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LABOR SUPPLY FLEXIBILITY AND PORTFOLIO CHOICE

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ABSTRACT

This paper develops a model showing that people who have flexibility in choosing how much to work will prefer to invest substantially more of their money in risky assets than if they had no such flexibility. Viewed in this way, labor supply flexibilty offers insurance against adverse investment outcomes. The model provides support for the conventional wisdom that the young can tolerate more risk in their investment portfolios than the old.

The model has other implications for the study of household financial behavior over the life cycle. It implies that households will take account of the value of labor supply flexibility in deciding how much to invest in their own human capital and when to retire. At the macro level it implies that people will have a labor supply response to shocks in the financial markets.

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1. Introduction

Conventional wisdom suggests that fluctuations in investment income influence the amount an individual may choose to work. We have all heard stories about the successful investor who retires at age 40 after having made a "killing" in the stock market. On the other hand, a hapless investor may be forced to delay retirement or take an extra job because an investment turned sour.

The purpose of this paper is to explore the interaction between labor supply and portfolio decisions -- a subject which has received little attention in the economics and finance literature. Finance theorists have studied the effect of human capital on portfolio choice but have taken the quantity of labor as given rather than as a choice variable.[1] Labor economists, on the other hand, have either treated the portfolio mix as given or ignored it altogether in their models of labor market behavior.[2]

While for many purposes it makes sense to compartmentalize these decisions, the interactions between them can be significant. Our analysis shows that an individual who has

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[1]See Mayers [1972] and Williams [1978].

[2]See Killingsworth [1983] for an extensive survey of life cycle labor supply models.

flexibility in choosing how much or how long to work later in life will prefer to invest substantially more of his money in risky assets than if he had no such flexibility. Viewed in this way, labor supply flexibilty is a kind of insurance against adverse investment outcomes.

An important corollary to this finding is that the interaction between investment and labor decisions depends strongly on the individual's stage in the life cycle. According to practitioners' conventional wisdom, the young can tolerate more risk in their investment portfolios than the old. Indeed, guides to personal investing usually advise that as one approaches retirement, investments should shift from riskier assets like common stocks to more conservative fixed-income securities. This conventional wisdom is well accepted but often for the wrong reasons.[3]

This contention finds support in our analysis. The young have a greater opportunity to insure against adverse portfolio outcomes through their future work effort. The main asset of young workers is their future earning power. For most young people, human capital is many times as great as non-human wealth held in the form of bank accounts, real estate, and securities. Flexibility in labor supply, therefore, plays a much greater role in the portfolio decisions of the young than of the old.

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[3] See Samuelson [1989].

The interaction of labor supply and portfolio choice has potentially important macroeconomic implications. With highly flexible labor supply, we would expect to observe stabilizing labor market responses to price shocks in securities markets. For example, we would expect to see an increase in labor supply in the aftermath of the October 1987 stock market crash.

The paper is organized as follows. In the next section we present a simple two-period model of individual labor supply and asset allocation. The model confirms that the ability to adjust labor supply after investment outcomes are known increases the amount a rational consumer/investor invests in risky assets. In the next section, we extend the model to a multiperiod setting to explore labor supply and portfolio choice over the life cycle. A concluding section considers the empirical implications of our model and discusses possible directions for future research.

2. A Two-Period Model.

Our purpose is to construct and analyze the simplest model that can capture the effect of ex post labor/leisure choice on investment decisions. Accordingly, the timing of the model involves only two stages.

In the first stage, the individual determines his current investment decision.[4] In the second stage, the rate of return on

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[4]He presumably is also working, but the model abstracts from any labor decisions at this point. That is, we presume that he has already made his optimal labor decision for this time period.

his investment is realized. Given his resulting wealth, he may then adjust his labor supply choice, increasing his labor earnings at the sacrifice of leisure or vice-versa.

For the moment, we abstract from additional time periods and from consumption and savings decisions. Thus, the individual's sole objective is to maximize his final period utility, which depends on his realized wealth and leisure.

The precise formulation of the individual's problem is as follows. In the first period, a risk-averse individual invests his wealth in two assets: a risk-free asset and a risky asset. Let r and z denote the realized returns of the respective assets, W_0 denote his initial wealth, and x the fraction of his wealth invested in the risky asset.

In addition to his investment income, the individual receives labor income from working in the second period. He allocates his time between work hours H and leisure hours L, subject to the (normalized) constraint, $L + H = 1$. Thus, his labor earnings are $W_H H = W_H(1-L)$, where W_H denotes the maximum wage income that the individual could earn if he consumed no leisure ($L = 0$). It is useful to think of W_H as the value of the individual's <u>human capital</u>, (all or some of which he may choose to take as leisure). This human capital is denominated in terms of the single consumption good and therefore embodies an assumption about the relative price of leisure.

