NBER WORKING PAPER SERIES

LABOR- AND CAPITAL-AUGMENTING TECHNICAL CHANGE

Daron Acemoglu

Working Paper 7544
http://www.nber.org/papers/w7544

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
February 2000

I thank Manuel Amador, Abhijit Banajee, Olivier Blanchard, and Jaume Ventura for useful comments. The views expressed herein are those of the authors and not necessarily those of the National Bureau of Economic Research.

Labor- and Capital-Augmenting Technical Change
Daron Acemoglu
NBER Working Paper No. 7544
February 2000
JEL No. O33, O14, O31, E25

ABSTRACT

I analyze an economy in which profit-maximizing firms can undertake both labor- or capital-augmenting technological improvements. In the long run, the economy looks like the standard growth model with purely labor-augmenting technical change, and the share of labor in GDP is constant. Along the transition path, however, there is capital-augmenting technical change and factor shares change. A range of policies may have counterintuitive implications due to their effect on the direction of technical change. For example, taxes on capital income reduce the labor share in the short run, but increase it in the medium/long run.

Daron Acemoglu
Massachusetts Institute of Technology
Department of Economics
E52-371
Cambridge, MA 02319
and NBER
daron@mit.edu

I. Introduction

Over the past hundred and fifty years of growth, the prices of the two key factors, labor and capital, have behaved very differently. While the wage rate, the rental price of labor, has increased at a rapid rate, the interest-rate, the rental price of capital, has remained approximately constant. This pattern appears remarkably stable across countries. Almost all models of growth and capital accumulation, of both endogenous and exogenous types, confront this fact using a special assumption on the *direction of technical change*: technical progress is assumed to be purely labor-augmenting.[1]

More specifically, consider an aggregate production function of the form $Y = F(MK, NL)$ where K is capital, and L is labor. The assumption of labor-augmenting technical change implies that new technologies only increase N, and do not affect M—or in other words, technical progress shifts the isoquants in a manner parallel to the labor axis. There is no obvious reason, however, why this should be so. Profit maximizing firms could invent or adopt technologies that increase M as well as N. Although starting with Romer's (1986) and Lucas' (1988) contributions a large literature has investigated the determinants of technological progress and growth, the direction of technical change—the reason why all progress takes the form of increases in N—has received little attention.

In this paper, I investigate this question. I show that in a standard model of endogenous growth, where firms invest in capital- and labor-augmenting technical change, all technical progress will be labor-augmenting along the balanced growth path. Therefore, given the usual assumptions for endogenous growth, the result that technical change will be purely labor-augmenting follows from profit maximizing incentives. Although in the long run the economy resembles the standard Solow model, along the transition path it will often experience capital-augmenting technical change, and as long as capital and labor are gross complements—i.e., the elasticity of substitution is less than 1—, it will converge to the balanced growth path. Intuitively, when the share of capital in GDP is too large, there will be further capital-augmenting technical change, and with the elasticity of substitution less than 1, this will push down the share of capital. Along the balance growth path, the share of capital and the interest rate will remain stable, while the wage rate will increase steadily due to labor-augmenting technical change.

[1]The other alternative is to assume that capital and labor enter in the aggregate production function with an elasticity of substitution that is identically equal to 1, which is clearly restrictive. Models of growth via capital deepening such as Jones and Manuelli (1990), Rebelo (1991), and Ventura (1997) are also consistent with increasing wage rates, but predict asymptotically declining interest rates and increasing capital share. Both the interest rate and the capital share in GDP have been approximately constant in the US over the past one hundred years (see, for example, Jorgensen, Gollop and Fraumeni, 1987, or the Economic Report of the President, 1998).

The ideas in this paper are closely related to the induced innovation literature of the 1960s. Fellner (1961) suggested that technical progress would tend to be more labor-augmenting because wages were growing, and were expected to grow, so technical change would try to save on this factor that was becoming more expensive. In an important contribution, Kennedy (1964) argued that innovations should occur so as to keep the share of GDP accruing to capital and labor constant.[2] Samuelson (1967), inspired by the contributions of Kennedy and Fellner, constructed a reduced form model where firms choose M and N in terms of the production function above in order to maximize the instantaneous rate of cost reduction. He showed that under certain conditions, this would imply equalization of factor shares. Samuelson also noted that with capital accumulation, technical change would tend to be labor-augmenting. Others, for example Nordhaus (1973), criticized this whole literature, however, because it lacked solid microfoundations. It was not clear who undertook the R&D activities, and how they were financed and priced.

My paper revisits this territory, but starts from a microeconomic model of technical change, as in, among others, Romer (1990), Segerstrom, Anant and Dinopoulos (1990), Grossman and Helpman (1991a,b), Aghion and Howitt (1992), and Young (1993), where innovations are carried out by profit maximizing firms. In contrast to these papers, I allow for both labor- and capital-augmenting innovations. There are in principle two ways of thinking about labor-augmenting technical progress; as the introduction of new production methods that directly increase the productivity of labor, or as the introduction of new goods and tasks that use labor. For concreteness, I take labor-augmenting progress to be "labor-using" progress, that is, the invention of new goods that are produced with labor. Similarly, I take capital-augmenting progress to be the invention of new goods using capital.[3] In this economy, new goods will be introduced because of the future expected profits from their sale. Intuitively, when there are n labor-augmenting goods, the profits from creating an additional one will be approximately proportional to $\frac{wL}{n}$ because each intermediate good producer will hire $\frac{L}{n}$ workers, and their profits will be given by a markup over the cost of production which depends on the wage rate, w. Similarly, when there are m capital-augmenting goods, the profits to further capital augmenting progress will be proportional to $\frac{rK}{m}$, where r is the rental rate of capital. When technical progress uses scarce factors such as labor, steady growth requires that further innovations build "upon the shoulders of giants", that is, the increase in n and m have to be proportional

[2]See also Ahmad (1965), Schmookler (1966), Hayami and Ruttan (1970), Habakkuk (1962), and David (1975).

[3]Later, I will show that the same results apply when labor-augmenting progress takes the form of "labor-enhancing" progress.

to their existing levels.[4] The return to allocating further resources to labor-augmenting innovation is therefore $n \cdot \frac{wL}{n}$ and the return to capital-augmenting innovation is $m \cdot \frac{rK}{m}$. These two returns will be balanced when factor shares are in line. Furthermore, when the elasticity of substitution between capital and labor is less than 1, a high level of n relative to m would imply that the share of capital is high compared to the share of labor. This will encourage more capital-augmenting technical progress. The converse will apply when m is too high. Equilibrium technical progress will therefore tend to stabilize factor shares.[5] Finally, since, with a stable interest rate, there will be capital accumulation along the balanced growth path, technical progress will increase n and the wage rate, while m remains stable.[6]

In addition to establishing the possibility of purely labor-augmenting technical change along the balanced growth path and analyzing the transitory dynamics, I use this model to study the impact of a range of policies on equilibrium factor shares. Minimum wages or other policies that lead to an adverse labor supply shift will increase the share of labor in GDP only in the short run, and will cause unemployment in the longer run. In contrast, subsidies to wages will increase the share of capital in the short run, but may reduce it in the medium/long run.

