Laboratory identification of the quality factor Q: comparative presentation and evaluation of spectral ratio and rise time methods

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Abstract

Two different methods for evaluating attenuation through the quality factor Q are presented, tested and compared. Both methods are based on laboratory experiments of ultrasonic P waves propagation at "ad-hoc" rock samples. The advantages of each experimental method are discussed . The spectral method is a rather laboratory oriented methodology providing attenuation information over a range of frequencies and seems to be well adapted for intermediate attenuative media of propagation. Its outcome is strongly influenced by parasitic signal reflections, coupling functions and scattering effects. On the other hand , the rise time method less influenced by scattering phenomena, is dependent on the coupling functions (role of interfaces) and the experimental set-up. For a given experimental set-up a linear relation can be determined between τ and T/Q when a known source signal is simulated by a realistic linear viscoelastic model of propagation.

1 Introduction

In recent years several methods in time and frequency domains have been developed to compute the attenuation of the media where a wave is propagated.

The concept of the attenuation is separated into two types i.e.:

a. "intrinsic attenuation" linked to the medium of propagation, representing the absorption due to the inelastic behaviour of rocks or soils
b. "extrinsic attenuation" representing the geometry effect of the medium of propagation, like geometrical spreading, scattering, interfaces losses, e.t.c.

Two main methods for attenuation measurements at laboratory scale, and based on the evaluation of the quality factor Q are presented and compared:

- the spectral ratio method studies attenuation in the frequency domain by matching both theoretical and experimental spectral ratios. In this technique it is

necessary to isolate a signal "window" of the wave propagation in a rock or soil sample.
- the rise time method is based on the broadening of the first pulse of a signal propagated in a medium of a linear viscoelastic character.

The purpose of this paper is to compare the rise time with the spectral ratio method based on experimental data collected from an adequate measuring set-up.

2 Wave propagation - Basic theoretical concepts

The "intrinsic attenuation", called everafter attenuation, is characterised by amplitude losses in the time domain and a preferential loss of the higher values of the frequency spectrum. In case of a wave propagation in an homogeneous, isotropic, infinite, attenuating medium for small strain amplitudes ($\varepsilon \leq 10^{-6}$) the linear viscoelastic model seems to be the most adequate model to simulate the constitutive law of the medium and the wave motion. The problem of the wave propagation faced as an 1-D problem it is described as the product of a convolution in time domain or a simple multiplication of the Fourier transforms in the frequency domain. Stresses and strains are also related by a modulus M(ω) like in a purely elastic case, the only difference being that this modulus is a complex number and frequency dependent. This relationship is commonly referred as the correspondence principle.

The one dimensional equation of motion in the frequency domain is:

$$M(\omega)\frac{d^2U(x,\omega)}{dx^2} + \rho\omega^2 U(x,\omega) = 0 \qquad (1)$$

where, M(ω) : a complex modulus
ρ : the medium density
U(x,ω) : the Fourier transform of the displacement u(x,t)

The general solution of eqn (1) for a plane wave is given as follows:

$$u(x,t) = u_0 \exp\left[i(\omega_0 t - k^* x)\right] \qquad (2)$$

k* : complex wave number (k*=k-ia)
k : a real wave number (k=w/v=2πf/v, v: wave celerity)

or

$$u(x,t) = u_0 \exp\left[-\alpha(\omega_0)x\right]\exp\left[i(\omega_0 t - kx)\right] = u_0 \exp\left[-\alpha(\omega_0)x\right]\exp\left[i\omega_0(t - x/V(\omega)\right] \qquad (3)$$

where, α(ω0) : an amplitude exponential decay factor: attenuation coefficient
V(ω0) : frequency dependent wave celerity.