Since, by assumption, there are no bequests, the individual spends all his accumulated wealth in the second (and final) period. He spends part of it on the single consumption good, C, and part on leisure. Thus spending on the consumption good is given by:

$$C = [1 + xz + (1-x)r]W_0 + W_H(1-L)$$
$$= [1 + r + x(z-r)]W_0 + W_H - W_HL$$
$$= W(z) - W_HL \qquad (1)$$

The last line treats the consumption/leisure choice in the standard way: the individual allocates his total wealth, $W(z) = [1 + r + x(z-r)]W_0 + W_H$, between leisure ("purchased" at price W_H) and consumption (the numeraire good).

The individual's objective is to maximize the expected value of his cardinal utility function, the arguments of which are consumption and leisure. That is, he maximizes $E[U(C,L)]$ subject to the budget constraint (1). Here, U is a concave function with second partial derivatives satisfying $U_{CC} < 0$ and $U_{LL} < 0$.

Two versions of the problem are of particular interest. In the ex ante version, the individual chooses x and L at the outset. Whatever the actual performance of his investment, the individual is committed to the fixed employment decision he entered into initially. In the ex post version, the individual chooses L after he observes the realized return z and, of course, his investment wealth.

These two versions are represented as follows:

$$\max_{x,L} E[U(C(z),L))] \qquad \text{s.t. (1)} \qquad \text{(V1)}$$

$$\max_{x} E[U(C(z),L(z)] \qquad \text{s.t. (1)} \quad L(z) = \text{argmax } U(C,L) \qquad \text{(V2)}$$

The notation, C(z), makes explicit the dependence of consumption on the realized investment return z. Note that in version two, the optimal consumption/leisure choice takes place under certainty -- that is, after the investment uncertainty has been resolved.

The paper's main hypothesis is:

H1: The individual's investment in the risky asset will be greater in V2 than in V1.

Roughly speaking, the individual invests more in the risky asset in V2 because his ability to earn discretionary labor income serves as a kind of <u>insurance</u> against bad investment outcomes. Our task is to examine the circumstances under which the stated result holds.

To investigate this hypothesis, we start with the simplest possible setting, described by the following assumption.

A1. The risky asset has only two possible return realizations, z_1 and z_2, occuring with probabilities p_1 and p_2, where $p_1 + p_2 = 1$.

<u>Remark</u>. For the investment problem to be non-trivial, the returns of the risky asset must neither dominate nor be dominated by the return of the risk-free asset. Thus, we assume:

$r_2 = z_2 - r > 0$ and $r_1 = z_1 - r < 0$.

In addition, to be attractive to the risk averse investor, the expected return of the risky asset must exceed the risk-free return: $p_1r_1 + p_2r_2 > 0$.

Equations 1 and 2 list the first-order conditions for version 1, while equations 3 and 4 list the first-order conditions for version 2.

$$V_L = p_1[-W_HU_C(C_1,L^*)+U_L(C_1,L^*)]+p_2[-W_HU_C(C_2,L^*)+U_L(C_2,L^*)] = 0 \quad (1)$$

$$x^* \text{ such that: } V_x = p_1r_1U_C(C_1,L^*) + p_2r_2U_C(C_2,L^*) = 0 \quad (2)$$

$$V_{L1} = -W_HU_C(C_1,L_1)+U_L(C_1,L_1) = 0;$$

$$V_{L2} = -W_HU_C(C_2,L_2)+U_L(C_2,L_2) = 0 \quad (3)$$

$$x' \text{ such that: } V_x = p_1r_1U_C(C_1,L_1) + p_2r_2U_C(C_2,L_2) = 0. \quad (4)$$

In the equations above, V denotes the individual's expected utility, subscripts x, C, and L denote partial derivatives, and C_i and L_i denote consumption and leisure when the return is z_i.

The first-order condition in (2) implies the well-known result that a risk-averse individual always makes a positive (perhaps very small) investment in the risky asset provided the expected excess return is positive. (Note that $C_1 = C_2$ at $x = 0$. Since $p_1r_1 + p_2r_2 > 0$, it follows that $V_x > 0$ at $x = 0$. Restoring the first-order condition requires raising x^* above zero.)

By carefully examining the first-order conditions, we can demonstrate the general result that labor flexibility induces the individual to invest more in the risky asset: $x' > x^*$. The proof is in the appendix.

