

Labor-Market Uncertainty and Portfolio Choice Puzzles*

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Abstract

The standard life-cycle model of household portfolio choice has difficulty generating a realistic age profile of risky share. Not only does the model imply a high risky share on average but also a steeply decreasing age profile, whereas the risky share increases with age in the data. In this paper, we show that age-dependent labor-market uncertainty is important in accounting for the observed age profile of risky share. A large uncertainty in the labor market—due to high unemployment risk, frequent job turnovers, and an unknown career path—prevents young workers from taking too much risk in the financial market. As the labor-market uncertainty is gradually resolved over time, workers can take more risk in their financial portfolios.

Keywords: Portfolio Choice, Labor-Market Uncertainty, Risky Share, Imperfect Information.

JEL Classification: G11, E21, J24, D14

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1 Introduction

According to the Survey of Consumer Finances (SCF), the risky share—the share of risky assets in total financial assets—increases with age at least until around retirement.¹ The participation rate—the fraction of households that participate in risky investment—is as low as 30% in the 21-25 age group and reaches its peak of 65% at ages 56-60. The conditional risky share (defined by the risky share among households that participate in risky investment) is 40% in the age group 21-25 and monotonically increases to 50% at ages 61-65. By contrast, the standard life-cycle models of household portfolio choice (e.g., Cocco, Gomes, and Maenhout (2005)) imply not only a very high average risky share but also a steeply decreasing age profile of risky share. In these models, households invest aggressively in stocks when young and gradually move toward a safer portfolio.

We argue that the household’s portfolio choice over the life-cycle is heavily influenced by earnings risk in the labor market. It is well known that young workers face larger uncertainty in the labor market—high unemployment rates, frequent job turnovers, and an unknown career path. For example, according to the 2013 Current Population Survey (CPS), the average unemployment rate of male workers ages 21-25 is as high as 14%, whereas that of workers ages 51-55 is 5.8%. According to Topel and Ward (1992), in the first 10 years after entering the labor market, a typical worker holds 7 jobs (about two-thirds of his career total). These transitions often take place between jobs with different career prospects. Moreover, workers have imperfect information about their true earnings ability. As a result, not only the *actual* but also the *perceived* uncertainty in the labor market is much larger for young workers.

Since the labor-market outcome is by and large uninsurable, young investors, despite a longer investment horizon, would not want to expose themselves to too much risk in the financial market. As the labor-market uncertainty is gradually resolved over time, households can take more risk in financial investments.

To quantitatively investigate this link between labor-market risk and financial investment, we introduce three types of age-dependent labor-market uncertainty—unemployment risk, occupational changes, and gradual learning about earnings ability—into an otherwise standard life-cycle model of household portfolio choices (e.g., Cocco, Gomes, and Maenhout (2005)). More specifically, we borrow the estimates on the life-cycle profile of unemployment risk from Choi, Janiak, and Villena-Roldan (2011) and adopt the age-dependent probability of occupational change documented by Kambourov and Manovskii (2008). In our model, each occupation has a different income path. Hence, occupational changes imply shifts in the income profile. The stochastic process of income profile changes is estimated based on the

¹The detailed definition of risky share is explained in Section 2.

wage variation of workers who switched occupations in the Panel Study of Income Dynamics (PSID). Finally, we introduce imperfect information and Bayesian learning about the income profile as in Guvenen (2007) and Guvenen and Smith (2014). We justify our calibration by showing that our model successfully matches the life-cycle pattern of consumption dispersion in the data. We also consider various specifications of the model to evaluate the marginal contribution of each component of labor-market uncertainty.

The model is calibrated to closely match four age profiles over the life cycle in the data: unemployment risk, occupational changes, earnings volatility, and cross-sectional dispersion of consumption. According to our model the average risky share is 56.3%, slightly higher than that in the SCF (46.5%). This reasonable value of risky share in our model is achieved under the relative risk aversion of 5, much lower than the typical value required in standard models. More important, the risky share increases, on average, with age: workers at ages 21-25 show an average risky share of 48%, while workers at 41-45 exhibit 59%. Thus, our model partially reconciles the large gap between the data and the standard model. The latter generates not only a steeply *decreasing* age profile but also a very high average risky share. For example, our model without labor-market uncertainty exhibits an average risky share of 83.4%.

One novel feature of our model is a realistic resolution of uncertainty. It is well known that uncertainty is resolved quickly under standard Bayesian learning. For example, almost all of the next period's income uncertainty is resolved within the first couple of years.² In our model, uncertainty is resolved at a much slower rate as workers have to learn about components that constantly move around. This interaction between Bayesian learning and occupational changes is important in accounting for the observed age profile of risky share. In particular, while occupational changes (actual risk) and imperfect information (perceived risk) have a small impact individually, when combined, they can substantially increase labor market uncertainty.

Our theory also predicts that workers in an industry (or occupation) with highly volatile earnings should take less risk in their financial investment. We test this implication using the industry volatility of labor income estimated by Campbell, Cocco, Gomes, and Maenhout (2001). We find that a household whose head is working in an industry where the labor-income volatility is 10% larger than the population average exhibits a risky share 0.7% lower than its population average. This result is consistent with Angerer and Lam (2009), who find a negative correlation between labor-income risk and risky share of workers among workers in the 1979 cohort of the National Longitudinal Survey of Young Men (NLSY).

Our work contributes to the large literature on the life-cycle portfolio choice in several

²Guvenen (2007) shows that an imperfect information model with heterogeneity in income growth can generate significant income risks over the long horizon. However, the uncertainty over the short horizon (1-2 years) is resolved very quickly.

ways. The closest paper to ours is Cocco, Gomes, and Maenhout (2005). We extend their analysis by introducing age-dependent labor-market uncertainty and show that the interaction with labor market risk is important for generating a more realistic age profile of risky share. Another closely related paper is Gomes and Michaelides (2005), who show, among others, that heterogeneity in risk aversion and Epstein-Zin preferences is not enough to account for the age profile of risky share. Chai, Horneff, Maurer, and Mitchell (2011) analyze portfolio choice with endogenous labor supply and uninsurable labor income risk. Wachter and Yogo (2010) account for the positive correlation between wealth and risky share in the data by using non-homothetic utility and decreasing relative risk aversion.

Our paper distinguishes itself from previous studies on the covariance between labor-market risk and stock returns. Benzoni, Collin-Dufresne, and Goldstein (2011) show how labor income and stock-market returns are likely to move together at a longer time horizon. As a result, stocks are riskier for young workers than for old. Storesletten, Telmer, and Yaron (2007) show that if labor income is perfectly correlated with stock returns, the age profile of risky share can exhibit a hump shape. Lynch and Tan (2011) show that the countercyclical volatility of labor-market income growth plays an important role in discouraging the stock-holdings motive for poor and young households. Huggett and Kaplan (2013) decompose human capital into safe and risky components and find that the level of human capital and stock returns have a small positive correlation.

Our results are also related to a growing literature analyzing age-dependent income risk. Karahan and Ozkan (2013) and Guvenen, Karahan, Ozkan, and Song (2015) show that the statistical properties of the income process (persistence and variance) can vary over the life cycle. Compared to these papers, we study the life-cycle income risk associated with both the extensive margin (unemployment risk) and the intensive margin (income process for the employed). In our model, the latter takes place explicitly through age-dependent occupation mobility shocks and implicitly through imperfect information. Guvenen (2007) and Guvenen and Smith (2014) examine the implications of imperfect information for the consumption profile over the life cycle. Consistent with their results, we find that imperfect information coupled with heterogeneous income profiles can match the *linearly* increasing dispersion of consumption along the life cycle. However, we take a step further and show that gradual learning about the income profile can also help to explain the portfolio choice puzzle.

Wang (2009) studies portfolio choice with income heterogeneity and learning within an infinite horizon model. In contrast, we employ a life-cycle model with a particular focus on the relation between risky share and age. Finally, Campanale (2011) develops a life-cycle model in which investors learn about stock-market returns. While uninformed investors can purchase information about the stock market from informed investors, it is impossible to know a priori

the unrealized path of lifetime earnings. Hence, our model makes a more realistic assumption about the investor’s earnings profile.

The paper is organized as follows. In Section 2, based on extensive data from the SCF, we document the stylized facts on household-portfolio profiles. We show that the increasing age profile of risky share is robust to various alternative measures. Section 3 develops a fully specified life-cycle model for our quantitative analysis. We then calibrate the model to match four age profiles over the life cycle: unemployment risk, occupational changes, earnings volatility, and consumption dispersion in the data. In Section 4, we consider various specifications of the model to evaluate the marginal contribution of each component of labor-market uncertainty newly featured. Section 5 tests the prediction of our theory using the cross-industry variation of income risks. Section 6 concludes.

2 Life-Cycle Profile of Households’ Portfolios

2.1 Definition of Risky Share

Based on the SCF for 1998-2007, we document several stylized facts on the life-cycle profile of households’ portfolio. The SCF provides detailed information on the households’ characteristics and their investment decisions. To be consistent with our model (where households face a binary choice between risk-free and risky investment), we classify assets in the SCF into two categories, namely, “safe” and “risky” assets. (The detailed description on how to classify assets into these two categories is presented below.) Several facts emerge:

1. *Participation*: On average, just a little over half (55.3%) of the population participates in investing in risky assets. This participation rate shows a hump shape over the life cycle, with its peak around the average retirement age (see Figure 1 below).
2. *Conditional Risky Share*: Households that participate in risky investment, on average, allocate about half (46.5%) of their financial wealth in risky assets. This conditional risky share increases monotonically over the life cycle.
3. *Unconditional Risky Share*: When participation and conditional risky share are combined, the unconditional risky share exhibits a hump shape over the life cycle.

In the SCF, some assets can be easily classified into one type or the other. For example, checking, savings, and money market accounts are safe investments while direct holding of stocks is risky. However, other assets (e.g., mutual funds and retirement accounts) are invested in a bundle of safe and risky instruments. Fortunately, the SCF provides some information

about how these accounts are invested. The respondents are asked not only how much money they have in each account but also where the money is invested. If the respondent reports that most of the money in the accounts is in bonds, money market, or other safe instruments, we classify them as safe investments. If the respondent reports that the money is invested in some form of stocks, we categorize them as risky investments. If he or she reports that the account involves investments in both safe and risky instruments, we assign half of the money in each category.³

The financial assets considered safe are checking accounts, savings accounts, money market accounts, certificates of deposit, the cash value of life insurance, U.S. government or state bonds, mutual funds invested in tax-free bonds or government-backed bonds, and trusts and annuities invested in bonds and money market accounts. The assets considered risky are stocks, stock brokerage accounts, mortgage-backed bonds, foreign and corporate bonds, mutual funds invested in stock funds, trusts and annuities invested in stocks or real estate, and pension plans that are a thrift, profit-sharing, or stock purchase plan. Also considered as a risky investment is the “share value of businesses owned but not actively managed excluding ownership of publicly traded stocks.” We exclude the share value of *actively managed* businesses from our benchmark definition of risky investments. We also present an alternative measure of risky share in which we include the value of actively managed businesses in the next subsection.

Table 1 shows a snapshot of households’ portfolios in the SCF. It reports the average amount (in 2009 dollars) held and the participation rate (the fraction of households that have a positive amount in that account) in each type of account. We restrict the sample to households that have a positive amount of assets. Nearly every household (99.8%) owns some form of safe assets, while only 55.3% of households invest in risky assets. For example, 87.9% of households hold a checking account and 58.3% hold a savings account, but only 21.2% directly own stocks. About half of households in the sample (51.9%) have some form of debt, such as consumer debt and education loans. However, the average amount is relatively small.⁴ House wealth constitutes 42.7% of total assets and 73.4% of households own house(s). Finally, 11.3% of households actively own business(es).

We define the risky share as the total value of risky financial assets divided by the total amount of financial assets, safe and risky. This definition is consistent with measures of

³The 1998 and 2001 SCF do not provide exact information on how pension plans are invested. In this case, we classify half of the money invested in these accounts as safe assets and the rest as risky assets (because the average risky share is close to 50%). In Appendix C we recalculate the risky share with different split rules between safe and risky assets such as 80-20 or 20-80, for example. The average of risky share is affected by the split rule, but the shape of the age profile is not.

⁴While 11.0% of households have negative net worth, only 2.9% of households have negative net worth and hold some amount of risky assets at the same time.

Table 1: Household Savings by Account

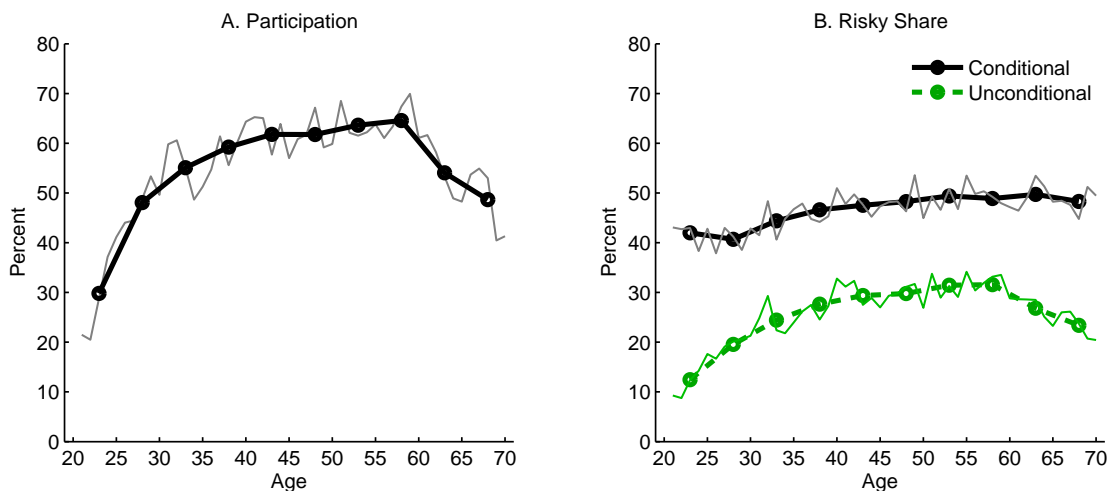
Account	Average Amount (in 2009 \$)	Participation (%)
<u>Total safe assets (S)</u>	106,187	99.8
Checking account	5,182	87.9
Savings account	11,357	58.3
Savings bond (safe)	9,576	19.6
Life insurance	9,509	27.8
Retirement accounts (safe)	26,879	42.5
<u>Total risky assets (R)</u>	135,356	55.3
Stocks	44,374	21.2
Trust (risky)	8,137	1.5
Mutual funds (risky)	21,702	15.1
Retirement accounts (risky)	40,403	45.9
<u>Total financial assets ($R + S$)</u>	241,543	100.0
<u>Debt (D)</u>	5,532	51.9
Consumer debt	2,965	47.5
Education loans	2,566	13.2
<u>Net house wealth ($NH = H - M$)</u>	177,141	73.4
House wealth (H)	250,867	73.4
Mortgages/Lines of credit (M)	73,726	49.2
<u>Total net wealth ($R + S - D + NH$)</u>	413,152	100.0
<u>Actively managed business (B)</u>	90,065	11.3

Note: The sample is restricted to households with a positive amount of financial assets in the Survey of Consumer Finances (1998-2007).

risky share found in numerous studies in the literature (Ameriks and Zeldes (2004), Guiso, Haliassos, and Jappelli (2002), and Gomes and Michaelides (2005), to name just a few). In Section 2.2 we explore alternative measures of risky share that include debt, houses, and own business investment.