The rest of the paper is organized as follows. The next section outlines the basic environment, and discusses two sets of assumptions that are consistent with steady technological progress. It demonstrates that when one of the factors—capital—can be accumulated, only one of these two sets of assumptions is consistent with balanced growth, and in this case equilibrium technical change will be purely labor-augmenting. Section III characterizes the balanced growth equilibrium and the transitory dynamics in this case. Section IV analyzes the consequences of a range of policies on the dynamics of the equilibrium. Section V extends the model to allow for the production and R&D sectors to

[4] Rivera-Batiz and Romer (1991) refer to this case as the knowledge-based specification. Empirical work in this area supports the notion of substantial spillovers from past research, e.g. Caballero and Jaffee (1993), or Jaffee, Trajtenberg and Henderson (1993).

[5] Alternatively, if only final output is used for R&D —as in the lab equipment specification of Rivera-Batiz and Romer (1991)—, technical change along the balanced growth path will be both labor and capital-augmenting. Yet this will imply steadily increasing rental rates, and so would be not consistent with balanced growth when one of the factors—capital— can be accumulated linearly. This is the case I analyzed in previous work, Acemoglu (1998, 1999), where the two factors were skilled and unskilled labor. I studied the degree to which technical change is skill-complementary. I argued that such a model implies that changes in the skilled-composition of the labor force could explain the increase skill premium over the past twenty years.

[6] An important question is what n and m correspond to in practice. Although it is difficult to answer this question precisely within the context of a stylized model, it seems plausible to think of many of the great inventions of the 20th century, including electricity, new chemicals and plastics, entertainment, and computers, as expanding the set of tasks that labor can perform and the types of goods that labor can produce.

compete for labor, and also shows that the same results obtain in a quality ladder model.

II. Modeling The Direction of Technical Change

I start with a simple model of the direction of technical change, and illustrate under what circumstances equilibrium technical change will be purely labor-augmenting.

Consider an economy that admits a representative consumer[7] with the usual constant relative risk aversion (CRRA) preferences

$$\int_0^\infty \frac{C^{1-\theta}-1}{1-\theta}e^{-\rho t}dt. \tag{1}$$

The budget constraint of the consumer is:

$$C+I+X \leq Y \equiv \left[\gamma Y_L^\alpha + (1-\gamma)Y_K^\alpha\right]^{1/\alpha} \tag{2}$$

where $-\infty < \alpha \leq 1$, I denotes investment, and X is total R&D expenditure, if any. Consumption, investment, and R&D expenditure come out of an output aggregate produced from a labor intensive and a capital intensive good, Y_L and Y_K, with elasticity of substitution $\varepsilon \equiv 1/(1-\alpha)$.

Total population is normalized to 1, with L unskilled workers, who will work in the production sector, and S "scientists" who will perform R&D. This implies that the production and R&D sectors do not compete for workers. This is only for simplification, and below I consider the case in which the two sectors compete for workers.

The optimal consumption path of the representative consumer satisfies the familiar Euler equation:

$$g_c = \frac{\dot{C}}{C} = \frac{1}{\theta}(r-\rho), \tag{3}$$

where r is the rate of interest, and the consumption sequence $C(t)\mid_0^\infty$ satisfies the standard transversality condition,

$$\lim_{t\to\infty} C(t)^{-\theta}e^{-\rho t} = 0.$$

Consumer maximization gives the relative price of the capital intensive good as:

$$p \equiv \frac{p_K}{p_L} = \frac{1-\gamma}{\gamma}\left(\frac{Y_K}{Y_L}\right)^{-(1-\alpha)}, \tag{4}$$

[7] The presence of two types of agents, skilled scientists and regular workers, causes no problem for the representative consumer assumption since with CRRA utility functions these preferences can be aggregated into a CRRA representative consumer. See, for example, Caselli and Venture (2000).

where p_K is the price of Y_K and p_L is the price of Y_L. To determine the level of prices, I choose the price of the consumption aggregate in each period as numeraire, i.e.:

$$\gamma p_L^{1-\varepsilon} + (1-\gamma) p_K^{1-\varepsilon} = \gamma p_L^{-\frac{\alpha}{1-\alpha}} + (1-\gamma) p_K^{-\frac{\alpha}{1-\alpha}} = 1. \tag{5}$$

The labor-intensive and capital-intensive goods are produced competitively from the constant elasticity of substitution (CES) production functions with elasticity $\nu \equiv 1/(1-\beta)$,

$$Y_L = \left[\int_0^n y_l(i)^\beta di\right]^{1/\beta} \text{ and } Y_K = \left[\int_0^m y_k(i)^\beta di\right]^{1/\beta}, \tag{6}$$

where $y(i)$'s denote the intermediate goods[8] and $\beta \in (0,1)$, so that $\nu > 1$ and different intermediate goods are gross substitutes. This formulation implies that there are two different sets of intermediate goods, n of those that are produced with labor, and m that are produced using only capital. An increase in n, an expansion in the set of goods that use labor, corresponds to *labor-augmenting technical change*, while an increase in m corresponds to *capital-augmenting technical change*.

Intermediate goods are supplied by monopolists who hold the relevant patent. A patent to produce an intermediate good is given to the first firm that invents that good, and lasts indefinitely. Intermediates are produced linearly from their respective factors:

$$y_l(i) = l(i) \text{ and } y_k(i) = k(i), \tag{7}$$

where $l(i)$ and $k(i)$ are labor and capital used in the production of good i.

The CES production functions in (6) yield isoelastic demands for intermediate goods, with elasticity $\nu \equiv 1/(1-\beta)$. Profit maximization by the monopolists then implies that prices are given by a constant markup over marginal cost,

$$p_l(i) = \left(1 - \frac{1}{\nu}\right)^{-1} w = \frac{1}{\beta} w \text{ and } p_k(i) = \left(1 - \frac{1}{\nu}\right)^{-1} r = \frac{1}{\beta} r, \tag{8}$$

and

$$y_l(i) = l(i) = \frac{L}{n} \text{ and } y_k(i) = k(i) = \frac{K}{m}, \tag{9}$$

where L is total workforce in production and K is the capital stock of the economy.

Substituting (9) into (6) and integrating yields

$$Y_L = n^{\frac{1-\beta}{\beta}} L \text{ and } Y_K = m^{\frac{1-\beta}{\beta}} K. \tag{10}$$

[8] Alternatively, preferences could be directly defined over the different varieties of $y(i)$, with identical results.