<u>Proposition One</u>. Given A1, the individual's investment in the risky asset is strictly greater when labor supply is flexible (and labor is actually varied ex post) than when it is fixed.

Remark. A useful way to highlight the intuition behind this result is to appeal to the well-known case of additive separable utility. Suppose utility is of the form: $U = g(C) + h(L)$, where g and h are concave in their respective arguments. Since the cross partial U_{CL} is zero, it follows that leisure is a normal good. Thus, the individual increases his consumption of leisure (reduces his working hours) upon a favorable wealth realization from his investment. The opposite outcome occurs for an unfavorable realization. To put this another way, the individual treats his discretionary labor income as a partial substitute for his investment income. Consequently, the individual invests a greater amount in the risky asset under flexible labor supply than under fixed supply (that is, Proposition One holds).

Four examples illustrate the implications of Proposition One.

Example 1. $U(C,L) = V[g(C) + \Gamma L]$, where V and g are concave functions. Assuming an interior optimum, the individual adjusts his leisure to exactly offset fluctuations in consumption that might result from fluctuations in investment income. That is, $C(z) = C^*$, for all z. The investor in effect enjoys perfect insurance. He adjusts L according to $L = (W(z) - C^*)/W_H$. As a result of Proposition One, it follows that $x' > x^*$.

If the function V is linear (so that the individual is risk neutral with respect to fluctuations in leisure), the investor will place the maximum amount in the risky asset. By contrast, under fixed labor supply, he will limit investment in the risky asset due to risk aversion (since g is concave).

Example 2. $U(C,L) = V(C + h(L))$, where V and h are increasing and concave functions. This is the converse case to Example 1. It follows that $L(z) = L^*$ for all z. That is, the individual's optimal choice of leisure is invariant to the investment outcome. Consequently, labor flexibility serves no insurance function and provides no advantage. The solutions to V1 and V2 are identical in all respects and so $x^* = x'$. Thus, this case offers an obvious counterexample to Proposition One. That is, the individual must actually exercise labor supply flexibility ex post in order for x' to exceed x^*.

Example 3. $U(C,L) = \log(C) + \Gamma\log(L)$.

In the version 1 solution, labor supply is fixed at L^*. From the first-order condition in (2), one finds:

$$x^*W_0 = [(p_1r_1 + p_2r_2)/(-r_1r_2)][[W_0(1+r) + WH(1-L^*)], \quad (5)$$

or more compactly: $x^*W_0 = \phi W_T$, where

$\phi = [(p_1r_1 + p_2r_2)/(-r_1r_2)]$ and $W_T = W_0(1+r) + W_H(1-L^*)$.

The result in (5) has an appealing interpretation. The dollar investment in the risky asset is proportional to W_T, the future value of the individual's total wealth (current wealth invested at the risk-free rate plus future labor earnings). The proportionality factor ϕ is a function of the expected excess return on the risky asset (the numerator) and a measure of variance (the denominator).

As one would expect, a higher mean or lower variance increases the investment in the risky asset. If one rewrites (5) as $x^* = \phi[1 + r + W_H(1 - L^*)/W_0]$, one sees that the proportion of current wealth invested in the risky asset increases with the level of human capital.

For logarithmic utility and flexible labor supply, one finds that:

$$L(z) = [\Gamma/(1+\Gamma)]W(z)/W_H \tag{6}$$

that is, leisure consumed is just proportional to realized total wealth, $W(z)$, and inversely proportional to W_H (the price of leisure). In turn, the indirect utility function takes the form $k\text{Log}(W(z))$, where k is a constant that depends on Γ and W_H.

Since the indirect utility functions for V1 and V2 have the same form, so too do the solutions for x. The explicit solution for x' is:

$$x'W_0 = [(p_1r_1 + p_2r_2)/(-r_1r_2)][W_0(1+r) + W_H] \tag{5'}$$

Though similar in form, equations (5) and (5') differ in a key respect. With flexible labor supply, the "full" or "maximal" value of the individual's human capital, W_H, is included in total wealth. By contrast, with labor supply fixed ex ante, the individual's total wealth includes only his actual earnings, $W_H(1-L^*)$. Consequently, $x' > x$ for all possible parameter values.

Note that for the flexible labor supply case L depends linearly on realized wealth (equation 6). It follows that investment returns and labor income are perfectly negatively correlated. The individual, in effect, uses his labor supply to provide himself insurance against investment risk.

