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LABORATORY INVESTIGATION OF RUBBLE-MOUND BREAKWATERS

Robert Y. Hudson,[1] F. ASCE

With Discussion by Messrs. José Reis de Carvalho and Daniel Vera-Cruz; William H. Booth, Jr.; Francis B. Slichter; Leland B. Jones; Omar J. Lillevang; Thorndike Saville, Jr.; and Robert Y. Hudson.

SYNOPSIS

This paper reports on a laboratory investigation conducted at the United States Army Engineer Waterways Experiment Station, Vicksburg, Mississippi, to determine criteria for the design and construction of rubble-mound breakwaters. Small-scale breakwater sections are hand-constructed in a concrete wave flume 119 ft long, 5 ft wide, 4 ft deep, and subjected to mechanically generated waves to determine the stability of the armor units.

A general stability equation has been derived and is being used to guide the experimental program and correlate the test data. From the test data obtained important unknown functions in the general stability equation have been determined for selected breakwater and test-wave conditions, and a new breakwater stability formula has been obtained.

In conjunction with the stability tests, wave run-up data are obtained for each breakwater section and wave condition tested. Also, measurements are obtained that enable the thickness and porosity of cover layers composed of different types of armor units to be determined.

The new stability formula and the experimental data obtained so far have provided essential information for an improved method of designing rubble-mound breakwaters with protective cover layers composed of quarry-stone and tetrapod armor units. Tests in progress (1959) to obtain experimental data for other special shapes of cast-concrete armor units (cubes, tetrahedrons, and tribars) should increase considerably the accuracy of rubble-mound breakwater design.

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1 Hydr. Engr., Chf. of Wave Action Sect., U. S. Army Engr. Waterways Experiment Sta., Vicksburg, Miss.

INTRODUCTION

Small-scale tests of rubble-mound breakwaters have been in progress at the U. S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi, almost continuously since 1942. During the period from 1942 to 1950, various phases of rubble-mound breakwater construction were investigated for the Bureau of Yards and Docks, Department of the Navy. The most important findings of that investigation concerned the accuracy of Iribarren's formula (1), (2), (3).[2] It was concluded (4) that the Iribarren formula can be used for the design of rubble-mound breakwaters only if experimental coefficients, of the kind developed during the investigation conducted for the Bureau of Yards and Docks, are available for the complete range of variables encountered in the design of full-scale structures.

In 1951, a comprehensive investigation of rubble-mound breakwaters (5) was begun at the Waterways Experiment Station for the Office, Chief of Engineers, U. S. Army. This investigation (in progress in 1959) is similar to the study conducted for the Bureau of Yards and Docks except that it is larger in scope; it includes the necessary range of important variables that affect the stability of rubble-mound breakwaters.

To insure optimum designs for breakwaters, design engineers should have accurate information concerning the required weight for the individual armor units in the protective cover layer, along the length of the structure, as a function of; (a) shape of unit, (b) specific weight of unit, (c) specific weight of water in which the structure will be situated, (d) beach slope seaward of the breakwater, (e) dimensions of waves at the location of the proposed structure, (f) seaside slope of breakwater, (g) porosity of protective cover layer, (h) thickness of cover layer, and (i) porosity and thickness of underlayers on which the armor units are to be placed. In addition, design engineers should be able to determine quantitatively; (a) the height of breakwater above still-water level necessary to prevent excessive overtopping by wave run-up, (b) the depths below still-water level to which the cover layer should extend, (c) the amount of damage that will be inflicted on a breakwater section not designed for overtopping when waves higher than the selected design wave occur, and (d) the best design of back slopes for preventing failure when overtopping of the breakwater is permitted. Information should also be available for designing the seaward end, or head, of the breakwater.

The test program under discussion includes tests to provide the design data and quantitative information that has been outlined. However, tests described in this paper are concerned, for the most part, with the types of rubble-mound breakwaters in which that part of the breakwater section subjected to the most intense wave action is composed of a pile of quarrystone armor units placed pell-mell, and those in which the protective cover layers are composed of two layers of cast-concrete armor units placed pell-mell over one or two quarry-stone underlayers.

2 Numerals in parentheses—thus, (1)—refer to corresponding items in the Bibliography—see Appendix I.

After the comprehensive investigation was begun, it was found that the Iribarren formula has limitations that render it unsatisfactory for use in correlating stability data from tests of small-scale rubble-mound breakwaters. Thus, it was necessary to reanalyze the phenomenon that results when waves attack a rubble-mound breakwater in order to develop a more general stability equation.

