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LABOR-MARKET MATCHING WITH PRECAUTIONARY SAVINGS AND AGGREGATE  
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Labor-Market Matching with Precautionary Savings and Aggregate Fluctuations  
Per Krusell, Toshihiko Mukoyama, and Aysegul Sahin  
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**ABSTRACT**

We analyze a Bewley-Huggett-Aiyagari incomplete-markets model with labor-market frictions. Consumers are subject to idiosyncratic employment shocks against which they cannot insure directly. The labor market has a Diamond-Mortensen-Pissarides structure: firms enter by posting vacancies and match with workers bilaterally, with match probabilities given by an aggregate matching function. Wages are determined through Nash bargaining. We also consider aggregate productivity shocks, and a complete set of contingent claims conditional on this risk.

We use the model to evaluate a tax-financed unemployment insurance scheme. Higher insurance is beneficial for consumption smoothing, but because it raises workers' outside option value, it discourages firm entry. We find that the latter effect is more potent for welfare outcomes; we tabulate the effects quantitatively for different kinds of consumers. We also demonstrate that productivity changes in the model---in steady state as well as stochastic ones---generate rather limited unemployment effects, unless workers are close to indifferent between working and not working; thus, recent findings are corroborated in our more general setting.

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# 1 Introduction

Over the last decades, a new theoretical strand of macroeconomic models has emerged that places individual households in the center of the analysis. In the new models, compared to earlier models relying on the representative-agent construct, households' decision problems and welfare determinants are much closer to the microeconomic reality of households as described and analyzed in the applied literatures using microeconomic data sets. The hope, of course, is that a more realistic microeconomic structure produces more reliable and robust macroeconomic implications. However, an arguably even more important value of the new models lies in their usefulness for analyzing household welfare. That is, how do macroeconomic events, and aggregate policy, influence individuals and their welfare, and in particular to what extent are different households affected differently? For example, a series of papers shed light on how business cycles influence the cross-section of households and report a range of estimates for the potential gains, for different subgroups of consumers, from stabilizing the cycle; some of these estimates are orders of magnitude larger than the estimates that Lucas (2003) provided and that even put in question the relevance of analyzing cycles at all.<sup>1</sup>

In the new workhorse model, often labeled the Bewley-Huggett-Aiyagari (BHA) model,<sup>2</sup> consumers face idiosyncratic earnings risks, are risk-averse, and can only insure partially—through saving—against these risks. The model delivers a stationary distribution over households and their asset, consumption, and income positions. Thus, risk plays a central role here, as does the idea that poorer consumers are less well insured and thus more vulnerable to adverse outcomes. In a typical BHA model, heterogeneity is in large part driven by exogenous employment shocks: consumers face risks of becoming unemployed (and opportunities of regaining employment) that are calibrated to data but not derived from first principles. The assumed exogeneity of these shocks is a potentially serious drawback, since it implies that neither policy nor events, such as changes in productivity, will influence them. For example, the adoption of generous unemployment benefits does not have an impact on individuals' employment opportunities in the BHA model. In contrast, a rich lit-

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<sup>1</sup>See, e.g., İmrohoroğlu (1989), Atkeson and Phelan (1994), Krusell and Smith (1999), Krebs (2006), Mukoyama and Şahin (2006), and Krusell et al. (2009c).

<sup>2</sup>See Bewley (undated), Huggett (1993), and Aiyagari (1994).

erature on matching frictions in the labor market—we label it the Diamond-Mortensen-Pissarides (DMP) search/matching framework—focuses precisely on the determinants of aggregate and individual unemployment.<sup>3</sup> Unfortunately, that literature generally relies on risk-neutral consumers: although households face significant risks, they do not suffer from these risks, and in no sense do consumer’s asset positions influence how they are affected by, or deal with, these risks.<sup>4</sup> The broad purpose of this paper is to bridge the gap between these two important literatures: we develop a model that literally combines the BHA model and the DMP model, without losing any features of either setting. The model we develop can be productively used in a number of applications, and indeed a major motivation behind our work has been to provide the necessary tools allowing these applications to be undertaken. Here, we address two questions that have been central in the two separate literatures but so far have delivered only partial answers.

Our first question regards the optimal level of government-provided unemployment insurance (UI). The literature on optimal unemployment insurance generally focuses on moral hazard concerns that affect workers’ willingness to search for jobs or work. The trade-off, thus, is between providing insurance and the incentives to work/search. Our framework, on the contrary, highlights the trade-off between insurance and job creation: high unemployment benefits provide better insurance for the unemployed but at the same time hurts their chances for reemployment since there is less job creation. As a result, less than perfect insurance is optimal even in the absence of moral hazard concerns. In our analysis, we assume that UI is financed by payroll taxes. In the BHA model, where markets for idiosyncratic risks are missing, such insurance would raise welfare; indeed the optimal level of insurance is full insurance. From the perspective of a labor market with matching frictions, however, wage and unemployment outcomes are influenced by workers’ welfare as unemployed: a higher “outside option” for workers raises the wage and lowers the incentives for firms to open new vacancies, thus leading to a rise in unemployment. Indeed, provided the Hosios condition holds, steady-state output in the Pissarides model is maximized when there is no UI at all. Using a

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<sup>3</sup>Classical contributions include Diamond (1981), Pissarides (1985), and Mortensen and Pissarides (1994).

<sup>4</sup>It is also possible to study models where workers are risk-averse but do not have any access to any asset or insurance markets; see, e.g., Wright (1986). However, such settings, when applied to the studies of the quantitative welfare consequences of labor-market policies, are likely to vastly exaggerate the welfare effects through risk channels.

steady-state version of our model, we study the welfare effects of UI.

Secondly, we ask how changes in productivity—permanent as well as temporary—influence labor-market outcomes. This is important not only as a background for analyzing household insurance when there are aggregate shocks but also in itself. There is a sharp distinction between how productivity shocks operate in representative-agent models of labor supply and in typical matching-friction models.<sup>5</sup> In the former, the focus is entirely on labor supply (which we abstract from in this paper): with consumer preferences that admit balanced growth, permanent productivity changes have no effect on hours worked or employment, though productivity fluctuations can lead to large employment fluctuations as consumers substitute leisure over time. In the latter, where employment is constrained by frictions, both permanent and temporary productivity increases would encourage vacancy creation and lead to lower unemployment, though a recent literature argues that these effects are much too small to explain the unemployment movements in the data.<sup>6</sup> How the productivity-unemployment channel operates when risk and consumption smoothing influence both individual wage formation and aggregate prices is thus the second question that we address.

Our findings are, first, that the optimal replacement rate is quite far from the 100% rate that the BHA setting would suggest. The precise value depends on what objective one uses, but as a rough approximation, no major change away from the 0 replacement rate assumed in a standard BHA model is called for from the perspective of optimal policy. Thus, the labor-market inefficiencies that would result from raising benefits are rather strong quantitatively as compared to the benefits from insurance over and beyond self-insurance. Second, we solve the model with aggregate shocks and find that productivity shocks generate unemployment fluctuations in about the same way as in the model without risk-averse consumers. Thus, in a calibration like that in Shimer (2005), unemployment fluctuations are small, and in one like that in Hagedorn and Manovskii (HM; 2008), they are large.

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<sup>5</sup>By representative-agent models here we refer to the broad class of recent models that encompass both real and monetary shocks and various assumptions about nominal price stickiness.

<sup>6</sup>See Hall (2005) and Shimer (2005).

In Section 2, we discuss some important background literature. The basic BHA dynamic general-equilibrium model without aggregate fluctuations is then set up in Section 3 and analyzed in Section 4. In Section 5, we extend our model to incorporate aggregate uncertainty. Section 6 concludes.

## 2 Some background

Methodologically, the main analytical challenge in solving our model is the interaction of two elements. First, there is a nontrivial and evolving distribution of wealth among consumers, which influences prices and aggregate quantities; and second, unlike in the one-firm neoclassical model, there is a distribution of wage outcomes influencing worker and firm decisions. Relative to previous work on the topic, this paper thus carefully addresses (i) wage determination, when a worker's outside option, because of incomplete insurance, depends on wealth; and (ii) how firms are valued in a context where there are missing markets against idiosyncratic risks. We deal with each of these issues directly. Thus, first, we consider Nash bargaining under which, since the worker's indirect utility depends on wealth, wealth will influence wages, and we characterize this dependence. Second, we consider complete markets with respect to aggregate risk, which means that all agents who hold equity—i.e., a share in firms—and therefore are concerned with firm valuation and firm decisions, are unconstrained and therefore—due to market completeness—in agreement.

Quantitative studies of the DMP matching model with risk-averse workers and savings are rare in the literature. Early and notable exceptions are Andolfatto (1996), Langot (1995), Merz (1995), and later Den Haan et al. (2000); these papers, however, all assume that idiosyncratic risks are fully insured.<sup>7</sup> Valdivia (1996) studies a model closer to ours in this regard, but restricts behavior (regarding portfolio choice and wage determination) in order to be able to solve the model. More recently, Costain and Reiter (2005, 2007), Kallcock (2006), Bils et al. (2009), Nakajima (2007), Rudanko (2006, 2009), Shao and Silos (2007), and Jung and Kuester (2008) all studied DMP models with risk aversion and imperfect consumption insurance. These papers vary considerably in the ways they deal with the aforementioned challenges, but none of the papers nests both the

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<sup>7</sup>For similar recent studies incorporating wage rigidity as well as other frictions, see Gertler and Trigari (2006) and Christiano et al. (2007).

DMP and BHA settings as we do here (we obtain the DMP setting if we simply set risk aversion to zero, and the BHA setting if the matching productivity is infinite).<sup>8</sup> On the other hand, some of these papers consider relevant features that we abstract from: Bils et al. (2009) and Shao and Silos (2007) allow for heterogeneous productivity of individuals, Rudanko (2006, 2009) considers long-term wage contracts, and Jung and Kuester (2008) has wage rigidity.<sup>9</sup>

Throughout the paper, and unlike in the large real-business-cycle and new-Keynesian literatures, we abstract from labor-supply issues: workers cannot choose hours, and they are always in a “corner” in that if offered employment, they would always choose it. This is clearly an important omission, because we believe that also the extensive margin is active for many households. However, we make the omission with a clear conscience, as we think that the model development we have offered in the present paper is a necessary first step; moreover, we have already begun researching, so far in the context of no aggregate shocks, the case with an active extensive margin.<sup>10</sup>

### 3 The steady-state model

In this section, we develop a model without aggregate shocks, and our focus is on a stationary equilibrium. Using the model, we then evaluate tax-financed unemployment insurance (UI).

#### 3.1 Population, preferences, and technology

Time is discrete. There is a continuum of consumers, with measure 1, in the economy; a consumer is either employed or unemployed. Production is decentralized; each worker produces  $zF(k)$ , where  $z$  represents the aggregate productivity level (constant in this section),  $F(\cdot)$  is an increasing and strictly concave production function, and  $k$  is the capital stock used by the worker. Capital de-

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<sup>8</sup>Costain and Reiter (2005) and Kallock (2006) do not consider capital as a neoclassical production input. Jung and Kuester (2008) assume that there are two types of agents: some cannot save or borrow, while the others have access to complete insurance. Bils et al. (2009), Shao and Silos (2007), and Costain and Reiter (2007) include some risk-neutral agents (this facilitates firm evaluation). Nakajima (2007) and Shao and Silos (2007) assume some form of “union bargaining”, thus circumventing the dependence of wages on wealth. Rudanko (2006, 2009) develops a framework where wages are determined via long-term wage contracts and risk-averse workers are excluded from asset markets completely.

<sup>9</sup>For a more detailed discussion of the related literature see Appendix A.

<sup>10</sup>See Krusell et al. (2008, 2009a, 2009b).

preciates geometrically at rate  $\delta$ . Workers have standard time-additive preferences with discount factor  $\beta$  and do not value leisure. Output is either consumed or invested or, as we shall see below, used for vacancy creation. The remainder of the model description focuses on the decentralized equilibrium.

### 3.2 Matching

There are many firms—one for each “job”—and these firms all act competitively. Due to symmetry, in equilibrium the same amount of  $k$  is employed at each filled job. Vacant jobs and unemployed workers are randomly matched each period according to an aggregate matching function. The aggregate matching function,  $M(u, v)$ , represents the number of matches in a period when there are  $u$  unemployed workers and  $v$  vacancies has standard features; we consider different functional forms below. The probability that a vacant job is filled in the current period,  $\lambda_f$ , thus equals  $M(u, v)/v = M(u/v, 1) = M(1/\theta, 1)$ , where  $\theta \equiv v/u$  is the vacancy-unemployment ratio. Similarly, the probability of an unemployed worker to be employed in the current period,  $\lambda_w$ , equals  $M(u, v)/u = (v/u)M(u/v, 1) = \theta\lambda_f$ . Thus,  $\lambda_f$  and  $\lambda_w$  are functions of  $\theta$ . We assume that a match is separated with probability  $\sigma$  in each period.<sup>11</sup>

From the above assumptions, the transition of the unemployment rate  $u$  can be described by

$$u' = (1 - \lambda_w)u + \sigma(1 - u),$$

where a next period variable is denoted by a prime ( $'$ ).