Our primary focus is how the risky share changes across different age groups. Figure 1 shows the participation rate, conditional (on participation) risky share, and unconditional risky share over the life cycle. In Appendix B we show that the age profile is robust to cohort

Figure 1: Risky Share over the Life Cycle



Note: Survey of Consumer Finances (1998-2007). The line with circles represents 5-year average. Panel A shows the participation rate (the fraction of households who participate in risky investment). Panel B shows the unconditional and conditional (on participation) risky shares.

or time effects.⁵ The line with circles represents the 5-year average (e.g., 21-25, 26-30, and so on). In Panel A, the participation rate (the fraction of households that participate in risky investment) exhibits a hump shape over the life cycle with its peak just before the average retirement age. It increases from 29.8% in the age group of 21-25 to 55.1% at ages 31-35, reaches its peak of 64.5% at ages 56-60 and then decreases to 54.0% at ages 61-65.

Panel B shows the conditional and unconditional risky shares. The conditional share—the share among the households that participate in risky investment—increases over the life cycle. It increases from 41.9% in the age group 21-25 to 47.5% at ages 41-45, and then to 49.7% at ages 61-65. Since our model abstracts from the participation decision, when we compare the model and the data we will focus on the conditional risky share only. The average conditional risky share is 46.5%. The unconditional risky share (participation rate times conditional risky share) also exhibits a hump shape. It rises from 12.4% in the age group 21-25 to its peak of 31.5% at ages 55-60, and then decreases to 26.8% at ages 61-65. In sum, these life-cycle patterns of risky share clearly suggest that younger investors are reluctant to take financial risks, despite longer investment horizons and higher average rate of returns to risky investment.

⁵Ameriks and Zeldes (2004) use earlier available SCF from 1983-1998. They find that both the unconditional and the conditional risky share weakly increase with age (or exhibit a hump shape) if time effects are controlled for but increase strongly with age if they control for cohort effects.

2.2 Robustness: House, Debt, and Business

In our benchmark definition the risky share is defined as the total value of risky assets divided by the total gross value of financial assets: $\frac{R}{R+S}$ where R and S are risky and safe assets, respectively. We examine whether the increasing age profile of risky share is robust to the inclusion of debt (D), house (H), and actively managed business (B).

According to Table 1, about half of households (51.9%) hold some amount of debt, such as credit card debt or education loans. It is possible that young households have low risky shares relative to their *gross* assets but high risky shares relative to *net* assets. Panel A of Figure 2 compares the risky shares relative to gross assets (our benchmark definition, $\frac{R}{R+S}$) to that relative to net assets ($\frac{R}{R+S-D}$ in the dotted line with squares).

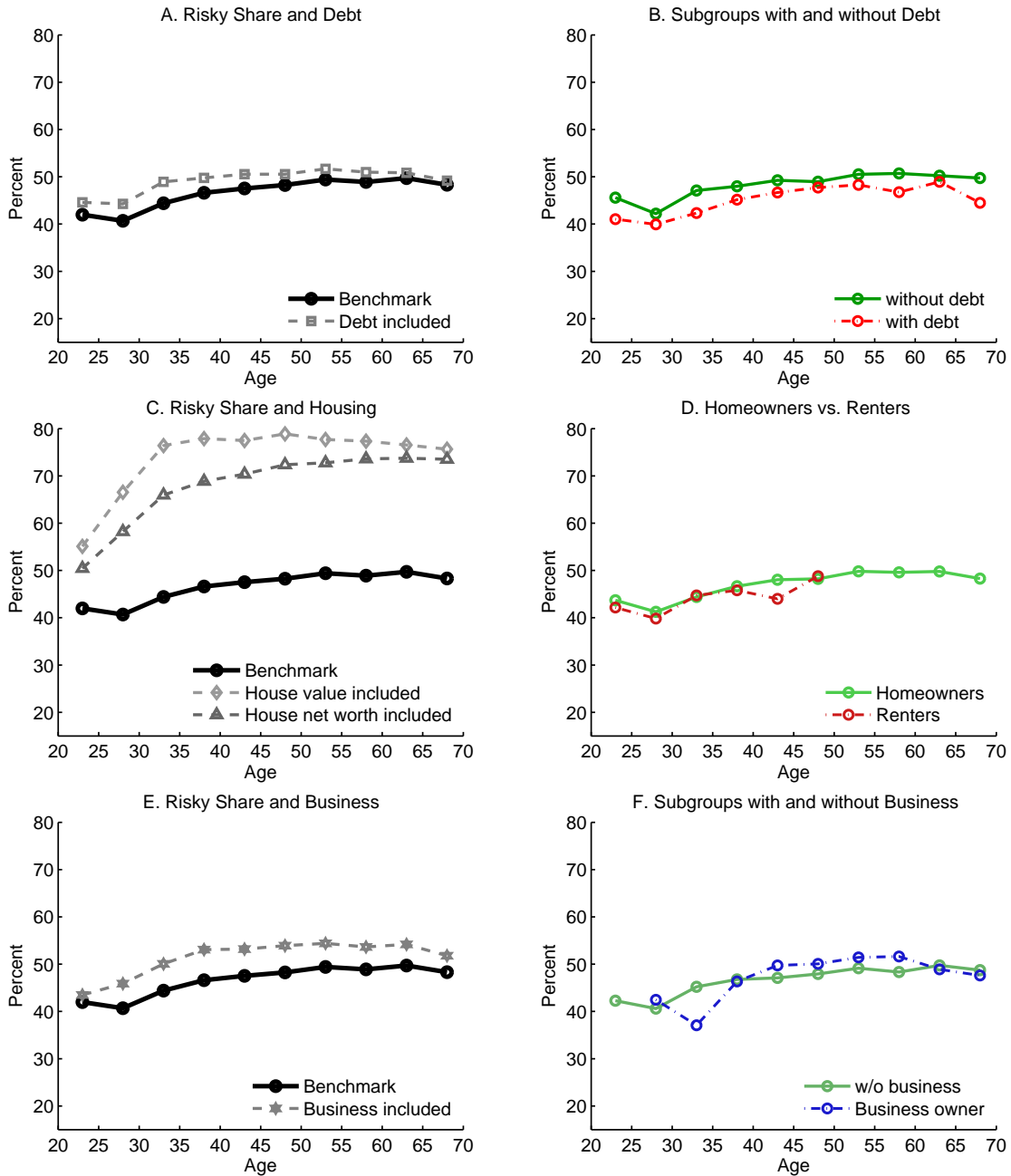
For an average household, consumer debt (\$5,532) is fairly small relative to its total financial assets (\$241,543). Thus, the difference between two measures is small: the average risky share increases from 46.5% to 50.5%. The shape of the age profile is little affected: it is increasing but at a slightly smaller rate. The risky share increases from 45.5% at ages 21-30 to 50.7% at ages 61-65. Panel B compares the risky shares of two subgroups based on our benchmark measure: those with some amount of debt and those without any debt. The age profiles of the two groups look similar.

Our benchmark definition of risky share also abstracts from an important asset of household wealth: houses. According to the SCF, 73.4% of households own a house. For the median household in the wealth distribution, house wealth is 52.4% of its total wealth. It is not obvious how to classify investment in houses. One approach is to include the total house(s) worth (as well as any investment in real estate, such as vacation houses) as part of risky assets: $\frac{R+H}{R+S+H}$. Panel C plots the risky share using this definition (the dotted line with diamonds). While the average risky share increases significantly to 75.7%, it rapidly increases up to age 35 and flattens until age 50 and then starts declining toward retirement.

Another way to treat house(s) is to include only the net worth of house(s) as a part of risky assets ($\frac{R+NH}{R+S+NH}$). The net worth of house(s) is the sum of the house(s) value minus the amount borrowed as well as other lines of credit or loans the household may have (i.e., $NH = H - M$ where H is the house value, and M represents mortgages as well as other lines of credit or loans for the house). Using this definition, the average risky share increases to 69.0% (the dotted line with triangles in Panel C). The risky share monotonically increases over the life cycle, similar to our benchmark definition.

Alternatively, one could view the total value of house(s) as risky assets but include the net value in the total wealth: $\frac{R+H}{R+S+NH}$. This is the definition used by Glover, Heathcote, Krueger, and Rios-Rull (2014). This measure produces a steeply decreasing risky-share profile. The average risky share is 189.0% at ages 21-30 and declines to 95.4% at ages 61-65. However, note

Figure 2: Conditional Risky Share: Alternative Definitions and Subgroups



Note: The left panels (A, C, and E) compare the risky shares under the benchmark definition to alternatives including debt (A), house value and net house value (C), and business worth (C). The right panels (B, D, and F) compare the risky shares across different groups under our benchmark definition: debtors and no-debt holders (B), renters and homeowners (D), and households that actively manage a business and that don't (F).

that this definition treats the house in an asymmetric way: total house value in the numerator and net house value in the denominator. According to this definition, the risky share decreases over the life cycle in a somewhat mechanical way. Most households buy a house at a relatively young age and pay their mortgage down over time. This leads to a rapidly decreasing risk share. In contrast, according to our first two measures which treat house(s) in a symmetric way, the risky share may increase or decrease over the life cycle depending on how fast (net) house wealth rises relative to total assets. We have shown that both our measures generate an increasing risky share over the life cycle.

There are also reasons to believe that homeownership may affect the risky share of financial assets. Based on a popular view, young households do not invest much in the stock market because their wealth is tied down to an illiquid asset, their house. Moreover, as noted by Cocco (2007), house price risk may crowd out stock holdings. Panel D of Figure 2 plots the risky shares (using our benchmark definition) of homeowners and renters, separately. In contrast to conventional wisdom, the two groups exhibit a remarkably similar age profile. The average conditional risky share for renters (43.3%) is slightly lower than that of homeowners (47.7%). These figures suggest that homeownership may not be a main reason why young households do not take more risk (than old) in financial investments.

Finally, our benchmark risky share does not reflect investment in households' own business. Panel E shows the risky share when the net value of actively managed businesses (B) is a part of risky assets: $\frac{R+B}{R+S+B}$. The net value of the business is the value of the business minus any amount the business owes plus any amount owed to the household by the business. With the value of actively managed business, the average risky share increases to 50.6% (from 46.5% according to our benchmark measure). However, the increasing pattern of the risky-share profile is unaffected. It increases from 42.6% at ages 21-25 to 52.7% at ages 61-65. Panel F compares the risky shares (using our benchmark measure) between households that do and do not actively run a business. While the average risky share is higher for business owners (48.0% vs. 46.6% for those who do not actively own a business), the increasing pattern of the age profile is similar for both groups.

3 Life-Cycle Model

3.1 Economic Environment

To quantitatively assess the link between labor-market uncertainty and portfolio choice, we develop a fully specified life-cycle model. We also provide a simple 3-period model in Appendix D to illustrate the effect of labor market uncertainty on risky share.

Demographics The economy is populated by a continuum of workers with total measure of one. A worker enters the labor market at age $j = 1$, retires at age j_R , and lives until age J . There is no population growth.

Preferences Each worker maximizes the time-separable discounted lifetime utility:

$$U = E \sum_{j=1}^J \delta^{j-1} \frac{c_j^{1-\gamma}}{1-\gamma} \quad (1)$$

where δ is the discount factor, c_j is consumption in period j , and γ is the relative risk aversion.⁶ For simplicity, we abstract from the labor effort choice and assume that labor supply is exogenous when employed.

Income Profile We assume that the log earnings of a worker i with age j , Y_j^i , are:

$$Y_j^i = z_j + y_j^i \quad \text{with} \quad y_j^i = a_j^i + \beta_j^i \times j + x_j^i + \varepsilon_j^i. \quad (2)$$

Log earnings consist of common (z_j) and individual-specific (y_j^i) components. The common component, z_j , represents the average age-earnings profile, which is assumed to be the same across workers and thus observable. The individual-specific component, y_j^i , consists of the income profile, $a_j^i + \beta_j^i \times j$, and stochastic shocks, $x_j^i + \varepsilon_j^i$. The income profile is characterized by the intercept, a_j^i , and the growth rate, β_j^i . Upon a worker's entering the labor market in period 1, these income profile parameters are drawn from the normal distribution: $a_1^i \sim N(0, \sigma_a^2)$ and $\beta_1^i \sim N(0, \sigma_\beta^2)$. If the worker stays in the same occupation, these parameters remain the same. However, with probability λ_j —which varies with age—workers change occupations (or jobs). Upon occupational change, each component of the income profile varies according to an AR(1) process:

$$a_j^i = \rho^a a_{j-1}^i + \nu_j^{ai}, \quad \text{with} \quad \nu_j^{ai} \sim \text{i.i.d. } N(0, \sigma_{av}^2) \quad (3)$$

$$\beta_j^i = \rho^\beta \beta_{j-1}^i + \nu_j^{\beta i}, \quad \text{with} \quad \nu_j^{\beta i} \sim \text{i.i.d. } N(0, \sigma_{\beta\nu}^2) \quad (4)$$

The persistence parameter reflects the fact that workers inherit some earnings prospect from previous occupations (or jobs).