This paper describes the apparatus and testing techniques used in the laboratory investigation, explains why it was considered necessary to abandon the use of Iribarren's formula in correlating test data, and presents the derivation of a more general stability equation that, with the experimental data obtained to date, was used to develop a simple formula for the weight of armor units necessary to insure the stability of rubble-mound breakwaters. Information concerning wave run-up, and the thickness and porosity of cover layer materials is also presented.

For this paper, a rubble-mound breakwater is considered to be one constructed with a core of quarry-run stones, sand, slag, or other suitable materials, protected from wave action by one or more stone underlayers and a cover layer of relatively large, selected quarry stones or specially-shaped concrete armor units.

Notation.—The letter symbols adopted for use in this paper are defined where they first appear, in the illustrations or in the text, and are arranged alphabetically, for convenience of reference, in Appendix II.

DISCUSSION OF IRIBARREN'S FORMULA

Iribarren's original formula for the weight of armor units in rubble-mound breakwaters, in its general form, revised (6) to make it dimensionally homogeneous, and retaining the coefficient of friction as a variable, reduces to

$$W_r = \frac{K' \gamma_r \mu^3 H^3}{(\mu \cos \alpha - \sin \alpha)^3 (S_r - 1)^3} \quad \ldots\ldots\ldots\ldots (1)$$

in which W_r is the weight of individual armor units, γ_r is the specific weight of the armor units, S_r is the specific gravity of the armor units relative to the water in which the breakwater is situated $(S_r = \gamma_r/\gamma_w)$, μ is the effective coefficient of friction between armor units, H is the height of wave attacking the breakwater, α is the angle, measured from the horizontal, of the exposed breakwater slope, and K' is an experimentally determined coefficient. The accuracy of this formula was discussed by Hudson and Jackson (4), and Hudson (6) in 1953. At that time it was concluded that the Iribarren formula could be used to correlate the test data, and that it could be made sufficiently accurate for use in designing full-scale rubble-mound breakwaters, if sufficient test data were available to evaluate the experimental coefficient (K').

After the comprehensive testing program was begun, and shortly after the conclusions concerning the adequacy of Iribarren's formula were published, preparations were initiated for tests to determine the stability of armor units as a function of armor-unit shape. These included a study to establish the values of the friction coefficient (μ) that should be used for the various shapes of armor units in the experimental determination of K' in Iribarren's formula. The first armor units of special shape for which friction coefficients were

measured were cubes and tetrapods. Tetrapod is the name of a patented armor unit of special shape that was developed at the Laboratoire Dauphinois d'Hydraulique Ets. Neyrpic, Grenoble, France (7). The tests showed that the friction coefficient in Iribarren's formula, as measured by the tangent of the angle of repose (ϕ), varied appreciably with the shape of armor unit and the method of placing these units in the cover layer. These results led to the realization that the experimental coefficient (K') in Iribarren's formula could not be determined accurately from small-scale breakwater stability tests unless accurate comparative values of the friction coefficient could be obtained for the different shapes of armor units. This realization was made more acute by the fact that Iribarren's force diagram, from which his basic stability equation was derived, is predicated on the assumption that the friction between armor units, specifically that component of the friction force parallel to the breakwater slope, is the primary force that resists the forces of wave action and determines the stability of the armor units.

Results of coefficient-of-friction determinations for three sizes of quarry stones, and for concrete cubes and tetrapods are shown in Table 1. Fig. 1 shows the shapes of these armor units. About seventy repeat tests of the

TABLE 1.—FRICTION COEFFICIENTS OF ARMOR UNITS

No.	Method of Measurement	Quarry Stone			Concrete Cubes	Concrete Tetrapods
	(1)	W_r = 0.10 lb (2)	W_r = 0.30 lb (3)	W_r = 0.62 lb (4)	W_r = 0.80 lb (5)	W_r = 0.21 lb (6)
1	Dumped in water	1.02	0.98	1.13	1.20	1.10
2	Dumped in air	0.79	0.90	0.87	1.34	---
3	Stacked in water	1.09	1.19	1.26	1.36	1.78
4	Stacked in air	0.97	1.12	1.22	1.75	---
	Avg (all methods)	0.97	1.05	1.12	1.41	---
	Avg (1 and 3)	1.06	1.09	1.20	1.28	1.44

0.30-lb, quarry-stone armor units were conducted to determine the range of μ for units of this type. It was found that μ varied from a low of 0.78 to a high of 1.28, with an average value of 0.98. Thus, μ varies not only with armor-unit shape and method of placing, but it also varies considerably from test to test for the same armor unit. The curves of Fig. 2 were prepared using the modified Iribarren formula (Eq. 1), and show the effects of variations in the measured value of μ on the computed values of K'. Because W_r is directly proportional to K', variations in μ have the same effect on computed values of W_r as they do on K'. It can be seen that for steep breakwater slopes, small variations in the measured value of μ cause large variations in the computed values of K' and W_r. This becomes more significant when it is recalled that the use of concrete armor units of special shape is more apt to be economically feasible only for the steeper breakwater slopes.