Three attenuation parameters are commonly used:

- the attenuation coefficient: $\alpha = -\frac{d}{dx}\ln\left[u(x)\right] = \frac{1}{x_2 - x_1}\ln\left[u(x_1)/u(x_2)\right]$, where u(x): the amplitude of a plane wave and $x_1 < x_2$: different distances
- the logarithmic decrement: $\delta = \ln[u(x_1)/u(x_2)]$, where u_1, u_2 are amplitudes of two successive maxima or minima in an exponentially decaying free vibration
- the quality factor Q, a dimensionless parameter defined by $Q = 4\pi E/\Delta W$, where E: the mean stored energy and ΔW: the energy loss per cycle.

The relationship between the factors Q, α and δ is described as below:

$$Q = 2\pi/\left[1 - \exp(-2\alpha\lambda)\right] \cong \pi/\alpha\lambda = \pi f / V\alpha = \delta/\pi \quad (4)$$

where, λ: wavelength (λ=V/f), V: velocity, f: main frequency.

3 Laboratory measurements for Q determination

A laboratory experimental set-up has been established in order to identify the attenuation characteristics of different rock samples by Q determination throughout two different methods:

- spectral ratio method (frequency domain - use of F.F.T.)
- rise time method (time domain).

3.1 Spectral ratio method

The principle of the spectral ratio method, as it has been applied in the case of our laboratory experiment on rock samples of cylindrical shape (length L=100 mm, diameter Φ=66 mm), lies on the normalisation of the spectral amplitude of a non attenuating reference (i.e. aluminium Q≅150000), to that of an identical geometry rock sample of high attenuation characteristics (Toksoz et al [1], Sears and Bonner [2], Klimis [3]). The signal (P wave), emitted by a piezoelectric bandwidth transducer of 25 mm diameter adequately coupled to the one edge of the sample, is directly propagated through the reference or the rock sample and collected by an identical piezoelectric transducer coupled on the opposite edge of the sample. The output signal is then transfered to an oscilloscope for digitisation and treatment of the signal. The whole procedure ends up to a PC where an appropriate program assigns the Q_R value. The general equation describing the output signal (R) in the frequency domain (use of Fast Fourier Transforms, F.F.T.) compared to the reference's (r) output signal, is given as follows:

$$I_n\left[\frac{U_R(f)}{U_r(f)}\right] = -\left[\alpha_R(f) - \alpha_r(f)\right] * L + I_n\left[\frac{U_{0R}(f)}{U_{0r}(f)} * \frac{C_{ER}(f)}{C_{Er}(f)} * \frac{C_{RR}(f)}{C_{Rr}(f)} * \frac{C_{TR}(f,L)}{C_{Tr}(f,L)}\right] \quad (5)$$

After a number of simplifications i.e.: perfect coupling between sample and transducers, identical source of emission for rock and reference sample, geometry effects independent of the frequency and a quality factor supposed nearly constant in the range of the examined frequencies, eqn (5) is transformed as below:

$$I_n\left[U_R(f)/U_r(f)\right] = -(\pi L / Q_R V_R)f + I_n(G_R / G_r) \quad (6)$$

The slope - $\pi L/Q_R{}^*V_R$ is then easily determined and so is the quality factor of the rock sample (Q_R).

3.2 Rise time method

The morphological parameter describing at the best the time domain signal "degradation" resulting from the preferential attenuation of high frequency components, is the rise time (τ).

Gladwin and Stacey [4] defined the rise time (τ) as the ratio of the maximum peak amplitude to the maximum slope of the first quarter - cycle of the pulse (Fig. 1). On the other hand, Blair and Spathis [5] defined the rise time (τ) as the time difference between the 90% and the 10% amplitude levels of the first peak pulse arrival (Fig. 1). They also show that the difference between the two definitions is

insignificant (deviation of about 5%), something that will be also later confirmed by Tarif [6] and Klimis [3].