Example 4. $U(C,L) = C^{\alpha}L^{\beta}/\alpha$. Here, α and β are restricted to be of the same sign and must satisfy $\alpha + \beta < 1$. One finds that:

$$L(z) = [(\beta/(\alpha+\beta)]W(z)/W_H.$$

It will be convenient to define:

$$\phi(\alpha) = [(-p_1r_1)^{-(\alpha-1)}-(p_2r_2)^{-(\alpha-1)}]/[r_2(p_2r_2)^{-(\alpha-1)}-r_1(-p_1r_1)^{-(\alpha-1)}].$$

Then, one finds,

$$x^* = \phi(\alpha)[W_0(1+r) + W_H(1-L^*)] \quad (7)$$

and $$x' = \phi(\alpha+\beta)[W_0(1+r) + W_H]. \quad (8)$$

Remarks. i) Together, the log and power (isoelastic) functions comprise the class of utility functions having constant relative risk aversion (CRRA). For this class, the individual's optimal investment in the risky asset is given by (7) or (8). (Note that the parameters α and β satisfy $\alpha = 1 - RRA_C$ and $\beta = 1 - RRA_L$, where RRA_C and RRA_L are the coefficients of relative risk aversion for consumption and leisure. In the log case, we have $RRA_C = RRA_L = 1$, so that $\alpha = \beta = 0$.)

ii) For the CRRA class, the individual's optimal investment is proportional to total wealth -- given by the respective bracketed terms in (7) and (8). For log utility, the "potential earnings effect" -- the gap between $W_H(1-L^*)$ and W_H -- is the sole difference between x^* and x'. For isoelastic utility, x^* and x' differ not only because of the wealth effect, but also due to the difference between $\phi(\alpha+\beta)$ and $\phi(\alpha)$. For reference, we note that ϕ is an increasing function.

Wage Uncertainty. So far our analysis has assumed a known future value for labor income. It is natural to ask how uncertainty about W_H affects the propensity to invest in the risky asset. In order to focus solely on the "uncertainty effect," we adopt the following assumptions:

A2. i) W_H is a random variable with the same expected value as in the certainty case.

ii) z and W_H are independent random variables.

Remark. Since capital and labor are complementary factors of production, it is likely that their returns are positively correlated rather than uncorrelated. Moreover, many employees own stock in their employer's firm. For them, there may even be a higher correlation between their labor income and the return on the risky asset. It is clear that this positive correlation will reduce the individual's demand for the risky asset, both in the fixed and flexible labor supply cases. We ignore this positive correlation, however, in order to focus on the effect of flexibility in labor supply on portfolio choice.

Under assumption A.2, let x^{**} and x'' denote the individual's optimal investment proportions under fixed and flexible labor supply respectively. One can demonstrate the following result.

__Proposition Two__. Suppose the investor's utility function displays constant relative risk aversion with respect to its arguments. The introduction of wage uncertainty lowers optimal investment in the risky asset regardless of whether labor supply is fixed or flexible (that is, $x^{**} < x^*$ and $x'' < x'$). Nonetheless, the proportion invested in the risky asset is greater in the case of flexible labor supply -- that is, $x'' > x^{**}$.

The proof rests on establishing that the first order term V_x is concave in W_B. For instance, in the flexible labor case, a straightforward (but tedious) calculation confirms that V_x is concave (evaluated at L(z) and x') for all parameter values. By definition, x' is optimal in the certainty case-- that is, $V_x(E[W_B]) = 0$ at x'. By Jensen's inequality and the concavity of V_x, it follows that $E[V_x(W_B)] < V_x(E[W_B]) = 0$ at x'. Thus, the optimal investment proportion must be lowered to restore the first-order condition, i.e. $x'' < x'$.

The argument for fixed labor supply is similar (V_x is concave in W_B) but contains one additional element. It is easy to check that the first-order condition V_L is convex in W_B. Thus, by Jensen's inequality, the introduction of a mean preserving spread in W_B raises the optimal consumption of leisure, $L^{**} > L^*$. Since V_x is decreasing in L, the optimal proportion invested in the risky asset must be lowered, $x^{**} < x^*$.

Finally, straightforward computation establishes that $x'' > x^{**}$. The difference between x'' and x^{**} is less than the difference between x' and x^*, but there remains a difference.

Remark. In general, one can construct utility functions such that the V_x and V_L are concave or convex in W_B. Thus, the unambiguous results obtained for the CRRA class need not hold for an arbitrarily chosen utility function.

3. A Life Cycle Model.

In this section we use the lifetime consumption and portfolio choice model of Samuelson [1969] and Merton [1969, 1971] to explore some of the implications of the interaction between portfolio choice and labor supply over the life cycle. Specifically, we are interested in testing the truth of the conventional wisdom that the young can tolerate more risk in their investment portfolios than the old.

