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L^1-STABILITY OF CONSTANTS IN A MODEL FOR RADIATING GASES*

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Abstract. In a previous work, the L^1-stability of constant states in a model of radiative gases, under a zero-mass initial disturbance, was left open. Actually, it was proved only for the Burgers flux and odd initial data which were non-negative on $\mathbb{R}^+$. We now prove this stability in full generality. This result is used, as usual, to prove the L^1-stability of shock profiles.

1. Introduction

The equations for radiative gases consist in the Euler equations of a perfect compressible fluid (conservation of mass, momentum, and energy), coupled with an elliptic equation for the temperature, which does not contain time derivatives. As such, its mathematical analysis is rather difficult. A baby model, consisting of a single conservation law coupled with an elliptic equation, has been studied by Kawashima and Nishibata [5, 6]. Their model reads

$$u_t + (u^2/2)_x = q_x, \quad -q_{xx} + q = u_x.$$

The right-hand side q_x may be viewed as a diffusion term Lu, where

$$Lu := K * u - u, \quad K(x) := \frac{1}{2}e^{-|x|}.$$

One notices that $K \geq 0$ and

$$\int_{\mathbb{R}} K(x)dx = 1,$$

so that L generates a linear semigroup on $L^1(\mathbb{R}) + L^\infty(\mathbb{R})$, which is L^1-contracting, preserves the total mass, and obeys to the maximum principle. Since the same (up to the linearity) hold true for the semigroup generated by a hyperbolic conservation law $u_t + f(u)_x = 0$, it is not surprising that the more general equation

$$u_t + f(u)_x = Lu \tag{1.1}$$

yields a well-posed Cauchy problem for data in $L^1(\mathbb{R}) + L^\infty(\mathbb{R})$ and that the corresponding semigroup $(S_t)_{t>0}$ satifies the three properties mentioned above, namely

Cntr if $b - a \in L^1(\mathbb{R})$, then $S_t b - S_t a \in L^1(\mathbb{R})$ and $\|S_t b - S_t a\|_1 \leq \|b - a\|_1$ (here, $\|\cdot\|_p$ denotes the L^p-norm on $\mathbb{R}$),

Cons if $b - a \in L^1(\mathbb{R})$, then

$$\int_{\mathbb{R}} (S_t b - S_t a)\, dx = \int_{\mathbb{R}} (b - a)\, dx,$$

Comp if $a \leq b$ a.e., then $S_t a \leq S_t b$ a.e.

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The notion of solution under consideration is that of "entropy solutions" which mimics the one of the hyperbolic situation. It was first considered by Kawashima and Nishibata [6], who proved that blow-up of derivatives may occur in finite time.

The existence and uniqueness of an entropy solution were first shown for data with bounded variations by Ito, in an unpublished work [4]. The case of data in $L^1 \cap L^\infty$ was treated recently by Lattanzio and Marcati [8], but the comparison principle (except with respect to constants) and the conservation of mass were not discussed. Up to our knowledge, the result has not yet been extended to the class $L^1 + L^\infty$, though we strongly believe that it still holds true. There even must be a general construction of a semigroup with the three properties above, provided that the linear semigroup generated by the dissipation operators L satisfies all of them; Trotter's product formula is a strong support for that conjecture. We leave that point for a future work, if no one addresses this question soon. We actually need here only the results proved in [8] (plus the conservation of mass) since we deal with integrable data. The extension to the class $L^1 + L^\infty$ is only needed in the proof that L^1 stability of constants implies that of shock profiles (see Theorem 2).

A natural question immediately arises from the contraction property. Assume that U is a steady solution (one may as well consider a travelling wave, by chosing a moving frame where it is steady), and let us consider the solution u associated to an initial data a which differs from U by an integrable disturbance. From above, we know that $u(t) - U$ remains integrable, with the properties that $\|u(t) - U\|_1$ is a nonincreasing function of time, while the mass m of $u(t) - U$ remains constant. Therefore, $\|u(t) - U\|_1$ tends to some limit ℓ, which is bounded by below by $|m|$. When U has finite total variation, and the limits $u_\pm = U(\pm\infty)$ (which exist in this case) are distinct, it is possible to reduce to the case $m = 0$, to the price of a shift of U. Thus we may ask whether $\ell = 0$ (a property called L^1-stability of U) or not. It has been found by H. Freistühler and the author [3] that a complete answer to this question needs the knowledge of the L^1-stability of constant states. In the latter case, the notion of L^1-stability has to be redefined since there is no possibility, when U is constant, of reducing to the case $m = 0$ by a shift. The "triangle" inequality $\ell \geq |m|$ shows that the condition $m = 0$ is necessary for this stability. Hence we say that a constant U is L^1-stable if $\|u(t) - U\|_1$ tends to zero whenever the initial disturbance is integrable and has "zero mass" (meaning that $m = 0$). The question is thus whether $m = 0$ implies L^1-stability of the constant. Of course, all these comments are valid for any conservation law with a dissipation compatible with the three properties **(Cntr, Cons, Comp)** above.