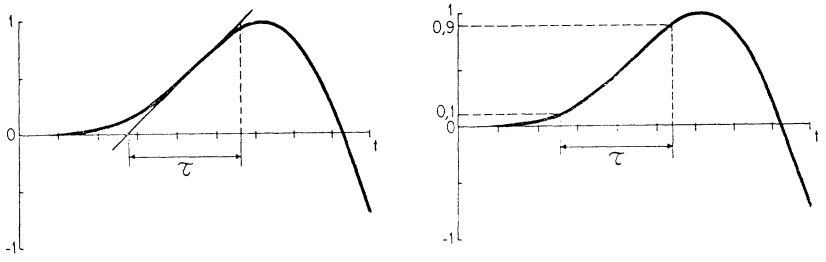

Figure 1: Definitions of the rise time (τ):
a) Gladwin and Stacey (1974), b) Blair and Spathis (1982)

From experimental studies Gladwin and Stacey [4] obtained a linear relation between the rise time and the ratio of traveltime over the quality factor (T/Q):

$$\tau = \tau_0 + c * (T/Q) \quad (7)$$

where: τ_0 : is the initial rise time or source rise time
T : is the traveltime (microseconds)
c : is a constant value

Eqn (7) has been discussed by many authors. Kjartansson [7] showed theoretically that for a constant-Q model and in the case of a delta function wavelet, the rise time was proportional to the traveltime, at least for Q greater than 20. Blair and Spathis [8] established experimentally that for seismic sources, τ_0 and c are source and Q dependent, but not strictly equal to the rise time at T=0, which means that eqn (7) is not valid for very short traveltimes. Their conclusions have been confirmed by the investigations of Campillo [9] and Jongmans [10]. All these results allow us to write: $\tau_0=\tau_0$(source, Q) and c=c(source, Q).

In the present paper, the determination of the parameters τ_0 and c is accomplished by the assignment of the reference (aluminium) output signal as our source signal. This signal is artificially propagated through different attenuating media of linear viscoelastic character on a distance L equal to the rock samples length, which are simulated by a NCQ (Nearly Constant-Q) model proposed by Azimi et al [11]. However, the quality factor can be considered as frequency independent for the frequency range where the experiments are operated. The use of a constant-Q model (CQ) would not change the results of the simulated signals. Therefore, eqn (7) for the reference (r) sample of aluminium is:

$$\tau_{(Alu)} = \tau_{0(Alu)} + C^*[L/(Q_{(Alu)}{}^*V_{(Alu)})] \quad (8)$$

For $Q_{Alu} \cong 150{,}000$, it is concluded that $\tau_{Alu} \cong \tau_{0,Alu}$. Considering that experimental procedure and apparatus are strictly the same for every measurement, and so does the samples' geometry, it can be assumed that: $\tau_{0(Rock)}=\tau_{(Alu)}$. This assumption has been confirmed by the experimental data of Klimis [3], allthough it is obvious that diffraction effects by apertures and interface losses, interfere in the experimental results.

4 Experimental comparison between the spectral ratio and the rise time method

Both methods have been applied at a considerable number of some geometry rock samples of different nature and "quality". For every signal recorded as an output of the experimental set-up, the quality factor Q is independently calculated using the spectral ratio method (see §3.1). The rise time (τ) and the traveltime (T) are directly measured on the signal. Then, the relationship τ versus T/Q is plotted (Fig. 2 and 3). The measurements are performed on about twenty-five different signals from Klimis [3] for two couples of bandwidth transducers of central frequency 0.5 and 1.0 MHz.

For the case of a source of 0.5 MHz a relation exists between rise time τ and the ratio T/Q=L/Q*V (where, L: sample length and V: P-wave celerity) issued from the experimental data:

$$\tau = 0{,}474 + 0{,}179 * (T/Q) \quad (9)$$

Q values derive from the spectral ratio method applied over a restrained frequency range where Q is almost constant. The correlation coefficient being r=0.960 it is concluded that the experimental data are well represented by this linear regression. This relation is compared with the following, obtained by using the NCQ model simulating our source signal:

$$\tau=0.435+0.185*(T/Q) \quad (10)$$

The slight discrepancy between eqns (9) and (10) could be attributed to different reasons:

- the source signal defined as the output signal of the reference, is not exactly the same examined for the rock samples since the role of interfaces cannot be neglected (influence of acoustic impedance Z),
- the various rock samples tested may not obey rigorously the NCQ model.