Merton (1971) has analyzed optimal portfolio and saving decisions in a life-cycle model where there is continuous trading and asset prices follow diffusion processes. This model can be directly applied to the present problem after extending it slightly to embrace two goods, consumption and leisure. As before, we limit attention to the case of two assets: one riskless and one risky.

The individual's problem is to choose (at each point in time during the life-cycle) the proportion of wealth invested in the risky asset, $x(t)$, his current rate of consumption, $c(t)$, and his current leisure, L(t). His objective is to maximize his discounted lifetime expected utility given by:

$$V = E[\int_0^T e^{-\delta t} u(c(t), L(t))dt$$

where δ is his rate of time preference.

One can apply the Mertonian framework and (under specific assumptions) derive closed form, analytic solutions for the individual's optimal portfolio and consumption choices. Toward this end, we limit our attention to utility functions which display constant relative risk aversion. For this family, it is well-known that optimal investment behavior over the life cycle is "myopic" -- at any point in time, the investor always invests the same proportion of his total wealth in the risky asset.

Indeed, the life-cycle investment rule is nearly identical to the behavior in the simple two period model presented earlier. Take the case of logarithmic utility: $U = \log(C) + \Gamma\log(L)$, as an example. When labor supply is fixed, the optimal proportion of wealth to invest in the risky asset at time t is:

$$x^* = \frac{(\alpha - r)}{\sigma^2}\left(1 + (1-L^*)\frac{W_H}{W}\right) \qquad (9)$$

where α and σ^2 are the instantaneous mean and variance of the rate of return on the risky asset, W is current financial wealth, and L* is the optimal amount of leisure chosen by the investor at the start of his career.

Note the close similarity between equations 9 and 5. In the continuous time model, the present value of future labor income is computed as:

$$W_H(t) = \frac{Y(1 - e^{r(t-T)})}{r},$$

where Y is the continuous stream of labor income, and T is the last year of labor income.

From equation 9, one notes that as human capital is expended over the course of the life cycle, the fraction of financial wealth invested in the risky asset declines. It follows that x* reaches its lowest value at the end of the individual's working life, when W_H is zero.

When there is labor supply flexibility, the problem is formally the same as a model in which there are two consumption goods. The amount invested in the risky asset is independent of how the individual chooses to divide his total consumption budget between the first consumption good and leisure (the second consumption good). The optimal proportion to invest in the risky asset is:

$$x' = \frac{(\alpha - r)}{\sigma^2}\left(1 + \frac{W_H}{W}\right) \tag{10}$$

A comparison of (9) and (10) shows that the proportion invested in the risky asset is unambiguously greater with flexible labor supply than with fixed labor supply. Note that as the individual grows older and W_H declines, the difference between x* and x' becomes smaller. This implies that flexibility in labor supply is more important in the portfolio decisions of the young than of the old.

A concrete example is a useful way to display exact life-cycle investment and saving dynamics. At age 31, an individual has $100,000 of financial wealth and has the opportunity to earn maximum labor income of $60,000 per year (if he were to consume no leisure). He can work until age 71. For the time being we abstract from his retirement decision altogether.[5] That is, his planning problem ends at age 71, when he plans to have zero financial wealth. (One may think of him as working until he dies or having retirement income provided by other means.)

Table 1 and Figure 1 summarize the individual's optimal life-cycle behavior in the case of logarithmic utility, $U = Log(C) + .5Log(L)$, and for real asset returns: $r = .03$, $\alpha = .09$, and $\sigma^2 = .12$. Labor is flexible in Part a of the table; it is fixed in Parts b and c. In Part b, the individual's leisure is fixed at $L = .55$, matching the <u>expected</u> leisure in the flexible labor case. In Part c, leisure is fixed at $L^* = .33$, the ex ante optimal value.

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[5] Our objective is to focus on the advantages of labor flexibility during the individual's working life. Obviously, choosing when to retire is another way of exercising labor supply flexibility. Research in progress by the authors analyzes the retirement decision as an optimal stopping problem. Since this latter approach is very different from our current concerns, it is omitted here.

Table 1. Life Cycle Portfolio Choice and Consumption of Goods and Leisure

	Proportion in Risky Asset x	Human Capital W_H	Financial Wealth W	Consumption C	Leisure L
a) Flexible					
Age 31	7.5	1398	100	65.9	.55
Age 41	3.7	1187	188	65.9	.55
Age 51	2.3	902	249	65.9	.55
Age 61	1.6	518	225	65.9	.55
b) Fixed (L = .55)					
Age 31	3.6	630	100	48.2	.55
Age 41	2.5	535	135	48.2	.55
Age 51	1.8	407	154	48.2	.55
Age 61	1.4	234	129	48.2	.55
c) Fixed (L* = .333)					
Age 31	5.1	931	100	68.1	.33
Age 41	3.0	791	156	68.1	.33
Age 51	2.1	601	191	68.1	.33
Age 61	1.5	346	166	68.1	.33

ASSUMPTIONS: $U = \log(C) + .5\log(L)$;
$r = .03$, $\alpha = .09$, and $\sigma^2 = .12$.
Maximum labor income is $60,000 per year, and initial financial wealth $100,000.