These questions received a complete answer in the case of a viscous diffusion u_{xx} (see [11, 3][1] and [10] for older partial results). The case of a semilinear relaxation was solved as well (see [9, 12]). For the radiative model, the stability of constants was proved only for the Burgers flux $f(u) = u^2/2$, assuming moreover that a be odd, non-negative on $\mathbb{R}^+$. Though this result is far from satisfactory, it was strong enough to imply the stability of shock profiles (see [12] for these two results). The goal of the present paper is to prove the stability of constants for the radiative model in full generality and to derive the stability of shock profiles.

Weak Solutions and L^2-Stability. It was pointed out in [5, 6] that the diffusion operator L is not strong enough to prevent shock formation, unlike the viscosity

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operator u_{xx}. Therefore weak solutions have to be considered.

Before describing this notion, we note that for every diffusion operator L_1 satisfying the contraction property, there holds an inequality "à la Kato" $(\mathrm{sgn} b)L_1 b \leq L_1|b|$, for every integrable b. In other words, $\eta'(u)L_1 u \leq L_1\eta(u)$ holds for every Kruzkhov entropy, and therefore for every convex function, by linearity[2]. Hence, when constructing the solution of the Cauchy problem by the vanishing viscosity method (for instance), we arrive to the following definition of a weak solution of

$$u_t + f(u)_x = L_1 u,$$

when the initial data a is bounded and measurable; the solution is the unique bounded measurable function $u : \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R}$ which satisfies, for every Kruzkhov entropy η (hence for every convex function) whose flux is Q, the inequality

$$\int_0^{+\infty}\int_{\mathbb{R}}(\eta(u)(\phi_t + L^*\phi) + Q(u)\phi_x)dxdt + \int_{\mathbb{R}}\eta(a)\phi(x,0)dx \geq 0,$$

for every non-negative test function ϕ.

The above definition is valid on L^∞. Using functions η of the form $|u|^p$, we see that the semigroup S_t preserves $L^p \cap L^\infty$ for every $p \geq 1$. Since $L^1 \cap L^\infty$ is dense in L^1, and since S_t is L^1-contracting, it uniquely extends as a contracting semigroup of $L^1(\mathbb{R})$. This will be our definition of L^1-solutions for L^1-data, though the equation need not make sense in $\mathcal{D}'(R \times (0,+\infty))$ for such solutions.

As mentioned above, the L^2-norm of the solution decays when $a \in L^2(\mathbb{R})$. We now show that, in the case $a \in L^1 \cap L^2(\mathbb{R})$, $u(t)$ tends to zero in the L^2-norm as t tends to infinity, with a rate $t^{-1/4}$. For that, we focus on the radiative operator L, though the argument is quite general. We shall only use the fact that $\widehat{Lu}(\xi) = -m(\xi)\hat{u}(\xi)$, where $m(\xi) \geq \omega \min\{1, \xi^2\}$ for some positive constant ω. As a matter of fact, we have $m(\xi) = \xi^2/(1+\xi^2)$. In practice, m is real, non-negative, even, and is increasing on R^+.

