in the fuzzy set WL_i of women whom $Name_i$ likes.

7) For each $Name_j$ find the degree to which $Name_i$ likes $Name_j$ and $Name_j$ is young:

$$\gamma_{ij} \triangleq \alpha_j \wedge \beta_{ij}.$$

Note: As in previous examples, we employ the aggregation operator min ($\wedge$) to represent the effect of conjunction. In effect, γ_{ij} is the grade of membership of $Name_j$ in the intersection of the fuzzy sets WL_i and *YW*.

8) Compute the relative sigma-count of young women among the women whom $Name_i$ likes:

$$\rho_i \triangleq \Sigma Count(YW/WL_i) = \frac{\Sigma Count(YW \cap WL_i)}{\Sigma Count(WL_i)} = \frac{\Sigma_j \gamma_{ij}}{\Sigma_j \beta_{ij}} = \frac{\Sigma_j \alpha_j \wedge \beta_{ij}}{\Sigma_j \beta_{ij}}.$$

9) Test the constraint induced by *MOST*:

$$\tau_i \triangleq {}_{\mu}MOST[Proportion = \rho_i].$$

This test score, then, represents the degree to which $Name_i$ has the property expressed by the predicate

$$P \triangleq \textit{likes mostly young women}.$$

Continuing the test procedure, we have the following.

10) Compute the relative sigma-count of men who have property P among young men:

$$\rho \triangleq \Sigma Count(P/YM) = \frac{\Sigma Count(P \cap YM)}{\Sigma Count(YM)} = \frac{\Sigma_i \tau_i \wedge \delta_i}{\Sigma_i \delta_i}.$$

11) Test the constraint induced by *MOST*:

$$\tau = {}_{\mu}MOST[Proportion = \rho].$$

This test score represents the overall test score for the disposition *young men like young women*.

IV. The Concept of a Canonical Form and Its Application to the Representation of Meaning

When the meaning of a proposition p is represented as a test procedure, it may be hard to discern in the description of the procedure the underlying structure of the process through which the meaning of p is constructed from the meanings of the constituent relations in the explanatory database.

A concept which makes it easier to perceive the logical structure of p, and thus to develop a better understanding of the meaning representation process, is that of a canonical form of p, abbreviated as $cf(p)$ [60], [65].

The concept of a canonical form relates to the basic idea which underlies test-score semantics, namely, that a proposition may be viewed as a system of elastic constraints whose domain is a collection of relations in the explanatory database. Equivalently, let $X_1, \cdots, X_n$ be a collection of variables which are constrained by p. Then, the canonical form of p may be expressed as

$$cf(p) \triangleq X \text{ is } F \qquad (4.1)$$

where $X = (X_1, \cdots, X_n)$ is the constrained variable which is usually implicit in p, and F is a fuzzy relation, likewise implicit in p, which plays the role of an elastic (or fuzzy) constraint on X. The relation between p and its canonical form will be expressed as

$$p \rightarrow X \text{ is } F \qquad (4.2)$$

signifying that the canonical form may be viewed as a representation of the meaning of p.

In general, the constrained variable X in $cf(p)$ is not uniquely determined by p, and is dependent on the focus of attention in the meaning-representation process. To place this in evidence, we shall refer to X as the *focal* variable.

As a simple illustration, consider the proposition

$$p \triangleq \text{Anne has blue eyes.} \qquad (4.3)$$

In this case, the focal variable may be expressed as

$$X \triangleq Color\,(Eyes\,(Anne))$$

and the elastic constraint is represented by the fuzzy relation *BLUE*. Thus, we can write

$$p \rightarrow Color(Eyes(Anne)) \text{ is } BLUE. \qquad (4.4)$$

As an additional illustration, consider the proposition

$$p \triangleq \text{Brian is much taller than Mildred.} \qquad (4.5)$$

Here, the focal variable has two components, $X = (X_1, X_2)$, where

$$X_1 = Height(Brian)$$
$$X_2 = Height(Mildred);$$

and the elastic constraint is characterized by the fuzzy relation *MUCH.TALLER* [*Height*1; *Height*2; μ], in which μ is the degree to which *Height*1 is *much taller* than *Height*2. In this case, we have

$$p \rightarrow (Height(Brian), Height(Mildred)) \text{ is } MUCH.TALLER. \qquad (4.6)$$