⁶Alternative preferences have also been proposed to address the portfolio choice puzzles. For example, Gomes and Michaelides (2005) use Epstein-Zin preferences with heterogeneity in both risk aversion and intertemporal elasticity of substitution. Wachter and Yogo (2010) use non-homothetic preferences. We adopt the standard preferences with constant relative risk aversion in order to highlight the role of labor-market uncertainty.

Workers also face idiosyncratic earnings shocks each period. These idiosyncratic shocks consist of the persistent (x_j^i) and purely transitory (ε_j^i) components. The persistent component follows an AR(1) process:

$$x_j^i = \rho x_{j-1}^i + \nu_j^i, \quad \text{with } \nu_j^i \sim \text{i.i.d. } N(0, \sigma_\nu^2) \quad (5)$$

where the transition probability is represented by a common finite-state Markov chain $\Gamma(x_j|x_{j-1})$. The transitory component follows an i.i.d. process: $\varepsilon_j^i \sim N(0, \sigma_\varepsilon^2)$, where the probability distribution of ε is denoted by $f(\varepsilon)$. In the calibration below, we ascribe the wage changes due to occupational switch to shocks to (a, β) and those within the occupation to shocks to (x, ε) . The stochastic movement in the income profile due to occupational switch is important for our model. Under imperfect information about the earnings profile (which is described below), the occupational (or job) change makes inference about the true parameters, a , β , and x harder. This helps us to generate a more realistic speed of Bayesian learning and consequently much larger uncertainty for young workers.

Unemployment Risk Each period, workers face age-dependent unemployment risk. With probability p_j^u , a worker becomes unemployed. We also assume that an unemployed worker switches occupations (when employed in the next period) with probability κ .

Savings Financial markets are incomplete in two senses. First, workers cannot borrow. Second, there are only two types of assets for savings: a risk-free bond b (paying a gross return of R in consumption units) and a stock s (paying $R_s = R + \mu + \eta$) where $\mu (> 0)$ represents the risk premium and η is the stochastic rate of return.⁷ Workers save for insuring themselves against labor-market uncertainty (precautionary savings) as well as for retirement (life-cycle savings).

Social Security The government runs a balanced-budget pay-as-you-go social security system. When a worker retires from the labor market at age j_R , he receives a social security benefit amount, ss , which is financed by taxing workers' labor incomes at rate τ_{ss} .⁸

Bayesian Learning In our benchmark model, workers do not have perfect knowledge about their income profile. While the individual-specific component of earnings, y , is observed, workers cannot perfectly distinguish each component (a , β , x , and ε). We assume that workers form their priors and update them in a Bayesian fashion. Given the normality assumption,

⁷For simplicity, we abstract from the general equilibrium aspect by assuming exogenous average rates of return to both stocks and bonds.

⁸Ball (2008) analyzes financial investments for different levels of the social security benefit. He finds that the generosity of the social security system has little impact on portfolio choice.

a worker's prior belief about the income profile is summarized by the mean and variance of intercept, $\{\mu_a, \sigma_a^2\}$, and those of slope, $\{\mu_\beta, \sigma_\beta^2\}$. Similarly, a worker's prior belief about the persistent component of the income shock is summarized by $\{\mu_x, \sigma_x^2\}$. When the prior beliefs over the covariances are denoted by σ_{ax} , $\sigma_{a\beta}$, and $\sigma_{\beta x}$, we can express the prior mean and variance matrices as:

$$\mathbf{M}_{j|j-1} = \begin{bmatrix} \mu_a \\ \mu_\beta \\ \mu_x \end{bmatrix}_{j|j-1} \quad \mathbf{V}_{j|j-1} = \begin{bmatrix} \sigma_a^2 & \sigma_{a\beta} & \sigma_{ax} \\ \sigma_{a\beta} & \sigma_\beta^2 & \sigma_{\beta x} \\ \sigma_{ax} & \sigma_{\beta x} & \sigma_x^2 \end{bmatrix}_{j|j-1} \quad (6)$$

where the subscript $j|j-1$ denotes information at age j before the actual earnings y_j is realized. The subscript $j|j$ denotes the information after earnings y_j is realized, i.e., posterior. The posterior means and variances at age j are given by:

$$\mathbf{M}_{j|j} = \mathbf{M}_{j|j-1} + \begin{bmatrix} \frac{\sigma_a^2 + \sigma_{a\beta} + \sigma_{ax}}{\sigma_a^2 + \sigma_\beta^2 j^2 + \sigma_x^2 + \sigma_\varepsilon^2 + \Gamma} \\ \frac{\sigma_{a\beta} + \sigma_\beta^2 j + \sigma_{\beta x}}{\sigma_a^2 + \sigma_\beta^2 j^2 + \sigma_x^2 + \sigma_\varepsilon^2 + \Gamma} \\ \frac{\sigma_{ax} + \sigma_{x\beta} j + \sigma_x^2}{\sigma_a^2 + \sigma_\beta^2 j^2 + \sigma_x^2 + \sigma_\varepsilon^2 + \Gamma} \end{bmatrix} (y_j - \mathbf{H}'_j \mathbf{M}_{j|j-1}) \quad (7)$$

$$\mathbf{V}_{j|j} = \mathbf{V}_{j|j-1} - \begin{bmatrix} \frac{\sigma_a^2 + \sigma_{a\beta} + \sigma_{ax}}{\sigma_a^2 + \sigma_\beta^2 j^2 + \sigma_x^2 + \sigma_\varepsilon^2 + \Gamma} \\ \frac{\sigma_{a\beta} + \sigma_\beta^2 j + \sigma_{\beta x}}{\sigma_a^2 + \sigma_\beta^2 j^2 + \sigma_x^2 + \sigma_\varepsilon^2 + \Gamma} \\ \frac{\sigma_{ax} + \sigma_{x\beta} j + \sigma_x^2}{\sigma_a^2 + \sigma_\beta^2 j^2 + \sigma_x^2 + \sigma_\varepsilon^2 + \Gamma} \end{bmatrix} \mathbf{H}'_j \mathbf{V}_{j|j-1} \quad (8)$$

where $\mathbf{H}_j = [1 \quad j \quad 1]'$ is a (3×1) vector and $\Gamma = 2\sigma_{a\beta}j + 2\sigma_{ax} + 2\sigma_{\beta x}j$.

After the posterior is formed, the worker forms a belief about his next period's income. For the worker who does not change his occupation, the belief (prior) about the next period's income is written by the conditional distribution function:

$$F(y_{j+1}|y_j) = N(\mathbf{H}'_{j+1} \mathbf{M}_{j+1|j}, \mathbf{H}'_{j+1} \mathbf{V}_{j+1|j} \mathbf{H}_{j+1} + \sigma_{\varepsilon_j}^2) \quad (9)$$

where

$$\mathbf{M}_{j+1|j} = \mathbf{R} \left[\mathbf{M}_{j|j-1} + \begin{bmatrix} \frac{\sigma_a^2 + \sigma_{a\beta} + \sigma_{ax}}{\sigma_a^2 + \sigma_\beta^2 j^2 + \sigma_x^2 + \sigma_\varepsilon^2 + \Gamma} \\ \frac{\sigma_{a\beta} + \sigma_\beta^2 j + \sigma_{\beta x}}{\sigma_a^2 + \sigma_\beta^2 j^2 + \sigma_x^2 + \sigma_\varepsilon^2 + \Gamma} \\ \frac{\sigma_{ax} + \sigma_{x\beta} j + \sigma_x^2}{\sigma_a^2 + \sigma_\beta^2 j^2 + \sigma_x^2 + \sigma_\varepsilon^2 + \Gamma} \end{bmatrix} (y_j - \mathbf{H}'_j \mathbf{M}_{j|j-1}) \right] \quad (10)$$

$$\mathbf{V}_{j+1|j} = \mathbf{R} \mathbf{V}_{j|j} \mathbf{R}' + \mathbf{Q} \quad (11)$$

with \mathbf{R} denoting a (3×3) matrix whose diagonal elements are $(1, 1, \rho)$ and \mathbf{Q} denoting a

(3 × 3) matrix whose diagonal element is (0, 0, σ_ν^2).

For the worker who changes his occupation next period, the belief about his next period's income is summarized by the following conditional distribution function:

$$F^0(y_{j+1}|y_j) = N(\mathbf{H}'_{j+1}\mathbf{M}^0_{j+1|j}, \mathbf{H}'_{j+1}\mathbf{V}^0_{j+1|j}\mathbf{H}_{j+1} + \sigma_{\varepsilon_j}^2) \quad (12)$$

where

$$\mathbf{M}^0_{j+1|j} = \mathbf{R}^0 \left[\mathbf{M}_{j|j-1} + \begin{bmatrix} \frac{\sigma_a^2 + \sigma_{a\beta} + \sigma_{ax}}{\sigma_a^2 + \sigma_\beta^2 j^2 + \sigma_x^2 + \sigma_\varepsilon^2 + \Gamma} \\ \frac{\sigma_{a\beta} + \sigma_\beta^2 j + \sigma_{\beta x}}{\sigma_a^2 + \sigma_\beta^2 j^2 + \sigma_x^2 + \sigma_\varepsilon^2 + \Gamma} \\ \frac{\sigma_{ax} + \sigma_x \beta j + \sigma_x^2}{\sigma_a^2 + \sigma_\beta^2 j^2 + \sigma_x^2 + \sigma_\varepsilon^2 + \Gamma} \end{bmatrix} (y_j - \mathbf{H}'_j \mathbf{M}_{j|j-1}) \right] \quad (13)$$

$$\mathbf{V}^0_{j+1|j} = \mathbf{R}^0 \mathbf{V}^0_{j|j} \mathbf{R}^0 + \mathbf{Q}^0. \quad (14)$$

In this case, \mathbf{R}^0 is a (3 × 3) matrix whose diagonal elements are (ρ^a, ρ^β, ρ) and \mathbf{Q}^0 is a (3 × 3) matrix with diagonal element of ($\sigma_{a\nu}^2, \sigma_{\beta\nu}^2, \sigma_\nu^2$).

Value Functions Let $k = \{e, u\}$ denote the employment status of a worker: employed or unemployed. It is convenient to collapse financial wealth into one variable, “cash in hand,” $W = bR + sR_s$. Then, the state variables include workers' wealth (W), the individual-specific component of labor income (y), the prior mean ($\mathbf{M}_{j|j-1}$), and the prior variance ($\mathbf{V}_{j|j-1}$).

One novel feature of our model is that we keep track of the prior variance ($\mathbf{V}_{j|j-1}$) as a state variable. A history of occupational changes will lead to different perceptions about one's future income. In a model without occupational change, age (j) is a sufficient statistic for the prior variance (e.g., Guvenen (2007) and Guvenen and Smith (2014)).

Now, the value function of a worker at age j is:

$$\begin{aligned} V_j^e(W, y, \mathbf{M}_{j|j-1}, \mathbf{V}_{j|j-1}) = \max_{c^k, s', b'} & \left\{ \frac{c_j^{1-\gamma}}{1-\gamma} + \delta p_j^u (1-\kappa) \int_{\eta'} V_{j+1}^u(W', y' = 0, \mathbf{M}_{j+1|j}, \mathbf{V}_{j+1|j}) d\pi(\eta') \right. \\ & + \delta p_j^u \kappa \int_{\eta'} V_{j+1}^u(W', y' = 0, \mathbf{M}_{j+1|j}^0, \mathbf{V}_{j+1|j}^0) d\pi(\eta') \\ & + \delta(1-p_j^u)(1-\lambda_j) \int_{\eta'} \int_{y'} V_{j+1}^e(W', y', \mathbf{M}_{j+1|j}, \mathbf{V}_{j+1|j}) dF_j(y'|y) d\pi(\eta') \\ & \left. + \delta(1-p_j^u)\lambda_j \int_{\eta'} \int_{y'} V_{j+1}^e(W', y', \mathbf{M}_{j+1|j}^0, \mathbf{V}_{j+1|j}^0) dF_j^0(y'|y) d\pi(\eta') \right\} \quad (15) \end{aligned}$$

$$\text{s.t.} \quad c^k + s' + b' = (1 - \tau_{ss}) \exp^{Y_j} \times \mathbf{1}\{k = e\} + ss \times \mathbf{1}\{j \geq j_R\} + W \quad (16)$$

where $\mathbf{1}\{\cdot\}$ is an indicator function, and income is $Y_j = z_j + y_j$.

Each period with probability p_j^u a worker becomes unemployed ($k = u$). Workers who remain employed draw the next period's income y' according to $F_j(y'|y)$, if they do not change occupations (with probability $1 - \lambda_j$). Those who do change occupations (with probability λ_j) draw the next period's income from $F_j^0(y'|y)$. With probability κ , an unemployed worker also changes occupations when he is employed next period.

Perfect Information Model (PIM) In order to evaluate the marginal contribution of each component of labor-market uncertainty, we consider various specifications differing with respect to assumptions about (i) unemployment risk, (ii) occupational change, and (iii) imperfect information about the income profile. The first alternative specification we consider is the standard life-cycle model without any of these three features. This specification is very similar to Cocco, Gomes, and Maenhout (2005). We will refer to this specification as the perfect information model (PIM). In this case, the value function of a j -year-old worker with an income profile of $\{a, \beta\}$ is:

$$V_j^{\{a, \beta\}}(W, x, \varepsilon) = \max_{c, s', b'} \left\{ u(c) + \delta \int_{\eta', x', \varepsilon'} V_{j+1}^{\{a, \beta\}}(W', x', \varepsilon') df(\varepsilon') d\Gamma(x'|x) d\pi(\eta') \right\} \quad (17)$$

$$\text{s.t.} \quad c + s' + b' = (1 - \tau_{ss}) \exp^{Y_j} + ss \times 1\{j \geq j_R\} + W.$$

The second alternative specification we consider is the standard model with age-dependent unemployment risk only, which is referred to as “PIM + U.” Finally, we consider the standard model with unemployment risk and occupational change (“PIM + U + O”).⁹

3.2 Calibration

The model is calibrated to closely match four age profiles over the life cycle in the data: unemployment risk, occupational changes, earnings volatility, and the cross-sectional dispersion of consumption.