Since factor markets are competitive, the wage and the rental rate of capital are

$$w = \beta n^{\frac{1-\beta}{\beta}} p_L \text{ and } r = \beta m^{\frac{1-\beta}{\beta}} p_K. \tag{11}$$

Substituting (10) into (4), the relative price of the capital intensive good is

$$p = \frac{1-\gamma}{\gamma}\left[\left(\frac{m}{n}\right)^{\frac{1-\beta}{\beta}}\frac{K}{L}\right]^{-(1-\alpha)}.$$

I define a balanced growth path (BGP) as an equilibrium path in which output grows at a constant rate. This implies that consumption also has to grow at a constant rate, so from the Euler equation, the rate of interest has to be constant. Therefore, in BGP the relative price of capital-intensive goods, p, has to remain constant, and hence p_K and p_L will also be constant.[9] Notice that for the relative price of capital goods to be constant, we need either m and n to grow at the same rate with no capital accumulation, or n to grow faster than m and K. Furthermore, notice that from equation (11), the interest rate will be constant in BGP only if m is constant. Therefore, BGP with a constant interest rate requires that only K and n grow.

The value of a monopolist who invents a new f-intermediate, for $f = l$ or k, is

$$V_f(t) = \int_t^{\infty} \exp\left[-\int_t^s \{r(\omega)+\delta_0\}d\omega\right]\pi_f(s)ds \tag{12}$$

where $r(t)$ is the interest rate at date t,

$$\pi_l = \frac{1-\beta}{\beta}\frac{wL}{n} \text{ and } \pi_k = \frac{1-\beta}{\beta}\frac{rK}{m} \tag{13}$$

are the flow profits from the sale of labor and capital-intensive intermediate goods, and δ_0 is the depreciation (obsolescence) rate of existing intermediates, for example because some new intermediates may be incompatible with the old ones.

To close the model, I need to specify how new intermediates are invented. Consider the following general form[10]

$$\dot{n} = x_l^{\eta}\left(n^{\upsilon}S_l\right)^{1-\eta} - \delta_0 n \text{ and } \dot{m} = x_k^{\eta}\left(m^{\upsilon}S_k\right)^{1-\eta} - \delta_0 m, \tag{14}$$

[9] This is because p_K cannot fall without bounds, so $\dot{m} > 0$ is inconsistent with BGP.

[10] There are obviously more involved versions of (14) which are also consistent with balanced growth, but are equivalent to (14) for the present purposes, e.g.

$$\begin{aligned} \dot{n} &= g_n\left(\frac{x_l}{n}\right)h_n(x_l, n^{\nu}S_l) - \delta_0, \\ \text{and } \dot{m} &= g_m\left(\frac{x_k}{m}\right)h_m(x_k, m^{\nu}S_k) - \delta_0, \end{aligned}$$

where g_n and g_m are weakly concave and increasing functions, and h_n and h_m exhibit constant returns to scale.

where x_l and x_k are the R&D expenditures in the two sectors in terms of the final good, and $S_l + S_k = S$ is the number of scientists.

The presence of x in the R&D equation implies that more goods can be invented by spending more on R&D, for example by using better equipment. The presence of n and m in these equations implies that scientists could potentially "stand upon the shoulders of giants", that is, current research benefits from past inventions. First, suppose $\upsilon < 1$ so that the extent of knowledge-based spillovers are limited. This immediately implies that balanced growth requires $\eta = 1$, in other words, only the final good should be used to create new intermediates as in the lab equipment specification of Rivera-Batiz and Romer (1991). With this assumption, BGP equilibrium implies that

$$\frac{rK}{m} = \frac{wL}{n},$$

which yields:

$$\frac{m}{n} = \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\beta}{\beta-(1-\beta)\alpha}} \left(\frac{K}{L}\right)^{\frac{\alpha\beta}{\beta-(1-\beta)\alpha}}.$$

This is isomorphic to the case previously discussed in Acemoglu (1998) where neither of the two factors could be accumulated. In this case, both m and n would grow at a constant rate, and the prices of both factors, r and w, would increase steadily. Although this case is appealing when the two factors are skilled and unskilled workers, with capital as one of the factors, the steady increase in the interest rate would encourage growth in the capital stock, which would further increase the interest rate. This case is therefore consistent neither with the constancy of the interest rate that we observe, nor with balanced growth.

An alternative specification for the R&D process involves $\upsilon = 1$ and $\eta = 0$ as in the knowledge-based R&D specification of Rivera-Batiz and Romer (1991), which implies[11]

$$\frac{\dot{n}}{n} = S_l - \delta_0 \text{ and } \frac{\dot{m}}{m} = S_k - \delta_0. \tag{15}$$

Zero-profits for R&D then determines the wages of scientists, ω_S, such that

$$\omega_S = \max\{nV_l, mV_k\}. \tag{16}$$

[11] The main results generalize to the case with $\upsilon = 1$ and $\eta \in [0, 1)$. Cost minimization would then imply that $\omega_S/n = nS_l/\ x_l$ and $\omega_S/m = mS_k/\ x_k$. This gives the same BGP results as (15), but transitory dynamics are more complicated because x_l and x_k change along the transition path.

Also note at this point another criticism of induced innovation models raised by Nordhaus (1973). Nordhaus criticized the absence of diminishing returns to labor-augmenting technical change. Modifying (15) such that $\frac{\dot{n}}{n} = f(S_l)$, that is introducing within period diminishing returns, will not affect the results. A formulation where $\dot{n} = g(n)f(S_l)$ without $g(n)$ being asymptotically linear would make it impossible for BGP technical change to be purely labor-augmenting, but would also be inconsistent with steady growth.

In BGP, incentives to innovate new goods using capital will be proportional to mV_k, i.e. to rK, and incentives to innovate new goods using labor will be proportional to nV_l or to wL. It is now possible to have a BGP equilibrium in which there is steady labor-augmenting technical change and capital accumulation, with no incentives to invent further capital-intensive goods. This is the case I will analyze in the rest of the paper.[12]

III. Characterization of Equilibrium

A. Balanced Growth Path (BGP)

I start with the analysis of BGP in the economy described above with the "innovations possibilities frontier" given by

$$\frac{\dot{n}}{n} = b(S_l - \delta) \text{ and } \frac{\dot{m}}{m} = b(S_k - \delta), \tag{17}$$

where, without loss of any generality, I introduced a constant coefficient b, which I will normalize to $b \equiv \frac{\beta}{1-\beta}$ below. In BGP, the interest rate, r, and the relative price, p, have to be constant, and the economy grows at some constant rate g. This implies that in BGP both wages and the capital stock of the economy also grow at this rate. Allowing for the depreciation of technologies at the rate δ, the value of inventing labor and capital-intensive goods are

$$V_l = \frac{1-\beta}{\beta}\frac{wL/n}{r-g+\delta} \text{ and } V_k = \frac{1-\beta}{\beta}\frac{rK/m}{r-g+\delta}.$$

Furthermore, since there has to be labor-augmenting technical change, the free entry condition (16) implies

$$nV_l = b\omega_S \text{ and } mV_k \leq b\omega_S.$$

So in BGP $\omega_S = \frac{wL}{r-g+\delta}$.