### 3.3 Asset structure

The consumers face idiosyncratic employment shocks, but we assume that there are no insurance markets for these idiosyncratic shocks. The consumers can hold only two kinds of asset: capital  $k$ , which is used as an input for production, and equity  $x$ , which is a claim to the firm’s profit. The total

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<sup>11</sup>The assumption of a constant and exogenous separation rate is made for convenience. It is potentially an important source, or propagator, of fluctuations. However, it is arguably not likely to be the most important one, at least not for generating the negative correlation between unemployment and vacancies. A temporary increase in the separation rate increases unemployment in a direct way but its impact on vacancies is less powerful than that resulting from increases in productivity or in “firm product demand.”



value of the firms is represented by  $p$ . Alternatively, we can say that there is one “representative firm”—with a continuum of jobs—whose value is  $p$ . It is assumed that the consumers are not allowed to hold the claims to the profit of individual jobs—they can only hold the claim to aggregate profits.<sup>12</sup> Let  $r$  be the return to capital and  $d$  be the dividend paid to the holders of equity. We normalize the total amount of equity to one. The equity price  $p$  has to satisfy the following equation:

$$p = \frac{d + p}{1 + r - \delta}, \quad (1)$$

where  $\delta$  is the depreciation rate of capital. This comes from the absence of arbitrage: one unit of capital generates  $1 + r - \delta$  unit of return next period and one unit of equity generates  $(d + p)/p$  unit of return (note that both assets are riskless). We use  $q$  to denote the the inverse of the gross real interest rate:  $q \equiv 1/(1 + r - \delta)$ .

Since capital and equity are equivalent from the consumer’s viewpoint, we do not have to keep track of the portfolio composition. In the following, we define

$$a \equiv (1 + r - \delta)k + (p + d)x$$

and use  $a$  as the consumer’s state variable.

## 3.4 Consumers

### 3.4.1 Employed consumers

The budget constraint for an employed consumer is

$$c + qa' = a + w, \quad (2)$$

where  $c$  is consumption and  $w$  is the wage rate. The wage is determined through Nash bargaining between the firm and the worker every period, and it turns out that the wage depends on the worker’s asset level. The details of Nash bargaining are motivated and explained later.

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<sup>12</sup>If the consumers are allowed to hold the claims to the individual job’s profit, there is an incentive for an employed consumer to hold a short position in its own job to hedge against the risk of separation.

For a given wage  $w$ , employed consumers choose their capital and equity holdings subject to (2) and the borrowing constraint  $\underline{a}$ :

$$\tilde{W}(w, a) = \max_{k', x'} u(c) + \beta [\sigma U(a') + (1 - \sigma)W(a')] \quad (3)$$

subject to

$$c + qa' = a + w \quad \text{and} \quad a' \geq \underline{a}.$$

Here,  $u(\cdot)$  is an increasing and concave utility function,  $U(\cdot)$  is the value function of an unemployed consumer, and  $W(\cdot)$  is the value function of an employed consumer, taking into account that the wage is renegotiated next period. Let the decision rule for  $a'$  for employed consumers be  $a' = \tilde{\psi}_e(w, a)$ . Denote the wage resulting from the Nash bargaining (detailed later) as  $w = \omega(a)$ .  $W(a)$  is formally defined as

$$W(a) \equiv \tilde{W}(\omega(a), a). \quad (4)$$

Also define

$$\psi_e(a) \equiv \tilde{\psi}_e(\omega(a), a). \quad (5)$$

### 3.4.2 Unemployed consumers

The budget constraint for an unemployed consumer is  $c + qa' = a + h$ , where  $h$  is the income for an unemployed worker.  $h$  can be thought as household production. This assumption is made so that an agent can earn some labor income even when she is unemployed. Alternatively,  $h$  can be interpreted as unemployment insurance, as in Hansen and İmrohoroğlu (1992); we use that interpretation in Section 4.3.

Unemployed consumers' optimization problem is:

$$U(a) = \max_{k', x'} u(c) + \beta [(1 - \lambda_w)U(a') + \lambda_w W(a')] \quad (6)$$

subject to

$$c + qa' = a + h \quad \text{and} \quad a' \geq \underline{a}.$$

Let the decision rule for  $a'$  for unemployed consumers be  $a' = \psi_u(a)$ .

### 3.5 Firms

Firms create jobs, rent capital from consumers, and produce. The firm's objective is to maximize the discounted present value of the profits of shareholders. To create a job, a firm first posts a vacancy. We assume that there is a flow cost of posting a vacancy and denote it by  $\xi$ . The value of posting a vacancy,  $V$ , is

$$V = -\xi + q \left[ (1 - \lambda_f)V + \lambda_f \int J(\psi_u(a)) \frac{f_u(a)}{u} da \right]. \quad (7)$$

The firm discounts the future at the rate  $q$ : this is not only the market rate but also the marginal rate of substitution of anyone with positive holdings of the firm.  $J(a)$  is the value of a job filled by a worker whose asset level is  $a$ . Since the matching process is random, the firm can be matched with any worker in the current period unemployment pool.  $f_u(a)$  is the population of unemployed workers with current asset level  $a$ . Thus,  $f_u(a)/u$  is the density function of the unemployed workers over  $a$ ; an unemployed worker with current asset level  $a$  will have asset level  $\psi_u(a)$  next period. In equilibrium, firms will post new vacancies until  $V = 0$ .

The value of a filled job, given the wage  $w$ , is

$$\tilde{J}(w, a) = \max_k zF(k) - rk - w + q \left[ \sigma V + (1 - \sigma)J(\tilde{\psi}_e(w, a)) \right]. \quad (8)$$

Note that  $\tilde{J}$  depends on  $a$  since with probability  $1 - \sigma$  the firm continues to be matched with the same worker, whose next-period asset level depends on  $a$ .  $\tilde{J}$  and  $J$  are related by

$$J(a) \equiv \tilde{J}(\omega(a), a). \quad (9)$$

The firm's first-order condition implies that  $r = zF'(k)$ .

In equilibrium, the period profit is equal to

$$\pi(a) = zF(\tilde{k}) - r\tilde{k} - \omega(a),$$

with  $\tilde{k} = \bar{k}/(1 - u)$ , where  $\bar{k}$  is the aggregate capital stock;  $\tilde{k}$  is the amount of capital per job.

The dividend is paid out as the sum of profit minus the total vacancy cost:

$$d = \int \pi(a) f_e(a) da - \xi v, \quad (10)$$

where  $f_e(a)$  is the population of the matched workers with wealth level  $a$ . Appendix B shows that this implies, using (1), that  $p + d$  is equal to the sum of  $J(a)$ .

### 3.6 Wage determination

The wage is determined by (generalized) Nash bargaining. We assume that the firms do not have commitment, so that wages are set period by period. The assumption of Nash bargaining facilitates the comparison with the early literature, since Nash bargaining is the most commonly used assumption under risk neutrality. The Nash-bargaining solution solves the problem

$$\max_w \left( \tilde{W}(w, a) - U(a) \right)^\gamma \left( \tilde{J}(w, a) - V \right)^{1-\gamma}; \quad (11)$$

$\gamma \in (0, 1)$  represents the bargaining power of the worker. We denote the solution  $w = \omega(a)$ ; the dependence of  $w$  on  $a$  stems from  $\tilde{W}(w, a) - U(a)$  depending on  $a$ .<sup>13</sup>

### 3.7 Computation

We formally define the recursive stationary equilibrium of this economy in Appendix C. In the standard BHA setting, computation of a steady state reduces to a one-dimensional fixed point problem in the value of the capital stock. An efficient algorithm for that model is to (i) guess on a value for the capital stock; (ii) obtain the prices (the wage and the rental rate) from the firm's first-order conditions for inputs (price equals marginal product); (iii) solve the consumer's dynamic-programming problem globally using nonlinear methods; (iv) simulate an agent's capital accumulation path for a large number of periods; and finally (v) compare the average capital stock held by the agent with the initial guess, and update.

The present model cannot be computed with an algorithm that reduces to a one-dimensional fixed-point problem; instead one needs to solve a functional fixed-point problem. The reason is that the consumer needs to know not just two prices but the entire wage function,  $\omega(a)$ , to maximize utility. Thus, the algorithm we use works as follows: (i) guess on  $\omega(a)$ , along with  $r$  and  $\theta$ ; (ii) using the matching function and the given  $\theta$ , compute the probability of finding a job; (iii)–(iv) as

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<sup>13</sup>Alexopoulos and Gladden (2006) examine the link between wealth and wages. They find, using data from the 1984 SIPP, that wealth increases reservation wages.

above, which also involves a straightforward contraction mapping and can be used to obtain the distributions of employed and unemployed consumers across capital holdings; (v) given the latter, obtain firm values by iterating on the firm’s value function (this involves no maximization and is a contraction mapping); and (vi) given all value functions, perform Nash bargaining. Now note that (vi) delivers an update for the wage function; (iv) gives an average total capital stock, which, since unemployment can be computed from  $\theta$ , allows us to find the per-firm capital stock, and hence we can obtain an update for  $r$  from the firm’s capital rental decision; and (v) gives a value of entry, which should be zero, and hence leads us to adjust the guess for  $\theta$ . The fixed-point problem in  $(\omega, r, \theta)$  could in principle be a very difficult one, but fortunately it turns out that it is easy to solve for this model. The reason for this is that the  $\omega$  function is mostly flat and that few agents have capital holdings over the range where it is not flat; see the results in Section 4.2 below. The detailed computational procedure can be found in Appendix D.

## 4 Steady-state experiments

We use our model economy for two main purposes: we look at the effects of unemployment insurance (UI) and we examine how changes in productivity influences labor-market outcomes. We therefore first calibrate the model and report its basic properties. We then look at UI policy and the effects of productivity change. Lastly, to illustrate how our model works, we compare it to some interesting similar models.

### 4.1 Calibration

One period is set to be six weeks. The production function is  $zF(k) = zk^\alpha$ . We choose  $\alpha = 0.3$ ,  $\delta = 0.01$  and  $\beta = 0.995$  using the following calibration targets: a capital share of 0.3, an investment-output ratio of 0.2, and annual real rate of return on capital of 0.04. The borrowing constraint  $\underline{a}$  is set at 0. In the benchmark, the aggregate productivity level  $z$  is set to 1 and we use the utility function  $u(c) = \log(c)$ ; in Appendix E, we also consider  $u(c) = c^{1-\zeta}/(1-\zeta)$  with  $\zeta = 5$ .

The above parameters are common to all the numerical simulations that we present throughout

this paper. However, for the matching-related parameters we consider two different calibrations, since the literature offers two different approaches. These calibrations mainly differ in how the utility or consumption value of the home activity, a parameter that is hard to measure directly, is set, and we therefore remain agnostic as to which route to take. Our benchmark calibration follows the calibration strategy presented in Shimer (2005) and it views the value of “home production” to be commensurate with the average level of UI. An alternative calibration, due to HM (2008), considers home production to be only slightly below the market wage; this calibration leads to a much stronger effect of productivity on outcomes, as we shall see.

**Benchmark calibration:** Shimer (2005) sets the value of the household production parameter  $h$  to be 40% of the wage. We thus set  $h$  to 0.99 which, in equilibrium, turns out to be about 40% of the average wage. The separation rate ( $\sigma$ ) is set to 0.05 based on the observation by Shimer that the monthly separation rate is 0.034. The matching function  $M(u, v)$  is  $M(u, v) = \chi u^\eta v^{1-\eta}$ , as in Shimer (2005). To calibrate  $\chi$  and  $\eta$ , Shimer (2005) targets the market tightness to be  $\theta = 1$  and sets  $\chi$  to match the job-finding probability. A worker finds a job with a 0.45 probability per month, so the flow arrival rate of job offers should average approximately  $1 - (1 - 0.45)^{1.5} = 0.592$  on a six-week basis. We follow Shimer and target  $\theta = 1$  in equilibrium, which results in  $\chi = 0.6$ . We set  $\xi = 0.532$  so that  $\theta = 1$  satisfies the equilibrium free-entry condition for posting vacancy. Shimer estimates the elasticity of the matching function as 0.72 and sets the bargaining parameter  $\gamma = 0.72$  by following the Hosios efficiency condition.<sup>14</sup> We follow his parametrization and set  $\eta = \gamma = 0.72$ .