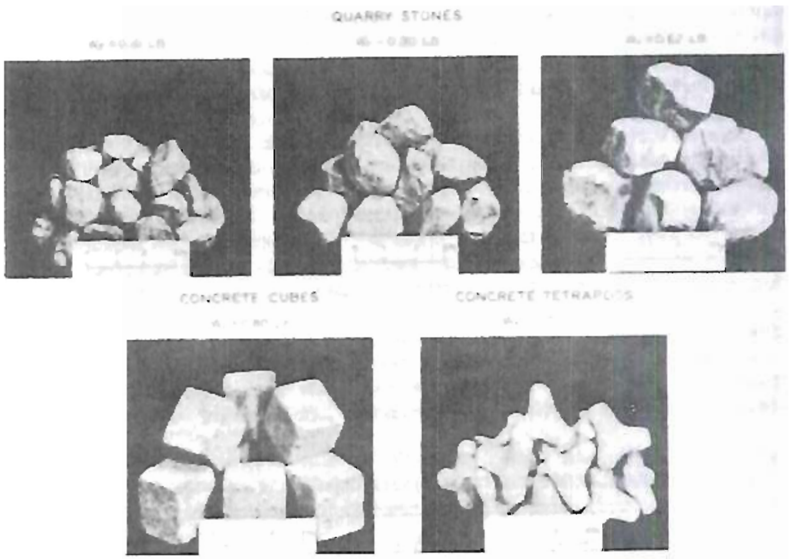

FIG. 1.—TYPES OF ARMOR UNITS FOR WHICH FRICTION COEFFICIENTS WERE DETERMINED

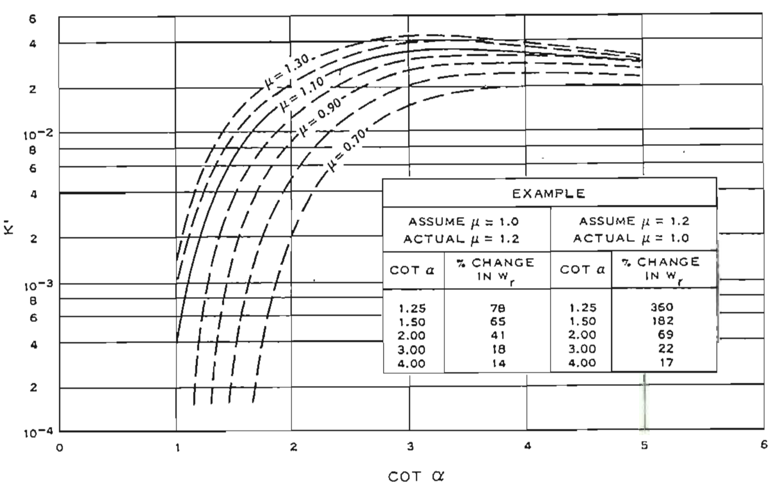

FIG. 2.—VARIATIONS OF K' WITH μ IN THE MODIFIED IRIBARREN FORMULA

Based on the results of the tests to determine friction coefficients, correlation of test data by the use of Iribarren's formula was abandoned, and a new stability equation, similar to the Iribarren formula but capable of more general application, was derived.

ANALYTICAL BASES OF STABILITY EQUATION

When short-period wind waves impinge on a pervious rubble-mound breakwater, the resulting interplay of forces developed by the wave-induced water motion and the resisting action of the armor units in the cover layer is extremely complex, and attempts to describe the phenomenon quantitatively by rigorous theoretical analyses have not, as yet, been successful. Waves at a breakwater may break completely, projecting a jet of water approximately perpendicular to the slope, break partially with a poorly defined jet, or establish an oscillatory motion of the water particles along the breakwater slope similar to the motion of a clapotis at a vertical wall. Characteristics of the motion of water particles when short-period wind waves encounter a rubble-mound breakwater are determined by the wave steepness (H/λ), the relative depth (d/λ), the relative height (H/d), the depth of water at the toe of the breakwater slope (d), the angle of the beach slope seaward of the breakwater (σ), angle of seaside slope of the breakwater with the horizontal (α), the angle of obliquity of the attacking waves (β), and the shape, thickness, and porosity of the cover layer and underlayer materials $(\Delta$, r, and P, respectively).