To keep interest rate constant, we must also have $\frac{\dot{m}}{m} = 0$, that is no net capital-augmenting technical change. This implies $S_k = \delta$. The remaining scientists will work on labor-augmenting technical change. The growth rate of the economy is therefore $g^{**} = \frac{1-\beta}{\beta}\frac{\dot{n}}{n} = \frac{1-\beta}{\beta}b(S-2\delta)$. Using the normalization $b \equiv \frac{\beta}{1-\beta}$, the growth rate is

$$g^{**} \equiv S - 2\delta.$$

[12] In Acemoglu (1998), I emphasized the presence of a market size effect that influences the direction of technical change; there will be more innovations directed at the more abundant factors because they constitute greater markets for new technologies. The lab equipment specification of R&D, with an elasticity of substitution, ε, greater than 1, exacerbates the market size effect. The market size effect is also present with the knowledge-based specification used in this paper, but is less pronounced. In particular, it is exactly balanced by a price effect that encourages more innovations towards more expensive factors, implying that incentives for further R&D are proportional to rK and wL.

I start the discussion with the case where $\delta = 0$, and define the growth rate in this case as $g^* \equiv S$.

When $\delta = 0$, in BGP $g_c = g^*$, and the Euler equation (3) gives the BGP interest rate as

$$r^* = \theta g^* + \rho$$

It is useful at this point to define

$$N \equiv n^{\frac{1-\beta}{\beta}} \text{ and } M \equiv m^{\frac{1-\beta}{\beta}},$$

which will simplify the notation below. With this normalization, the relative price of capital-intensive goods is $p = \frac{1-\gamma}{\gamma}\left(\frac{MK}{NL}\right)^{-(1-\alpha)}$, and the interest rate is:

$$r = \beta M \left[\gamma p^{\frac{\alpha}{1-\alpha}} + (1-\gamma)\right]^{\frac{1-\alpha}{\alpha}}.$$

Additionally, I define a normalized capital stock,

$$k \equiv \frac{MK}{NL},$$

which yields

$$r = R(M,k) \equiv \beta M \left[\gamma \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\alpha}{1-\alpha}} k^{-\alpha} + (1-\gamma)\right]^{\frac{1-\alpha}{\alpha}}. \tag{18}$$

This equation implies that there are many combinations of M and k (and hence p) that are consistent with BGP when $\delta = 0$.

Let $k = G(M)$, $G' > 0$, such that M and k are consistent with BGP (i.e., $r^* = R(M,k)$). Define the "relative share of capital", σ_K, as[13]

$$\sigma_K = \frac{rK}{wL} = p\frac{MK}{NL} = \frac{1-\gamma}{\gamma}\left(\frac{MK}{NL}\right)^{\alpha} = \frac{1-\gamma}{\gamma}k^{\alpha}.$$

So the relative share of capital, σ_K, will also differ in different BGPs. In particular, when M is higher, k will also be higher. The implication for the share of capital depends on the elasticity of substitution. When k increases, σ_K will also increase if $\alpha > 0$ (i.e. if the elasticity of substitution, $\varepsilon \equiv 1/(1-\alpha)$, is greater than 1), and will decrease if $\alpha < 0$.

[13] Strictly speaking, this is not the relative share capital as it leaves out the share of income accruing to scientists.

Now define $\overline{k}$ as the level of normalized capital such that it is equally profitable to invent new capital and labor-intensive goods, that is[14]

$$k = \overline{k} \equiv \left(\frac{\gamma}{1-\gamma}\right)^{1/\alpha} \iff \sigma_K = 1.$$

Finally, let M^* be such that $\overline{k} = G(M^*)$. We can now state (proof in the text):

Proposition 1 In the case where $\delta = 0$, there exists a BGP for each $M \geq M^*$. In all BGPs, output, consumption, wages, and the capital stock grow at the same rate g^*, and the share of labor is constant. Each BGP has a different relative capital share, σ_K.

This proposition is one of the main results of the paper. It demonstrates that, for a natural formulation of the technological process, BGP will be characterized with purely labor-augmenting technical change, even though profit maximizing firms could also invent new capital-augmenting technologies. Moreover, in all BGPs, the share of labor (or capital) remains constant as the economy grows. However, there are many different levels of the labor share consistent with balanced growth. The intuition for the multiplicity of BGPs is that, without depreciation, all that is required for a BGP is that investment in labor-augmenting technical change should be more profitable than capital-augmenting improvements, i.e. $V_k \leq V_l$, and this can happen for a range of capital (labor) shares. We will see next that starting from given conditions, the equilibrium is nevertheless unique, so the initial conditions determine the long run equilibrium factor shares. Furthermore, we will see that policy can affect the factor distribution of income, though in a somewhat paradoxical manner.

Next consider the case in which there is depreciation of new technologies, i.e. $\delta > 0$. If there is no capital-augmenting technical change, M will now decrease over time, and eventually it will reach too low a level. Therefore, balanced growth requires that, in addition to the equations above, we must have

$$nV_l = mV_k \text{ or } \sigma_K = 1.$$

So in equilibrium there must be both labor- and capital-augmenting R&D. However, capital-augmenting R&D only keeps the level of capital-augmenting technology, m, constant. Since some of the scientists now have to work to invent new capital-augmenting

[14] The incentives to carry out capital- and labor-augmenting improvements are balanced when $\sigma_K = 1$ because both types of innovations are equally difficult. It is straightforward to modify equation (17) such that $\frac{\dot{n}}{n} = b_l(S_l - \delta)$ and $\frac{\dot{m}}{m} = b_k(S_k - \delta)$, in which case $\sigma_K = b_k/b_l$ would ensure equal profits from the two types of innovations.

technologies to replenish those that depreciate, the growth rate of the economy will be lower, g^{**} instead of g^{*}. The BGP interest rate will then be

$$r^{**} = \theta g^{**} + \rho,$$

and we require

$$r^{**} = R(M, \overline{k}),$$

which defines a unique M^{**} consistent with BGP, i.e., $\overline{k} = G(M^{**})$. Hence, the BGP is now unique. I state this as a proposition (proof in the text).

Proposition 2 When $\delta > 0$, there is a unique BGP where $M = M^{**}$, $r = r^{**}$, and output, consumption, and wages grow at the rate g^{**}.

This proposition demonstrates that as long as there is some technological depreciation, the balanced growth path is unique. Nevertheless, I expect δ to be small; that is, plausibly there should be only limited exogenous technological depreciation.[15] So the medium run behavior will be similar to the dynamics of the economy with $\delta = 0$. For this reason, below I focus more on the dynamics when $\delta = 0$.

B. Transitory Dynamics

I now analyze the transitory dynamics of the economy with $\delta = 0$. Although there are multiple steady states, starting from any initial conditions, there is a unique equilibrium that converges to one of the steady states. However, in some cases the economy will not converge to any of the steady states. As I go along, I will also state the results for case in which $\delta > 0$.