In terms of the possibility distribution of X, the canonical form of p may be interpreted as the assignment of F to Π_X. Thus, we may write

$$p \rightarrow X \text{ is } F \rightarrow \Pi_X = F \qquad (4.7)$$

in which the equation

$$\Pi_X = F \qquad (4.8)$$

is termed the *possibility assignment equation* [60]. In effect, this equation signifies that the canonical form $cf(p) \triangleq X \text{ is } F$ implies that

$$Poss\,\{X = u\} = \mu_F(u), \qquad u \in U \qquad (4.9)$$

where μ_F is the membership function of F. It is in this sense that F, acting as an elastic constraint on X, restricts the possible values which X can take in U. An important implication of this observation is that a proposition p may be interpreted as an implicit assignment statement which characterizes the possibility distribution of the focal variable in p.

As an illustration, consider the disposition

$$d \triangleq \text{overeating causes obesity} \qquad (4.10)$$

which upon explicitation becomes

$$p \triangleq \text{most of those who overeat are obese.} \qquad (4.11)$$

If the focal variable in this case is chosen to be the relative sigma-count of those who are obese among those who overeat, the canonical form of p becomes

$$\Sigma Count(OBESE/OVEREAT) \text{ is } MOST \qquad (4.12)$$

which in virtue of (4.9) implies that

$$Poss\left\{\Sigma Count(OBESE/OVEREAT) = u\right\} = \mu_{MOST}(u) \qquad (4.13)$$

where μ_{MOST} is the membership function of *MOST*. What is important to note is that (4.13) is equivalent to the assertion that the overall test score for p is expressed by

$$\tau = \mu_{MOST}(\Sigma Count(OBESE/OVEREAT)) \quad (4.14)$$

in which *OBESE*, *OVEREAT*, and *MOST* play the roles of the constituent relations in *ED*.

It is of interest to observe that the notion of a semantic network may be viewed as a special case of the concept of a canonical form. As a simple illustration, consider the proposition

$$p \triangleq Richard\ gave\ Cindy\ a\ red\ pin. \quad (4.15)$$

As a semantic network, this proposition may be represented in the standard form:

$$\begin{aligned} &Agent(GIVE) = Richard\\ &Recipient(GIVE) = Cindy\\ &Time(GIVE) = Past\\ &Object(GIVE) = Pin\\ &Color(Pin) = Red. \end{aligned} \quad (4.16)$$

Now, if we identify X_1 with *Agent*(*GIVE*), X_2 with *Recipient*(*GIVE*), etc., the semantic network representation (4.16) may be regarded as a canonical form in which $X = (X_1, \cdots, X_5)$, and

$$\begin{aligned} &X_1 = Richard\\ &X_2 = Cindy\\ &X_3 \text{ is } Past\\ &X_4 \text{ is } Pin\\ &X_5 \text{ is } Red. \end{aligned} \quad (4.17)$$

More generally, since any semantic network may be expressed as a collection of triples of the form (Object, Attribute, Attribute Value), it can be transformed at once into a canonical form. However, since a canonical form has a much greater expressive power than a semantic network, it may be difficult to transform a canonical form into a semantic network.

V. Inference

The concept of a canonical form provides a convenient framework for representing the rules of inference in fuzzy logic. Since the main concern of this paper is with knowledge representation rather than with inference, our discussion of the rules of inference in fuzzy logic in this section has the format of a summary.

In the so-called categorical rules of inference, the premises are assumed to be in the canonical form *X is A* or the conditional canonical form *X is A if Y is B*, where *A* and *B* are fuzzy predicates (or relations). In the syllogistic rules, the premises are expressed as *Q A's are B's*, where *Q* is a fuzzy quantifier and *A* and *B* are fuzzy predicates (or relations).

The rules in question are the following.