**Alternative calibration:** HM (2008) calibrate the standard matching model by using a different approach. They choose a matching function of the form  $M(u, v) = \frac{uv}{(u^l + v^l)^{1/l}}$ , implying a job-filling rate of  $\frac{1}{(1+\theta^l)^{1/l}}$ . HM choose  $\theta = 0.634$  since the monthly job-filling rate is 0.71 and the monthly job-finding rate is 0.45.<sup>15</sup> We convert these rates to a six-week basis (which is our model period). The corresponding job-filling rate is  $1 - (1 - 0.71)^{1.5} = 0.844$  and the job-finding rate is  $1 - (1 - 0.45)^{1.5} = 0.592$  which implies that  $\theta = 0.7$ . Thus we set  $\theta = 0.7$ . Then we choose  $l$  to

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<sup>14</sup>In our setting, the Hosios condition does not guarantee the constrained efficiency. This is due to the BHA structure of the model. Dávila et al. (2005) show that the externality in capital accumulation process makes the outcome of the standard BHA model constrained inefficient, and the same effect will be at work in our setting.

<sup>15</sup>Note that  $\lambda_w/\lambda_f = \theta$ .

match the job-finding rate in the data, which gives us  $l = 2.2$ . HM compute the capital cost of posting a vacancy as 47.4% of average weekly labor productivity and the labor cost of posting a vacancy as 11.0% of average weekly labor productivity. In total, the vacancy cost is 58.4% of the average wage. We set the vacancy cost  $\xi$  to be 60% of the average productivity in our model. The corresponding value is  $\xi = 2.165$ . We choose the bargaining parameter to be  $\gamma = 0.052$  following HM. We set  $h$  to 2.29 which is the value that satisfies the free-entry condition given  $\theta$  and  $\xi$ .

## 4.2 Characterization of the steady state

We first display some key model features and then discuss our experiments. Table 1 presents the summary statistics for different calibrations.

	$\xi$	$u$	$v$	$\theta$	$\bar{k}$	$p$	$d$	$w$
Shimer	0.5315	7.69%	0.0769	1.0	66.54	0.82	0.004	2.48
HM	2.1648	7.81%	0.0546	0.7	66.42	2.37	0.012	2.38

Table 1: Summary statistics for the Shimer and HM calibrations

Both calibrations have similar values of the unemployment rate, aggregate capital, and the average wage. The vacancy-unemployment ratio is different for these two calibrations, reflecting the differences in the calibration strategies of Shimer (2005) and HM (2008), as are the stock price and dividends: the vacancy cost in the HM calibration is much higher.  $w$  corresponds to the average wage realized in the economy by taking into account the asset distribution. Despite the high value of unemployment income in the HM calibration, the average wage is higher in the Shimer calibration, again as an artifact of the much higher vacancy cost in the HM calibration.

Figure 1 shows the wage functions.<sup>16</sup> The observed concavity of  $\omega(a)$  follows, intuitively, from two features: (i) the function being increasing, which is due to the outside option being worse for consumers with a low stock of assets (since their buffer against unemployment shocks is lower), and (ii) a natural upper bound, which is given by that value which a risk-neutral agent would obtain in

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<sup>16</sup>Note that the range of the vertical axis is different on the left and right panels to highlight the differences in wage functions for the two calibrations.

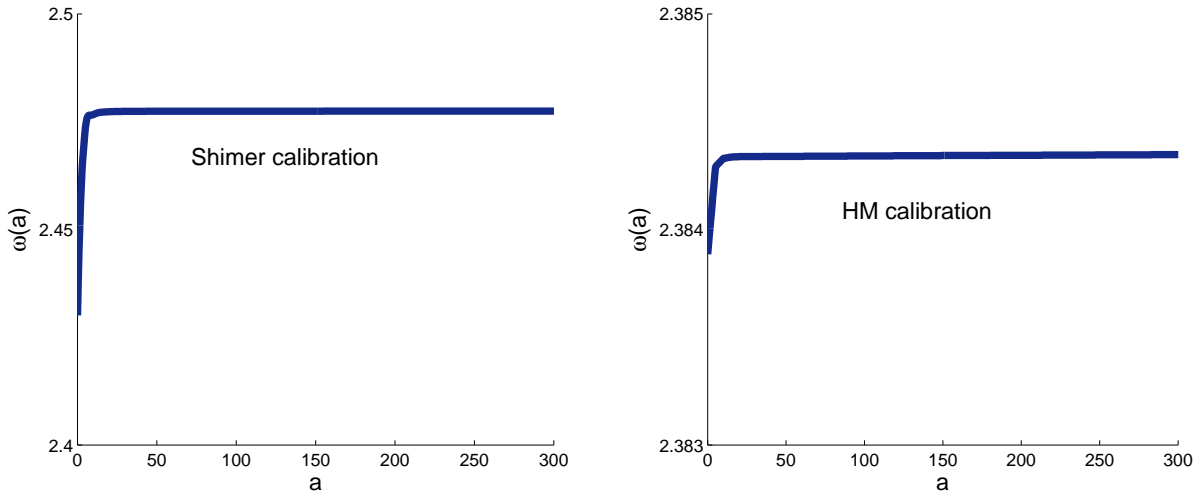


Figure 1: Wage function for the Shimer calibration (left panel) and the HM calibration (right panel).

the bargaining (as in the DMP model), and this value is approached here for consumers with infinite asset holdings since they are “perfectly insured.”<sup>17</sup> It turns out that with our parametrization, the wages are only increasing significantly for the  $a$ s that are very close to the borrowing constraint. We provide more detailed discussion on the wage function in Appendix E where we also consider different preferences with higher risk aversion.

How much wage dispersion is created by asset inequality jointly with our wage-determination process? Very little, especially with the HM calibration. Using the mean-min ratio as a measure, the Shimer calibration delivers 1.02 and the HM calibration 1.0002.<sup>18</sup> These numbers would go up in a model where asset inequality is more pronounced.

<sup>17</sup>Acemoglu and Shimer (2000) build a model of risk-averse workers searching for different types of jobs. They assume that high-paying jobs are more difficult to find. With the assumption of CRRA utility, wealthier workers are less risk-averse and they tend to apply for higher-paying jobs. As a result, there is a positive association between wealth and wages through a different mechanism than ours.

<sup>18</sup>Hornstein et al. (2006) suggest the mean-min ratio—the ratio of the mean wage to the minimum wage—as a useful measure of frictional wage inequality.



### 4.3 Welfare effects of UI

From the perspective of the precautionary-savings literature—the BHA model—direct insurance against unemployment is clearly welfare-improving from the individual’s perspective. Is it desirable, taking general-equilibrium effects into account? In the BHA model, yes, but not necessarily in the present model, since such insurance will influence wage formation and, therefore, firms’ incentives to enter. In particular, wages will rise, deterring entry. We now study how these two effects play out.

We assume that financing of the UI payments using taxes that are proportional to wages. We also assume that there is no home production so that all the income for the unemployed workers comes from the unemployment insurance. The budget constraints for the employed and unemployed workers now become

$$c + qa' = a + (1 - \tau)w \quad \text{and} \quad c + qa' = a + (1 - \tau)h,$$

respectively, where  $h$  is the government-financed unemployment insurance. For a given  $h$ , the government sets  $\tau$  to balance its period-by-period budget constraint:

$$\tau \int \omega(a) f_e(a) da = (1 - \tau)uh.$$

#### 4.3.1 Effects on quantities

In this section, we use the Shimer calibration. Figure 2 reveals how wages, capital, and unemployment are influenced by UI in the long run. A higher  $h$  increases the relative value of the outside option for the workers and increases the wage. This reduces the firm’s incentive to post vacancies, and hence unemployment rises. The improvement in insurance, however, also reduces the capital-labor ratio,  $\bar{k}/(1 - u)$ , through the decline in precautionary saving: the capital-labor ratio falls from 72.17 to 72.07 when  $h$  increases from 0.00 to 1.50. This calculation suggests that steady-state output is the highest with zero (or even negative) UI; indeed, this is the case for our calibration.

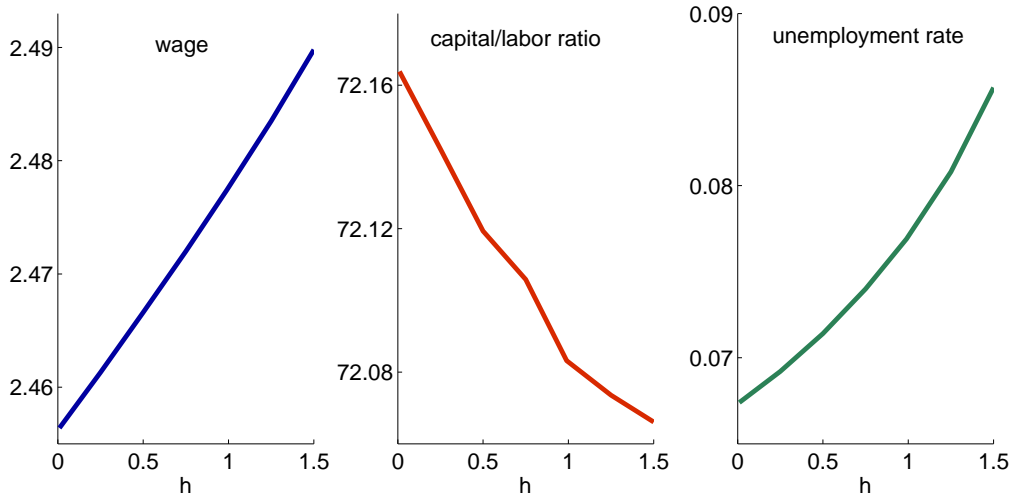


Figure 2: Wages and capital/labor ratio and the unemployment rate as a function of  $h$ .

### 4.3.2 Welfare

Rather than compute full transition dynamics as a result of a change in the level of UI, we compare steady states with different levels of UI. As is clear from above, however, a straight comparison of, say, average utility across steady states will not do justice to UI, since high UI levels are associated with low long-run levels of capital. Therefore, we report an alternative welfare measure that is more neutral with regard to these comparisons. Assume, therefore, that we are looking at a number of different “countries,” each with a different UI level and each in a steady state, but otherwise identical. One of these countries is our benchmark economy with  $h = 0.99$ . Now imagine moving each of the consumers in the benchmark economy, along with her employment status and asset holdings, into each of the other economies and then comparing utilities. For example, if an unemployed consumer moves to a high-UI economy, she will benefit in the short run from higher income, but she will also suffer in the future from having a lower wage when employed; a very rich unemployed consumer might also gain a significant amount since interest rates are higher in the high-UI economy. Our welfare measure in these comparisons,  $\lambda$ , is defined from

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \log((1 + \lambda)c_t) \right] = E_0 \left[ \sum_{t=0}^{\infty} \beta^t \log(\tilde{c}_t) \right],$$

where  $c_t$  is the consumption under the benchmark case ( $h = 0.99$ ) and  $\tilde{c}_t$  is the consumption under a particular experiment.<sup>19</sup>

Table 2 summarizes the average values of  $\lambda$  in the total population, among all the unemployed, and among all the employed. It also shows the ratio of people who gain ( $\lambda > 0$ ). The distribution of the individual states is taken from the invariant distribution of the benchmark.

$h$ from 0.99 to	total	unemployed	employed	% gaining	poorest unemployed	poorest employed
$h = 0.01$	0.11%	-0.09%	0.13%	92.10%	-4.0%	-1.0%
$h = 0.25$	0.12%	-0.02%	0.14%	92.29%	-1.2%	-0.3%
$h = 0.50$	0.11%	0.01%	0.12%	99.22%	-0.54%	-0.09%
$h = 0.75$	0.07%	0.02%	0.07%	99.96%	-0.2%	0.00%
$h = 1$	0%	0%	0%	0%	0%	0%
$h = 1.25$	-0.14%	-0.09%	-0.15%	0.00%	0.06%	-0.1%
$h = 1.50$	-0.38%	-0.27%	-0.39%	0.00%	-0.04%	-0.32%

Table 2: Average values of  $\lambda$ , compared to the benchmark of  $h = 0.99$ .

It turns out that almost everyone (everyone except for the very poorest unemployed workers, who are borrowing-constrained) gains from lowering  $h$  to 0.75. The average gain can be increased by lowering  $h$  further to 0.25—in fact, the amount of  $h$  that maximizes the average gain is about 0.30. Recall that as we lower  $h$ , the profitability of a match increases and this induces more vacancy posting and reduces unemployment. This favors a lower value of  $h$ . However, there are two negative welfare effects coming from a lower  $h$ . The first is that the (before-tax) wage rate declines with lower  $h$ —this is a consequence of Nash bargaining. The second is the direct effect of lowering the UI: a lower  $h$  means less insurance for unemployed workers. Almost for all values of  $h$ , the positive effect of lowering  $h$  dominates, and welfare increases as we lower  $h$ . However, if  $h$  is lowered to values close enough to zero, the welfare starts to decline because of the negative effects. We show the  $\lambda$

<sup>19</sup>  $\lambda$  can be solved as  $\lambda = \exp((\tilde{V} - V)(1 - \beta)) - 1$ , where

$$V = E \left[ \sum_{t=0}^{\infty} \beta^t \log(c_t) \right] \quad \text{and} \quad \tilde{V} = E \left[ \sum_{t=0}^{\infty} \beta^t \log(\tilde{c}_t) \right].$$

values for the employed (on average), the unemployed (on average), the constrained unemployed, and the constrained employed workers in Figure 3.