I will analyze the dynamics of the system by studying the behavior of three variables, $c \equiv \frac{MC}{NL}$, M, and $k \equiv \frac{MK}{NL}$. The Euler equation for the representative consumer, (3), can be written as

$$\frac{\dot{C}}{C} = \frac{1}{\theta}\left(\beta M\left[\gamma\left(\frac{1-\gamma}{\gamma}\right)^{\frac{\alpha}{1-\alpha}} k^{-\alpha} + (1-\gamma)\right]^{\frac{1-\alpha}{\alpha}} - \rho\right).$$

Transforming this using $c \equiv \frac{MC}{NL}$, we have

$$\frac{\dot{c}}{c} = \frac{\dot{C}}{C} + \frac{\dot{M}}{M} - \frac{\dot{N}}{N},$$

[15]The parameter δ measures purely technological obsolescence, *not* creative destruction. Section V.B analyzes a model of quality ladders and creative destruction where without technological obsolescence, there will again exist multiple BGPs.

$$= \frac{1}{\theta}\left(\beta M\left[\gamma\left(\frac{1-\gamma}{\gamma}\right)^{\frac{\alpha}{1-\alpha}}k^{-\alpha}+(1-\gamma)\right]^{\frac{1-\alpha}{\alpha}}-\rho\right)+S_k-S_l. \tag{19}$$

The law of motion of normalized capital stock is

$$\begin{aligned}\frac{\dot{k}}{k} &= \frac{\dot{K}}{K}+\frac{\dot{M}}{M}-\frac{\dot{N}}{N}, \\ &= \frac{\left[\gamma(NL)^{\alpha}+(1-\gamma)(MK)^{\alpha}\right]^{1/\alpha}-C}{K}+S_k-S_l, \\ &= \frac{\left[\gamma+(1-\gamma)k^{\alpha}\right]^{1/\alpha}-c}{k}+S_k-S_l. \end{aligned} \tag{20}$$

Finally, we have the law of motion of M as

$$\frac{\dot{M}}{M}=S_k. \tag{21}$$

Equations (19)-(21) determine the transitory dynamics of the economy. First, notice that $S_k>0$ and $S_l=0$ if and only if $\sigma_K>1$; and $S_l>0$ and $S_k=0$ if and only if $\sigma_K<1$.[16] Next notice that this system has a special feature in that the growth rate of M is constant. Either we have $\sigma_K<1$, and M will be constant, or it will grow at a constant rate (though it may endogenously switch from one regime to the other). This implies that, for the most part, the dynamics will be determined by the standard forces of the neoclassical growth model.[17] In particular, as soon as $\sigma_K\leq 1$ and M is consistent with balanced growth (i.e. $M\geq M^*$), the system has $\dot{M}=S_k=0$, so will converge to the balance growth path. This enables a simple characterization of the dynamics of the system.

For ease of exposition I break the initial conditions into a number of different cases. Recall also that in BGP, we must have $r=r^*$ and $k=G(M)$.

Cases:

1. $k<\overline{k}$, $\alpha<0$. In this case, capital and labor-intensive goods are gross complements ($\varepsilon\equiv 1/(1-\alpha)<1$). The fact that $k<\overline{k}$ implies that the share of capital is greater

[16] Observe that $S_l=S_k$ only if $V_l(t)=V_k(t)$, which is only possible if $\dot{V}_l(t)=\dot{V}_k(t)=0$. This, in turn, would imply $\pi_l=\pi_k$ over the same interval of time, hence is impossible given $\sigma_K\neq 1$.

[17] Notice for example that around the BGP, we have

$$\begin{pmatrix}\dot{c}/c\\ \dot{k}/k\end{pmatrix}=\begin{pmatrix}0 & -\\ - & +\end{pmatrix}\begin{pmatrix}c\\ k\end{pmatrix},$$

which has one negative and one positive eigenvalue, so exhibits a unique saddle path.

than the labor share ($\sigma_K > 1$). Therefore, there will only be capital-augmenting technical change at first, i.e. $\dot{M}/M = S$ and $\dot{N}/N = 0$. However, both capital accumulation and the increase in M imply that k is also growing, so it will reach $\overline{k}$ in finite time, say at t'. At this point, M stops growing and remains at some level $M' \geq M^*$, and the economy will have reached a BGP with k approaching $k' = G(M') > \overline{k}$ and N growing at a constant rate. Figure 1 illustrates this case diagrammatically.

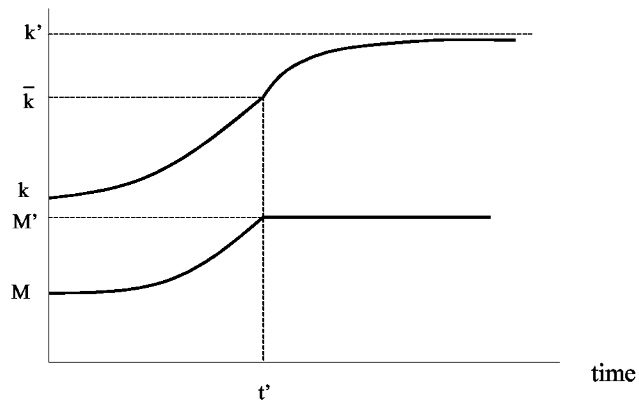

Figure 1: Dynamics in case 1.

In contrast, if $\delta > 0$, because $k' > \overline{k}$ and $\sigma_K < 1$, we will have $S_k = 0$, and M will fall steadily after time t' (i.e. after reaching M') due to natural depreciation at the rate δ, so $\dot{M}/M < 0$. Also we will have $\dot{k}/k < 0$. The transitory dynamics will eventually take us to M^{**} and $k = \overline{k}$, which is the unique steady state.

2. $M < M^*$, $k > \overline{k}$, $\alpha < 0$. The share of labor is now greater than the share of capital ($\sigma_K < 1$), so $\dot{M} = 0$ to start with, and k will decline (since N grows faster than K). When k reaches $\overline{k}$ say at time t', $M < M^*$, so $r < r^*$. Hence M has to grow for the interest rate to increase to r^*. So starting at $k = \overline{k}$, both M and N grow, and this continues until M reaches M^*, say time t''.[18] After this point, M stays constant,

[18] To see why t'' has to be finite, notice that otherwise we would have $\dot{M}/M \to 0^+$, so S_k would be declining and S_l would be increasing towards S asymptotically. Since $k = \overline{k}$, this implies that $\dot{K}/K$ has to increase. Moreover, $k = \overline{k}$ and $\dot{M} > 0$ also imply from (19) that $\dot{C}/C$ is increasing, or that C is accelerating. But (20) implies that as $t \to \infty$, $\dot{K}/K$ has to increase, leading to a contradiction. Hence, t'' has to be finite.

and N starts growing at a constant rate. After t', k remains constant. Figure 2 depicts this case diagrammatically.

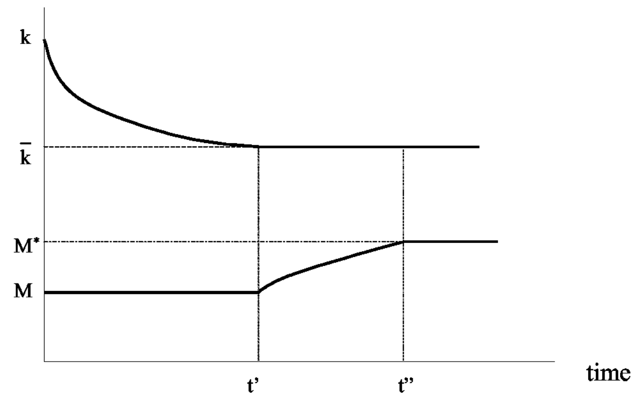

Figure 2: Dynamics in case 2.