There are six sets of parameters: (i) life-cycle parameters $\{j_R, J\}$, (ii) preferences $\{\gamma, \delta\}$, (iii) asset returns $\{R, \mu, \sigma_\eta^2\}$, (iv) labor-income process $\{z_j, \rho, \rho_a, \rho_\beta, \sigma_a^2, \sigma_\beta^2, \sigma_\nu^2, \sigma_{a\nu}^2, \sigma_{\beta\nu}^2, \sigma_\varepsilon^2\}$, (v) unemployment risk and occupational changes $\{p_j^u, \lambda_j, \kappa\}$, and (vi) the social security system $\{\tau_{ss}, ss\}$. Table 2 reports all parameter values for the benchmark case.

Life Cycle, Preferences, and Social Security The model period is one year. Workers are born and enter the labor market at $j = 1$ and live for 60 periods, $J = 60$. This life cycle corresponds

⁹The value function of these alternative specifications can be written by extending Equation (17) to contain unemployment risk p_j^u and occupational change λ_j , similar to Equation (15).

to ages 21-80. Workers retire at $j_R = 45$ (age 65) when they start receiving the social security benefit, ss . The social security tax rate $\tau_{ss} = 13\%$ is chosen to target the replacement ratio of 40% for a worker with average productivity. The relative risk aversion, γ , is set to 5. Note that this value is much lower than those typically adopted to match the average risky share in the literature. As shown below, our benchmark model is able to generate the average risky share of about 56%, close to that in the data, with this value of risk aversion. The discount factor, $\delta = 0.92$, is calibrated to match the capital-to-income ratio of 3.2, the value commonly targeted in the literature.¹⁰

Asset Returns The gross rate of return to the risk-free bond $R = 1.02$ is based on the average real rate of return to 3-month US Treasury bills for the post-war period. Following Gomes and Michaelides (2005), we set the equity premium, μ , to 4%. The standard deviation of the innovations to the rate of return to stocks, σ_η , is 18%, also based on Gomes and Michaelides (2005).¹¹ We assume that the stock returns are orthogonal to labor-income risks.¹²

Unemployment Risk Based on the CPS for 1976-2013, Choi, Janiak, and Villena-Roldan (2011) estimate the transition rates from employment to unemployment over the life cycle. Panel A of Figure 3—reproduced based on their estimates—clearly shows that the probability of becoming unemployed decreases with age. For example, a 21-year-old worker faces a 3.5% chance of becoming unemployed, whereas a 64-year-old worker faces a much smaller risk, less than 1%. We use these estimates for the age-dependent unemployment risk, p_j^u .

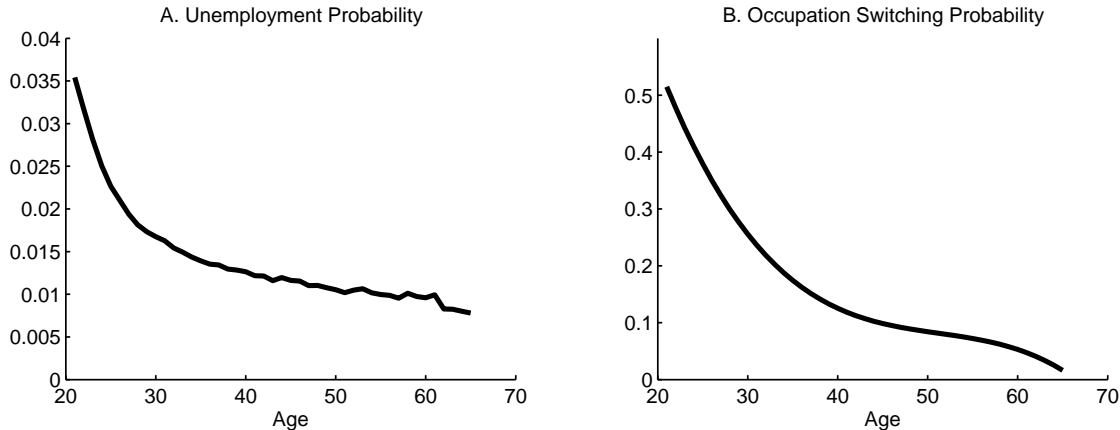
Occupational Changes According to Topel and Ward (1992), the average number of jobs held by workers within the first 10 years of entering the labor market is 7. Kambourov and Manovskii (2008) estimate that the average probability that workers ages 23-28 switch occupations (at the 3-digit occupation-code level) is 39% for workers without college education and 33% for those with some college education. For workers ages 47-61, these numbers significantly decline to 7% and 9%, respectively. Panel B of Figure 3 plots the age-dependent probability of switching occupations, λ_j , based on their estimates. It is important to emphasize that occupational switch provides an additional source of uncertainty in the labor market, which is

¹⁰In the perfect information model (PIM) we set $\delta = 1.01$. In this case, the model requires a large discount factor to match the capital-to-income ratio observed in the data because (i) the precautionary savings motive against labor-market uncertainty is small and (ii) an increasing profile of earnings induces workers to borrow heavily early in life.

¹¹Jagannathan and Kocherlakota (1996) report that for the period between 1926 and 1990, the standard deviation of annual real returns in the S&P stock price index was 21% as opposed to 4.4% in T-bills.

¹²The empirical evidence on the correlation between labor-income risk and stock market returns is mixed. While Davis and Willen (2000) find a positive correlation, Campbell, Cocco, Gomes, and Maenhout (2001) find a positive correlation only for specific population groups.

Figure 3: Unemployment Risk and Occupational Mobility over the Life Cycle



Note: Panel A plots the age profile of the probability of becoming unemployed from Choi, Janiak, and Villena-Roldan (2011). Panel B plots the probability of switching occupation by age from Kambourov and Manovskii (2008).

reflected in the variance-covariance matrix $\mathbf{V}_{j+1|j}^0$ in Equation (12). This interaction between occupational change and Bayesian learning distinguishes our model from those of Guvenen (2007) and Guvenen and Smith (2014).

Labor-Income Process The deterministic age-earnings profile, which is common across workers, z_j , is taken from Hansen (1993). For the stochastic process of idiosyncratic productivity shock (x, ε) , we use the estimates of Guvenen and Smith (2014), according to which $\rho = 0.756$ and $\sigma_v^2 = 5.15\%$ for the persistent component (x) and $\sigma_\varepsilon^2 = 1\%$ for the purely transitory component (ε) .

Regarding the income profile (a, β) , we follow Guvenen and Smith’s (2013) strategy which uses consumption dispersion to infer the uncertainty that workers face under imperfect information. The initial variance of the intercept in the income profile, σ_a^2 is chosen to match the cross-sectional consumption variance at age 27. The initial variance of the slope of the profile, σ_β^2 , is chosen to match the cross-sectional variance of log consumption at age 57. Thus, our model almost exactly reproduces the observed increasing age profile of the consumption variance as reported by Heathcote, Storesletten, and Violante (2014). (See Figure 6 below.)

A worker switches his occupation with probability λ_j . Upon occupational change, the income profile may change as well. We assume that this occurs according to an AR(1) process. We estimate this stochastic process for the profile shift, $\{\rho_a, \rho_\beta, \sigma_{a\nu}^2, \sigma_{\beta\nu}^2\}$, based on the individual wage data from the PSID 1970-2005.¹³ First, we run the regression of log hourly

¹³Following the convention in the literature, we restrict the data sample to not-self-employed male workers between the ages 21-60 who work more than 250 hours annually and earn more than half the minimum wage

Table 2: Benchmark Parameters

Parameter	Notation	Value	Target / Source
Life Cycle	J	60	–
Retirement Age	j_R	45	–
Risk Aversion	γ	5	–
Discount Factor	δ	0.92	Capital to Income Ratio
Risk-free Rate	R	1.02	Gomes and Michaelides (2005)
Equity-Risk Premium	μ	0.04	Gomes and Michaelides (2005)
Stock-Return Volatility	σ_η	0.18	Gomes and Michaelides (2005)
Social Security Benefit	ss	0.40	Replacement Ratio
Social Security Tax	τ_{ss}	0.13	Balanced Social Security Budget
Persistence of a	ρ_a	0.50	PSID
Variance of innovation to a (intercept)	$\sigma_{a\nu}^2$	3.5%	PSID
Persistence of β (slope)	ρ_β	0.17	PSID
Variance of innovation to β	$\sigma_{\beta\nu}^2$	0.006%	PSID
Population Variance of a	σ_a^2	16%	Consumption Variance for Age 27
Population Variance of β	σ_β^2	0.012%	Consumption Variance for Age 57
Persistence of x	ρ	0.756	Guvenen and Smith (2014)
Variance of innovation to x	σ_ν^2	5.15%	Guvenen and Smith (2014)
Variance of i.i.d. component ϵ	σ_ϵ^2	1.0%	Guvenen and Smith (2014)
Common Age-Earnings Profile	$\{z_j\}_{j=21}^{65}$	–	Hansen (1993)
Unemployment Risk	$\{p_j^u\}_{j=21}^{65}$	Figure 3	Choi, Janiak, and Villena-Roldan (2011)
Prob of Occupational Change	$\{\lambda_j\}_{j=21}^{65}$	Figure 3	Kambourov and Manovskii (2008)
Prob of Occupational Change (Unemp.)	κ	0.51	PSID

wages ($\ln w_{it}$) on 3-digit occupation dummies (OCC_s), time dummies (D_t), as well as age and age squared:

$$\ln(w)_{it} = b_0 + b_1 age_{it} + b_2 age_{it}^2 + \sum_{s=1}^S b_s^o \times OCC_s + \sum_{t=1970}^{2005} b_t \times D_t + e_{it}$$

The occupation dummies capture the average wage in each occupation (occupation-specific ability). The estimated occupation-specific ability is assigned to each worker in the corresponding occupation as a measure of a_i . We estimate an AR(1) process of changes in a_i , Equation (3), using the sample of workers who switch occupations between time t and $t + 1$. This yields our estimates of an AR(1) process of a upon occupational change: $\rho_a = 0.5$ and $\sigma_{av}^2 = 3.5\%$. For the growth component (β_i), we first calculate the growth rate in the hourly wage for each occupation between ages 25 and 55. We then calculate the occupation-specific slope coefficient using the average growth rates of each occupation. As in the case of the intercept, we assign the occupation-specific slope component to each worker in the corresponding occupation. Equation (4) is estimated using the sample of workers who switch occupations between time t and $t + 1$. This yields our estimates for β_{it} : $\rho_\beta = 0.17$ and $\sigma_{\beta v}^2 = 0.006\%$.

Finally, according to the PSID, 51% of unemployed workers (being unemployed for longer than 3 months during the year) who find a job in the following year reported that they changed occupations. This gives us $\kappa = 0.51$.¹⁴

Initial Priors We assume that workers do not have any prior knowledge regarding their income profile upon entering the labor market. Thus, we set their initial prior variances to those of unconditional population variances. While we view this assumption as a useful benchmark, we also consider the case where workers have some information about their income profile as in Guvenen (2007) and Guvenen and Smith (2014). We find that our main results are robust to this assumption.

4 Results

4.1 Policy Functions

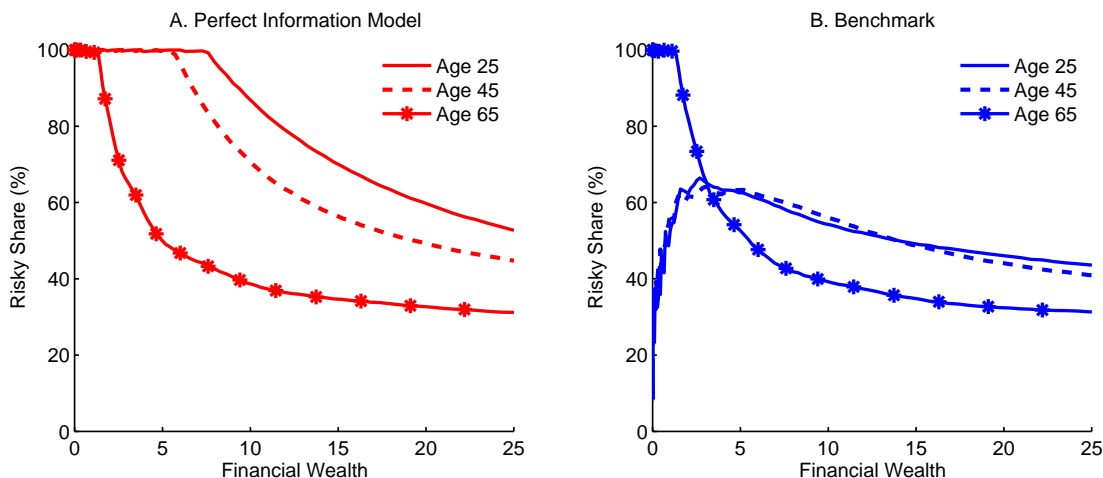
In order to understand the basic economic mechanism of the model, we first illustrate the portfolio decision in the model without any age-dependent labor-market uncertainty (such as unemployment risk, occupational changes and imperfect information). We call this specifica-

for the given year. We calculate the hourly wage by dividing annual labor earnings by annual working hours.

¹⁴If we use 1 month as a threshold for being unemployed, this value is 47%. With 6 months, this value is 54%.

tion perfect information model (PIM).¹⁵ All other parameter values in the PIM remain the same except for the discount factor, which is adjusted to match the capital-to-income ratio. Thus, the PIM still contains the idiosyncratic productivity shocks (which we calibrated to the standard values in the literature).