Up to a rescaling, we may assume that $\omega = 1$. We now use Plancherel's formula, with the convention that

$$\hat{u}(\xi) = \frac{1}{\sqrt{2\pi}}\int_{\mathbb{R}} e^{-ix\xi}u(x)dx.$$

We have

$$\|u\|_2^2 = \|\hat{u}\|_2^2 = \left(\int_{|\xi|>\alpha} + \int_{|\xi|<\alpha}\right)|\hat{u}|^2 d\xi,$$

from which there follows

$$\|u\|_2^2 \leq \frac{1}{m(\alpha)}\int_{\mathbb{R}} m(\xi)|\hat{u}|^2 d\xi + 2\alpha\|\hat{u}\|_\infty^2 \leq \frac{1}{m(\alpha)}N(u)^2 + \frac{\alpha}{\pi}\|u\|_1^2,$$

where $N(u)$ denotes the norm

$$\left(-\int_{\mathbb{R}} u(Lu)dx\right)^{1/2}.$$

[2]That fact can be proved first for smooth η and u, then by a density argument.

We now minimize the right-hand side with respect to the parameter $\alpha > 0$. We find

$$\|u\|_2^2 \leq 3 \max\{N(u)^2, (2\pi)^{-2/3} \|u\|_1^{4/3} N(u)^{2/3}\}. \tag{1.2}$$

Applying this inequality to the solution of (1.1), and recalling that $\|u\|_1$ is decaying, we have

$$\frac{\|u\|_2^2}{\|a\|_1^2} \leq \phi\left(\frac{N(u)^2}{\|a\|_1^2}\right), \quad \phi(\sigma) := 3 \max\{\sigma, (2\pi)^{-2/3} \sigma^{1/3}\}. \tag{1.3}$$

Let us denote by ψ the inverse of ϕ:

$$\psi(\tau) = \min\left\{\frac{\tau}{3}, \frac{2\pi}{27}\tau^3\right\}.$$

Applying the entropy inequality to $\eta(u) = u^2/2$, we have

$$\frac{d}{dt}\|u\|_2^2 + 2N(u)^2 \leq 0$$

(remember that, since the solutions are not necessarily smooth, the inequality may be strict). Then, thanks to (1.3), we obtain the differential inequality

$$Y' + \psi(Y) \leq 0, \quad Y := \frac{\|u\|_2^2}{\|a\|_1^2}.$$

From this, there comes

$$R(Y(t)) \leq R(Y(0)) - t, \quad R(y) := \int_{3/2\pi}^{y} \frac{dz}{\psi(z)}. \tag{1.4}$$

Since $R(0+) = -\infty$ (for $y < 3/2\pi$, we have $R(y) = 3\pi - 27/(4\pi y^2)$), the inequality (1.4) is an *a priori* estimate for Y: We find $Y(t) \leq R^{-1}(R(Y(0)) - t)$. Since $R(Y(0)) - t$ tends to $-\infty$ as $t \to +\infty$, the right-hand side decays like $2\sqrt{\pi t/27}$. Hence the following statement.

PROPOSITION 1. *Given an initial data $a \in L^1 \cap L^2(\mathbb{R})$, the weak solution of (1.1) decays in L^2 with*

$$\frac{\|u(t)\|_2}{\|a\|_1} \leq \left(\frac{4\pi t}{27}\right)^{-1/4} + \mathcal{O}(t^{-1/2}).$$

COMMENTS: It is remarkable that this inequality does not depend on the flux f in equation (1.1). Importantly, this decay is valid even if the total mass $\int a(x)\,dx$ is nonzero. Here, the $\mathcal{O}$ involves the norm $\|a\|_2$. A stronger property holds in the case of the viscous diffusion u_{xx}, where $m(\xi) \equiv \xi^2$. Then $R(+\infty)$ is finite and the conclusion holds with the weaker assumption $a \in L^1(\mathbb{R})$: there is an immediate L^2-regularization effect which is usually called dispersion. This regularization does not hold for the radiative diffusion. Indeed, if $f \equiv 0$, (1.1) is nothing but an ordinary differential equation in each space L^p, since L is a continuous operator on L^p. In particular, if the solution belongs to L^p at some time t, then $a \in L^p$. However, we do not exclude a dispersion effect due to the nonlinearity when $f'' > 0$, as in the case of the Burger's equation ($f(u) = u^2/2$, $L \equiv 0$, see [1, 2]). We leave open this question. The decay rate $t^{-1/4}$ is sharp under our assumptions. Indeed, if $f \equiv 0$, then the solution can be computed explicitly via Fourier transform. If, for instance, $\hat{a}$ is real valued, even, non-negative and compactly supported, then $\|u(t)\|_1$ turns out to be equivalent to a constant time $t^{-1/4}$. We do not expect a different behavior for a nonzero flux f, since a formal asymptotics yields diffusion waves of the form $t^{-1/2}h(xt^{-1/2})$ when a is integrable, the profile h being fully determined by the initial mass

$$m_0 := \int_{\mathbb{R}} a(x)dx.$$

Our proof is specific to the scalar case since it makes use of the control of the L^1-norm. It does not give a better decay than the one known for the case of systems, as described in [7], but it has the advantage to concern every weak solution instead of being restricted only to small and smooth solutions.