In order to quantify the influence of scattering phenomena, samples of which the grain or pore size is comparable to the wavelength λ have been excluded. In that case the linear regression of the experimental data is slightly improved:

$$\tau = 0.462 + 0.180*(T/Q) \quad (11)$$

with a correlation coefficient r = 0.967.

The importance of the influence of the interfaces is shown by abstracting all samples not satisfying the inegality: $Z_{Alu} / Z_R \leq 1.7$

$$\tau = 0.445 + 0.189 * (T/Q) \quad (12)$$

with a correlation coefficient r= 0.9883.

Nevertheless,both eqns (11) and (12) are very close to the "theoretical" one, providing almost identical Q values. If a variation of about $\pm$20% on Q values is imposed on the "theoretical" relation, then almost all experimental data are included (Fig. 2). It is also observed (Fig. 3) that for low T/Q values the dispersion of experimental data is more pronounced. The accuracy of the traveltime (T) and the rise time (τ) measurements being of almost 2%, it is concluded that the rise time method appears more precise to the spectral ratio method for low attenuative media (i.e. Q>100). In the same way the rise time method appears more privileged for high attenuative media (Q<5).

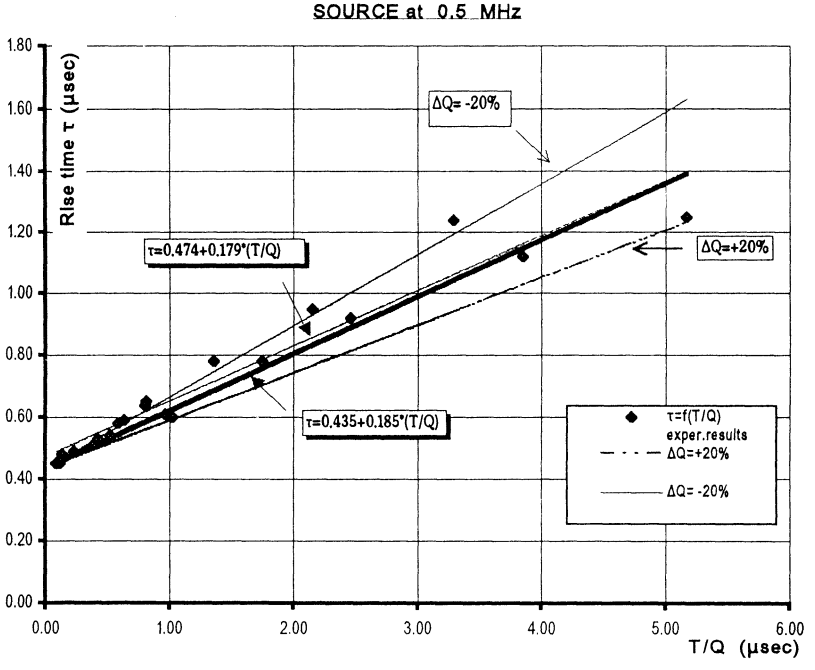

Fig. 2: Relationship between rise time (τ) and (T/Q) for source at 0.5 MHz

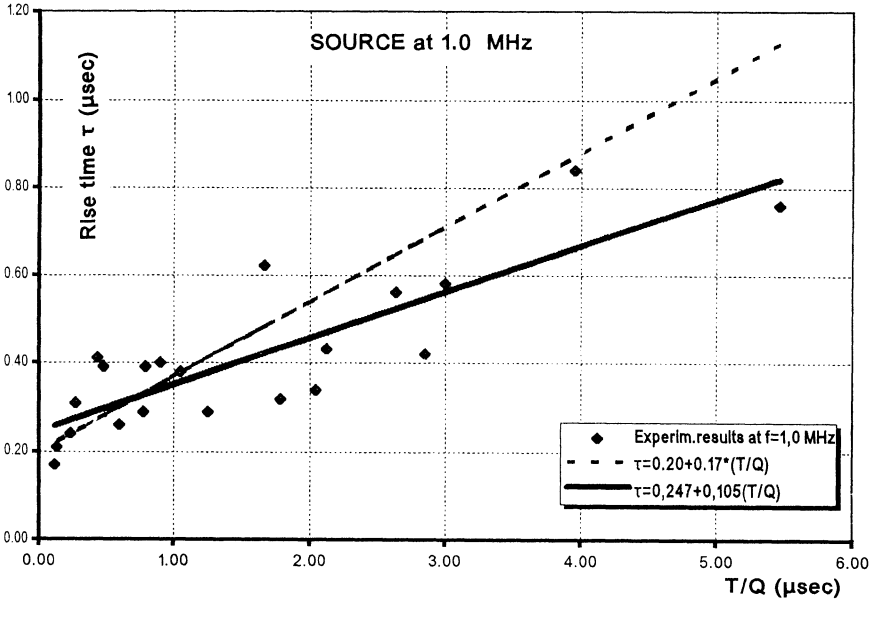

Fig. 3: Relationship between rise time (τ) and (T/Q) for source at 1.0 MHz

On the other hand, when the couple of transducers at central frequency of 1,0 MHz is used , then the "theoretical" relation of $\tau=f(T/Q)$ is :

$$\tau= 0.200 + 0.170 * (T/Q) \quad (13)$$

The experimental data are poorly related (r=0.842) by a linear relation:

$$\tau= 0.247+ 0.105 * (T/Q) \quad (14)$$

The quality factors Q deduced by the spectral ratio method are somehow different between sources at 0.5 and 1.0 MHz. On the opposite the rise time method provides identical Q values for 0.5 and 1.0 MHz sources. Trials to improve the experimental relationship by minimising the influence of the interfaces as in the case of 0.5 MHz improves very little the correlation coefficient (r=0.855). After a thorough examination is has been found that a restriction of the spectral "window" imposed on the time domain signal provides Q values similar to those obtained by the rise time method. In this way undesired reflections and parasitic phenomena are excluded or minimised in the spectral ratio method.