Figure 4: Optimal Portfolio Choice for a Worker with Median Income



Panel A of Figure 4 shows the optimal portfolio choice (i.e., policy function) of a worker with the median income for three age groups: 25, 45, and 65 in the PIM. The horizontal axis represents the wealth, from 0 to 25, where the average wealth is about 6 in our model. Without any age-dependent uncertainty in the labor market, the risky share falls with age—opposite to what we see in the SCF—as young workers face much longer investment horizons to take advantage of a high equity premium. For example, a 25-year-old worker with median labor income and average wealth would like to allocate almost all financial wealth to risky assets. The risky share decreases with wealth for all three age groups. Despite the presence of idiosyncratic productivity risk, workers can predict the future labor-market outcome fairly well in the PIM model. Thus, having a future labor-income stream is similar to holding a low-risk asset. A worker with little wealth allocates almost all his savings to risky investments. This is because “safe” labor income makes up a large portion of his total wealth, which is the sum of financial wealth and the present value of lifetime labor income (i.e., the value of human capital). But, for wealthier workers, “safe” labor income is a small portion of total wealth. Hence, wealthier investors exhibit a low risky share in terms of their financial wealth.

However, in our benchmark model (Panel B) young workers face much larger uncertainty in the labor market, discouraging them from taking further risk in the financial market. A

¹⁵In Appendix D we illustrate how the risky share varies with wealth and age using a simple 3-period model.

25-year-old with average wealth (about 6 in the model) shows a risky share of 61% in the benchmark as opposed to that of 100% in the PIM. A 45-year-old with average wealth is also somewhat conservative: his risky share is 62%, while it is 96% in the PIM. A 65-year-old worker who retires next period exhibits a portfolio choice almost identical to that in the PIM because the labor-market uncertainty is irrelevant.

Unlike the PIM, the risky share is not monotonic in wealth in the benchmark. This is because workers face two conflicting incentives for taking risk in financial investments. On the one hand, they would like to hedge against the large labor-market uncertainty. On the other hand, they would like to build up wealth quickly by taking advantage of the equity premium (life-cycle savings motive). For both 25- and 45-year-old workers, the risky share increases with wealth when the wealth level is close to 0, indicating that the life-cycle savings motive dominates the desire to hedge against labor-market uncertainty for wealth-poor workers. The risky share starts declining around 3, which is one-half of the average wealth in our model.

4.2 Comparison to Survey of Consumer Finances

Table 3 presents the average risky share and the slope of the age profile from the data (SCF), the benchmark model, and the PIM.¹⁶ Our benchmark model generates a risky share of 56.3% close to the 46.5% in the data. This is generated with a relative risk aversion of 5, much lower than values typically assumed in the literature. In the PIM, which is similar to the standard life-cycle model without age-dependent labor market uncertainty, this ratio is 83.4%. If the PIM were to match the average risky share of 46.5%, it would require a value of relative risk aversion above 15 under the same parameterization of the income process. Even in this case, however, the PIM fails to generate an increasing profile of risky share over the life cycle.

We next turn our attention to the age profile. Financial advisors often recommend that young investors, facing a longer investment horizon, take more risk in financial investments. However, our data based on the SCF show a pattern opposite to this advice: the risky share on average increases by 0.12 percentage point each year between ages 21 and 65 (Table 3). In our benchmark model, on average, the risky share increases by 0.36 percentage point. Young workers, faced with large uncertainty in the labor market, would not want to take too much risk in the financial market. As the labor-market uncertainty gradually resolves over time—through (i) decreased unemployment risk, (ii) decreased probability of occupational switch, and (iii) learning about one’s true earnings ability, they can afford taking more risk in financial investments. By stark contrast, the PIM (which does not have any of these features) generates a risky-share profile that steeply *decreases* by 1.22 percentage points each year between ages

¹⁶The model statistics are based on the simulated panel of 10,000 households.

21 and 65. This is because younger workers expect a long stream of (relatively safe) labor income so they can afford to take more financial risk.

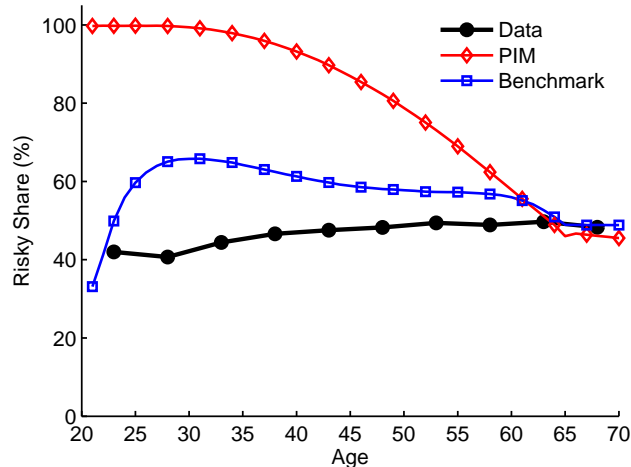
Table 3: Risky Shares: Data vs. Models

Statistic	Data (%)	Benchmark	PIM
Average	46.5	56.3	83.4
ages 21-25	41.9	47.9	99.7
ages 41-45	47.5	59.7	89.6
ages 61-65	49.7	52.3	51.0
Slope of age profile (in percentage points)	0.12	0.36	-1.22

Note: The slope of the age profile refers to the average increase of the risky share (in percentage points) over the life cycle (from age 21 to 65). PIM refers to the perfect information model.

Figure 5 plots the risky shares of the PIM and the benchmark over the life cycle. In the PIM, the risky share starts with 99.7% at age 21, gradually decreases to 86.8% at age 45, and declines sharply to 46.0% at age 65. In our benchmark model, however, the age profile of the risky share is not monotonic. It starts with a low level of 33.1% at age 21, increases to 58.8% at age 45, and decreases gradually to 48.9% at age 65.

Figure 5: Risky Share over the Life Cycle: Data vs. Model



Note: Data are based on the Survey of Consumer Finances. PIM refers to the perfect information model.

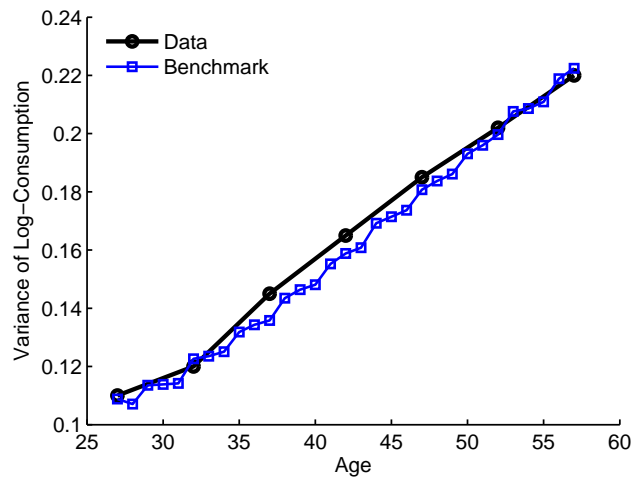
This is because a young worker faces two conflicting incentives to take risks in making investments. On the one hand, he would like to hedge against the large labor-market uncer-

tainty. On the other hand, he would like to build up his savings (life-cycle savings motive) quickly by taking advantage of the risk premium. When the worker enters the labor market, the former effect dominates, suppressing the risky share, but gradually the latter (life-cycle savings) effect comes in, generating a non-monotonic shape. Overall, our model is able to track the age profile of the risky share in the SCF. We view this as a partial resolution in reconciling the tension between the data and theory on the households' portfolio choice over the life cycle.

4.3 Dispersion of Consumption by Age

It is well known that the cross-sectional dispersion of consumption increases over the life cycle. For example, Heathcote, Storesletten, and Violante (2014) find that the variance of log consumption increases from 0.10 at age 25 to 0.20 at age 55. We chose the parameters for the heterogeneous income profile (dispersion of a and β) to match these values. As Guvenen (2007) points out, a gradual learning about income profile can generate a *linearly* increasing dispersion in consumption: a household's consumption depends on its permanent income, which is gradually revealed over time. Figure 6 shows that the age profile of the cross-sectional variance of log consumption in our model closely tracks that reported in the literature, confirming that our heterogeneous income profile and learning are well specified.

Figure 6: Cross-Sectional Variance of Log Consumption by Age



Note: The age profile of the data is based on the estimate in Heathcote, Storesletten, and Violante (2014).

4.4 Speed of Learning: Short- vs. Long-Run Uncertainty

One novel feature of our model is a realistic speed of learning. Guvenen (2007) shows that an imperfect information model with heterogeneous income profiles can generate significant income risks over long horizons. However, the uncertainty over the short horizon (e.g., 1-2 years) is resolved very fast under Bayesian learning. For example, as shown below, within a couple of years after entering the labor market, almost 90% of one-period income uncertainty is resolved. We find this rate of learning unrealistic. We argue that not only the long-run but also the short-run risk is particularly important for the portfolio choice because portfolio decisions can take place at frequent time intervals. By introducing occupational switch—which is associated with potential shifts in the income profile—the uncertainty is resolved at a more realistic slower rate. We show that this interaction between learning and job changes is particularly important for generating a realistic age profile of risky share.

To distinguish between short-run and long-run income risks, we compute the forecast error variance or mean squared error (MSE)—also used in Guvenen (2007) and Guvenen and Smith (2014)—at various horizons. The forecast error variance is defined as:

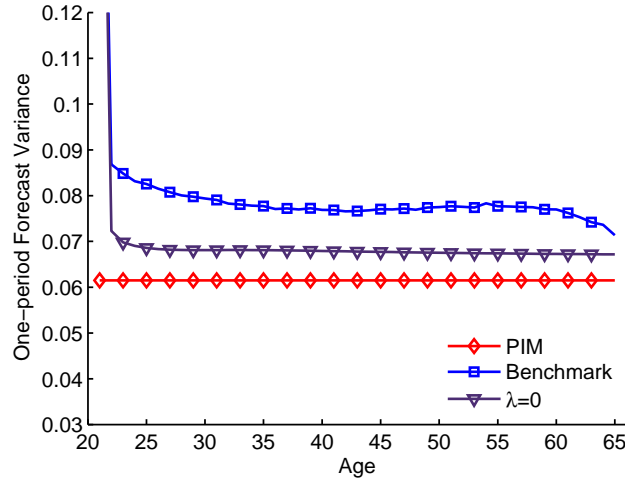
$$\text{MSE}_{j+s|j} = \mathbf{H}'_{j+s} \mathbf{V}_{j+s|j} \mathbf{H}_{j+s} + \sigma_{\varepsilon_j}^2 \quad \text{with} \quad \mathbf{V}_{j+s|j} = \mathbf{R}^s \mathbf{V}_{j|j} \mathbf{R}'^s + \sum_{i=0}^{s-1} \mathbf{R}^i \mathbf{Q} \mathbf{R}'^i.$$

The speed of learning is measured by how fast MSE converges to that under perfect information.

Figure 7 shows the *one-period* forecast error variance of income, $\text{MSE}_{j+1|j}$, for three model specifications: the PIM (plotted with diamonds), benchmark (squares) and the benchmark without occupation changes, $\lambda = 0$ (triangles). In the PIM, MSEs reflect the uncertainty due to stochastic income shocks only (x and ε). Thus, it is not age dependent by construction. When there is no occupational switch ($\lambda = 0$), the MSE converges to that of the PIM within almost a year. That is, the short-run uncertainty related to the income profile is quickly resolved right after the worker enter the labor market. Considering the number of job turnovers and the time it takes for workers to settle into a long-term career (e.g., Topel and Ward (1992)), this speed of learning seems too fast. However, in our benchmark model, since young workers face a high probability of occupational change, the short-run uncertainty is resolved gradually: the MSE is significantly larger than that of the PIM and is resolved at a much slower rate.

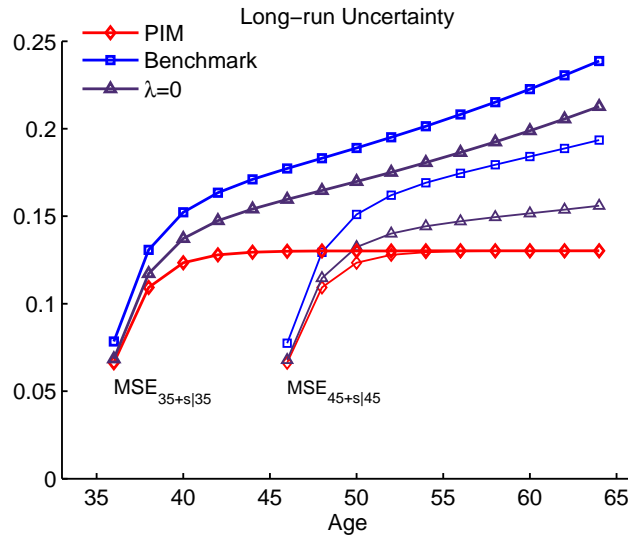
We have just shown that without occupational change the income uncertainty over a short horizon is resolved very quickly. This is not true for the uncertainty over longer time horizons. Figure 8 shows the MSE over various horizons for workers ages 35 and 45, for example. In both benchmark models with and without an occupational switch, uncertainty about the slope of the income profile, β_j , translates into a substantial amount of risk over longer horizons, as was

Figure 7: Short-Run Uncertainty: One-Period Forecast Error Variance of Income



Note: We plot average one-year forecast-error variance of income, $MSE_{j+1|j}$ where “ $\lambda = 0$ ” represents the benchmark model without occupational switches.

Figure 8: Forecast Error Variance of Income over Various Horizons



Note: We plot $MSE_{j+s|j}$ for two age groups $j = 35$ and 45 for various horizons s for the PIM, benchmark, and the benchmark model without occupational changes ($\lambda = 0$).

emphasized by Guvenen (2007).¹⁷ This distinction between short- and long-run uncertainty is

¹⁷In the case of an occupational switch, priors about the variance evolve based on $\mathbf{V}_{j+1|j}^0 = \mathbf{R}^0 \mathbf{V}_{j|j}^0 \mathbf{R}^{t0} + \mathbf{Q}^0$ where \mathbf{R}^0 is a (3×3) matrix whose diagonal elements are $(\rho^a, \rho^\beta, \rho)$ and \mathbf{Q}^0 is a (3×3) shock matrix with diagonal elements $[\sigma_{a\nu}^2, \sigma_{\beta\nu}^2, \sigma_\nu^2]$. While innovations $\sigma_{\beta\nu}^2$ add noise to the system, the relatively small persistence $\rho^\beta = 0.17$ decreases the prior uncertainty. Over long time horizons the latter effect is stronger, resulting in a smaller variance—for this specific case—compared to the one with $\lambda_j = 0$.

subtle but important for the portfolio choice. The lifetime uncertainty about earnings ability is important for *total* savings, which is well illustrated by Guvenen (2007). However, for the portfolio choice, labor-market uncertainty over the short horizon is also important because workers are able to adjust their financial portfolios frequently (e.g., every year in our model).