In this case, the dynamics are also quite similar when $\delta > 0$.

3. $M > M^*$, $k > \overline{k}$, $\alpha < 0$. The share of labor is again greater than the share of capital ($\sigma_K < 1$), so $\dot{M} = 0$ and $\dot{N}/N = S$. But the interest rate, r, is less than r^*. The dynamics of the system are now identical to the standard neoclassical model starting with a level of capital greater than the steady state level and with labor-augmenting technical change at a constant rate. The economy converges to a BGP with $k = G(M) > \overline{k}$.

   Once again, if $\delta > 0$, then M will fall steadily due to natural depreciation at the rate δ, i.e. $\dot{M}/M < 0$, and also $\dot{k}/k < 0$. The transitory dynamics will eventually take us to the unique steady-state, M^{**} and $k = \overline{k}$.

4. Finally, when we have $\alpha > 0$ (i.e., $\varepsilon > 1$), the system will explode with asymptotically faster capital accumulation. If $k > \overline{k}$, the share of labor is less than the share of capital (i.e. $\sigma_K > 1$). This implies $\frac{\dot{N}}{N} = 0$, and $\dot{M} > 0$. But as M increases so does σ_K, encouraging further increase in M. Alternatively, if $k < \overline{k}$, first $\frac{\dot{N}}{N} = S$ and $\dot{M} = 0$, but $k < \overline{k}$ is not consistent with balanced growth, so k will also grow. When it exceeds $\overline{k}$, capital-augmenting technical change will become more profitable, and

the system will again explode.[19]

Therefore, when $\alpha < 0$, i.e. capital and labor-intensive goods are gross complements, starting from any initial condition, there is a unique equilibrium that converges to a balanced growth path, and the initial conditions determine the long run factor shares. In contrast, when $\alpha > 0$, the economy will never converge to a balanced growth path. In the rest of the paper, I focus on the case with $\alpha < 0$ where the BGP is always stable.

IV. Comparative Dynamics

I now discuss the impact of a range of policies on the factor distribution of income. My focus is again on the medium run behavior, so I explicitly discuss the case with $\delta = 0$. Once again, when $\delta > 0$, these results apply in the medium run, but the economy returns to its balanced growth path in the very long run. I first analyze the impact of a shift in the labor supply schedule of the economy, and then discuss the implications of government policy. Recall that from now on, I assume $\alpha < 0$, so labor and capital are gross complements.

Suppose that the economy has a static labor supply equation given by

$$L = \lambda \phi_0 \left(\frac{w}{Y} \right)$$

where ϕ_0 is an increasing function.[20] This equation links the supply of labor to the

[19] The exception is when $M = M^*$ and $k < \overline{k}$ initially. In this case, the economy will asymptotically approach $\overline{k}$, with $\frac{\dot{N}}{N} = S$ and $\dot{M} = 0$.

[20] This type of equation follows from a variety of microfoundations, including the neoclassical model of labor supply, or efficiency wage models. For example, preferences could be extended to

$$\int_0^\infty \frac{(C \cdot h^\eta)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt$$

where h is leisure. Alternatively, the labor market could be modeled as in Shapiro and Stiglitz (1984). Firms would always be on their demand curve, but have to satisfy an incentive compatibility constraint (for workers who share risk and hence act risk neutral),

$$w \geq b + e + \frac{e}{\psi} \left[(r - g) + s + s \frac{L}{\bar{L} - L} \right]$$

where ψ is the probability that a shirking worker is caught, g is the growth rate of the economy, e is cost of effort, b is unemployment benefit, and $\bar{L}$ is total labor force. Let $b = b_0 w$ and $e = e_0 Y$. This implies that the cost of effort increases in income and benefits increase with wages. Therefore, in BGP, the non-shirking condition takes the form:

$$\frac{wL}{Y} \geq \frac{e_0}{1 - b_0} \left[L + \frac{L}{\psi} \left(r + s - g + s \frac{L}{\bar{L} - L} \right) \right]$$

wage normalized by the level of income, so that steady growth does not lead to a steady increase in labor supply. This equation can be rearranged to express the supply of labor as a function of the labor share in GDP, or an inverse function of the relative share of capital, σ_K, that is

$$L = \lambda\phi(\sigma_K) \tag{22}$$

where ϕ is an decreasing function.

Now consider a decline in λ starting from a BGP with some level of capital-augmenting technology M'. This corresponds to an adverse supply shock, so at a given level of capital stock, it will reduce employment, increase wages, and reduce the interest rate. Since $\alpha < 0$, it will also reduce the relative share of capital σ_K. As in BGP $\sigma_K < 1$ anyway, this will not affect the direction of technical change, but will only slow down capital accumulation until the interest rate is restored to its steady state level, r^*. Therefore, eventually the capital share will return to its initial level, and employment will fall along the transition path. So the adverse supply shift has no effect on the factor distribution of income in the medium run. Figure 3 draws the dynamics of employment and labor share in this case.

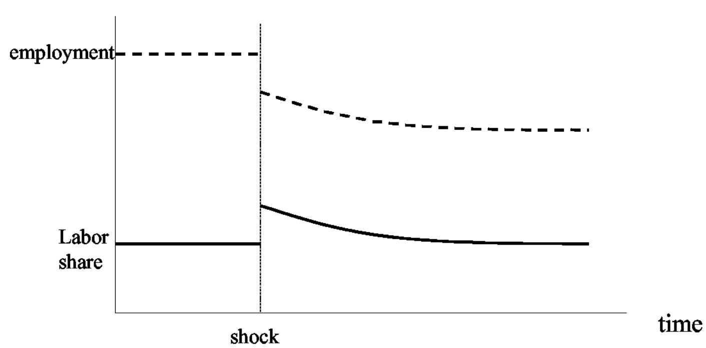

Figure 3: Employment and labor share dynamics in response to an adverse labor supply shock.

The implications of an increase in λ are quite different. Such a favorable labor supply shock will increase employment, reduce wages, and increase the interest rate. If

Therefore, the non-shirking condition is expressed as a relation between the labor share and the level of employment as in the reduced form equation (22).

I chose the reduced form formulation in the text because it ensures that there are no further dynamics coming from labor supply decisions and simplifies the discussion.

M' is close to M^*, so that σ_K is close to 1, this change will increase σ_K above 1, and encourage capital-augmenting technical change. This implies, however, that when the economy converges back to balanced growth, the level of M will be higher, so σ_K, the relative share of capital, will be lower. Figure 4 draws the dynamics of employment and labor share in this case. When $\delta > 0$, of course the relative share of capital has to return to 1, but the analysis here suggests that in response to such a labor supply shock, it will first increase, then fall below 1 for a while, and then increase again.