Categorical rules

$X, Y, Z, \cdots \triangleq$ variables taking values in $U, V, W, \ldots$

Examples

$X \triangleq Age(Mary)$, $Y \triangleq Distance(P1, P2)$
$A, B, C, \cdots \triangleq$ fuzzy predicates (relations)

Examples

$A \triangleq small$, $B \triangleq much\ larger$

Entailment rule

X is A
$A \subset B \rightarrow \mu_A(u) \leq \mu_B(u), u \in U$

X is B

Example

Mary is very young
very young $\subset$ *young*

Mary is young

Conjunction rule

X is A
X is B

X is $A \cap B \rightarrow \mu_{A \cap B}(u) = \mu_A(u) \wedge \mu_B(u)$

$\cap$ = intersection (conjunction)

Example

pressure is not very high
pressure is not very low

---
pressure is not very high and not very low

Disjunction rule

X is A
or *X is B*

X is $A \cup B \rightarrow \mu_{A \cup B}(u) = \mu_A(u) \vee \mu_B(u)$

$\cup$ = union (disjunction)

Projection rule

(X, Y) *is R*

X is ${}_XR \rightarrow \mu_{{}_XR}(u) = sup_v \mu_R(u, v)$

${}_XR \triangleq$ projection of R on U

Example

(X, Y) *is close to* $(3, 2)$

X is close to 3

Compositional rule

(X, Y) *is R* $\rightarrow$ *binary predicate*
Y is B

X is $A \circ R \rightarrow \mu_{A \circ R}(u) = sup_v(\mu_R(u, v) \wedge \mu_B(v))$

Example

X is much larger than Y
Y is large

X is much larger ∘ *large*

Negation rule

not (X is A)

$X \text{ is } \neg A \rightarrow \mu_{\neg A}(u) = 1 - \mu_A(u)$

$\neg \triangleq$ negation

Example

not (Mary is young)

Mary is not young

Extension principle

$$\frac{X \text{ is } A}{f(X) \text{ is } f(A)}$$

$$A = \mu_1/u_1 + \mu_2/u_2 + \cdots + \mu_n/u_n$$

$$f(A) = \mu_1/f(u_1) + \mu_2/f(u_2) + \cdots + \mu_n/f(u_n)$$

Example

X is small

X^2 *is* ^{2}*small*

$^2small \triangleq very\ small,\ \mu_{very\ small} = (\mu_{small})^2$

It should be noted that the use of the canonical form in these rules stands in sharp contrast to the way in which the rules of inference are expressed in classical logic. The advantage of the canonical form is that it places in evidence that inference in fuzzy logic may be interpreted as a propagation of elastic constraints. This point of view is particularly useful in the applications of fuzzy logic to control and decision analysis (*Proc. of the 2nd IFSA Congress, 1987; Proc. of the International Workshop*, Iizuka, 1988).

As was pointed out already, it is the qualitative mode of reasoning that plays a key role in the applications of fuzzy logic to control. In such applications, the input-output relations are expressed as collections of fuzzy if-then rules [32].

For example, if X and Y are input variables and Z is the output variable, the relation between X, Y, and Z may be expressed as

Z *is* C_1 if X *is* A_1 and Y *is* B_1

Z *is* C_2 if X *is* A_2 and Y *is* B_2

Z *is* C_n if X *is* A_n and Y *is* B_n

where C_i, A_i, and B_i, $i = 1, \cdots, n$ are fuzzy subsets of their respective universes of discourse. For example,

Z is small if X is large and Y is medium
Z is not large if X is very small and Y is not large

Given a characterization of the dependence of Z on X and Y in this form, one can employ the compositional rule of inference to compute the value of Z given the values of X and Y. This is what underlies the Togai–Watanbe fuzzy logic chip [50] and the operation of fuzzy logic controllers in industrial process control [48].

In general, the applications of fuzzy logic in systems and process control fall into two categories. First, there are those applications in which, in comparison to traditional methods, fuzzy logic control offers the advantage of greater simplicity, greater robustness, and lower cost. The cement kiln control pioneered by the F. L. Smidth Company falls into this category.

Second, are the applications in which the traditional methods provide no solution. The self-parking fuzzy car conceived by Sugeno [48] is a prime example of what humans can do so easily and is so difficult to emulate by the traditional approaches to systems control.

Syllogistic Rules: In its generic form, a fuzzy syllogism may be expressed as the inference schema

Q_1*A's are B's*

Q_2*C's are D's*

Q_3*E's are F's*

in which A, B, C, D, E, and F are interrelated fuzzy predicates and Q_1, Q_2, and Q_3 are fuzzy quantifiers.