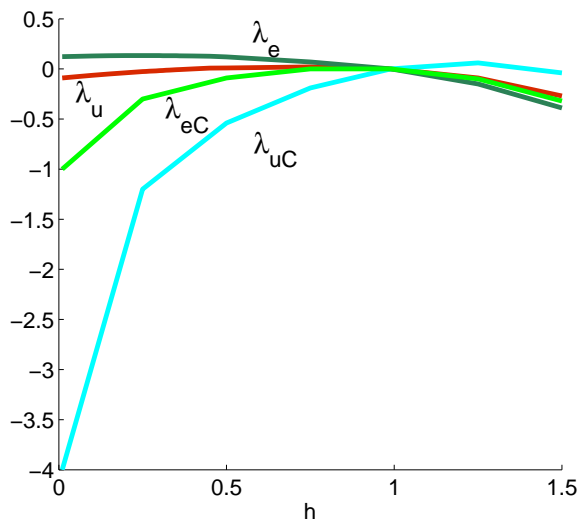


Figure 3:  $\lambda$ s for employed, unemployed, unconstrained/unemployed, and constrained/employed workers

It is straightforward to conduct the same welfare analysis using a standard Aiyagari (1994) model. The welfare outcomes are not entirely trivial, since a change in UI does influence capital accumulation and hence prices; moreover, it involves redistribution. We find, not surprisingly, that welfare is monotonically increasing in  $h$  for the unemployed in that economy, whereas the employed experience very small welfare changes. That is, we assume that the employment process is exogenously given with the job finding rate at 0.6 and the job separation rate at 0.05. (Therefore, the unemployment rate is fixed at 7.69% regardless of the value of  $h$ .) Since the employment process is given exogenously, unemployment insurance and taxes are non-distortionary. Thus, perfect insurance ( $w = h$ ) achieves a Pareto efficient outcome. There is still some disagreement among the workers, since the insurance involves income transfer between the unemployed and the employed, and since a change in precautionary savings influence the wage and the interest rates, which the people with different wealth positions benefit differentially from. The comparison of our

model with the Aiyagari model reveals the importance of incomplete asset markets and endogenous labor-market frictions. If a policy maker used the average  $\lambda$  to measure the desirability of the policy, a lower unemployment insurance level would be recommended if our model were used, whereas a much higher level would be recommended if the Aiyagari model were used.

#### 4.4 Productivity and labor-market outcomes

Permanent changes in productivity have no effect on employment in macroeconomic models built around labor supply, as long as the preferences over consumption and leisure admit balanced growth. In contrast, in models where employment is determined by labor-market frictions and labor supply plays no role, productivity does have an effect. The question is how large the effect is, both in the long and in the short run. In this section, we answer the former of these questions.

We consider a 2% deviation in aggregate productivity. Table 3 presents the summary statics for different  $z$  values for the Shimer calibration.

	$u$	$v$	$\theta$	$\bar{k}$	$p$	$d$	$w$
$z = 0.98$	7.79%	-3.6%	-4.8%	-3.0%	-3.9%	-3.9%	-2.8%
$z = 1.02$	7.60%	+3.6%	+4.9%	+3.0%	+3.4%	+3.4%	+2.9%

Table 3: Summary statistics for different  $z$  values (the Shimer calibration).

In the table (and all the following tables that involves the comparison across different  $z$ ), we show the percentage deviations from the  $z = 1.00$  case, except for  $u$  (whose absolute level is displayed). Aggregate capital increases with  $z$ :  $\bar{k} = 64.58$  for  $z = 0.98$  and  $\bar{k} = 66.54$  for  $z = 1.00$  and  $\bar{k} = 68.52$  for  $z = 1.02$ . The vacancy-to-unemployment ratio ( $\theta$ ) increases with  $z$  as well, as does the employment rate, equity prices, and dividends. Note that quantitatively, the differences in  $u$  are very small compared to what we see in business cycle data. This echoes Shimer's (2005) finding that the DMP model with linear utility generates very small unemployment and vacancy fluctuations. Figure 4 (left panel) shows the wage functions for different values of  $z$  and assets; the wage schedule moves up and down with  $z$ .

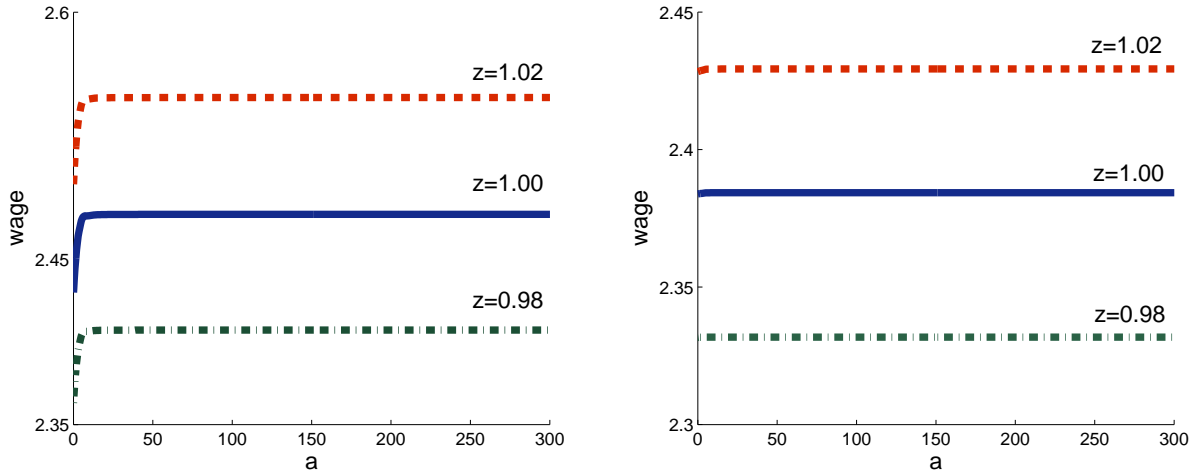


Figure 4: Wages for different  $z$ s for the Shimer calibration (left panel) and the HM calibration (right panel).

HM (2008) suggests that when the parameter values are calibrated differently from Shimer (2005), the DMP model exhibits much larger labor-market fluctuations in response to changes in  $z$ . HM demonstrated this in a model with linear utility. For comparison purposes, we present the results from our model with a HM-type calibration.

Table 4 presents the summary statics for different  $z$  values for the HM calibration. As in HM (where a linear utility function was used), the response of unemployment and vacancies is much larger with this calibration. Firm profits change more with  $z$ , leading to a larger change in  $\theta$  and  $v$ , resulting in a large response in  $u$ . When  $u$  changes substantially, the marginal product of capital changes and, as a result,  $\bar{k}$  also changes. Figure 4 (right panel) shows the change in wages. First,

	$u$	$v$	$\theta$	$\bar{k}$	$p$	$d$	$w$
$z = 0.98$	16.09%	-21.3%	-61.8%	-11.6%	-21.4%	-21.4%	-2.2%
$z = 1.02$	6.23%	+21.6%	+52.4%	+4.6%	+21.5%	+21.5%	+1.9%

Table 4: Summary statistics for different  $z$  values (HM calibration).

note that the wage functions (as functions of  $a$ ) are flatter than the benchmark wage functions.

Second, here the wage responds less to the change in  $z$ , compared to the benchmark case. A smaller response in the wage implies a larger response in the firm's profit, contributing to a larger change in  $\theta$ .

## 4.5 Alternative models

In this section, we compare our present model to two similar models that are of interest.

### 4.5.1 Linear utility

Consider the linear DMP model with a linear utility function, which by definition abstracts from risk. In Appendix F, we show that the solution of this model delivers

$$\frac{y - h}{r - \delta + \sigma + \gamma\chi\theta^{1-\eta}} = \frac{\xi}{(1 - \gamma)\chi\theta^{-\eta}}, \quad (12)$$

where  $y$  is defined as

$$y = \arg \max_{\tilde{k}} z\tilde{k}^\alpha - r\tilde{k},$$

that is,

$$y = z \left( \frac{r}{\alpha z} \right)^{\frac{\alpha}{\alpha-1}} - r \left( \frac{r}{\alpha z} \right)^{\frac{1}{\alpha-1}}, \quad (13)$$

since

$$\tilde{k} = \left( \frac{r}{\alpha z} \right)^{\frac{1}{\alpha-1}}. \quad (14)$$

The idea here is to compare the effects of productivity across the linear model and the benchmark model. More precisely, we set all the parameters except for  $\xi$  to be the same as in our original models (with the benchmark Shimer calibration and the HM calibration). First, let  $z = 1.00$  and set  $\xi$  so that (12) holds with  $\theta = 1$ .<sup>20</sup> Note that  $y$  is defined in (13) and  $r = 1/\beta - 1 + \delta$ . The steady-state condition for  $u$

$$u = \frac{\sigma}{\sigma + \chi\theta^{1-\eta}} \quad (15)$$

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<sup>20</sup>This results in  $\xi = 0.529$  in the benchmark calibration. The value of  $\xi$  is the same for the HM calibration for both the incomplete markets model and the linear model.

can be solved for  $u$ , from  $\theta = 1$ . From (14) and  $\tilde{k} = \bar{k}/(1 - u)$ , we obtain aggregate capital,  $\bar{k}$ :

$$\frac{\bar{k}}{1 - u} = \left(\frac{r}{\alpha z}\right)^{\frac{1}{\alpha-1}}. \quad (16)$$

		$y$	$u$	$v$	$\theta$	$\bar{k}$
Shimer calibration	$z = 0.98$ -linear	-2.94%	7.79%	-3.6%	-4.8%	-3.0%
	$z = 0.98$ -incomplete	-2.94%	7.79%	-3.6%	-4.8%	-3.0%
	$z = 1.02$ -linear	+2.97%	7.60%	+3.6%	+4.9%	+3.0%
	$z = 1.02$ -incomplete	+2.97%	7.60%	+3.6%	+4.9%	+3.0%
HM calibration	$z = 0.98$ -linear	-11.2%	16.10%	-21.4%	-62.0%	-11.6%
	$z = 0.98$ -incomplete	-11.2%	16.10%	-21.3%	-61.8%	-11.6%
	$z = 1.02$ -linear	4.0%	6.23%	+21.5%	+52.4%	+4.6%
	$z = 1.02$ -incomplete	4.0%	6.23%	+21.6%	+52.4%	+4.6%

Table 5: Summary statistics for the linear model and for the incomplete markets model.

Now, we change  $z$ . (12) can be solved for  $\theta$  (note that the value of  $y$  is different for each  $z$ ), then (15) can be solved for  $u$ , and finally (16) can be solved for  $\bar{k}$ . Table 5 summarizes the result. The results are remarkably similar to those in our incomplete-markets model with log utility (reproduced here from Tables 3 and 4). Moreover, this result is not an artifact of having a low number of agents on the curved part of the wage function. In Appendix G, we look at a slightly different model in order to be able to accommodate more larger dispersion in wealth.<sup>21</sup> In particular, we generate wealth dispersion that is such that a much larger fraction of workers are in the upward-sloping part of the wage curve. We find that, despite a much larger mass of agents in the curved part of the wage distributions, the responses of the economy to changes in  $z$  are remarkably similar to those in the linear model. The intuition is that, with Nash bargaining, the wage function moves with  $z$  in a parallel manner, and thus the impact of  $z$  on a firm's profit is uniform across different matches. We also consider a model with higher risk aversion in Appendix E and find that the incomplete-markets model under higher risk aversion behaves similarly to the linear model.

<sup>21</sup>The previous literature, including Huggett (1996) and Krusell and Smith (1998), discuss the importance of, and the difficulties in, generating realistic wealth heterogeneity in models with realistic income/employment processes.



#### 4.5.2 UI as a utility benefit

Recall that unemployed consumers' optimization problem is

$$U(a) = \max_{k', x'} u(a + h - qa') + \beta [(1 - \lambda_w)U(a') + \lambda_w W(a')].$$

All the benefit while unemployed is thus given to the unemployed agents in terms of consumption goods. In this section we consider a variation of this model by lowering  $h$  by 0.01 (from 2.29 to 2.28) and instead giving the unemployed consumers some (separable) utility  $u_l$  from leisure so that the total utility for the unemployed worker is the same in both economies. With  $h = 2.28$ , this requires  $u_l = 0.004$ . Thus for our experiments here we use

$$U(a) = \max_{k', x'} u(a + h - qa') + u_l + \beta [(1 - \lambda_w)U(a') + \lambda_w W(a')].$$

	$u$	$v$	$\theta$	$\bar{k}$	$p$	$d$	$w$
$z = 0.98$	15.96%	-21.2%	-61.4%	-11.4%	-20.6%	-20.6%	-2.2%
$z = 1.00$	7.81%	0.0546	0.7	66.49	2.37	0.012	2.39
$z = 1.02$	6.23%	+21.5%	+52.4%	+4.6%	+22.4%	+22.5%	+1.9%

Table 6: Summary statistics for different  $z$  values (alternative HM calibration).