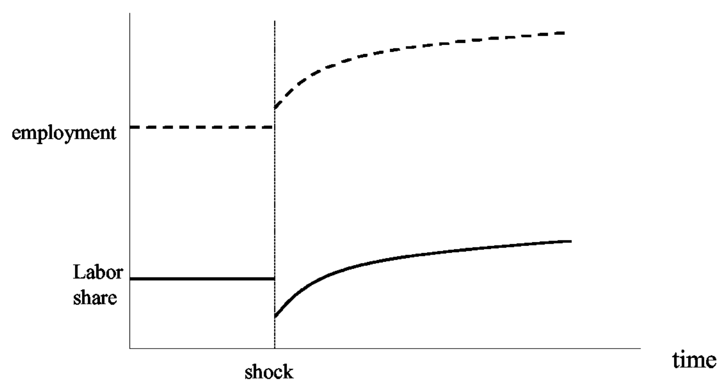

Figure 4: Employment and labor share dynamics in response to a favorable labor supply shock.

The implications of redistributive policies are also similar. First consider a mandated minimum wage w_M greater than the equilibrium wage (and growing at the same rate). This can be interpreted as an adverse labor supply shift, so will have exactly the same effects; it will increase the labor share in the short run, but leave it unchanged in the medium run.

Suppose next that the government imposes a tax τ_K on capital income, and a tax τ_L on labor income. This changes the equilibrium conditions above as follows

$$g_c = \frac{\dot{C}}{C} = \frac{1}{\theta}(r(1-\tau_K) - \rho)$$

and the labor supply schedule changes to

$$L = \lambda\phi\left((1-\tau_L)^{-1}\sigma_K\right)$$

So an increase in τ_L corresponds to a decline in λ, i.e. an adverse labor supply shock. It will therefore increase the labor share in the short run, but leave it unchanged in the medium run, as shown in Figure 3. In contrast, an increase in τ_K or a decline in τ_L will reduce the labor share in the short run, but will increase it in the medium/long-run. These policies will also increase employment. Therefore, in this economy, subsidies to labor and capital taxes may have very different long run implications than their short run consequences and than their implications in models where all technological change is assumed to be labor-augmenting.

V. Extensions

A. Unskilled Labor in R&D

The fact that there are two types of workers, unskilled labor and scientists, in the above model may be viewed as unattractive feature, since relative price of labor does not affect the cost of R&D. This assumption is made only for simplicity, and I now modify the model to allow the production and R&D sectors to compete for labor. For brevity, I only discuss the case without technological depreciation (i.e. $\delta = 0$). In particular, equation (15) changes to

$$\frac{\dot{n}}{n} = bL_l \text{ and } \frac{\dot{m}}{m} = bL_k, \tag{23}$$

with

$$L + L_l + L_k = 1,$$

so that new goods are invented by workers employed in the R&D sector. Most of the analysis from Sections II and III apply, but the free entry condition in BGP is modified to

$$nV_l = b\frac{1-\beta}{\beta}\frac{wL}{r-g} = w,$$

so that the marginal product of a worker in production is equated to his marginal product in R&D. This equation implies that in BGP

$$r - g = L.$$

Now using the Euler equation for consumption, (3), we have

$$(\theta - 1)g^* + \rho = L.$$

Furthermore, since in BGP $L_k = 0$,

$$g^* = \frac{1-\beta}{\beta}\frac{\dot{n}}{n} = L_l = 1 - L,$$

so

$$g^* = \frac{1-\rho}{\theta}.$$

The rest of the analysis is unchanged. In particular, in BGP there is only labor-augmenting technical change. Unfortunately, in this case, transitory dynamics are more complicated because both the number of production workers and the speed of technical progress change along the transition path.

B. Quality ladders

Labor-augmenting technical change has so far been interpreted as "labor-using" change, that is the introduction of new goods and tasks that use labor. I now show that the results of the above analysis generalize to different formulations of the technological change process. More specifically, I discuss the case where technical change takes the form of quality improvements as in Grossman and Helpman (1991a,b) and Aghion and Howitt (1992), and directly increases the productivity of labor and/or capital.

Preferences are still defined over the output aggregate given by (2). Suppose the two goods are produced competitively with the production functions

$$\begin{aligned} Y_L &= \frac{1}{1-\beta}\left(Q_L^\beta {z_L}^{1-\beta}\right) L^\beta \\ Y_K &= \frac{1}{1-\beta}\left(Q_K^\beta {z_K}^{1-\beta}\right) K^\beta \end{aligned} \tag{24}$$

where z_L and z_K are quantities of machines that complement labor and capital, and Q_L and Q_K denote the qualities of these machines. Technical progress takes the form of improvements over existing machines. For example, an R&D firm may discover a new vintage of labor-complementary machines, and this vintage would have productivity $Q'_L = (1+\lambda)Q_L$, where $\lambda > 0$. This R&D firm would be the monopoly supplier of this vintage, and it would dominate the market until a new, and better, vintage arrives. I assume that a scientist who works to discover a new vintage of Q_L (or Q_K) does so at the flow rate φ. Notice that this assumption already builds in knowledge-based spillovers that were required for the results: research on a vintage of quality Q_L leads to proportionately better machine, so the greater is Q_L, the greater is the resulting improvement in the "level" of productivity (i.e. λQ_L).

Final good producers maximize:

$$p_L Y_L - wL - \chi_L z_L \text{ and } p_K Y_K - rK - \chi_K z_K$$

where χ's denote the prices of machines. Without loss of a generality, I normalize the marginal cost of producing z to $1/(1+\lambda)$, and assume that λ a small enough that the leading monopolist will set a limit price to ensure that the next best vintage breaks even (see, for example, Grossman and Helpman, 1991b). This limit price is $\chi_K = \chi_L = 1$. Hence,

$$z_L = p_L^{1/\beta} Q_L L \text{ and } \; z_K = p_K^{1/\beta} Q_K K.$$

Substituting these into (24) yields

$$Y_L = \frac{1}{1-\beta} p_L^{(1-\beta)/\beta} Q_L L \text{ and } Y_K = \frac{1}{1-\beta} p_K^{(1-\beta)/\beta} Q_K K,$$

and the equilibrium interest and wage rates are:

$$\begin{aligned} r &= \beta(1-\beta)^{-1} p_K^{1/\beta} Q_K, \\ w &= \beta(1-\beta)^{-1} p_L^{1/\beta} Q_L. \end{aligned} \tag{25}$$

The values of a new (higher) quality intermediate good are given by standard Bellman equations (similar to (12) above):

$$\begin{aligned} (r+\delta_l)V_l - \dot{V}_l &= \frac{\lambda}{1+\lambda} p_L^{1/\beta} Q_L L \\ (r+\delta_k)V_l - \dot{V}_k &= \frac{\lambda}{1+\lambda} p_K^{1/\beta} Q_K K \end{aligned} \tag{26}$$

In BGP, $\dot{V}_l = \dot{V}_k$, so we have

$$V_l = \frac{\lambda w L}{(1+\lambda)(r+\delta_l)} \text{ and } V_k = \frac{\lambda r K}{(1+\lambda)(r+\delta_k)},$$

where δ_l and δ_k are the endogenous rates of creative destruction. From the above assumptions, we have $\delta_l = \varphi S_l$ and $\delta_k = \varphi S_k$. These equations immediately imply that only $V_l \geq V_k$ (or $V_l = V_k$) is consistent with stable factor shares.[21] Therefore, along the BGP, there will only be labor-augmenting technical change, i.e. $S_l = S$ and $S_k = 0$. So, the result that equilibrium technical change will be purely labor-augmenting does not rely on a specific formulation of the technological change process.