L^2-Decay for Zero-Mass Solutions. We now consider the case $m_0 = 0$, for which we shall prove a stronger decay rate of the L^2-norm, namely of order $t^{-1/2}$. As mentioned above, this improved decay cannot hold for a nonzero-mass solution.

Our strategy consists in introducing the potential $p(x,t)$ by

$$p_x = u, \quad p_t = Lp - f(u),$$

thanks to equation (1.1). We normalize p by $p(-\infty, 0) = 0$, so that

$$p(x,t) = \int_{-\infty}^{x} u(y,t)dy.$$

Hence, p is bounded, continuous in the space variable. The conservation of mass then implies

$$p(\pm\infty, t) = 0, \quad t \geq 0.$$

Using entropy inequality for u and the improved regularity for p, we have

$$\left(\frac{1}{2}(u^2 + p^2)\right)_t + (g(u))_x \leq uLu + pLp - pf(u),$$

with $g'(r) := rf'(r)$ and $g(0) = 0$. Let us now denote by r the solution of the elliptic equation $-r_{xx} + r = u$, so that $Lu = r_{xx} = r - u$ and $Lp = r_x$. Then $uLu + pLp = u(r-u) + pr_x = (pr)_x - u^2$. We end up with

$$\left(\frac{1}{2}(u^2 + p^2)\right)_t + (g(u) - pr)_x + u^2 \leq -pf(u).$$

Integrating with respect to x, taking into account that $g(u)$ and pr vanish at infinity, we obtain

$$\frac{d}{dt}\int_{\mathbb{R}} \frac{1}{2}(u^2 + p^2)dx + \int_{\mathbb{R}} u^2 dx \leq -\int_{\mathbb{R}} pf(u). \tag{1.5}$$

At this stage, we are free to assume $f(0) = 0$, since the addition of a constant does not modify the equation. Using a moving frame, which does not affect the L^p norms which we are dealing with, we may also assume that $f'(0) = 0$. Hence, assuming that f is twice differentiable, we have $f(\sigma) = \mathcal{O}(\sigma^2)$ as σ tends to zero.

We now suppose that a is bounded, an assumption that we shall get rid of later on. Since $\|u(t)\|_\infty \le \|a\|_\infty$, we obtain $|f(u)| \le c(\|a\|_\infty)u^2$, and we may bound the right-hand side of (1.5) by

$$c(\|a\|_\infty)\|p(t)\|_\infty\|u(t)\|_2^2.$$

We now use the fact that $\|p(t)\|_\infty \le \|u(t)\|_1 \le \|a\|_1$ and we conclude that

$$\frac{d}{dt}\int_{\mathbb{R}} \frac{1}{2}(u^2 + p^2)dx + \|u\|_2^2 \le c(\|a\|_\infty)\|a\|_1\|u\|_2^2. \tag{1.6}$$

Let us begin with the case where $c(\|a\|_\infty)\|a\|_1 < 1$. Under this assumption, (1.6) shows that if $p(\cdot, 0) \in H^1(\mathbb{R})$, then $t \mapsto \|u(t)\|_2^2$ is integrable on $\mathbb{R}^+$. Since it is also a nonincreasing function, it must be bounded above by βt^{-1} for some constant β. Therefore, $\|u(t)\|_2$ decays as $t^{-1/2}$, a better decay than the one found for nonzero mass data.

At this point, it is worth noticing that this decay will also hold true provided $p(\cdot, 0) \in H^1(\mathbb{R})$, $a \in L^\infty(\mathbb{R})$ and $\|u(t)\|_1$ tends to zero, since then we shall be allowed to apply the same argument from the data $u(T)$, T being large enough so as to satisfy $c(\|a\|_\infty)\|u(T)\|_1 < 1$. Since we shall show eventually that $\|u(t)\|_1$ tends to zero for every zero-mass initial data, we see that the decay rate $t^{-1/2}$ will hold true whenever $p(\cdot, 0) \in H^1(\mathbb{R})$ and $a \in L^\infty(\mathbb{R})$.