The interrelations between A, B, C, D, E, and F provide a basis for a classification of fuzzy syllogisms. The more important of these syllogisms are the following.

a) *Intersection/Product Syllogism:*

$$C = A \wedge B,\ E = A,\ F = C \wedge D.$$

b) *Chaining Syllogism:*

$$C = B,\ E = A,\ F = D.$$

c) *Consequent Conjunction Syllogism:*

$$A = C = E,\ F = B \wedge D.$$

d) *Consequent Disjunction Syllogism:*

$$A = C = E,\ F = B \vee D.$$

e) *Antecedent Conjunction Syllogism:*

$$B = D = F,\ E = A \wedge C.$$

f) *Antecedent Disjunction Syllogism:*

$$B = D = F,\ E = A \vee C.$$

In the context of expert systems, these and related syllogisms provide a set of inference rules for combining evidence through conjunction, disjunction, and chaining [62].

One of the basic problems in fuzzy syllogistic reasoning is the following: given A, B, C, D, E, and F, find the maximally specific (i.e., most restrictive) fuzzy quantifier Q_3 such that the proposition Q_3*E's are F's* is entailed by the premises. In the case of a), b), and c), this leads to the following syllogisms.

Intersection/Product Syllogism:

Q_1*A's are B's*

Q_2(*A* and *B*)*'s are C's*

($Q_1 \otimes Q_2$)*A's are* (*B* and *C*)*'s* (5.1)

where $\otimes$ denotes the product in fuzzy arithmetic [24]. It should be noted that (5.1) may be viewed as an analog of the basic probabilistic identity

$$p(B, C/A) = p(B/A)\, p(C/A, B).$$

A concrete example of the intersection/product syllogism is the following:

most students are young

most young students are single

most^2 *students are young* and *single* (5.2)

where *most*^{2} denotes the product of the fuzzy quantifier *most* with itself.

Chaining Syllogism:

Q_1*A's are B's*

Q_2*B's are C's*

($Q_1 \otimes Q_2$)*A's are C's*

This syllogism may be viewed as a special case of the intersection product syllogism. It results when $B \subset A$ and Q_1 and Q_2 are monotone increasing, that is, $\geq Q_1 = Q_1$, and $\geq Q_2 = Q_2$, where $\geq Q_1$ should be read as *at least* Q_1. A simple example of the chaining syllogism is the following:

most students are undergraduates

most undergraduates are single

most^2 *students are single*

Note that *undergraduates* $\subset$ *students* and that in the conclusion $F =$ *single*, rather than *young and single*, as in (5.2).

Consequent Conjunction Syllogism: The consequent conjunction syllogism is an example of a basic syllogism which is not a derivative of the intersection/product syllogism. Its statement may be expressed as follows:

Q_1*A's are B's*

Q_2*A's are C's*

Q A's are (*B* and *C*)*'s* (5.3)

where Q is a fuzzy quantifier which is defined by the inequalities

$$0 \circledvee (Q_1 \otimes Q_2 \ominus 1) \leq Q \leq Q_1 \circledwedge Q_2 \qquad (5.4)$$

in which $\circledvee$, $\circledwedge$, $\oplus$, and $\ominus$ are the operations of $\vee$ (max), $\wedge$ (min), $+$, and $-$ in fuzzy arithmetic.

An illustration of (5.3) is provided by the example

most students are young

most students are single

Q students are single and *young*

where

$$2most \circledvee 1 \leq Q \leq most.$$

This expression for Q follows from (5.4) by noting that

$$most \circledvee most = most$$

and

$$0 \circledvee (2most \ominus 1) = 2most \ominus 1.$$

The three basic syllogisms stated above are merely examples of a collection of fuzzy syllogisms which may be developed and employed for purposes of inference from common sense knowledge. In addition to its application to common sense reasoning, fuzzy syllogistic reasoning may serve to provide a basis for combining uncertain evidence in expert systems [62].

VI. Concluding Remarks

One of the basic aims of fuzzy logic is to provide a computational framework for knowledge representation and inference in an environment of uncertainty and imprecision. In such environments, fuzzy logic is effective when the solutions need not be precise and/or it is acceptable for a conclusion to have a dispositional rather than categorical validity. The importance of fuzzy logic derives from the fact that there are many real world applications which fit these conditions, especially in the realm of knowledge-based systems for decision-making and control.

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