VI. Conclusion

Almost all models of economic growth rely on a very specific assumption; all technological change is assumed to be labor-augmenting. Recent years have witnessed important advances in our understanding of the determinants of technological change at the

[21] Notice also that despite the possibility of creative destruction, if there is no additional technological obsolescence, $V_l \geq V_k$ and there are again multiple BGPs.

aggregate level (see, for example, Aghion and Howitt, 1998, for a summary of much of the research to date), but the question of why all technical change appears to be labor-augmenting has received no attention. There seems to be no compelling reason why new ideas and better production methods have to help labor only, and the standard assumption of growth models appears highly ad hoc.

I studied the determinants of the direction of technical change in a model where the invention of new production methods is a purposeful activity. Profit maximizing firms can introduce capital- and/or labor-augmenting technological improvements. The major result is that, with the standard assumptions used to generate endogenous growth, technical change will in fact be purely labor-augmenting along the balanced growth path. Although in steady state the economy looks like the standard model with a steadily increasing wage rate and a constant interest rate, out of steady state there is often capital-augmenting technical change. Furthermore, I showed that a range of policies will have unusual implications because they induce capital-augmenting technical change.

The analysis of the direction of technical change introduces a range of novel questions. Tax policies, international trade, and large shocks, such as oil price increases, can all have important effects on what factors new technologies complement, and therefore very different macroeconomic consequences. The study of these issues is a fruitful area for future research. Understanding the nature of the factor bias of technologies can also shed new light on some important debates. For example, over the past twenty years, wage inequality increased rapidly in the US, UK, and Canada, with little or no change in European economies. In contrast, Blanchard (1998) has shown that over the same period, the share of capital has increased rapidly in the European economies while remaining constant in the Anglo-Saxon countries. I suspect that the behavior of wage inequality in the Anglo-Saxon economies and the capital share in Europe are related, and result, at least in part, from the differences in the type of technologies adopted and developed in these economies. An analysis of these issues therefore requires models in which the direction of technical change is endogenous.

VII. References

Acemoglu, Daron, "Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality" *Quarterly Journal of Economics*, CXIII (1998), 1055-1090.

Acemoglu, Daron, "Patterns of Skill Premia" NBER Working Paper no. 7018 (1999)

Aghion, Philippe and Peter Howitt, "A Model of Growth Through Creative Destruction" *Econometrica*, LX (1992), 323-351.

Aghion, Philippe and Peter Howitt, *Endogenous Growth Theory*, Cambridge, MA, MIT Press, 1998.

Ahmad, Syed, "On The Theory of Induced Invention," *Economic Journal* LXXVI, (1966), 344-357.

Blanchard, Olivier, "The Medium Run" *Brookings Papers on Economic Activity*, (1998).

Caballero, Ricardo J. and Adam B. Jaffe, "How High are the Giants' Shoulders: An Empirical Assessment of Knowledge Spillovers and Creative Destruction in a Model of Economic Growth" *NBER Macroeconomics Annual 1993,* Cambridge, MA, MIT Press, 1993, 15-74.

Caselli, Francesco and Jaume Ventura "A Representative Consumer Theory of Distribution" forthcoming *American Economic Review* (2000).

David, Paul, *Technical Choice, Innovation and Economic Growth: Essays on American and British Experience in the Nineteenth Century*, London, Cambridge University Press, 1975.

Grossman, Gene and Elhanan Helpman, "Quality Ladders in the Theory of Growth" *Review of Economic Studies*, LIIX (1991a), 43-61.

Grossman, Gene and Elhanan Helpman, *Innovation and Growth in the Global Economy,* Cambridge, MA, MIT Press, 1991b.

Habakkuk, H. J., *American and British Technology in the Nineteenth Century: Search for Labor Saving Inventions*, Cambridge University Press, 1962.

Hayami, Yujiro and Vernon Ruttan, "Factor Prices and Technical Change in Agricultural Development: the U.S. and Japan, 1880-1960" *Journal of Political Economy*, LXXII (1970), 1115-1141.

Jaffe, Adam B., Manuel Trajtenberg, and Rebecca Henderson, "Geographic Localizaion of Knowledge Spillovers as Evidenced by Patent Citations" *Quarterly Journal of Economics*, CVIII (1993), 577-598.

Jones, Larry E. and Rodolfo E. Manuelli, "A Convex Model of Equilibrium Growth: Theory and Policy Implications" *Journal of Political Economy*, 98 (1990), 1008-1038.

Jorgenson, Dale W., Frank M. Gollop, and Barbara M. Fraumeni, *Productivity and U. S. Economic Growth*, Cambridge, MA, Harvard University Press, 1987.

Kennedy, Charles, "Induced Bias in Innovation and the Theory of Distribution" *Economic Journal,* LXXIV (1964), 541-547.

Lucas, Robert, "On the mechanics of economic development." *Journal of Monetary Economics*, 22 (1988), 3-42.

Rebelo, Sergio, "Long-Run Policy Analysis and Long-Run Growth" *Journal of Political Economy* 99 (1991), 500-521.

Rivera-Batiz, Luis A. and Paul M. Romer, "Economic Integration and Endogenous Growth" *Quarterly Journal of Economics*, 106 (1991), 531-555.

Romer, Paul M., "Increasing Returns and Long-Run Growth" *Journal of Political Economy* 94 (1986), 1002-1037.

Romer, Paul M., "Endogenous Technological Change" *Journal of Political Economy,* IIC (1990), S71-S102.

Salter W. E.G., *Productivity and Technical Change*, 2nd edition, Cambridge University Press, (1966) Cambridge, United Kingdom.

Samuelson, Paul, "A Theory of Induced Innovations Along Kennedy-Weisacker Lines" *Review of Economics and Statistics*, XLVII (1965), 444-464.

Schmookler, Jacob, *Invention and Economic Growth*, Cambridge, Harvard University Press, 1966.

Segerstrom, P. S., T. Anant, and E. Dinopoulos, "A Schumpeterian Model of the Product Life Cycle" *American Economic Review* 80 (1990), 1077-1092.

Shapiro, Carl and Joseph Stiglitz; "Unemployment as Worker Discipline Device" *American Economic Review,* 73 (1984), 433-444.

Ventura, Jaume, "Growth and Interdependence" *Quarterly Journal of Economics*, CXII (1997), 57-84.

Young, A. "Invention and Bounded Learning by Doing" *Journal of Political Economy* 101 (1993), 443-472.