L^1-Stability for Small Zero-Mass Data. By "small data" we mean those which satisfy the assumptions $p(\cdot, 0) \in H^1(\mathbb{R})$, $a \in L^\infty(\mathbb{R})$ and $c(\|a\|_\infty)\|a\|_1 < 1$ above. For these, we already know the decay rate $t^{-1/2}$ of the L^2-norm. We now find another dispersion inequality.

We start from the inequality

$$|u|_t + (f(u)\mathrm{sgn} u)_x \le L|u|.$$

Multiplying by $|x|$ and integrating by parts, we obtain

$$\frac{d}{dt}\int_{\mathbb{R}} |xu|dx \le \int_{\mathbb{R}} |u|(L^*|x|)dx + \int_{\mathbb{R}} f(u)\mathrm{sgn}(xu)\,dx.$$

Actually, L is self-adjoint, and an elementary computation gives $L^*|x| = L|x| = e^{-|x|}$, which is square integrable. Hence, we deduce the following inequality:

$$\frac{d}{dt}\int_{\mathbb{R}} |xu|dx \le c_1\|u\|_2 + c(\|a\|_\infty)\|u\|_2^2.$$

Thanks to the known L^2-decay, we derive that, provided xa is integrable, then xu is integrable too, and we have other proof corrections:

$$\|xu\|_1 = \mathcal{O}(\sqrt{t}).$$

We now apply Cauchy-Schwarz inequality:

$$\|u\|_1^2 \le \|(1 + |x|)u\|_1 \int_{\mathbb{R}} \frac{|u|}{1 + |x|}dx = \mathcal{O}(\sqrt{t}) \int_{\mathbb{R}} \frac{|u|}{1 + |x|}dx,$$

and then once more:

$$\|u\|_1^4 \le \mathcal{O}(t)\|u\|_2^2.$$

Since $t \mapsto \|u\|_2^2$ is integrable (see the former section), we conclude that

$$\int_0^{+\infty} \|u\|_1^4 \frac{dt}{t} < +\infty. \tag{1.7}$$

Now, reminding that $t \mapsto \|u\|_1$ is nonincreasing, (1.7) implies that its limit is zero.

Let us summarize what we have proved here.

LEMMA 1. *Let $p_0 \in H^1(\mathbb{R})$ be given and define $a := (p_0)_x$. Assume that $xa \in L^1(\mathbb{R})$, $a \in L^1 \cap L^\infty(\mathbb{R})$ and that*

$$c(\|a\|_\infty)\|a\|_1 < 1. \tag{1.8}$$

Then the solution tends to zero in L^1 :

$$\lim_{t\to+\infty} \|u(t)\|_1 = 0.$$

L^1-Stability for General Zero-Mass Data. We now get rid of all nonessential assumptions. These are of two distinct natures. One of them is quantitative (the inequality (1.8)), while the other ones are qualitative.

For a general data $a \in L^1(\mathbb{R})$, we define

$$\ell(a) := \lim_{t\to+\infty} \|S_t a\|_1.$$

From the contraction property, we immediately obtain the Lipschitz inequality

$$|\ell(b) - \ell(a)| \le \|b-a\|_1. \tag{1.9}$$

On the other hand, $\ell(S_t a) = \ell(a)$ for every $t > 0$.

Let $R > 0$ be given and define X_R the convex subset of $L^1(\mathbb{R})$, of functions v satisfying $\|v\|_\infty < R$ and

$$\int_{\mathbb{R}} v(x)dx = 0.$$

The set of data a which satisfy the assumptions of Lemma 1 is a dense subset[3] in

$$Y_R := \{b \in X_R\,;\, c(R)\|b\|_1 < 1\}$$

for the L^1-topology. Therefore, L^1-Lipschitz continuity of ℓ implies that $\ell \equiv 0$ on Y_R.