The ability of an armor unit in the cover layer to resist the forces caused by wave action is determined by the buoyant weight of the armor unit (W_r'), the position of the unit relative to the still-water level (z), the angle of seaside slope (α), the height of breakwater crown above still-water level (h), the width of breakwater crown (m), the shape of unit (Δ), porosity of the armor units in place (P), thickness of the cover layer (r), the porosities and thicknesses of the underlayers, and the method of placing the breakwater material, especially the armor units in the cover layer (dumped pell-mell, placed in some orderly manner to obtain wedging action, or stacked without wedging action).

Short-period wind waves incident on a rubble-mound breakwater develop dynamic forces that tend to lift and roll the armor units from the breakwater slope. These forces consist of a drag force

$$F_d = \frac{1}{2} C_d k_a l^2 \frac{\gamma_w}{g} V^2 \qquad (2)$$

and an inertia force

$$F_m = C_m k_v l^3 \frac{\gamma_w}{g} \frac{\partial V}{\partial t} \qquad (3)$$

in which C_d is a drag coefficient, C_m is a virtual-mass coefficient, l is a characteristic linear dimension of the unit such that the projected area of the unit perpendicular to the velocity is $k_a l^2$, and the volume of the unit is $k_v l^3$, γ_w is the specific weight of the water in which the breakwater is to be situated, g is acceleration due to gravity, and V is the velocity of the water flowing around or impinging on the armor units in the cover layer. Because of the difficulties inherent in an attempt to evaluate the separate sets of coefficients,

$C_d\ k_a$ and $C_m\ k_v$, that would involve either direct measurement or a derived expression of the acceleration $(\partial V/\partial t)$ in terms of the wave characteristics, and in order to simplify the force equation used to correlate test data, the effects of acceleration are combined with the drag force. The resulting equation is

$$F_q = C_q\ l^2\ \frac{\gamma_w}{g}\ V^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which C_q, the total coefficient, is a function of the terms $\frac{1}{V^2}\frac{\partial V}{\partial t}$, $C_d\ k_a$, and $C_m\ k_v$.

The velocity of the water jet resulting from a breaking wave (V_b) is equal to the particle velocity at the wave crest that, at the instant of breaking, is equal to the celerity of the wave form. Thus, for shallow-water waves, as $d/\lambda \longrightarrow 0$,

$$V_b^{\,2} = g\ d_b \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Also at breaking, $H_b = k\ d_b$, in which $k = f(H/\lambda)$. Therefore, by substitution,

$$V_b^{\,2} = \frac{g}{k}\ H_b \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Substituting this value of velocity in Eq. 4, the expression for the force exerted on an armor unit by a breaking wave, in terms of wave height, is

$$F_q = C_q\ l^2\ \frac{\gamma_w}{k}\ H_b \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

For breakwaters constructed by dumping or by placing armor units essentially pell-mell, the forces resisting displacement are the buoyant weight of the individual units and the friction between units. Except for isolated instances in which wedging action is involved, friction between armor units can be neglected, and the principal resisting force for pell-mell-constructed cover layers can be assumed to be

$$W_r' = k_v\ l^3(\gamma_r - \gamma_w) \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

in which γ_r is the specific weight of the armor units.

For incipient instability of armor units in a rubble-mound breakwater, or fill slope, subjected to breaking waves, $W_r' = F_q$, or

$$k_v\ l^3(\gamma_r - \gamma_w) = C_q\ l^2\ \frac{\gamma_w}{k}\ H_b \quad \ldots\ldots\ldots\ldots (9)$$

Letting $S_r = \gamma_r/\gamma_w$, and substituting in Eq. 9,

$$k_v\ l(S_r - 1) = \frac{C_q\ H_b}{k}$$

or

$$\frac{H_b}{l(S_r - 1)} = \frac{k\ (k_v)}{C_q} \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

The weight of an armor unit in air is $W_r = k_v\ l^3\ \gamma_r$, or

$$l = \left(\frac{W_r}{k_v\ \gamma_r}\right)^{1/3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Substituting this value of l in Eq. 10,

$$\frac{\gamma_r^{\,1/3}\ H_b}{(S_r - 1)\ W_r^{\,1/3}} = \frac{k\ (k_v)^{2/3}}{C_q} \quad \ldots\ldots\ldots\ldots (12)$$

in which

$$\frac{k\ (k_v)^{2/3}}{C_q} = f\left(C_d,\ k_a,\ C_m,\ k_v,\ \frac{1}{V^2}\frac{\partial V}{\partial t},\ d/\lambda,\ H/\lambda\right)$$