Table 6 shows the statistics when we vary  $z$ . Figure 5 shows the wage function for the baseline and alternative HM calibrations. Clearly, the wage is increasing over the entire domain of  $a$  when there is utility from leisure. The intuition is that the utility from leisure is constant for everyone, while the utility from goods is subject to diminishing marginal utility. Since wages are given by goods, for each given wage, the utility from unemployment is relatively high for wealth-rich consumers. Therefore, they have higher bargaining power. When a large value is assigned for  $u_l$ , the surplus from very wealthy consumer's match becomes negative, so employed workers choose to separate once they become wealthy. Such endogenous separation (and labor supply decision) is an interesting mechanism, but beyond the scope of the current paper.<sup>22</sup>

<sup>22</sup>Krusell et al. (2008, 2009a, 2009b) explore endogenous separation in an "island" model of unemployment, and Bils et al. (2009) look at a similar model.

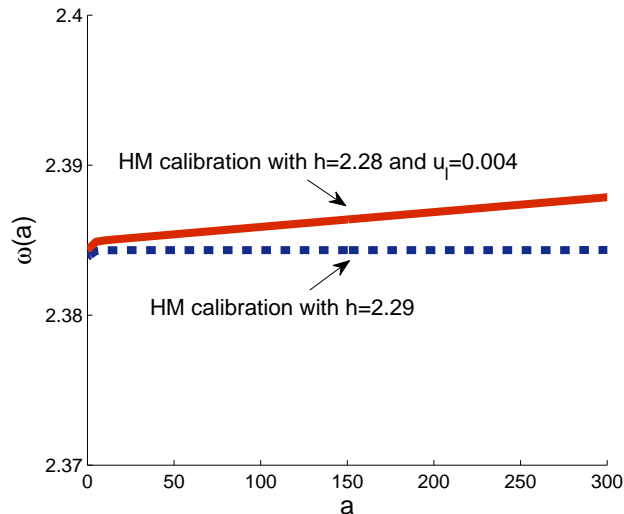


Figure 5: Wages for the HM and alternative HM calibrations.

## 5 The model with aggregate shocks

We now incorporate aggregate uncertainty: aggregate productivity shocks. There are two broad reasons for introducing aggregate shocks. One is substantive: the steady-state analysis above only gives an indication of aggregate dynamics, since a model with capital and consumption smoothing implies that labor-market tightness will have nontrivial short-run dynamics. Indeed, as indicated above, the model will generate stochastic processes for aggregates that, at least under some parameter configurations, are broadly in line with available U.S. data. The second reason for looking at the model with aggregate shocks is methodological: it provides a “how-to” for what we believe is a range of potentially very interesting applications. There are two methodological issues of importance: the evaluation of firm profits (how are they priced, and how are dynamic firm decisions made?) and the numerical implementation, which is significantly more involved than that in Krusell and Smith (1998, 1997).<sup>23</sup>

Like in Krusell and Smith (1998), we assume that the aggregate productivity  $z$  is either good

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<sup>23</sup>We do not look at a version with larger wealth dispersion for the aggregate-uncertainty economy, since the steady-state analysis suggests that the aggregate results would not change much.

( $z = g$ ) or bad ( $z = b$ ), with  $g > b$ , and follows a first-order Markov process, with the probability of moving from state  $z$  to state  $z'$  denoted by  $\pi_{zz'}$ . Unlike in Krusell and Smith, however, we do not need to make additional assumptions about individual employment shocks (and their correlation with aggregates) since they are endogenous here.

## 5.1 Asset structure

Again, we assume that there are no insurance markets for the idiosyncratic shocks. Agents can hold only two kinds of asset—capital  $k$  and equity  $x$ . Here, again, the equity  $x$  is the claim for the aggregate profit. We continue to assume that the consumers are not allowed to hold claims to the profits from individual jobs. Now, the difference from the previous section is that capital and equity have different return structures, so the consumers face a portfolio choice problem.

Let  $S$  be the joint distribution of asset and employment across the consumers. Then the aggregate state at any given period can be described by  $(z, S)$ .

The distribution of assets across the consumers in the next period is determined in the current period, since it depends on the consumers' asset accumulation and portfolio choice decisions, which are governed by the current state  $(z, S)$ . The *distribution* of the employment states at the aggregate level in the next period is determined in the current period by the number of vacancies (the firm's decision) and unemployment, which are also governed by  $(z, S)$ . Thus, next period's state  $S'$  is determined by  $(z, S)$ . Let us write this dependence as  $S' = \Omega(z, S)$ . It is important to note that even though  $S'$  is already determined by the current state, the employment state of each individual in the next period is still uncertain.

By above argument, there are only two possible (and uncertain) values of the *aggregate states* next period:  $(g, S')$  and  $(b, S')$ . We can span these states by the two “aggregate” assets that we have—capital and equity. Note that the asset markets are still incomplete, since there are no assets for insuring against idiosyncratic risks. Below, when we consider the consumer's decisions, we will consider a portfolio choice between two “Arrow securities”: securities each of which provides one unit of consumption good in one of the aggregate states and nothing in the other.<sup>24</sup> This is

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<sup>24</sup>Krusell and Smith (1997) also have two assets but do not use contingent claims in their implementation.

without loss of generality, since we can create these securities by combining capital and equity. For ease of exposition, below we introduce an entity called an “investment firm” that performs this transformation.

Suppose that the current state is  $(z, S)$ . Let  $Q_{z'}(z, S)$  be the price of an Arrow-security that pays out one unit of consumption good when the next-period state is  $z'$ . Let the interest rate be  $r(z, S)$  and the equity price be  $p(z, S)$ . Then, from no arbitrage, the asset prices have to satisfy

$$Q_g(z, S)(1 - \delta + r(g, S')) + Q_b(z, S)(1 - \delta + r(b, S')) = 1 \quad (17)$$

and

$$p(z, S) = Q_g(z, S)[p(g, S') + d(g, S')] + Q_b(z, S)[p(b, S') + d(b, S')], \quad (18)$$

where  $d(z, S)$  is the dividend. (17) and (18) show that there is a one-to-one mapping between  $\{r(z, S), p(z, S)\}$  and  $\{Q_g(z, S), Q_b(z, S)\}$  for a given  $d(z, S)$ .

## 5.2 Consumers

Consumers in the economy choose their demand for Arrow securities subject to a budget constraint. We impose an exogenous borrowing constraint for each contingent claim at  $\underline{a}$ . Consumers in the economy differ in their employment status and asset holdings.

### 5.2.1 Employed consumers

Let  $a'_{z'}$  be the demand of an Arrow security that pays out one unit of consumption good in the next period if next period's state is  $z'$ .

Let  $\tilde{W}(w, a; z, S)$  be the value of being an employed consumer, given the wage  $w$ . An employed worker's optimization problem is

$$W(a; z, S) = \max_{a'_g \geq \underline{a}, a'_b \geq \underline{a}} u(c) + \beta [ \pi_{zg}(\sigma U(a'_g; g, S') + (1 - \sigma)W(a'_g; g, S')) + \pi_{zb}(\sigma U(a'_b; b, S') + (1 - \sigma)W(a'_b; b, S')) ] \quad (19)$$

subject to

$$c + Q_g(z, S)a'_g + Q_b(z, S)a'_b = a + w$$

and

$$S' = \Omega(z, S).$$

Here,  $c$  is the consumption level. Let the decision rule for  $a'_{z'}$  be  $\tilde{\psi}_e^{z'}(w, a; z, S)$ . Here,  $U(a; z, S)$  is the value of being an unemployed consumer with asset holding  $a$  and  $W(a; z, S)$  is the value of being an employed consumer, taking into account that the wage depends on  $a$  and  $(z, S)$  through Nash bargaining. More formally, denoting the wage function as

$$w = \omega(a; z, S),$$

$W(a; z, S)$  is defined as

$$W(a; z, S) = \tilde{W}(\omega(a; z, S), a; z, S) \quad (20)$$

and we define the decision rule  $\psi_e^{z'}(a; z, S)$  as

$$\psi_e^{z'}(a; z, S) = \tilde{\psi}_e^{z'}(\omega(a; z, S), a; z, S). \quad (21)$$

### 5.2.2 Unemployed consumers

The unemployed worker's optimization problem is

$$U(a; z, S) = \max_{a'_g \geq \underline{a}, a'_b \geq \underline{a}} u(c) + \beta [ \pi_{zg}((1 - \lambda_w(z, S))U(a'_g; g, S') + \lambda_w(z, S)W(a'_g; g, S')) + \pi_{zb}((1 - \lambda_w(z, S))U(a'_b; b, S') + \lambda_w(z, S)W(a'_b; b, S')) ] \quad (22)$$

subject to

$$c + Q_g(z, S)a'_g + Q_b(z, S)a'_b = a + h$$

and

$$S' = \Omega(z, S).$$

Here,  $h$  is the income that the consumer receives when she is unemployed. The job-finding probability  $\lambda_w(z, S)$  is defined as in Section 3.2. Let the decision rule for  $a'_{z'}$  be  $\psi_u^{z'}(a; z, S)$ .

### 5.3 Firms

In order to be able to get matched with a worker and produce, a firm posts a vacancy.<sup>25</sup> Let the vacancy cost be  $\xi$ . The value of a vacancy  $V(z, S)$  is

$$V(z, S) = -\xi + Q_g(z, S)((1 - \lambda_f(z, S))V(g, S') + \lambda_f(z, S) \int J(\psi_u^g(a; z, S); g, S')[f_u(a; S)/u]da) \\ + Q_b(z, S)((1 - \lambda_f(z, S))V(b, S') + \lambda_f(z, S) \int J(\psi_u^b(a; z, S); b, S')[f_u(a; S)/u]da), \quad (23)$$

where  $J(a; z, S)$  is the value of a matched job (taking into account the wage bargaining) and  $f_u(a; S)$  is the population of unemployed workers with asset  $a$ . The worker-finding probability  $\lambda_f(z, S)$  is defined as in Section 3.2. The firm will post vacancies  $v(z, S)$  until  $V(z, S) = 0$ . Note that the firm discounts the future value by Arrow security prices. This is because these values correspond to the rates at which the non-constrained consumers discount each future state. In our numerical exercise below, it turns out that (almost) no one is at the borrowing constraint; therefore, this way of discounting is (almost) unanimously supported.

Let us consider a job matched with a worker with asset holdings  $a$ . The firm rents capital from the consumers at a rental rate of  $r(z, S)$  and pays the worker a wage of  $\omega(a; z, S)$ . The output is determined by the production function  $zF(k)$ . The value of a filled job given the wage  $w$ ,  $\tilde{J}(w, a; z, S)$ , is

$$\tilde{J}(w, a; z, S) = \tilde{\pi}(w; z, S) + Q_g(z, S)(\sigma V(g, S') + (1 - \sigma)J(\tilde{\psi}_e^g(w, a; z, S); g, S')) \\ + Q_b(z, S)(\sigma V(b, S') + (1 - \sigma)J(\tilde{\psi}_e^b(w, a; z, S); b, S')), \quad (24)$$

where the flow profit  $\tilde{\pi}(w; z, S)$  is defined as

$$\tilde{\pi}(w; z, S) = \max_k zF(k) - r(z, S)k - w. \quad (25)$$

From the first-order condition,

$$r(z, S) = zF'(k)$$

holds. In equilibrium, the capital stock per job is  $\tilde{k} = \bar{k}/(1 - u)$ , where  $\bar{k}$  is the aggregate capital stock. Thus, the equilibrium profit is

$$\pi(a; z, S) = zF(\tilde{k}) - r(z, S)\tilde{k} - \omega(a; z, S).$$

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<sup>25</sup>We envision a “representative firm” as a collection of jobs, and it thus posts several vacancies.

$\tilde{J}$  and  $J$  are related by

$$J(a; z, S) = \tilde{J}(\omega(a; z, S), a; z, S). \quad (26)$$

The dividend is determined by aggregating the profits of all matched firms in the economy as

$$d(z, S) = \int \pi(a; z, S) f_e(a; S) da - \xi v, \quad (27)$$

where  $f_e(a; S)$  is the population of employed workers with asset  $a$ . Note that since wages paid by firms depend on the asset positions of the workers that they are matched with, dividends depend on the wealth distribution of the employed consumers in the economy.

#### 5.4 Wage determination

Vacant jobs and unemployed workers are randomly matched each period according to the aggregate matching function  $M(u, v)$ , which is identical to the one we defined in the previous section. A realized match produces some pure economic rent that is shared by the firm and the worker through Nash bargaining. The wage that the firm pays a worker with asset holdings  $a$  is determined by

$$\max_w \left( \tilde{W}(w, a; z, S) - U(a; z, S) \right)^\gamma \left( \tilde{J}(w, a; z, S) - V(z, S) \right)^{1-\gamma}. \quad (28)$$

#### 5.5 Investment firms

There are competitive investment firms who sell contingency claims to consumers by rearranging capital and equities. For the asset market to clear, it has to be that, for each  $z'$ ,

$$\int \psi_e^{z'}(a; z, S) f_e(a; S) da + \int \psi_u^{z'}(a; z, S) f_u(a; S) da = (1 - \delta + r(z', S')) \bar{k}' + p(z', S') + d(z', S'). \quad (29)$$

Asset prices, moreover, have to satisfy (17) and (18) have to hold.