Let now a be in X_R and satisfy

$$c(R)\|a\|_1 < 2.$$

Then $b := a/2 \in Y_R$ and therefore $\ell(b) = 0$. From (1.9), we deduce that $\ell(a) \le \|a-b\|_1 = \|a\|_1/2$. This implies that there exists some large enough time T, such

[3]Because it contains the set of those $a \in X_R$ which have compact support.

that $S_T a \in Y_R$. Hence $\ell(S_T a) = 0$, meaning that $\ell(a) = 0$. Arguing by induction, the same conclusion is true under the assumption

$$a \in X_R, \quad c(R)\|a\|_1 < 2^k$$

for some integer k. Hence, every element a of X_R satisfies $\ell(a) = 0$. Using again (1.9) and the density of $\cup_{R>0} X_R$ in the zero-mass hyperplane of $L^1(\mathbb{R})$, we finish with the following theorem.

THEOREM 1. *Let $a \in L^1(\mathbb{R})$ have zero-mass. Then*

$$\lim_{t \to +\infty} \|S_t a\|_1 = 0.$$

As mentionned above, we also have the following result.

PROPOSITION 2. *Let $p_0 \in H^1(\mathbb{R})$ be given and let define $a := (p_0)_x$. If either a is bounded or f is uniformly an $\mathcal{O}(u^2)$, then*

$$\int_0^{+\infty} \|S_t a\|_2^2 dt < +\infty,$$

and in particular $\|S_t a\|_2 = \mathcal{O}(t^{-1/2})$.

It is not known whether this L^2-decay result can be extended to more general data and fluxes, for instance in the context of Theorem 1.

L^1-Stability of Shock Profiles. We recall briefly in this paragraph how Theorem 1 implies the L^1-stability of shock profiles.

We may first restrict to a steady profile, up to the choice of a moving frame. Thus let U be such a steady shock profile. We denote by $u_- < u_+$ the lower and upper bounds of U, which are end values since U is monotonous (see [5]). It has been shown in [12] that the stability holds true for every initial data a such that

$$\int_{\mathbb{R}} (a - U)dx = 0, \quad U(\cdot - \alpha) \le a \le U(\cdot - \beta)$$

for some constants α and β. The idea follows that developped first in [11], which uses the Lasalle's invariant principle and techniques of dynamical systems.

Because of the L^1-contraction property, the set $\mathcal{A}$ of data a for which $\|S_t(a) - U\|_1$ tends to zero as $t \to +\infty$ (that is, the basin of attraction of U) is closed under the L^1-distance. Thus, the former result implies that this set contains all data such that

$$\int_{\mathbb{R}} (a - U)dx = 0, \quad u_- \le a \le u_+.$$

Denoting by $\mathcal{B}$ the subset of $U + L^1$, defined by these (in)equalities, the L^1-contraction tells us even more: $\mathcal{A}$ contains every a such that the L^1-distance of $S_t a$ to $\mathcal{B}$ tends to zero.

There remains to show that for a general data with $a - U \in L^1$ and $\int_{\mathbb{R}} (a - U)dx = 0$, this distance $d(t)$ tends to zero. For that purpose, we follow the strategy of [3]: there exist two functions $a_\pm$, with $a_\pm - u_\pm \in L^1$ and such that

$$\int_{\mathbb{R}} (a_\pm - u_\pm)dx = 0, \quad a_- \le a \le a_+.$$

Applying Theorem 1 to $a_\pm$, we have

$$\lim_{t\to+\infty} \|S_t a_\pm - u_\pm\|_1 = 0.$$

However, the comparison principle tells that $S_t a_- \le S_t a \le S_t a_+$ and therefore

$$d(t) \le \|S_t a_- - u_-\|_1 + \|S_t a_+ - u_+\|_1.$$

This shows that $d(t)$ tends to zero and hence that a belongs to $\mathcal{A}$.

In conclusion, assuming that the Cauchy problem is well-posed in $L^1 + L^\infty$ and satisfies the three main properties (no doubt about that), we have the following result.

THEOREM 2. *Let U be a shock profile, meaning that $u(x,t) := U(x-st)$ is a travelling wave of*

$$u_t + f(u)_x = Lu,$$

as in [5]. Let a be given in $U + L^1$ and h be defined by

$$h := \frac{1}{U(+\infty) - U(-\infty)} \int_{\mathbb{R}} (a - U)dx.$$

Then

$$\lim_{t\to+\infty} \|S_t a - U(\cdot - st - h)\|_1 = 0.$$

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