The forces that tend to displace armor units from breakwater slopes when the waves do not break, or break only partially, are not the same as those forces that result from breaking waves, nor do they act in the same directions. However, the order of magnitude of the nonbreaking wave forces, and the effects of these forces on the stability of rubble-mound breakwaters, should be approximately the same as those caused by breaking waves. It is believed, therefore, that Eq. 12 adequately represents, at least in the first approximation, the major forces of both breaking and nonbreaking waves. Thus, for both types of short-period wave motions on rubble-mound breakwaters, and introducing those variables that were not included in the derivation of Eq. 12, the most general equation used in this investigation to guide the testing program and correlate test data is

$$\frac{\gamma_r^{\,1/3}\ H}{(S_r - 1)\ W_r^{\,1/3}} = f\begin{pmatrix} \alpha,\ C_d,\ C_m,\ k_a,\ k_v,\ \frac{1}{V^2}\frac{\partial V}{\partial t},\ H/\lambda,\ d/\lambda, \\ H/d,\ d,\ \sigma,\ P,\ r,\ h,\ m,\ z,\ \beta, \text{ and} \\ \text{the method of placing armor units} \end{pmatrix} \quad \ldots (13)$$

In Eq. 13, C_d and C_m are functions of Δ and the Reynolds number (**R**), and k_a and k_v are functions of Δ. The term $\frac{1}{V^2}\frac{\partial V}{\partial t}$, that is a form of Iversen's modulus for accelerated motion (8), is omitted from the list of variables tested in this investigation because of the difficulty of obtaining accurate velocity-time histories of the flow around individual armor units.

In the first phase of this testing program, the upper portion of the small-scale breakwaters was constructed of rocks simulating quarry stones, all pieces of which were of nearly the same weight, specific weight, and shape. In addition, the crown width of the breakwater test sections was standardized at three times the average diameter of the armor units; the angle of obliquity of the test waves was 0°, and the cover layer was extended to a depth below still-water level sufficient to insure that the stability of the structure would not be influenced by the stones used in the lower portion of the test section. For those tests in which the no-damage criterion was used in the selection of design-wave heights, the crown heights above still-water level were sufficient to prevent overtopping by the test waves. For those tests in which the wave heights used were greater than the previously selected design-wave heights, the crown heights above still-water level, and the depths to which the cover layers extended below still-water level, were equal to the previously selected design-wave heights. For all tests, the water depth between the wave generator and the breakwater was constant, and was sufficient to prevent the ratio H/d from influencing the action of waves on the structure. For the tests conducted, the variation in Reynolds number was comparatively small. Tests in a larger wave flume at the laboratory of the Beach Erosion Board, Washington, D. C., are being conducted to determine the effects of this variable on the stability of armor units in rubble-mound breakwaters.

When damage is allowed to occur to the breakwater (by use of wave heights greater than the design-wave height), the geometry of the structure, the motion of the water particles, and the resulting forces on the breakwater differ from those resulting from tests in which the no-damage criterion is used. Thus, a damage parameter, D, defined as the percentage of armor units displaced from the cover layer by wave action, is included as a prime variable.

For the breakwater sections investigated in the first phase of the testing program, in which the armor units were rocks simulating rounded and smooth quarry stones placed pell-mell

$$\frac{\gamma_r^{1/3}\, H}{\left(S_r - 1\right) W_r^{1/3}} = f(\alpha,\ H/\lambda,\ d/\lambda,\ \text{and } D) \quad \ldots\ldots\ldots\ (14)$$

In the second phase of the testing program the armor units used were patterned after the tetrapod, and the rubble mound was protected by two or more layers of armor units placed over one or two quarry-stone underlayers. For these tests

$$\frac{\gamma_r^{1/3}\, H}{\left(S_r - 1\right) W_r^{1/3}} = f(\alpha,\ H/\lambda,\ d/\lambda,\ r) \quad \ldots\ldots\ldots\ (15)$$

The dimensionless parameter on the left side of Eqs. 13 through 15 is designated the stability number (N_S) for rubble-mound breakwaters.

EXPERIMENTAL EQUIPMENT AND PROCEDURE

Test Apparatus.—The breakwater stability tests are conducted in a concrete flume 5 ft wide, 4 ft deep, and 119 ft long, equipped with a plunger-type wave generator. Wave heights are measured with a parallel-rod-type wave gage, and recorded on a direct-writing oscillograph. The wave-height measuring apparatus consists of the wave gage (two 1/8-in. stainless steel, parallel rods 1.2 ft long, spaced 2 in. apart), a balancing circuit, a Brush universal analyzer, and a magnetic oscillograph.

Cross-section measurements of the small-scale breakwaters are obtained with a sounding rod equipped with a circular spirit level for plumbing, a scale graduated in thousandths of a foot, and a ball-and-socket foot that facilitates adjustment to the irregular surface of the breakwaters. The foot is circular, and for each test the diameter of the foot is equal to one-half the average diameter of the armor units.