5 Conclusions

Commenting the aforementioned results we can draw the following concluding remarks:

- A linear relation exists between the rise time and the ratio T/Q (Q measured from the spectral method). According to the results presented both experimental methods are equally precise.
- For low attenuative media (Q>100) the rise time method appears more precise compared to the spectral ratio method.
- For high attenuating materials (Q<5) the rise time method seems to be also more advantageous since the first quarter period is free of any other contaminating reflections.
- The major draw-back of the rise time method lies on the determination of the relation between the rise time τ and the travel time over quality factor ratio (T/Q). Nevertheless, for a known source using NCQ model, it has been possible to obtain a reliable "theoretical" relation.
- Another difficulty attributed to the rise time method is the determination of the so-called "constants" τ_0 and c depend out upon the experimental set-up and the coupling functions between emitter and the examined medium. For a given experimental set-up and a known source signal it is possible to approximate this relation throughout a simulation using a given model of propagation (i.e. constant-Q or NCQ model).
- Finally, by means of the divergent results between the two methods for a source of 1.0 MHz it is shown that the spectral ratio is more vulnerable to scattering phenomena and undesired reflections, therefore the Q values calculated from this method can be different when the length of the "window" where the spectral ratio is calculated, is modified.

In conclusion, both methods can be reliable and relatively accurate at different circumstances dependent on the geometry, the frequency range of operation and the scale of the experiment (laboratory or on field). It would be therefore preferable that the attenuation measurements are obtained by both methods if possible, or otherwise suitably used each of them.

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