NOTE: All variables are in thousands of dollars except the proportion invested in risky assets which is a multiple of financial wealth.

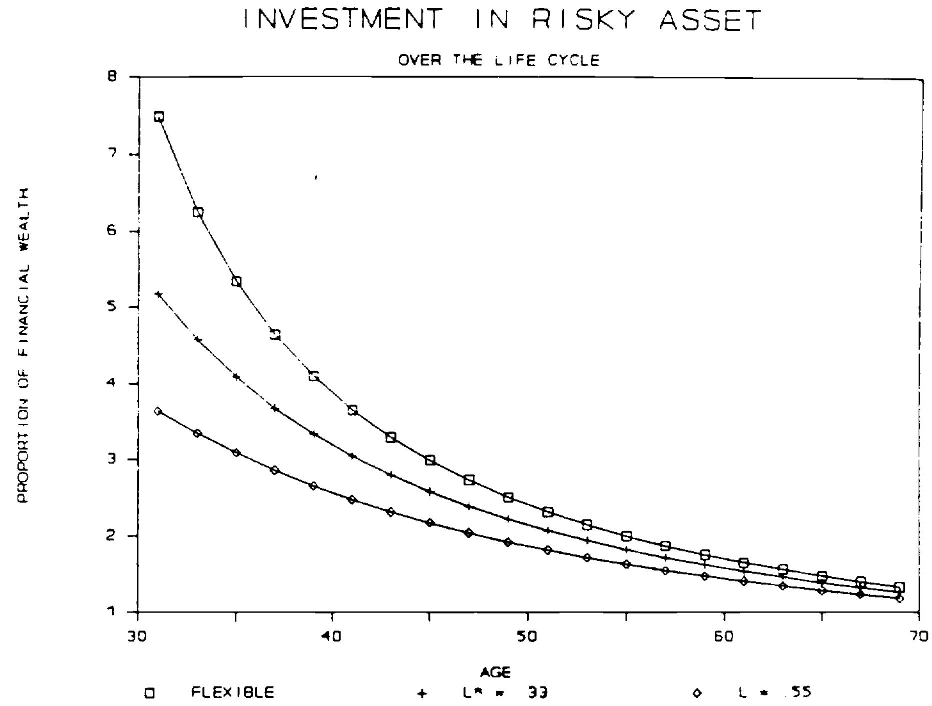

Figure 1. Investment in the Risky Asset as a Multiple of Financial Wealth

ASSUMPTIONS: $U = \log(C) + .5\log(L)$;
$r = .03$, $\alpha = .09$, and $\sigma^2 = .12$.

The table's lead column shows the individual's optimal investment proportion as a multiple of his current financial wealth.[6] Observe that under either labor supply regime, the values of x are well in excess of one -- that is, the individual is borrowing at the risk-free rate to finance his investment in the risky asset. As the example illustrates, the individual's degree of leverage is greatest early in the life cycle and when labor supply is flexible.

Casual empiricism suggests that young workers do tend to have highly leveraged portfolios. The major asset held by the young is residential real estate financed in large part with mortgage loans. The model also predicts that households with greater labo~ flexibility will tend to have riskier investment portfolios. This hypothesis is a subject for future research.

The other columns show expected values of the key wealth and consumption variables.[7] The table's second column tracks the individual's human capital -- the present value of his future labor earnings-- over the life cycle. This component of wealth is non-stochastic. Under flexible labor, human capital embodies the individual's maximum labor income ($60,000 per annum), before

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[6] It is straightforward to confirm that for the given asset returns, the agent optimally invests 50% of his current total wealth ($W + W_H$) in the risky asset. Table 1's lead column shows the corresponding proportion of the individual's financial wealth (W) going to the risky asset.

[7] Analytic expressions for the expected investment and consumption behavior can be found in Merton (1969 and 1971). Extending these results in the case of flexible labor supply (i.e. two consumption goods) is straightforward.

income is withdrawn to purchase the consumption good and leisure. In the case of fixed labor, human capital measures the present discounted value of the individual's actual yearly earnings, $(1-L)W_H$. As emphasized earlier, this is the key difference between the fixed and flexible cases. In the former, investments in the risky asset are based on actual human capital; in the latter, they are based on maximum, potential human capital.