4.5 Decomposing the Contribution of Three Types of Uncertainty

We have introduced three types of labor-market uncertainty into the standard life-cycle model: (i) age-dependent unemployment risk, ii) age-dependent occupational mobility, and (iii) imperfect information about earnings ability. We decompose the contribution of each component by considering various specifications of the model economy.

The first model specification we consider is the PIM. The second model is the PIM with age-dependent unemployment risk only, referred to as “PIM+U.” The comparison of this model with the PIM will isolate the contribution of age-dependent unemployment risk. The third model is the PIM with age-dependent unemployment risk and age-dependent probability of occupational switch, referred to as “PIM+U+O.” The comparison of this model with “PIM+U” will isolate the marginal role of occupational switch. This specification is also equivalent to the benchmark model without imperfect information about true earnings ability. Thus, the comparison of this specification with the benchmark will provide a marginal contribution of imperfect information. Table 4 summarizes the labor-market uncertainty of these 4 specifications. For each specification, we recalibrate the discount factor to match the capital-to-income ratio of 3.2 and keep all other parameters the same.

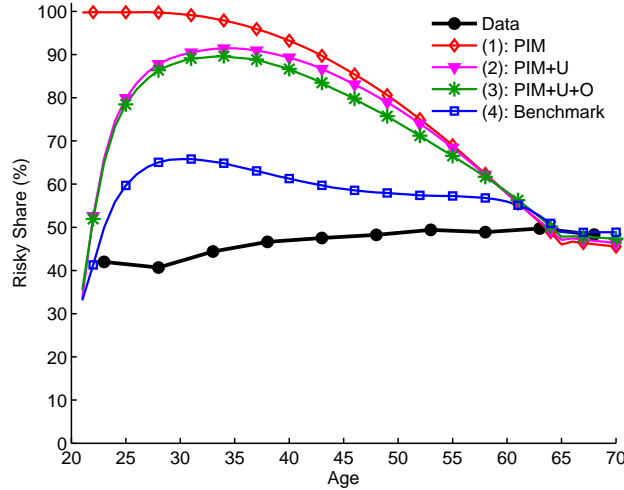
Table 4: Labor-Market Uncertainty Across Models

	(1) PIM	(2) PIM+U	(3) PIM+U+O	(4) Benchmark
Unemployment Risk	No	Yes	Yes	Yes
Occupational Switch	No	No	Yes	Yes
Imperfect Information	No	No	No	Yes

Figure 9 shows the age profile of the risky share for all 4 model specifications along with that from the data. Adding the age-dependent unemployment risk to the PIM decreases the average risky share from 83.4% to 75.8%. Figure 9 shows that the impact of unemployment risk on risky share is most important for young workers (line with “ ∇ ”). For example, a 25-year-old worker who faces a 3% unemployment risk decreases the risky share from 99.8% to 79.9%. The impact of unemployment risk on the portfolio choice becomes negligible after

age 40 when the annual unemployment risk becomes close to 1%.

Figure 9: Risky Share Profiles from Models



Note: “Benchmark” features all three types of labor-market uncertainty: unemployment risk, occupational change and imperfect information about the income profile. “PIM+U” refers to the PIM with unemployment risk. “PIM+U+O” refers to the PIM with unemployment risk and occupational switch.

Introducing the probability of occupation switch (thus moving from PIM+U to PIM+U+O) by itself has little impact on the risky-share profile. It slightly decreases the average risky share to 74.1%. This is because any additional risk of occupational switch is completely resolved once a worker observes his new income profile in the new occupation. However, as we introduce imperfect information into the model, which becomes our benchmark, the average risky share decreases to 56.3%. Overall, in accounting for the total decrease in average risky share from 83.4% (PIM) to 56.3% (Benchmark), (i) age-dependent unemployment risk has contributed 25%, (ii) occupational mobility contributed 8%, and imperfect information the most, 69%. We would like to note, however, that imperfect information alone is not sufficient to decrease the risky share by this magnitude. As we have shown in Figure 7, absent the probability of occupational switch, the uncertainty about the income profile is resolved quickly. In fact, the benchmark model without occupational switch—i.e., the $\lambda = 0$ case we have shown above—generates an average risky share of 69%, just 6 percentage points lower than that of PIM+U (75%). Hence, only when coupled with occupational switch does imperfect information substantially decrease the risky share.

4.6 Sensitivity Analysis

We perform various sensitivity analyses to see whether our main results are robust with respect to different parameterizations. In particular, we are concerned with the robustness in 6 dimensions. First, we examine the case where workers have some private information about their ability upon entering the labor market. Second, we consider two alternative values of relative risk aversion: $\gamma = 3$ and $\gamma = 4$. Third, we see how the initial distribution of wealth (the wealth distribution of 21-year-old workers) affects the results. Fourth, we consider the model with a smaller dispersion in the intercept of earnings profiles, $\sigma_a^2 = 0.08$, the value used in Guvenen and Smith (2014) for a direct comparison to their results. Finally, we examine the case where workers draw a completely new income profile (a, β) from the unconditional population distribution upon occupational change. We view this as an upper bound case for the role of imperfect information and slow learning. In each sensitivity analysis, we keep all other parameters of the model the same as those in our benchmark specification. Table 5 reports the results of these sensitivity analyses.

Table 5: Risky Shares: Sensitivity Analysis

Model	Average Risky Share (%)	Slope of Profile (pp)
Data	46.5	0.12
Benchmark	56.3	0.36
$\psi_a = 0.80, \psi_\beta = 0.80$	56.5	0.31
$\gamma = 4$	78.7	-0.26
$\gamma = 3$	94.7	-0.42
Initial Assets = $0.1 \times \bar{W}$	59.1	0.07
$\sigma_a^2 = 0.08$	59.3	0.58
Priors Fully Reset	51.2	0.53

Note: The benchmark features $\psi_a = 0, \psi_\beta = 0, \gamma = 5, \sigma_a^2 = 0.16$, zero initial assets, and priors evolve according to AR(1) process in Equation (13).

In our benchmark model we assumed that workers are not fully informed about their initial earnings ability upon entering the labor market and their prior variances start with the population variance of the unconditional distribution of a and β . This might be too extreme given that workers might have some private information about themselves. Indeed, Guvenen (2007) and Guvenen and Smith (2014) find that workers know a significant fraction of their

lifetime income. In our model, the amount of prior knowledge is given by the matrix:

$$\mathbf{V}_{1|0} = \begin{bmatrix} (1 - \psi_a)\sigma_a^2 & \sigma_{a\beta} & \sigma_{ax} \\ \sigma_{a\beta} & (1 - \psi_\beta)\sigma_\beta^2 & \sigma_{\beta x} \\ \sigma_{ax} & \sigma_{\beta x} & \sigma_x^2 \end{bmatrix}$$

where the benchmark corresponds to $\psi_a = 0$ and $\psi_\beta = 0$. Following Guvenen (2007) and Guvenen and Smith (2014), we set: $\{\psi_a = 0.80, \psi_\beta = 0.80\}$.¹⁸ It turns out that the amount of information upon labor market entry has little impact on our results (Table 5). Even if young workers completely know their initial income profiles, they may face new uncertainty once they change occupations and draw a new (unobserved) profile. Hence, the initial amount of uncertainty makes a difference in a model with constant (a, β) but not in our benchmark, where the income profile may change upon occupational switch.

The relative risk aversion in our benchmark model is 5. We consider somewhat smaller values of relative risk aversion: $\gamma = 4$ and $\gamma = 3$. As we lower the value of γ , the risky share significantly increases to 78.7% and 94.7%, respectively. The increasing pattern of the age profile is also affected, while the risky share is increasing at ages 21-24 only. On average, the risky share decreases by 0.26 and 0.42 percentage point when $\gamma = 4$ and $\gamma = 3$, respectively.

Young workers enter the labor market with zero assets in our benchmark. While most workers enter the labor market with little wealth or debt, many can borrow or rely on family financing. The ability to borrow should affect financial decisions toward risk. To reflect this, we consider the case where workers enter the labor market with a small amount of wealth—10% of the economy-wide average wealth. This has a small impact on the result. Since they have some wealth, the average risky share slightly increases to 59.1% and the risky share is increasing very mildly with age by 0.07 percentage point on average over the life cycle.

In the benchmark, we chose the initial dispersion of ability, $\sigma_a^2 = 0.16$, to match the cross-sectional variance of log consumption of 27-year-old workers in the data (from Guvenen (2007)). We now consider the case with a smaller initial ability dispersion: $\sigma_a^2 = 0.08$, the value used in Guvenen and Smith (2014). The average risky share increases slightly to 59.3% and the risky share increases at a faster rate, by 0.58 percentage point per year.

Our final sensitivity analysis concerns how the priors are formed upon an occupational switch. In our benchmark model, the income profile follows an AR(1) process and worker's perception reflect this actual shift of income profile. Thus, the prior also follows AR(1) and is reflected in the variance-covariance matrix $\mathbf{V}_{j+1|j}^0$ in Equation (12). Sometimes, a job change

¹⁸Guvenen (2007) and Guvenen and Smith (2014) examine prior uncertainty with respect to σ_β^2 . Since in our parameterization σ_a^2 is set to a larger value, we also experiment with the prior uncertainty regarding this parameter.

across very different industries or occupations may generate considerable new uncertainty. Now, consider a somewhat extreme case where upon occupational change workers “incorrectly” believe that they would draw completely new values of (a_{j+1}, β_{j+1}) from the unconditional distribution, independently of their current (a_j, β_j) . Thus, the (subjective) priors about the next period’s income profile are fully reset upon occupational change. We call this specification as the “full reset” model. This model sets the diagonal elements of \mathbf{R}^0 and \mathbf{Q}^0 to $(0, 0, \rho)$ and $(\sigma_a^2, \sigma_\beta^2, \sigma_\nu)$, respectively, in the prior updating rule in Equation (13). This specification can be considered an upper bound for the uncertainty created by the occupational change. The average risky share decreases to 51.2%, almost the same as that in the data. Moreover, the age profile tracks that in the data very closely as the risky share increases by 0.53 percentage point per year on average.

4.7 Risky Share and Wealth

While the primary focus of our model is to account for the age profile of the risky share of households, Guiso, Haliassos, and Jappelli (2002) highlight one more stylized fact that is hard to reconcile with standard models: the correlation between wealth and risky share. The risky share is disproportionately larger for richer households in the data. Wachter and Yogo (2010) address this puzzle using a non-homothetic utility with a decreasing relative risk aversion. In this section, we show that our model can also help us to partially close the gap between the theory and the data by generating a moderately positive correlation between risky share and wealth.

Table 6: Risky Share by Wealth

Wealth Quintile	Data (%)	PIM	Benchmark
1 st	35.9	88.4	41.5
2 nd	40.5	99.0	63.2
3 rd	44.4	94.5	65.3
4 th	51.7	77.6	59.4
5 th	66.6	52.3	46.8
Average	46.5	83.4	56.3

Table 6 reports the average (conditional) risky share across 5 quintile groups in the distribution of household wealth in the SCF. The risky share clearly shows a strong positive correlation with household wealth: rich households take much more risk in financial investments. The conditional risky share increases from 35.9% in the 1st quintile to 44.4% in the

3rd, and 66.6% in the 5th. The participation rate (not reported in the table) monotonically increases with wealth. For example, in the 5th quintile of the wealth distribution, almost everyone (97.5%) participates in risky investment. We report these statistics for the PIM and the benchmark. In the PIM, the risky share *decreases* from 88.4% in the first quintile to 52.3% in the 5th, which is completely opposite to that in the data. According to the benchmark the risky share increases with wealth, although it is not monotonic: it is 41.5% in the first quintile, increases to 65.3% in the third quintile, and then decreases to 46.8% in the fifth.

5 Industry Income Volatility and Risky Share

Our theory predicts that workers in jobs (e.g., industries or occupations) with highly volatile earnings should be conservative in financial investments. Testing this implication is not simple because workers also self-select into industries across which income volatilities are systematically different (e.g., agriculture vs. education). Despite this limitation, we examine the partial correlation between the risky share and industry-specific income risk (measured by the average volatility of individual income shocks).

For the industry-specific labor-income risk, we use the estimate by Campbell, Cocco, Gomes, and Maenhout (2001), which is based on the PSID.¹⁹ According to these estimates, workers in agriculture face the largest uncertainty in income with an average variance of income shock of 31.7%, whereas those in public administration face the smallest variance, 4.7%. Across industries, the variances of income shocks are high in construction (10.8%) and business services (11.8%); moderate in wholesale and retail trade (8.9%) and transportation and finance (9%); and small in communication (6.7%) and manufacturing (5.2%).

We run the regression of households' risky shares on the industry-specific income risk, the industry of the household's main job, and other individual characteristics, such as total income, age, education, number of children, and marital status. Since we are using the conditional risky share, we include households with a positive amount of risky investment only. Table 7 reports the estimated coefficients and their standard errors from this regression. The statistically significant (at 5%) negative coefficient on labor-income risk confirms that larger labor-market risk crowds out financial risk. As the risk in the labor market (the variance of the labor-income shock) increases by 1 percentage point, the household's risky share decreases by 0.077 percentage point (with a standard error of 0.035). This is consistent with Angerer and Lam

¹⁹The income specification used by Campbell, Cocco, Gomes, and Maenhout (2001) is $\log(Y_{it}) = f(t, Z_{i,t}) + \nu_{i,t} + \varepsilon_{i,t}$ where $f(t, Z_{i,t})$ is a deterministic function of age and other characteristics, $\nu_{i,t}$ represents a permanent shock that evolves based on $\nu_{i,t} = \nu_{i,t-1} + u_{i,t}$, with $u_{i,t} \sim N(0, \sigma_u^2)$ while $\varepsilon_{i,t}$ is a temporary shock with $\varepsilon_{i,t} \sim N(0, \sigma_\varepsilon^2)$. The variances reported here are the sum of the estimated variances for σ_u^2 and σ_ε^2 for every industry.