#### 5.6 Computation

Given all the above, the definition of a recursive competitive equilibrium is straightforward.<sup>26</sup> The computation of our model, however, is considerably more complex than for standard incomplete

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<sup>26</sup>It is stated in Appendix H. Appendix I also shows that when the above conditions are satisfied, the resource balance condition (goods market clearing condition) is satisfied.

markets models. In addition to the problems we faced in the model without aggregate shocks, now the distribution of asset and employment moves around endogenously over time, and this information is an input in the consumer’s optimization problem. The complexity is similar to that in Krusell and Smith (1998, 1997). In our setting, however, since unemployment is not summarized by an exogenously given stochastic process, the aggregate productivity shock  $z$  is no longer the sole determinant of the unemployment rate. As a consequence, we need more aggregate state variables than in previous settings.

Following Krusell and Smith (1998), we study a setting where consumer have boundedly rational perceptions of the evolution of the aggregate state, since we hope that “approximate aggregation” will hold. Thus, we assume that the consumer perceives next period’s aggregate capital stock,  $\bar{k}'$ , to be a (log-)linear function of  $(z, \bar{k}, u)$ , where  $u$  is an additional state relative to the setting without labor-market frictions. Since there are other nontrivial market-clearing conditions, we also need to model the resulting outcomes—for  $\theta$ ,  $p$ ,  $d$ , and  $Q_z$ —as simple (typically linear) functions of  $(z, \bar{k}, u)$ .<sup>27</sup> After computing the equilibrium based on these assumptions and simulating the economy, we check whether these perceptions are accurate. This indeed turns out to be the case, as we will show below.

The main steps of our algorithm, using the  $z = g$  case for illustration, are as follows.

1. Postulate the law of motion for  $\bar{k}$  by assuming that  $\bar{k}'$  is a function of  $(z, \bar{k}, u)$ .
2. Assume that  $\theta$ ,  $p$ ,  $d$ , and  $Q_g$  are functions of  $(z, \bar{k}, u)$  and guess on coefficients of prediction rules.
3. Calculate  $u'$  from  $u' = (1 - \lambda_w(\theta))u + \sigma(1 - u)$ .

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<sup>27</sup>An alternative procedure is to include an extra step, where the consumers solve an auxiliary optimization problem that include  $\theta$ ,  $p$ ,  $d$ , and  $Q_g$  as additional state variables and make sure that the equilibrium conditions regarding these variables hold at this stage. This procedure is suggested by, for example, Ríos-Rull (1999). In our current setting, this will increase computational burden substantially. It turns out that (as is stated in Section 5.8 and detailed in Appendices L and M) our prediction rules are very accurate so that we believe that including this extra step will not alter the results. Chang and Kim (2006) utilize a computational method that is similar to ours.



4. Calculate  $Q_b$  by using  $Q_g$  from

$$Q_g(z, \bar{k}, u)(1 - \delta + r(g, \bar{k}', u')) + Q_b(z, \bar{k}, u)(1 - \delta + r(b, \bar{k}', u')) = 1.$$

5. Perform the individual optimization and Nash bargaining.

6. Simulate the economy using the result from the previous step. Generate data on  $\bar{k}$ ,  $p$ ,  $d$ ,  $Q_g$ ,  $\theta$ . Check if it is consistent with the predictions in the first step. If not, revise the prediction rules and continue until convergence.

The detailed algorithm that we use is described in Appendix J.

## 5.7 Calibration

We follow the same calibration as in Section 4.1. Aggregate shocks take the values  $z \in \{b, g\} = \{0.98, 1.02\}$  following Cooley and Prescott (1995).<sup>28</sup>  $\pi_{ij}$  is the probability of the transition from state  $i$  to state  $j$ . Following Krusell and Smith (1999) and Krusell et al. (2009c), we set the average business-cycle duration to 2 years. Our model period is six weeks; therefore the average duration is 16 periods. From  $1/(1 - \pi_{bb}) = 1/(1 - \pi_{gg}) = 16$ ,  $\pi_{bb} = \pi_{gg} = 0.9375$ .

## 5.8 Results: the Shimer calibration

The laws of motion for  $\bar{k}$ ,  $\theta$ ,  $p + d$ ,  $Q_g$ , and  $Q_b$  are presented in Appendix K. Approximate aggregation seems to hold up very well in the present setting. The  $R^2$  of this prediction rules are above 0.999, i.e., very high. We also report additional accuracy checks in Appendix L. The average aggregate capital is  $\bar{k} = 66.65$  for  $z = b$  and  $\bar{k} = 67.22$  for  $z = g$ .

Table 7 summarizes the statistics for each state from our simulations. All the values are shown as percentage deviations of the average value in each state from the total average, except for  $u$  (which is in levels). The vacancy-to-unemployment ratio ( $\theta$ ) increases with  $z$ , but the fluctuations in  $\theta$  are not realistic in magnitude; this is the fact discussed in Shimer (2005) and Hall (2005). Similarly, the unemployment rate is negatively correlated with the aggregate productivity shock,

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<sup>28</sup>This is consistent with the unconditional standard deviation of the Solow residual in Cooley and Prescott (1995).

	$u$	$v$	$\theta$	$\bar{k}$	$p$	$d$
$z = b$	7.73%	-2.6%	-3.3%	-0.5%	-2.2%	-58.3%
$z = g$	7.64%	+2.1%	+2.7%	+0.4%	+1.9%	+48.7%

Table 7: Summary statistics for the model with aggregate shocks (the Shimer calibration).

but the magnitude of the fluctuations is much below what we observe in the data. Equity prices are higher on average for the  $z = g$  states, although the magnitude of these fluctuations is small.

Figure 6 shows a sample path for aggregate capital obtained from simulating our model. We see

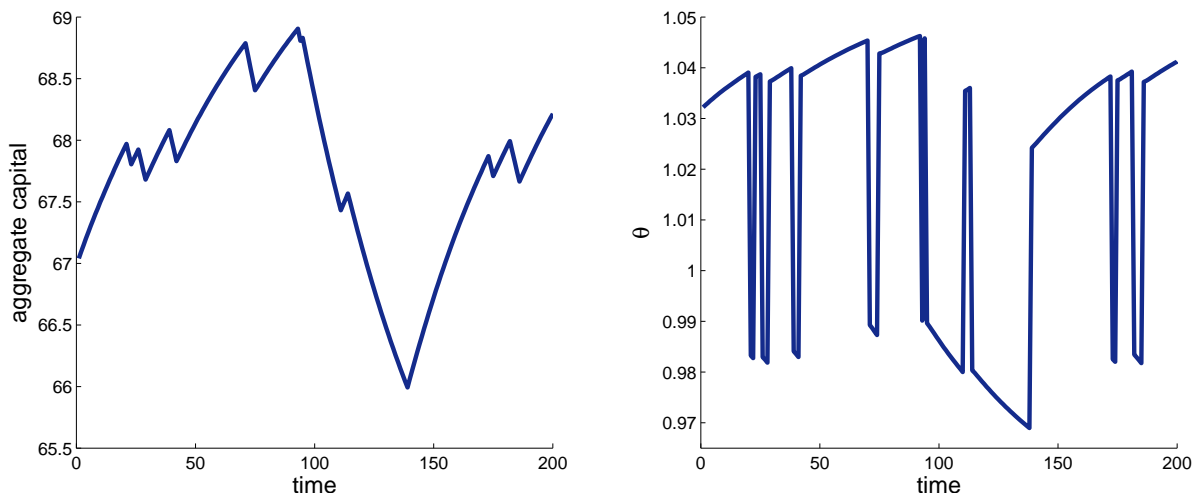


Figure 6: Sample paths of  $\bar{k}$  and  $\theta$ , Shimer calibration.

that  $\bar{k}$  increases when  $z = g$  and decreases when  $z = b$ , and  $\theta$  is not constant over time conditional on an aggregate state: as the capital stock adjusts, the profitability of firm entry changes. For example, if  $z$  moves from  $b$  to  $g$  and there is no immediate switch back,  $\theta$  jumps; the capital stock will start increasing, and as it increases,  $\theta$  will keep increasing further.<sup>29</sup> The reason is that more capital availability will make capital cheaper to rent.

Figure 7 (left panel) shows a sample path for the unemployment and vacancy rates obtained

<sup>29</sup>The asset price follows a sample path very similar to that of  $\theta$ , since it too is a jump variable.

from simulating our model. When the aggregate state switches from  $z = b$  to  $z = g$ ,  $\theta$  jumps

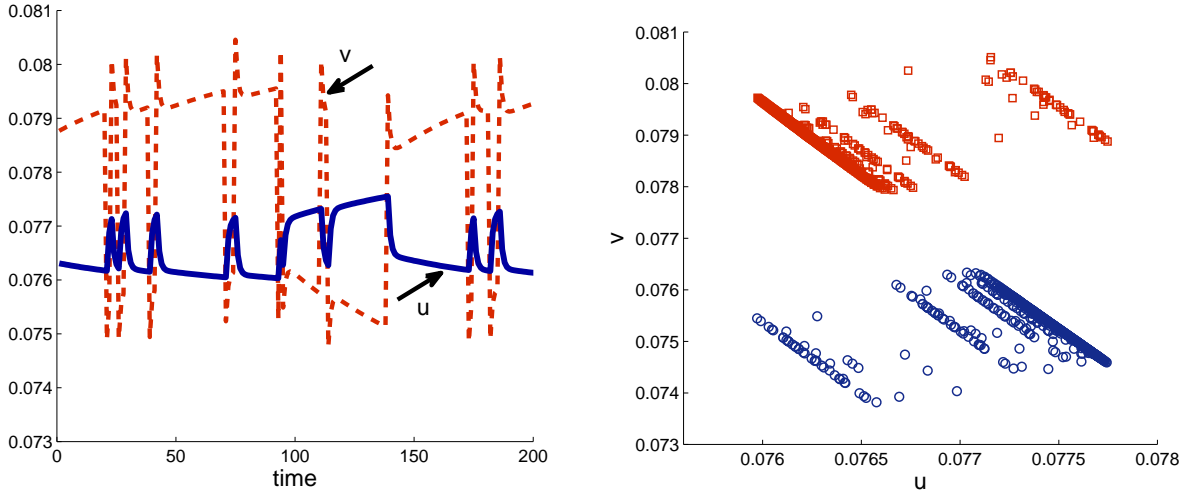


Figure 7: Sample paths of  $u$  and  $v$  (left panel) and unemployment rate ( $u$ ), vacancy rate ( $v$ ) plot (right panel), Shimer calibration. On the right panel,  $u$  and  $v$  values for  $z = b$  are plotted as “circles” and  $u$  and  $v$  values for  $z = g$  are plotted as “squares.”

up. For a given  $u$ , this means a large increase in  $v$ . This will make  $u$  go down significantly in the following period. If the aggregate state is still  $g$  in the following period,  $\theta$  will remain high (and even increase somewhat), but since  $u$  is now lower,  $v$  must fall as well (but remain higher than prior to the  $z$  switch). Subsequently, if  $z$  continues to be  $g$ ,  $v$  and  $u$  will keep moving, and in opposite directions, since a rising  $\theta$  reflects higher entry and a lower jobless rate.<sup>30</sup> When  $z$  switches from  $g$  to  $b$ , we see the opposite pattern. Note that the comparative steady states in the previous section describe the situation after the adjustment of  $k$  (and  $u$ ) was completed, that is, after  $z$  has continued to be at the same state for a long time. The results in this section suggest that this adjustment is fairly slow. This explains why the unemployment fluctuations here are not as large as those we saw across steady states with different productivities.

Figure 7 (right panel) shows combinations of the unemployment rates ( $u$ ) and vacancy rates

<sup>30</sup>A decrease in  $u$  has the opposite effect, since the capital stock per job is  $\tilde{k} = \bar{k}/(1 - u)$ , but this effect turns out to be smaller than the profitability effect.

( $v$ ) obtained from simulating our model for 2000 periods.<sup>31</sup> Most of the points in the figure are on either of the two “thick,” circled (low- $v$ , or  $z = b$ ) or squared (high- $v$ , or  $z = g$ ), lines; they correspond to the paths  $u$  and  $v$  travel once the economy has been in one of the  $z$  states for several consecutive periods. On these thick lines, as time passes without a new  $z$  switch,  $u$  and  $v$  move in opposite directions, as  $\theta$  adjusts. This can be compared with the  $\theta$  movements in a model without capital and concave utility: there,  $\theta$  can only take on as many values as there are values for  $z$ , and so will capital (if there is capital in the model); here, because of consumption smoothing, capital must move slowly, as must  $\theta$ , within each aggregate state. The other lines in the figure are points the economy reaches in the period of a switch.<sup>32</sup>

## 5.9 Results: the Hagedorn and Manovskii calibration

In this section, we examine the calibration by Hagedorn and Manovskii (2008). The laws of motion for  $\bar{k}$ ,  $\theta$ ,  $p + d$ ,  $Q_g$ , and  $Q_b$  are presented in Appendix K.