The table's third column shows the <u>expected</u> value of financial wealth (a stochastic variable) over the life cycle. Note that the (average) growth in financial wealth is much greater under flexible labor than under fixed (for either L = .55 or L = .33). This is a direct result of the greater investment in the risky asset when labor is flexible.

The table's final two columns show the life-cycle pattern of consumption and leisure. For convenience, we have assumed a particular rate of time preference: $\delta = .06$. This knife-edge value insures that the individual's expected consumption and leisure behavior is constant over the life cycle.[8] In the flexible labor case, the combination of labor income and investment income support an expected annual consumption flow of $65,900 and expected leisure of .55. By contrast, when labor and leisure are fixed (at L = .55), the supportable steady-state consumption stream is only $48,200 per year.

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[8] Since the agent invests 50% of his total wealth in the risky asset, the overall expected return on his (total) wealth is E(z) = (.5)(3%) + (.5)(9%) = 6%. Choosing a matching rate of time preference insures a level consumption stream on average.

It is important to recognize that Table 1's consumption and leisure entries (since they are _expected_ values) exaggerate the _utility_ differences between the fixed and flexible cases. Fortunately, exact analytic expressions for the individual's lifetime expected utility can be readily developed. A natural way to express the welfare cost associated with fixed labor supply is to compute the proportional increase in lifetime total wealth (defined as the present value of maximum labor income plus financial wealth at age 31) necessary to leave the individual as well off under fixed labor as under flexible.

For Part b, the proportional increase in wealth is 12%. This is to say that on top of his initial lifetime total wealth ($1,498,000), the individual would need an additional $175,000 to bring him the same level of utility as he'd enjoy with flexible labor. In Part c, the individual makes an ex ante optimal leisure choice, $L^* = .33$. Here, one computes the compensating differential to be $133,000 (9% of lifetime total wealth).

It is interesting to observe that the individual's optimal ex ante choice of leisure ($L^* = .33$) is considerably smaller than the average amount of leisure ($L = .55$) chosen in the case of flexible labor. It is straightforward to carry out the requisite optimization in each case to obtain closed-form expressions for decision variables of interest.

The simplest expressions emerge when initial financial wealth is zero. In this case, the optimal ex ante choice of leisure is simply $L^* = \Gamma/(1+\Gamma)$. Leisure is determined once and for all (as if it were a "stock" variable). Here, L* depends only on the individual's utility trade-off, not on any aspects of security returns.[9] In turn, the individual's lifetime consumption stream is (1-L*)S = [1/(1+Γ)]S, where S is the level of consumption supportable by the flow of future labor income, Y, in combination with optimal investment behavior.

By contrast, in the case of flexible labor supply leisure is determined optimally as a <u>flow</u> variable. Again, let S denote the level of total spending (on consumption and leisure) supported by the (maximum) flow of labor income Y. It is easy to check that (on average) a fraction Γ/[(1+Γ)Y] of this spending is on leisure, implying that L' = [Γ/(1+Γ)][S/Y]. (Recall that L' is the <u>expected</u> fraction of time consumed as leisure; actual leisure choice varies with investment performance.) As long as the risky asset's expected return exceeds the risk-free return, the sustainable flow of total consumption exceeds the flow of labor earnings: S > Y. Consequently, L' is greater than L*. In turn, expected spending on the consumption good is given by [1/(1+Γ)]S -- <u>identical</u> to expected consumption in the case of fixed labor supply. (In the example above, if initial financial wealth is zero instead of $100,000, one finds L* = .33, L' = .51, and the annual rate of consumption is $61,500 under either fixed or flexible labor.)

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[9] In the case of logarithmic utility, the individual's expected lifetime utility (under optimal decisions) is proportional to $\log((1-L)W_B) + \Gamma\log(L)$. Thus, L* depends only on Γ, measuring the trade-off between the <u>stocks</u> of wealth and leisure.