Table 7: Regression of Risky Share on Income Risk of Industry

Dependent Variable = Household's Risky Share		
Constant	0.0291***	(0.0113)
Industry income risk	-0.0769**	(0.0347)
Log Income	0.0303***	(0.0009)
Age	0.0015***	(0.0001)
Education	0.0313***	(0.0035)
Number of children	-0.0044***	(0.0009)
Marriage dummy	-0.0066**	(0.0011)

Notes: The numbers in parentheses are standard errors. Industry income risk measures are based on Campbell, Cocco, Gomes, and Maenhout (2001).

(2009), who find a negative correlation between labor-income risks and the share of risky assets from the NLSY 1979 cohort. The other coefficients are consistent with our economic priors. Workers with more education (a proxy for permanent income) and total income take more risks in making financial investments. So do older workers.

6 Conclusion

Despite a longer investment horizon, the average young household maintains a conservative financial portfolio, not aggressively taking advantage of high rates of return from risky investment; old households invest more aggressively, showing a much higher risky share in their financial portfolios. We argue that the increasing age profile of risky share has to do with labor-market uncertainty over the life cycle. It is well known that young workers face larger uncertainty in the labor market—high unemployment risks, frequent job turnovers, unknown future career, and so forth. Young workers—faced with much larger uncertainty in the labor market—are not willing to take too much risk in financial investments. As the labor-market uncertainty is gradually resolved over time, they can afford to take more risks in the financial market.

To assess the quantitative importance of the link between labor-market risk and financial investment, we introduce three types of age-dependent labor-market uncertainty into an otherwise standard life-cycle model of household portfolio choices: unemployment risk, occupational changes, and gradual learning about the true income profile. When the model is calibrated to match the life-cycle patterns of income volatility, unemployment risk, occupational changes, and consumption dispersion in the data, the model is able to generate the

age profile of risky share that is consistent with what we found from the Survey of Consumer Finances.

According to our benchmark specification, the average risky share is 56%, close to that in the data (47%). This low risky share is obtained under the relative risk aversion of 5, which is a much smaller value than that required in the standard model. According to the SCF, the risky share increases on average by 0.12 percentage point each year between ages 21 and 65. Our model also generates an increasing age profile of the risky share: it increases by 0.36 percentage point, on average. The standard life-cycle model without age-dependent labor-market uncertainty generates a counterfactual rapidly decreasing age profile of risky share.

Our theory also predicts that workers in an industry with highly volatile earnings should take less risk in their financial portfolios. Based on the regression of the household's risky share on the industry's labor-income risk, we find that a household working in an industry where income volatility is 10% larger than the mean exhibits a risky share that is 0.7% lower than the average.

We argue that a complete theory of households' portfolio choice should consider the risk not only in financial investments but also elsewhere, especially in the labor market.

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Appendix

A Data: Survey of Consumer Finances

General Description In our data analysis we use available surveys from the Survey of Consumer Finances (SCF) for the periods 1998-2007. The SCF is a cross-sectional survey conducted every 3 years. It provides detailed information on the finances of US families. Respondents are selected randomly, with a strong attempt to select families from all economic strata. The “primary economic unit” consists of an economically dominant single individual or couple (married or living as partners) in a household and all other individuals who are financially dependent on that individual or couple. In a household with a mixed-sex couple the “head” is taken to be the male. One set of the survey cases was selected from a multistage area-probability design and provides good coverage of characteristics broadly distributed in the population. The other set of survey cases was selected based on tax data. This second sample was designed to disproportionately select families that were likely to be relatively wealthy. Weights compensate for the unequal probabilities of selection. To deal with respondents who were unable to provide a precise answer the survey gives the option of providing a range. In the surveys, variables that contained missing values have been imputed five times drawing repeatedly from an estimate of the conditional distribution of the data. Multiple imputation offers a couple of advantages over singly-imputed data. Using all surveys we are left with a total of 88,415 observations.

Example of Survey We provide an example of the questionnaire related to checking accounts. The following questions are being asked, among others. 1) Do you have any checking accounts at any type of institution? 2) How many checking accounts do you have? 3) How much is in this account? (What was the average over the last month.) For some other accounts like individual retirement accounts, the respondent is asked specifically how the money is invested. The questions are: 1) Do you have any individual retirement accounts? 2) How much in total is in your IRA(s)? 3) How is the money in this IRA invested? Is most of it in certificates of deposit or other bank accounts, most of it in stocks, most of it in bonds or similar assets or what? The possible answers are 1) CDs/ Bank accounts; money market, 2) Stock; Mutual funds, 3) Bonds/ Similar assets; T-Bills; Treasury notes, 4) Combinations of 1, 2, 3, 5) Combinations of 2, 3, 6) Combinations of 1, 2, 7) Universal life policy or other similar insurance products, 8) Annuity, 9) Commodities, 10) Real estate/mortgages, 11) Limited partnership/Other similar investments, 12) Brokerage accounts, 13) Split/Other.

Construction of Variables In this section we explain the type of assets we categorize as safe and risky. Most SCF surveys code variables under the same name, with few exceptions. We will describe variables based on 1998 and note any changes with respect to the other years: 2001, 2004, 2007. In all our definitions, we make use of weights, variable *X42001*.

— Checking accounts, Money Market Accounts: The variables *X3506*, *X3510*, *X3514*, *X3518*, *X3522*, *X3526* report the amount of money the respondent has in six different accounts. The respondent is

asked whether each of these accounts is a checking account or a money market account. Responses can be found in variables X3507, X3511, X3515, X3519, X3523, X3527. We define **Checking Accounts** (and respectively **Money Market Accounts**) as the sum of these accounts.

— Savings accounts: We define the sum of variables X3804, X3807, X3810, X3813, X3816 as **Savings Accounts**.

— Certificates of Deposit: The variable X3721 gives the amount of money in certificates of deposit. We define **Certificates of Deposit** as equal to this variable as long as the account does not belong to someone unrelated to the household (variable $X7620 < 4$).

— Saving bonds: We define as **Savings Bonds(safe)** the sum of variables X3902 (money saved in U.S. government savings bonds), variable X3908 (face value of government bonds) and variable X3910 (money in state and municipal bonds). We define as **Savings Bonds(risky)** the sum of variables X3906 (face value of Mortgage-backed bonds), variable X7934 (face value of Corporate bonds) and variable X7633 (face value of Foreign bonds).

— Life Insurance: Variable X4006 gives the cash value of life insurance policies while variable X4010 the amount currently borrowed using these policies. We define as **Life Insurance** the amount given by X4006-X4010.

— Credit card debt: Variables X413, X421, X424, X427, X430, X7575 gives the amounts owed on credit card loans. We define **Credit Card Debt** as the sum of these variables.

— Miscellaneous assets and debts: This category gives the amount of money the respondent is owed by friends, relatives or others, money in gold or jewelry and others. Variable X4018 gives the total amount owed and X4022, X4026, X4030 the dollar value in these types of assets. Variable X4032 is the amount owed by the respondent. We define **Miscellaneous Assets** as $X4018 + X4022 + X4026 + X4030 - X4032$.

— Other Consumer Loans: Variables X2723, X2740, X2823, X2840, X2923, X2940 give the amount still owed on loans like medical bills, furniture, recreational equipment or business loans. Using variables X6842-X6847 we make sure these loans are not part of business loans and we define the variable **Other Consumer Loans** equal to $X2723 + X2740 + X2823 + X2840 + X2923 + X2940$.

— Education Loans: Variables X7824, X7847, X7870, X7924, X7947, and X7970 give the amount still owed on education loans. We define the variable **Education Loans** equal to the sum of these variables.

— Debt: We define variable **Debt** as equal to the sum of Credit card debt, other consumer loans, and education loans.

— Brokerage Accounts: Variable X3930 gives the amount the total dollar value of all the cash or call money accounts, and the variable X3932 the current balance of margin loans at a stock brokerage. We define **Brokerage Accounts** equal to X3039-X3932.

— Mutual Funds: Variable X3822 gives the total market value of all the Stock Funds, variable X3824 the total market value of all of the Tax-free Bond Funds, variable X3826 the total market value of all Government-Backed Bonds, variable X3828 the total market value of Other Bond Funds, and variable X3830 the total market value of all of the Combination funds or any other mutual funds of the respon-

dent. We define as **Mutual Funds(safe)** the sum of variables $X3824+X3826+X3828+0.5\times X3830$ and as **Mutual Funds(risky)** the sum of variables $X3822+0.5\times X3830$.

— Publicly Traded Stocks: Variable $X3915$ gives the total market value of stocks owned by the respondent, and variable $X7641$ the market value of stocks of companies outside the U.S. We define **Stocks** as equal to $X3915+X7641$.

— Annuities: Variable $X6820$ gives the total dollar value of annuities. Variable $X6826$ reports how the money is invested. We define **Annuities(safe)** equal to $X6820$ if $X6826=2$ (Bonds/interest; CDS/Money Market) and equal to $0.5\times X6820$ if $X6826=5$ (Split between Stocks/Interest; Combination of Stocks, Mutual Fund, CD). We define **Annuities(risky)** equal to $X6820$ if $X6826=1$ or $=3$ (Stocks; Mutual Funds or Real Estate) and equal to $0.5\times X6820$ if $X6826=5$.

— Trust: Variable $X6835$ gives the total dollar value of assets in a trust. Variable $X6841$ reports how the money is invested. We define **Trust(safe)** equal to $X6835$ if $X6841 = 2$ (Bonds/interest; CDS/Money Market) and equal to $0.5\times X6835$ if $X6841=5$ (Split between Stocks/Interest; Combination of Stocks, Mutual Fund, CDS). We define **Trust(risky)** equal to $X6835$ if $X6841=1$ or $=3$ (Stocks; Mutual Funds or Real Estate) and equal to $0.5\times X6835$ if $X6841=5$.

— Individual Retirement Accounts: Variables $X3610, X3620, X3630$ report how much money in total is in individual retirement accounts. Variable $X3631$ reports how the money is invested. We define the variable **IRA(safe)** equal to $X3610 + X3620 + X3630$ if $X3631 = 1$ (money market) or $X3631 = 3$ (Bonds/ Similar Assets; T-Bills) or $X3631=11$ (Universal life policy). **IRA(safe)** equals $\frac{2}{3}(X3610 + X3620 + X3630)$ if $X3631=4$ (combination of money market-stock mutual funds-bonds and T-bills), equal to $\frac{1}{2}(X3610 + X3620 + X3630)$ if $X3631=5$ (combination of stock mutual funds-bonds and T-bills), and equal to $\frac{1}{2}(X3610 + X3620 + X3630)$ if $X3631=6$ (combination of money market-stock mutual funds) or $X3631=7$ (split). Similarly we define the variable **IRA(risky)** equal to $X3610 + X3620 + X3630$ if $X3631 = 2$ (stocks) or $X3631 = 14$ (Real Estate/Mortgages) or $X3631 = 15$ (Limited Partnership) or $X3631 = 16$ (Brokerage account). **IRA(risky)** equals $\frac{1}{3}(X3610 + X3620 + X3630)$ if $X3631 = 4$ (combination of money market-stock mutual funds-bonds and T-bills), equal to $\frac{1}{2}(X3610 + X3620 + X3630)$ if $X3631 = 5$ (combination of stock mutual funds-bonds and T-bills), and equal to $\frac{1}{2}(X3610 + X3620 + X3630)$ if $X3631 = 6$ (combination of money market-stock mutual funds) or $X3631 = -7$ (split).

— Pensions: The variables $X4226, X4326, X4426, X4826, X4926, X5026$ give the total amount of money at present in pension accounts. We subtract any possible loans against these accounts by using the variables $X4229, X4328, X4428, X4828, X4928, X5028$. Variables $X4216, X4316, X4416, X4816, X4916, X5016$ provide information on how the money is invested. We define **Pensions(risky)** if any of the latter variables equal 3 (Profit-Sharing Plan) or 4 (Stock purchase plan). Other than these two options the SCF does not provide many details regarding pension plans. For example, respondents can report that the money is invested in a 401K without further information on how the money is invested. In this case, we split the money half in **Pensions(safe)** and the other half in **Pensions(risky)**. As mentioned in the text, we experiment with other split rules and show our findings in Table C-1 of Appendix C.

— Business: Variables $X3129, X3229, X3329$ report the net worth of business, variables $X3124,$

X3224, X3324 and X3126, X3226, X3326 the amount owed to the business and the amount owed by the business, respectively. Finally, variable X3335 gives the share value of any remaining businesses. We define **Actively Managed Business** as equal to $X3129 + X3229 + X3329 + X3124 + X3224 + X3324 - X3126 - X3226 - X3326 + X3335$. Similarly we define **Non-Actively Managed Business** as the sum of $X3408 + X3412 + X3416 + X3420 + X3424 + X3428$.

— Housing: Variable X513, X526 gives the value of the land the respondent (partially) owns, variable X604 the value of the site, and variable X614 the value of the mobile home, the respondent owns. Variable X623 is the total value of home and site if he owns both. Variable X716 is the value of home/apartment/property that the respondent owns (partially). Variables X1706, X1806, X1906 give the total value of property such as vacation houses or investment in real estate. We define **Value of the Home** as the sum of the above variables. Variables X805, X905, X1005, X1044 and X1715, X1815, X1915 are the amounts of money owed on loans associated with these properties. Finally, variables X1108, X1119, X1130, X1136 are other lines of credit. We define the variable **Mortgages** as equal to the sum of these variables.