Approximate aggregation seems to obtain here as well for the main variables, though there are larger errors for  $\theta$  and for the stock price. Table 8 summarizes the statistics for each state from the simulations. We can see that the cyclical properties of the variables are qualitatively the same as

	$u$	$v$	$\theta$	$\bar{k}$	$p$	$d$
$z = b$	8.75%	-10.2%	-20.5%	-0.4%	-7.5%	-134.7%
$z = g$	7.17%	+8.5%	+17.1%	+0.4%	+6.3%	+112.6%

Table 8: Summary statistics for the model with aggregate shocks (HM calibration)

for the benchmark calibration, but quantitatively the fluctuations are much larger. The responses of variables to  $z$  are smaller than in the comparative statics of the model without aggregate shocks, except for  $d$ .  $d$  tends to be very volatile since the total profit is small and very volatile (much larger for  $z = g$  than for  $z = b$ ), while vacancy-posting does not decline much even when  $z = b$ . In

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<sup>31</sup>The first 500 periods are discarded.

<sup>32</sup>The north-eastern-most and the south-western-most lines are lines the economy reaches after one switch, and the other lines (with a similar thickness to these lines) reflect the adjustment after the switch. The other scattered dots are the cases where a switch occurs during the adjustment.

particular, the level of  $d$  can become negative when  $z = b$ .

Figure 8 illustrates the behavior of unemployment and vacancies. The qualitative properties

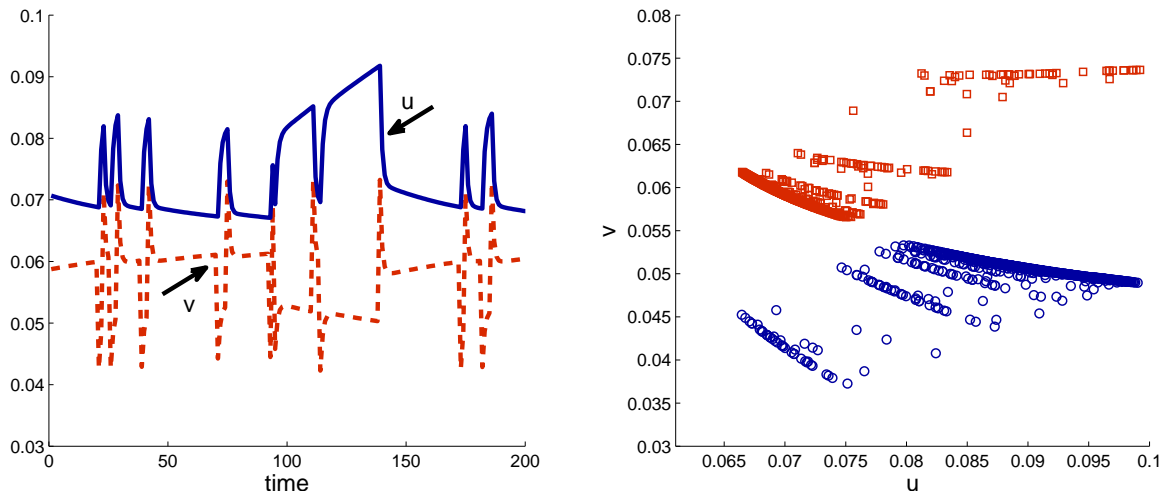


Figure 8: Sample paths of  $u$  and  $v$  (left panel) and unemployment rate ( $u$ ), vacancy rate ( $v$ ) plot (right panel), HM calibration. On the right panel,  $u$  and  $v$  values for  $z = b$  are plotted as “circles” and  $u$  and  $v$  values for  $z = g$  are plotted as “squares.”

of these paths are very similar to those for the Shimer calibration, but the magnitude of the fluctuations is much larger.

### 5.10 Aggregate statistics more broadly

In this subsection, we evaluate the model’s performance to match the fluctuations in aggregate variables. The standard deviation of output is 0.0138 for the Shimer calibration and 0.0159 for the HM calibration. In the U.S. economy, the unemployment rate and vacancies are negatively correlated. Shimer (2005) reports that the correlation of HP-filtered unemployment rate and vacancies is  $-0.894$  for U.S. data.<sup>33</sup> In our model with a Shimer calibration, the correlation coefficient of the cyclical components of unemployment rate and vacancies is  $-0.84$ , while for the HM calibration this statistic is  $-0.55$ . Figure 9 shows the HP-filtered Beveridge curves for both calibration exercises.

<sup>33</sup>Shimer (2005) uses  $10^5$  as the smoothing parameter.

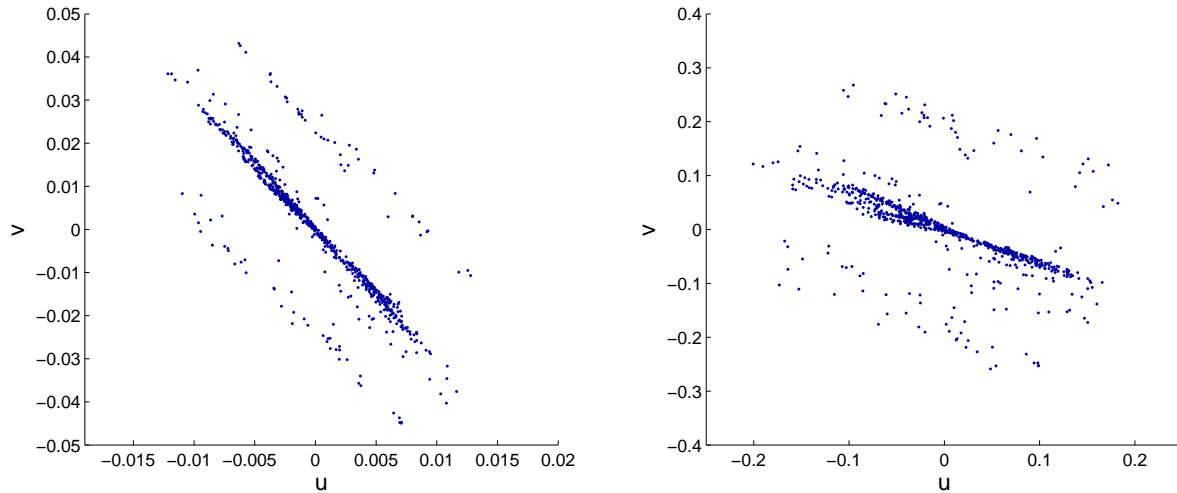


Figure 9: Beveridge curves for the Shimer calibration (left panel) and the HM calibration (right panel).

Table 9 shows the standard deviation of investment, consumption, the labor share, wages, and the vacancy-unemployment ratio relative to the standard deviation of output for the data and our model (both Shimer calibration and HM calibration).<sup>34</sup>

	U.S. Economy	Model Shimer	Model HM
Investment	3.14	4.52	3.60
Consumption	0.56	0.18	0.11
Labor share	0.43	0.05	0.36
Wage	0.44	0.93	0.29
Vacancy-unemployment ratio	16.27	1.42	8.38

Table 9: Standard deviation of detrended series divided by the standard deviation of output. All variables are logged and HP-filtered. Note that standard deviation of output is 0.0158 for the U.S. data, 0.0138 for the Shimer calibration and 0.0159 for the HM calibration.

<sup>34</sup>The details of the calculation of the reported statistics can be found in Appendix M. This appendix also contains a table that reports the standard deviations of stock prices and dividends relative to the standard deviation of output.

Our Shimer calibration produces less output fluctuations and much lower fluctuations in the vacancy-unemployment ratio. This type of finding has been documented by Andolfatto (1996), Hall (2005), and Shimer (2005), among others. They find that the standard DMP model cannot generate the observed fluctuations in unemployment and vacancies. Our finding is similar—our results suggest that that market incompleteness does not improve the performance of the DMP model in matching the magnitude of unemployment fluctuations. Since the benchmark result falls short in generating fluctuations in unemployment, it also cannot match the fluctuations in output. The HM calibration generates much larger fluctuations in unemployment and vacancies, and the result comes closer to the actual data.

Table 19 in the Appendix M also reports that our benchmark model underpredicts the fluctuations in stock prices. That is, it is also subject to the stock-market volatility puzzle. The HM calibration leads to much larger fluctuations in stock prices as measured by  $p$ . Measured by  $p + \bar{k}$ , the fluctuations in stock prices are much smaller. Dividends (measured by either  $d$  or  $d + (1 + r - \delta)\bar{k} - \bar{k}'$ ) fluctuate much more than in the data, and the puzzle of the relative volatility between stock prices and dividends remains unsolved. In our framework, the two volatility puzzles (those for the stock-market volatility and for labor market fluctuations) seem closely related. As can be seen from the results based on the HM calibration, if we can generate fluctuations in stock prices of a realistic magnitude, we will be able to generate more realistic labor-market fluctuations. This is because the stock price reflects future profits ( $p + d$  is the sum of  $J$ s in the economy), and future profits drive the firm’s vacancy-posting decision ( $v$  increases as  $J$  increases). The HM calibration achieves this, while it overpredicts the volatility of the dividends.

Table 10 reports the correlation between a range of variables (investment, consumption, the labor share, wages, and the vacancy-unemployment) and output for the data and our model (for both calibrations). In the U.S. economy, the labor share is countercyclical. Ríos-Rull and Santaeulàlia-Llopis (2007) reports a correlation coefficient of  $-0.24$  between the labor share and output, while Andolfatto (1996) reports this number to be  $-0.38$ . Our model generates a countercyclical labor share with a correlation of  $-0.99$  for the Shimer calibration and  $-0.98$  for the HM calibrations.

Clearly, this correlation would fall substantially if the model had more than one aggregate shock.

	U.S. Economy	Model Shimer	Model HM
Investment	0.90	0.99	0.98
Consumption	0.74	0.62	0.37
Labor share	-0.13	-0.99	-0.98
Wage	0.04	0.99	0.96
Vacancy-unemployment ratio	0.90	1.00	0.93

Table 10: Correlation of investment, consumption, labor share, wage, and vacancy-unemployment with output. All variables are logged and HP-filtered.

## 5.11 The alternative models again

Here we revisit the alternative models from Section 4.5 to see how they behave when they are subject to aggregate shocks; we also consider a model with complete markets.

### 5.11.1 Linear utility and a complete-markets model

The comparison to known versions of this model should also be of interest. We outline these in the Appendix, sections N and O.<sup>35</sup> The first version is the linear version, i.e., that with linear utility but otherwise like the present model. The second version is a model with complete consumption insurance across households: a Merz-Andolfatto model. As suspected from the above analysis of steady states, the labor-market fluctuations are very similar in the main model in this paper and in the linear model. These models differ greatly, however, in their implications for consumption and investment, whose fluctuations in the linear model are far larger (than in our benchmark model and than in data). As for the Merz-Andolfatto model, it behaves very similarly to our benchmark model, both in terms of its labor-market features and its implications for consumption and investment.<sup>36</sup>

<sup>35</sup>In the case of the complete-markets model, we define equilibrium and solve for it differently than in the original Merz-Andolfatto papers; we adhere to individual Nash bargaining, but treat workers as “assets.” This formulation allows the study of equilibria where the wage setting does not satisfy the Hosios condition.

<sup>36</sup>In Appendix P we provide a detailed comparison of our model with the linear and complete-markets versions of the DMP model. Note that in the linear model, consumption and investment are allowed to be negative. For that



### 5.11.2 UI as a utility benefit

In this section, we consider the alternative specification in Section 4.5.2—that is, we give the consumers utility from leisure. The results are summarized in Tables 11 and 12. The model behavior is similar to the baseline HM calibration case.

	$u$	$v$	$\theta$	$\bar{k}$	$p$	$d$
$z = b$	8.75%	-10.2%	-20.4%	-0.4%	-7.6%	-134.9%
$z = g$	7.17%	+8.5%	+17.1%	+0.3%	+6.3%	+112.8%

Table 11: Summary statistics for the model with aggregate shocks (alternative HM calibration)

	Baseline	Alternative
Investment	3.60	3.60
Consumption	0.11	0.11
Labor share	0.36	0.36
Wage	0.29	0.29
Vacancy-unemployment ratio	8.38	8.37

Table 12: Standard deviation of detrended series divided by the standard deviation of output. All variables are logged and HP-filtered. Note that standard deviation of output is 0.0159 for for the HM calibration and 0.0158 for the alternative HM calibration.

Approximate aggregation holds similarly—Table 13 shows similar  $R^2$  fits.