Types of Tests Conducted.—Two primary types of stability tests are being conducted in this investigation. First, design-wave heights are determined for breakwater sections of sufficient height to prevent overtopping by the test waves. Design-wave height is defined as the maximum wave height, measured at the location of a proposed breakwater before it is constructed, that will not damage the cover layer. The removal of as much as 1% of the total number of armor units in the cover layer is considered to be "no damage."

The second type of tests being conducted is concerned with determination of safety factors for breakwater sections designed on the basis of the criteria

established from results of the no-damage and no-overtopping tests. For the safety-factor tests, breakwater sections are constructed in the wave flume in accordance with the results of the no-damage and no-overtopping tests, and the amount of damage, as determined by the percentage of armor units removed from the cover layer, is obtained as a function of wave height. Wave heights greater than the previously selected design-wave height for the no-damage and no-overtopping criteria are used in these tests.

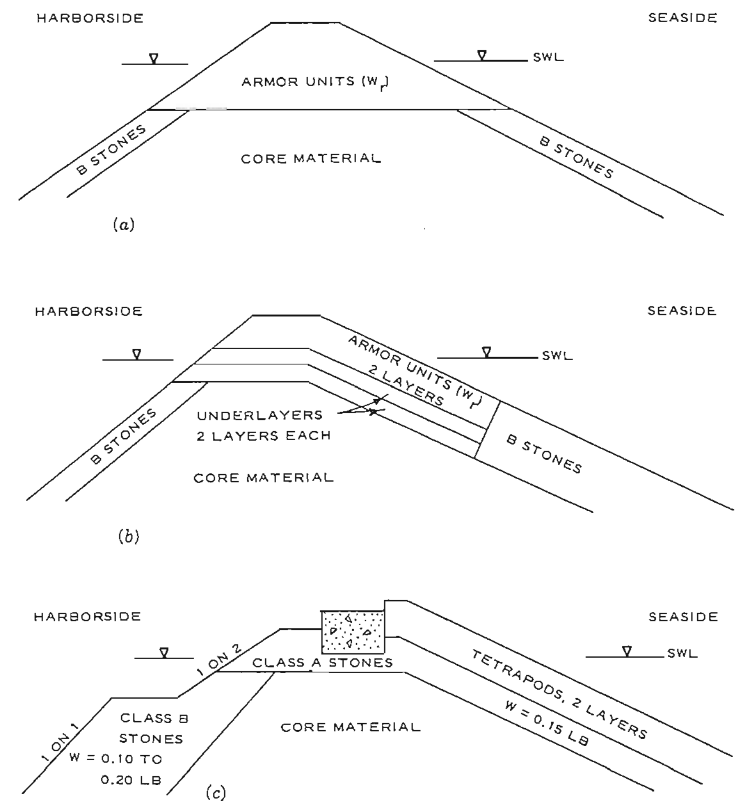

FIG. 3.—ELEMENTS OF BREAKWATER SECTIONS TESTED

In addition to the two previously mentioned types of tests, special tests are conducted from time to time to determine optimum designs for specific breakwaters. In these tests, design-wave heights may be determined for conditions other than no-damage and no-overtopping.

Breakwater Sections Tested.—Rubble-mound breakwaters of the types shown schematically in Figs. 3(a) and 3(b) have been used in most of the stability tests. In the no-damage and no-overtopping tests, the crown heights were sufficient to prevent overtopping, and the cover layer was extended a sufficient distance below still-water level to prevent damage to the class B stones used below the armor units. The distance below still-water level to which the armor units extended, as well as the height of the breakwater crown above still-water level, was equal to or greater than the wave heights used to test the breakwater sections. In the safety-factor tests of quarry-stone armor units, the crown heights above still-water level, and the maximum distances below still-water level to which the armor units in the cover layers extended, were numerically equal to the design-wave heights previously selected in the corresponding no-damage and no-overtopping tests.

In a few tests to determine the stability of the Crescent City Harbor breakwater (9), the type of breakwater section shown in Fig. 3(c) was used. This breakwater section was designed for overtopping.

Types of Breakwater Materials Used.—

Quarry-stone armor units and class B stones.—In each stability test the quarry-stone armor units were as nearly the same weight, specific weight, and shape as possible. Both the armor stones and class B stones were sized from crushed basalt. The weights of class B stones were approximately the same as those of the armor stones; however, the class B stones were sized by means of sieves, whereas each armor stone was sized and shaped by hand and weighed on a torsion balance having a sensitivity of 0.1 g. Two sizes of armor stones were used to insure that the design-wave heights, and the heights of waves used in the safety-factor tests, would be within the range of wave dimensions that the wave machine can generate. Approximately 2800 pieces of the larger-size armor stones were used. Based on a representative sample of 175 pieces, the average weight and specific weight of the larger-size armor stones were 0.30 lb and 176.0 lb per cu ft, respectively. Based on a representative samples of 475 pieces, the average weight and specific weight of the smaller-size armor stones were 0.10 lb and 174.7 pcf, respectively. The core material, that was the same for all tests conducted, consisted of crushed basalt with a mean particle diameter of 1/8 in.