Summary. The analysis of this section shows that the results from the life-cycle model closely resemble those of the simpler two-period model. For the class of utility functions displaying constant relative risk aversion, flexible labor supply generates a strong investment effect, $x' > x*$, -- a far greater proportion of assets is invested in the risky asset. In the life-cycle model, there is also a compounding effect: with flexible labor, larger initial investments in the risky asset lead to more rapid accumulation of wealth on average, leading to still greater risky investments. The difference in investment behavior between the fixed and flexible labor cases is greatest early in the life-cycle when the individual's stock of human capital is greatest. Finally, the welfare advantage of labor flexibility is significant for typical numerical examples.[10]

4. Summary and Conclusion.

The model developed in this paper suggests that labor supply flexibility can play an important role in household asset allocation. Based on this model we expect to find that in a

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[10] Life-cycle behavior for utility functions of the form, $U(C,L) = C^{\alpha}L^{\beta}/\alpha$, closely resembles that displayed for the logarithmic case. The sole difference is an effect owing to the degree of risk aversion. For utility functions more risk averse than the logarithmic (α and β negative), the difference in investment behavior and welfare between the flexible and fixed cases is diminished. For example, with $\alpha = -2$ and $\beta = -1$, the necessary compensating differentials in wealth fall to 2% and 3% when labor is fixed ex ante respectively at $L*$ and L' (the expected value under flexible leisure). For utility functions more risk tolerant than the logarithmic (α and β positive), the differences between the fixed and flexible cases are magnified.

cross-section of households the proportion of assets held in risky investments will increase with the degree of labor supply flexibility. No empirical study of household financial behavior of which we are aware has tried to test this hypothesis.

How can we measure labor supply flexibility for this purpose? First, it is probably true that households with more than one adult have more flexibility than single people. So family status is a potential measure of flexibility. Second, occupation is another potential indicator of flexibility. Many occupations offer opportunities for working extra hours, taking extra jobs, or delaying retirement.

Our life cycle model suggests that age is another important determinant of household portfolio behavior. A well-specified empirical model would have to take age into account.

Labor supply flexibility is valuable. In future research we intend to measure just how valuable. People can increase their labor supply flexibility by investing in education and training (in an effort to make their skills more transferable). Thus the value of this flexibility is crucial for determining the optimal investment in human capital. Most studies of investment in human capital ignore this insurance motive.

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APPENDIX

Proof of Proposition One: $x' > x^*$.

Step 1) In the flexible case, we first identify the individual's leisure choice in response to fluctuations in wealth. To do this, take the total differential of the first-order condition in (3) with respect to W and L and rearrange to obtain:

$dL/dW = [W_H U_{CC} - U_{CL}]/[(W_H)^2 U_{CC} - 2W_H U_{CL} + U_{LL}]$.

The denominator is simply V_{LL} (the second-order condition) and is, of course, negative. For the moment, consider the case that the numerator is also negative, so that $dL/dW > 0$ -- that is, leisure is a normal good.

Step 2) Confirm that $L_1 < L^* < L_2$. To see this, evaluate V_L in (1) first at L_1 (where it is positive) and then at L_2 (where it is negative). Thus, the optimal ex ante choice of leisure lies between the ex post leisure decisions.

Step 3) Show that $x' > x^*$. To see this, evaluate V_x in (2) and (4) at x^*, the ex ante optimal level of investment in the risky asset. By definition, at x^*, $V_x = 0$ in (2).

By comparison, in (4) we find that because of ex post labor flexibility, $V_x > 0$ at x*. The argument is straightforward. If the higher return (z_2) is realized, the individual's marginal utility of consumption (the second term in 4) is greater at L_2 than at L*. The derivative of this term with respect to L is $p_2r_2[-W_HU_{CC} + U_{CL}] > 0$. (The bracketed term is identical, except for sign, to the numerator of dL/dW in step 1. Since leisure is a normal good, the bracketed term is positive.) Thus, the increase in leisure from L* to L2 raises the second term in (4). If the lower return is realized, marginal utility (the first term in 4) increases as well. Here, the derivative with respect to L is $p_1r_1[-W_HU_{CC} + U_{CL}] < 0$, since $r_1 < 0$. Thus, the decrease from L* to L_1 increases the first term. Therefore, we have shown that $V_x > 0$, at x* in expression (4). To restore the first-order condition, the investment in the risky asset must be increased. Consequently, x' > x*.

Step 4) If leisure is an inferior good, repeat steps 2 and 3 changing the direction of the inequalities where appropriate: $L_1 > L^* > L_2$. Next, examine the effect of flexible leisure on V_x in (4) evaluated at x*. Again, one finds that, term by term, marginal utility is increased. (Note that relative to the normal case, the signs of the derivative are reversed but so are the changes in leisure.) Again we find x' > x*, investment in the risky asset increases due to labor flexibility.[11]

[11] The interested reader may wish to test this result with an example. Let $U(C,L) = [.5C + kC^2 + L - .5L^2]^\beta$, where β is in the unit interval. When k is in the positive neighborhood of zero, leisure is an inferior good. The combination of negative U_{CL} and inferior leisure results in x' > x*.