	Baseline	Alternative
$\bar{k}$	0.99999	0.99999
$\theta$	0.99999	0.99999
$p + d$	0.99997	0.99998
$Q_g$	0.99992	0.99991
$Q_b$	0.99914	0.99920

Table 13: The  $R^2$ s of prediction rules for the HM and alternative HM calibrations.

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reason, to make the comparison with the linear model possible, we apply the HP filter without taking the natural logarithm of the model-generated time series in Appendix P.

## 6 Conclusion

We introduce one additional element into the Pissarides (1985) model: workers are risk-averse (and cannot perfectly insure consumption). This elaboration on the standard matching model can equivalently be described as a BHA incomplete-markets model with aggregate shocks and with one additional element: labor-market frictions. We find that approximate aggregation holds, making model solution feasible despite a number of new elements relative to earlier models with aggregate shocks and nontrivially varying wealth distributions.

Our positive findings are that for all the parameter configurations considered, labor-market aggregates behave almost as in the linear-utility model counterparts, and that the consumption and investment fluctuations behave as in the typical representative-agent real-business-cycle model. If we adopt Hagedorn and Manovskii's (2008) view, that is, we set the monetary value of unemployment to be very high, then the resulting setting has significant fluctuations in unemployment and vacancies, and the other aggregate variables behave in realistic ways as well (e.g., the labor share is countercyclical and real wages fluctuate relatively little). If, in contrast, we adopt the calibrations in Hall (2005) or Shimer (2005), the fluctuations in unemployment and vacancies are significantly lower than in the data.

Our model can serve as a framework for analyzing various stabilization and social-insurance policies. We consider a simple example in Section 4.3 and analyze the welfare effects of unemployment insurance in our model. We find that endogenizing unemployment by using a search-matching framework results in significantly different implications than those found in standard BHA models with exogenously given employment processes: UI is much less desirable than it would be in a model where partial insurance is the only friction.

What would the implications of UI have in economies with aggregate fluctuations? We speculate that under the Shimer calibration, since aggregate shocks do not influence individual risks much at all, the value of making UI cyclical is likely small; moreover, its optimal level—we presume that UI is funded by constant taxes (i.e., the government budget balances on average but not period by period)—would look very similar to that in the steady-state version of the model. Under the HM

calibration, individual unemployment risk is, in contrast, highly influenced by aggregate shocks, but because the value of being unemployed is so close to the value of being employed in this kind of calibration, there is very little to insure, and UI policy is not likely to generate significant welfare gains.

One dimension in which the benchmark model is not empirically satisfactory is in the dispersion of wealth and wages. Wages differ only because workers' bargaining power depend on their asset holdings; moreover, the effect of wealth on wages is very slight for most asset levels. Wealth, therefore, differs among workers only due to past employment luck. Since many elements of heterogeneity are left out—worker ability, match quality, preferences, etc.—it is an open question as to what our target wealth dispersion should be. Future versions of the present setting ought to incorporate these elements, and it is an open question whether approximate aggregation will continue to hold then, and how the positive findings for aggregates that we obtain here will change. The analysis of steady states with multiple discount factors, however, indicates that at least some models with large wealth dispersion will (i) be feasible to study and (ii) yield similar predictions to those obtained here.

## References

- [1] Acemoglu, Daron and Robert Shimer (2000). “Productivity Gains from Unemployment Insurance,” *European Economic Review* 44, 1195-1224.
- [2] Aiyagari, S. Rao (1994). “Uninsured Idiosyncratic Risk and Aggregate Saving,” *Quarterly Journal of Economics* 109, 659-684.
- [3] Alexopoulos, Michelle and Tricia Gladden (2006). “Wealth, Reservation Wages, and Labor Market Transitions in the U.S.: Evidence from the Survey of Income and Program Participation,” mimeo.
- [4] Andolfatto, David (1996). “Business Cycles and Labor Market Search,” *American Economic Review* 86, 112-132.
- [5] Atkeson, Andrew and Christopher Phelan (1994). “Reconsidering the Costs of Business Cycles with Incomplete Markets,” in Stanley Fischer and Julio J. Rotemberg (eds.) *NBER Macroeconomics Annual 1994*, National Bureau of Economic Research, Cambridge, MA.
- [6] Bewley, Truman F. (undated). “Interest Bearing Money and the Equilibrium Stock of Capital,” mimeo.
- [7] Bilts, Mark, Yongsung Chang, and Sun-Bin Kim (2009). “Heterogeneity and Cyclical Unemployment,” NBER Working Paper 15166.
- [8] Castañeda, Ana, Javier Díaz-Giménez, and José-Víctor Ríos-Rull (2003). “Accounting for Earnings and Wealth Inequality,” *Journal of Political Economy* 111, 818-857.
- [9] Christiano, Lawrence, Cosmin Ilut, Roberto Motto, and Massimo Rostagno (2007). “Monetary Policy and Stock Market Boom-Bust Cycles,” mimeo. Northwestern University and European Central Bank.

- [10] Chang, Yongsung and Sun-Bin Kim (2006). "From Individual to Aggregate Labor Supply: A Quantitative Analysis Based on a Heterogeneous Agent Macroeconomy," *International Economic Review* 47, 1-27.
- [11] Cooley, Thomas F., and Edward C. Prescott (1995). "Economic Growth and Business Cycles," in Thomas F. Cooley (ed.) *Frontiers of Business Cycle Research* Princeton University Press, Princeton, NJ.
- [12] Costain, James S. and Michael Reiter (2005). "Stabilization versus Insurance: Welfare Effects of Procyclical Taxation under Incomplete Markets," mimeo. Universidad Carlos III de Madrid and Universitat Pompeu Fabra.
- [13] Costain, James S. and Michael Reiter (2007). "On the Relevance of Saving for Labor Market Fluctuations," mimeo. Bank of Spain and Institute for Advanced Studies, Vienna, and UPF, Barcelona.
- [14] Dávila Julio, Jay Hong, Per Krusell, José-Víctor Ríos-Rull (2005). "Constrained Efficiency in the Neoclassical Growth Model with Uninsurable Idiosyncratic Shocks," mimeo. University of Pennsylvania and Princeton University.
- [15] Den Haan, Wouter J., Gary Ramey, and Joel Watson (2000). "Job Destruction and Propagation of Shocks," *American Economic Review* 90, 482-498.
- [16] Diamond, Peter A. (1981). "Mobility Costs, Frictional Unemployment, and Efficiency," *Journal of Political Economy* 89, 798-812.
- [17] Gertler, Mark and Antonella Trigari (2006). "Unemployment Fluctuations with Staggered Nash Wage Bargaining," *Journal of Political Economy*, forthcoming.
- [18] Hagedorn, Marcus and Iourii Manovskii (2008). "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited," *American Economic Review*, 1692-1706.

- [19] Hall, Robert E. (2005). "Employment Fluctuations with Equilibrium Wage Stickiness," *American Economic Review* 95, 50-65.
- [20] Hansen, Gary D. and Ayşe İmrohoroğlu (1992). "The Role of Unemployment Insurance in an Economy with Liquidity Constraints and Moral Hazard," *Journal of Political Economy* 100, 118-142.
- [21] Hornstein, Andreas, Per Krusell, and Giovanni L. Violante (2005). "Unemployment and Vacancy Fluctuations in the Matching Model: Inspecting the Mechanism," *Economic Quarterly (Federal Reserve Bank of Richmond)* 91, 19-51.
- [22] Hornstein, Andreas, Per Krusell, and Giovanni L. Violante (2006). "Frictional Wage Dispersion: A Quantitative Assessment," mimeo. Federal Reserve Bank of Richmond, Princeton University, and New York University.
- [23] Huggett, Mark (1993). "The Risk-Free Rate in Heterogeneous-Agent Incomplete-Insurance Economies," *Journal of Economic Dynamics and Control* 17, 953-969.
- [24] Huggett, Mark (1996). "Wealth Distribution in Life-Cycle Economies," *Journal of Monetary Economics* 38, 469-494.
- [25] İmrohoroğlu, Ayşe (1989). "The Cost of Business Cycles with Indivisibilities and Liquidity Constraints," *Journal of Political Economy* 97, 1364-1383.
- [26] Jung, Philip and Keith Kuester (2008). "The (Un)Importance of Unemployment Fluctuations for Welfare," Philadelphia Fed Working Paper 08-31, 2008.
- [27] Kallcock, Kevin R. "Cyclical Behavior of Labor Markets: The Role of Risk Aversion and Savings," Ph.D. Thesis, Yale University.
- [28] Krebs, Tom (2006). "Multi-Dimensional Risk and the Cost of Business Cycles," *Review of Economic Dynamics* 9, 640-658.

- [29] Krusell, Per, Toshihiko Mukoyama, Richard Rogerson, and Ayşegül Şahin (2008). “The Aggregate Implications of Indivisible Labor, Incomplete Markets and Search Frictions,” *Journal of Monetary Economics* 55, 961-979.
- [30] Krusell, Per, Toshihiko Mukoyama, Richard Rogerson, and Ayşegül Şahin (2009a). “A Three State Model of Worker Flows in General Equilibrium,” mimeo.
- [31] Krusell, Per, Toshihiko Mukoyama, Richard Rogerson, and Ayşegül Şahin (2009b). “Aggregate Labor Market Outcomes: The Role of Choice and Chance,” mimeo.
- [32] Krusell, Per, Toshihiko Mukoyama, Ayşegül Şahin, and Anthony A. Smith, Jr. (2009c). “Revisiting the Welfare Effects of Eliminating Business Cycles,” *Review of Economic Dynamics*, 12, 393-404.
- [33] Krusell, Per and Anthony A. Smith, Jr. (1997). “Income and Wealth Heterogeneity, Portfolio Choice, and Equilibrium Asset Returns,” *Macroeconomic Dynamics* 1, 387-422.
- [34] Krusell, Per and Anthony A. Smith, Jr. (1998). “Income and Wealth Heterogeneity in the Macroeconomy,” *Journal of Political Economy* 106, 867-896.
- [35] Krusell, Per and Anthony A. Smith, Jr. (1999). “On the Welfare Effects of Eliminating Business Cycles,” *Review of Economic Dynamics* 2, 245-272.
- [36] Langot, François, (1995). “Unemployment and Business Cycles: A Matching Model,” in: P.Y. Henin (ed.) *Advances in Business Cycle Research* Springer-Verlag, New York, NY.
- [37] Lucas, Robert E., Jr. (2003). “Macroeconomic Priorities,” AEA 2003 Presidential Address.
- [38] Merz, Monika (1995). “Search in the Labor Market and the Real Business Cycle,” *Journal of Monetary Economics* 36, 269-300.
- [39] Mortensen, Dale T. and Christopher A. Pissarides (1994). “Job Creation and Job Destruction in the Theory of Unemployment,” *Review of Economic Studies* 61, 397-415.

- [40] Mukoyama, Toshihiko and Aysegül Şahin (2006). “Costs of Business Cycles for Unskilled Workers,” *Journal of Monetary Economics* 53, 2179-2193.
- [41] Nakajima, Makoto (2007). “Business Cycles in the Equilibrium Model of Labor Market Search and Self-insurance,” mimeo. University of Illinois, Urbana-Champaign.
- [42] Pissarides, Christopher A. (1985). “Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages,” *American Economic Review* 75, 676-690.
- [43] Ríos-Rull, José-Víctor (1999). “Computation of Equilibria in Heterogeneous-Agents Models,” in Ramon Marimon and Andrew Scott (eds.) *Computational Methods for the Study of Dynamic Economies* Oxford University Press, New York.
- [44] Ríos-Rull, José-Víctor and Raül Santaeuàlia-Llopis (2007). “Redistributive Shocks and Productivity Shocks,” mimeo. University of Pennsylvania.
- [45] Rudanko, Leena (2006). “Idiosyncratic and Aggregate Risk in a Frictional Labor Market,” mimeo. University of Chicago.
- [46] Rudanko, Leena (2009). “Labor Market Dynamics under Long Term Wage Contracting and Incomplete Markets,” *Journal of Monetary Economics*, 56, 170-183.
- [47] Shao, Enchuan and Pedro Silos (2007). “Uninsurable Individual Risk and the Cyclical Behavior of Unemployment and Vacancies,” mimeo. University of Iowa and Federal Reserve Bank of Atlanta.
- [48] Shimer, Robert J. (2005). “The Cyclical Behavior of Equilibrium Unemployment and Vacancies,” *American Economic Review* 95, 25-49.
- [49] Shiller, Robert J. (1981). “Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?,” *American Economic Review* 71, 421-436.
- [50] Valdivia, Victor (1996). “Policy Evaluation in Heterogeneous Agent Economies: The Welfare Impact of Unemployment Insurance,” Ph.D. Thesis, Northwestern University.



- [51] Wright, Randall (1986). “The Redistributive Roles of Unemployment Insurance and the Dynamics of Voting,” *Journal of Public Economics* 31, 377-389.