Tetrapod armor units.—Tests have been conducted using tetrapod-shaped armor units molded of both concrete and leadite. Leadite is the trade name for a caulking compound that has a specific weight nearly the same as that of the concrete used to mold the tetrapod armor units. Based on representative samples of 125 pieces, the average weight and specific weight of the concrete tetrapods were 0.21 lb and 142.3 pcf, respectively, and the average weight and specific weight of the leadite tetrapods were 0.22 lb and 140.4 pcf, respectively.

Method of Constructing Test Sections.— The breakwater test sections were constructed in the wave flume on a sand base 85 ft from the wave generator. The core material and class B stones from the base of the test section to the crown of the core were placed with the flume dewatered. The core material was wetted with a hose and then compacted with hand trowels to simulate the natural consolidation effected by wave action during construction of full-scale structures. The class B stones were then placed by shovel and dressed by hand, after which the flume was flooded to the desired still-water level. For the type of breakwater shown in Fig. 3(a), the quarry-stone armor units from the crown of the class B stones and core-material section to the still-water level (swl) were placed by dumping, pell-mell, from a container at the water surface. Above the still-water level the quarry-stone armor units were placed

by hand. For the types of breakwater illustrated in Figs. 3(b) and 3(c), the class A and class B stones and the core material were placed in the manner described for placement of the class B stones, and the armor units, both above and below the water surface, were placed by hand. These methods of constructing the breakwater test sections were adopted so as to reproduce, as nearly as possible, the usual methods of constructing full-scale structures.

Selection of Design-Wave Heights.—Design-wave heights for the no-damage $(H_D = 0)$ criterion were determined by subjecting the test sections to waves made successively higher, in approximately 0.02-ft increments, until the maximum wave height was found that would not remove more than 1% of the armor units from the cover layer. Each size wave was allowed to attack the breakwater for a cumulative period of 30 min, after which the test sections were rebuilt prior to attack by the next added-increment wave.

TABLE 2.—RANGES OF WAVE AND BREAKWATER CHARACTERISTICS TESTED

Characteristic (1)	Range of Test Conditions (2)
Wave height (H)	0.28 to 0.69 ft
Water depth (d)	1.26 and 2.00 ft
Wave period (T)	0.88 to 2.65 sec
Wave length (λ)	4.0 to 20.0 ft
Relative depth (d/λ)	0.10 to 0.50
Wave steepness (H/λ)	0.015 to 0.128
Specific weight of:	
Quarry stones (γ_r)	166.0 to 191.6 lb per cu ft
Concrete tetrapods (γ_r)	135.0 to 154.0 lb per cu ft
Leadite tetrapods (γ_r)	134.0 to 142.0 lb per cu ft
Water (γ_w)	62.4 lb per cu ft
Weight of:	
Quarry stones (W_r)	0.09 to 0.31 lb
Concrete tetrapods (W_r)	0.18 to 0.24 lb
Leadite tetrapods (W_r)	0.21 to 0.23 lb
Breakwater slope ($\tan \alpha$)	1 on 1.25 to 1 on 5

Range of Test Conditions.—The tests involved the ranges of wave and breakwater characteristics listed in Table 2.

Test Waves.—During the tests, the wave generator was stopped as soon as reflected waves from the breakwater reached it, and the waves were allowed to decay in order to prevent the test section from being exposed to a multiple, undefined wave system. Accurate determination of the height of test waves was complicated by the presence of waves of abnormal height in the train of waves, caused by the starting and stopping of the generator. Usually there were one or two large waves at the end of each cycle. The larger waves, which occurred approximately 1% of the time that waves attacked the test structure, averaged about 12% higher than the average height of the highest one-third of the waves in the wave trains ($H_{1/3}$). Waves of height $H_{1/3}$ are called the "significant" waves of fully established wave trains in nature. It has been determined (10) that storm-wave trains in nature contain waves about 25% larger than the significant wave 5% of the time, 33% larger 3% of the time, and 58% larger 1% of the time. The importance of these facts with respect to the design of rubble-mound breakwaters is not fully understood at the present time. However, it is believed that the existence of these larger-size waves in natural wave trains must be considered in the selection of design-wave heights and factors of safety.