— **Safe Assets** = Checking Accounts + Money Market Accounts + Savings Accounts + Certificates of Deposit + Savings Bonds(safe) + Life Insurance + Miscellaneous Assets + Mutual Funds(safe) + Annuities(safe) + Trust(safe) + IRA(safe) + Pensions(safe)

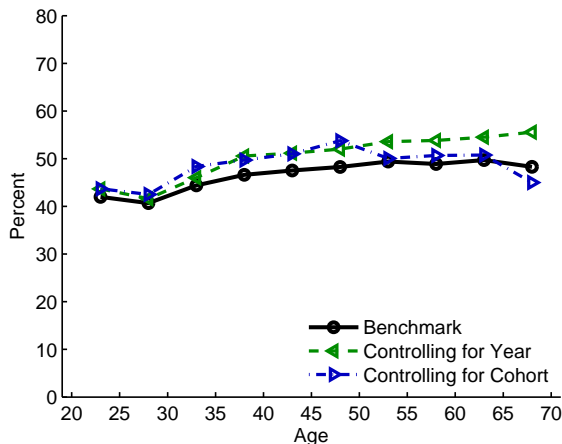
— **Risky Assets** = Savings Bonds(risky) + Brokerage Accounts + Stocks + Mutual Funds(risky) + Annuities(risky) + Trust(risky) + IRA(risky) + Pensions(risky) + Non-Actively Managed Business

Our benchmark definition is $\frac{R}{R+S} = \frac{\text{Risky Assets}}{\text{Risky Assets} + \text{Safe Assets}}$. When we include debt in our definition we calculate $\frac{R}{R+S-D} = \frac{\text{Risky Assets}}{\text{Risky Assets} + \text{Safe Assets} - \text{Debt}}$. To calculate the risky share including housing we follow three different approaches using the house worth (H=Value of the Home) and net house worth (NH=Value of the Home - Mortgages). Finally to calculate the risky share including business we use $\frac{R+B}{R+S+B} = \frac{\text{Risky Assets} + \text{Business}}{\text{Risky Assets} + \text{Safe Assets} + \text{Business}}$.

Differences in variables definitions across surveys: The 2001 survey asks more detailed questions about other future retirement benefits. We use information from variables X6491, X6492, X6493, X6494, X6495, X6496 to allocate these pensions to safe and risky categories. The 2004 and 2007 surveys code variables X6577 and X6587 for money invested in annuities and trusts, respectively. These last two surveys convey much more detailed information regarding pension plans. Variables X11032, X11132, X11232, X11332, X11432, X11532 report how much money in total is in pension funds. Variables X11036, X11136, X11236, X11336, X11436, X11536 report how the money is invested. We add to the variable **Pension(safe)** the amount in any account if any of X11036 – X11536 is equal to 2 (interest-earning assets). We add to the variable **Pension(risky)** if these variables equal 1, 4 or 5 (stocks, real estate, hedge fund). If they equal 3 (split) we split the money half in each category.

B Risky Share: Year and Cohort Effects

Figure B-1: Risky Share: Year and Cohort Effects



Note: Survey of Consumer Finances: We plot the raw risky share as in our benchmark definition and compare it with the risky share controlling for year and cohort effects.

Our benchmark definition of the risky share calculated the raw risky share averaged across age. Our data include information from four different SCF surveys (1998-2007). It is of interest to check whether the increasing pattern remains intact if we control for year or cohort effects. Ameriks and Zeldes (2004) use earlier available SCF surveys from 1983-1998. They find that both the unconditional and the conditional risky share weakly increase with age (or exhibits a hump shape) if time effects are controlled for but increase strongly with age if they control for cohort effects.

Figure B-1 plots the results from regressing risky shares to age dummies and either year or cohort dummies. It appears that the risky share profile is in general robust to these two specifications. In particular, if time effects are controlled for, the risky share increases a little more sharply from to 42% to 55%. If cohort effects are controlled for, then the risky share exhibits a small hump as it peaks at age 50 but remains stable between ages 55-65. Overall, cohort and time effects do not seem to affect the conditional risky share in a significant way.

C Division of Pension Plans between Safe and Risky Assets

As mentioned in the main text, the 1998 and 2001 SCF do not provide exact information on how pension plans, such as a 401(k), are invested. For our benchmark definition of the risky share, we categorized half of the money invested in these accounts as safe asset holdings and half as risky assets. Our choice of an equal split related to the average risky share is close to 50%. Based on Munnell (2012), investors typically hold around 65% of their pension plans in equities. To this end,

we re-calculate the risky share of financial assets using alternative split rules. In particular, we experiment with two extreme cases: a rule that allocates 80% of the money in these accounts to safe assets (and 20% in risky), and a rule that allocates 20% of these money to safe assets (and 80% to risky). We report our findings in Table C-1. The average risky share is sensitive to our choice. Naturally, if we allocate most of the money to risky assets, the risky share will increase to 51.0%. If we allocate most of the money to safe assets, the risky share will decrease to 42.7%. However, the increasing age profile documented under our benchmark definition remains intact.

Table C-1: **Portfolio Choice for Different Split Rules**

Age group	Benchmark	50-50	20-80	80-20
21-30	40.9%	45.4%	36.4%	
31-40	45.7%	50.7%	40.6%	
41-50	47.9%	52.5%	43.3%	
51-60	49.1%	52.4%	45.9%	
61-65	49.4%	51.2%	47.7%	
Average	46.5%	51.0%	42.7%	

D A Simple Portfolio Choice Theory

Using a simple 3-period model, we illustrate how portfolio choice is affected by age, labor-market risk, and wealth.

A worker lives for three periods. Each period he receives income y_t , which is an i.i.d. random variable with a probability function $f(y_t)$. Preferences are given by

$$U = E \sum_{t=1}^3 \beta^{t-1} \frac{c_t^{1-\gamma}}{1-\gamma}$$

where γ is the coefficient of relative risk aversion and c_t is consumption in period t . Two types of financial assets are available for savings. One is a risk-free bond, b_t , that pays a fixed gross return, R and the other is a stock, s_t , that pays a stochastic gross return, $R_s = R + \mu + \eta$, where μ is the risk premium and η is excess return drawn from a normal distribution of $N(0, \sigma_\eta^2)$. The probability density function associated with η is denoted by $\pi(\eta)$. On average, the stock yields a higher rate of return than the bond to compensate for the risk associated with η : $\mu > 0$.

Current income is divided between consumption, c , and savings, $b' + s'$. It is convenient to collapse total wealth into a single state variable $W = bR + sR_s$. Borrowing is not allowed for each investment ($b \geq 0$ and $s \geq 0$). The present value of utility in period j , V_j , can be written recursively when the next period's value is denoted with a prime ($'$):

$$V_j(W, y) = \max_{c, s', b'} \left\{ \frac{c^{1-\gamma}}{1-\gamma} + \beta \int_{\eta'} \int_{y'} V_{j+1}(W', y') df(y') d\pi(\eta') \right\} \quad j = 1, 2, 3$$

$$\text{s.t.} \quad c + s' + b' = W + y$$

$$c \geq 0, \quad s' \geq 0, \quad b' \geq 0$$

where $V_4(\cdot, \cdot) = 0$.

Case 1: No labor income (Samuleson Rule).

Under the CRRA preferences, with no labor income in the future, a worker allocates savings according to the constant share between risky and safe assets, the so-called Samuelson (1969) Rule, so that the risky share is:²⁰

$$\frac{s'}{s' + b'} \approx \frac{1}{\gamma} \frac{\mu}{\sigma_\eta^2}$$

This rule is intuitive. The risky share (i) increases in the risk premium, μ , (ii) decreases in the risk aversion, γ , and (iii) decreases with the risk of stock returns, σ_η . According to this rule, wealth, W , and the investment horizon (age), j , are irrelevant for the portfolio decision, inconsistent with advice often provided by financial analysts. The risky share is independent of wealth because of CRRA preferences. While the longer horizon provides an opportunity to weather the risk in stock returns, the variance of *total returns* also increases with the horizon. With CRRA preferences and i.i.d. stock returns the two effects cancel each other so that the risky share remains independent of the investment horizon.

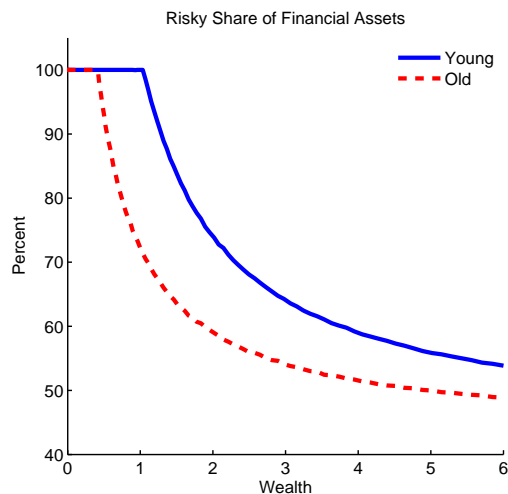
Case 2: Deterministic labor income

We now illustrate how labor-market uncertainty affects the risky share in a three-period example. In this example, the first period corresponds to “Young” worker, the second to the “Old” worker, and the last to “Retired.”

First, consider the case where labor income is deterministic ($y > 0, \sigma_y^2 = 0$) so that there is no uncertainty in the labor-market outcome. Figure D-2 plots the risky share ($\frac{s'}{s'+b'}$) of “Young” and “Old” for various levels of wealth. For both “Young” and “Old,” the risky share decreases with wealth, completely opposite to what we saw in Section 2. When the labor income is deterministic, having a job is equivalent to holding a risk-free fixed-income asset. A worker with little wealth, because risk-free labor income makes up a large portion of total wealth, would like to allocate most of his savings to risky investments. In fact, according to the optimal policy function, whether young or old, a wealth-poor worker, whose W is close to 0, allocates all his savings to stocks. As wealth increases, the risky share decreases. When wealth is large relative to labor income (where labor income becomes a negligible portion of total wealth), the risky share converges to the value implied by the Samuelson Rule.

²⁰“No labor income” refers to the case where tomorrow’s income $y' = 0$. Today’s labor income y is a part of “cash in hand,” $W + y$.

Figure D-2: Risky Share: Deterministic Labor Income



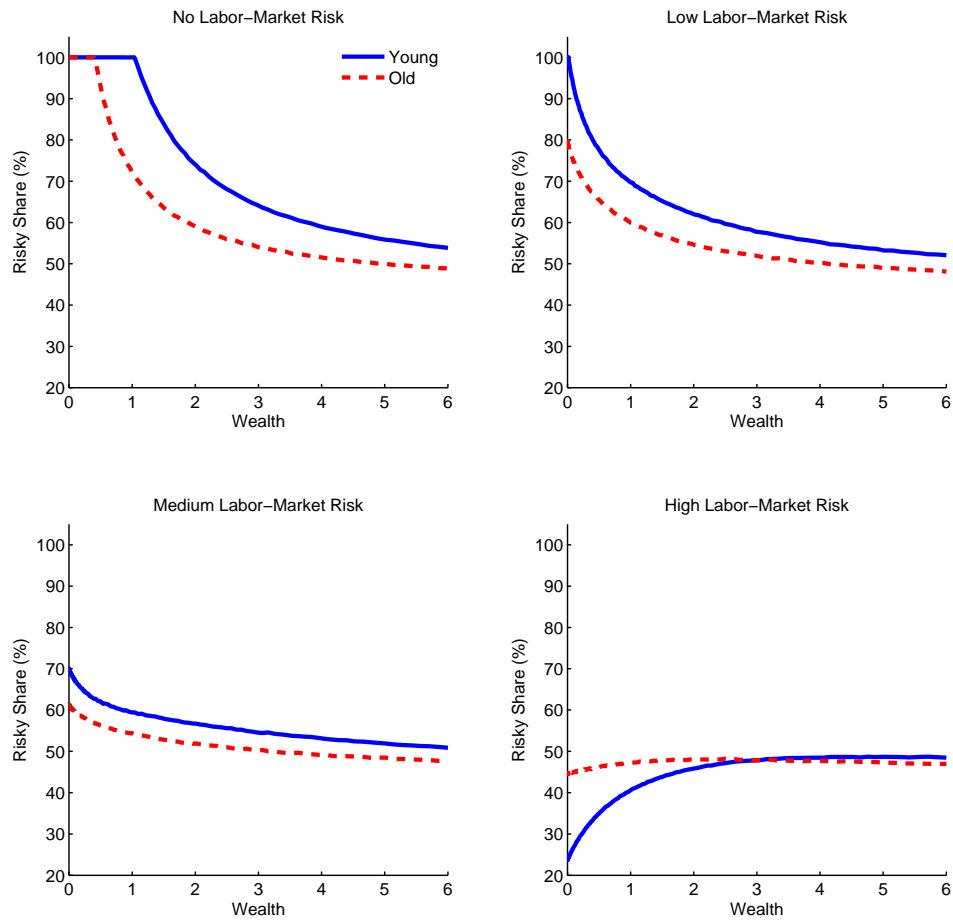
Note: Risky Share ($\frac{s'}{s'+bt}$) for “Young” and “Old”

The risky share decreases with age, again opposite to what we saw in the data. Since the young anticipate a longer stream of deterministic labor income (for the two remaining periods)—which is equivalent to holding a fixed-income asset—they are willing to take more risk in their financial investments. Figure D-2 shows that this is true for any given level of wealth, unless the wealth is close to zero where the risky share is 100% for both young and old. In sum, with no uncertainty in the labor market, the risky share *decreases* with age and wealth, both of which are opposite to what we find in the SCF.

Case 3: Stochastic labor income

We now consider the case where labor income is stochastic. Figure D-3 shows the optimal risky share for 4 different values of labor income risk, σ_y : zero (deterministic), small, medium, and high. As the labor-income risk increases, a worker becomes less willing to make risky financial investments. The risky share declines for both young and old. Now, the large uncertainty in the labor market discourages workers from making further risky financial investments. With a high enough labor-market risk (the last panel in Figure D-3), (i) the young’s risky share is lower than the old’s, and (ii) it is increasing in financial wealth. Young investors, on average wealth poor, are highly exposed to risk, since their income consists mainly of highly volatile labor income. This example clearly illustrates that labor-market uncertainty is crucial for the relationship between the risky share and investors’ age and financial wealth.

Figure D-3: Risky Share: Stochastic Labor Income



Note: Risky shares ($\frac{s'}{s'+b'}$) of “Young” and “Old” for four values of labor-market income variance (σ_y).