RESULTS OF STABILITY TESTS

No-Damage Conditions.—Data obtained from stability tests of quarry-stone and tetrapod-shaped armor units for the no-damage criterion are shown in Fig. 4 in the form of a log-log plot, with the stability number as the ordinate,

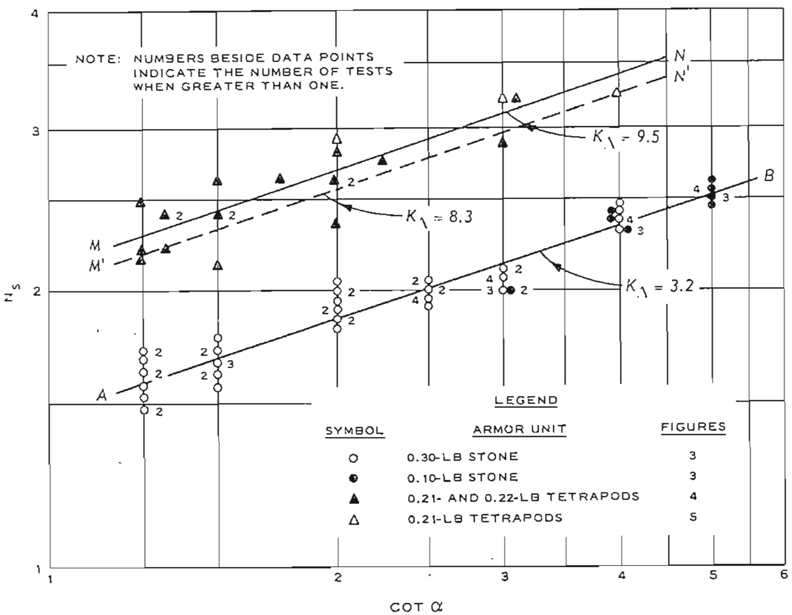

FIG. 4.—STABILITY OF QUARRY-STONE AND TETRAPOD ARMOR UNITS: N_s AS A FUNCTION OF Δ AND α FOR THE NO-DAMAGE AND NO-OVERTOPPING CRITERIA

cot α as the abscissa, and the shape of the armor unit as the parameter. These data consist of experimentally determined design-wave heights and corresponding computed stability numbers, as functions of breakwater slope and shape of armor unit. Data concerning quarry-stone armor units were obtained for breakwater sections of the type shown in Fig. 3, and the design-wave heights were determined for the no-damage and no-overtopping criteria. Data concerning tetrapods, using the no-damage and no-overtopping criteria, were obtained for breakwater sections of the type shown in Fig. 3(b). Data

were also obtained for a breakwater section of the type shown in Fig. 3(c), using the no-damage criterion. The crown of the latter breakwater section was designed for overtopping.

Analysis of the test data indicated that, for the conditions tested, the effects of the variables d/λ and H/λ on the stability of armor units are of second order in importance compared with the effects of breakwater slope and shape of armor unit. A formula for determining the weight of armor units necessary to insure stability of rubble-mound breakwaters of the types tested, and in relatively deep water, can be obtained from the equation of the approximate best-fit lines in Fig. 4. The lines AB and MN were drawn through the data points using a slope of one-third to simplify the derived formula. The equation of a straight line on log-log paper is of the form $y = a\,x^b$, in which a is the y intercept at x = 1, and b is the slope of the line. The equation of lines AB and MN, therefore, is

$$N_s = a\,(\cot\,\alpha)^{1/3} \quad \ldots\ldots\ldots\ldots (16)$$

or

$$\frac{\gamma_r^{1/3}\,H_{D=0}}{W_r^{1/3}\left(S_r - 1\right)} = a\,(\cot\,\alpha)^{1/3} \quad \ldots\ldots\ldots\ldots (17)$$

from which, if $K_\Delta = a^3$,

$$W_r = \frac{\gamma_r\left(H_{D=0}\right)^3}{K_\Delta\left(S_r - 1\right)^3\,\cot\,\alpha} \quad \ldots\ldots\ldots\ldots (18)$$

This is the desired stability formula for quarry-stone and tetrapod-shaped armor units for the no-damage and no-overtopping conditions. The test data indicate that, for pell-mell placing of armor units, the experimentally determined coefficient (K_Δ) varies primarily with shape of the armor units. The values of K_Δ for quarry-stone and tetrapod-shaped armor units, corresponding to the best-fit lines AB and MN of Fig. 4, are 3.2 and 9.5, respectively.

Tests conducted previously showed that, for the type of breakwater tested and for breakwater slopes flatter than 1-on-2, the stability number increases slightly as the number of layers of armor units is increased from 2 to 4. Although an increase in stability number means a decrease in weight of armor unit for the same wave height, the saving in volume of material per armor unit is more than offset by the increased thickness of the cover layer. These tests indicated that n = 2 is the optimum for tetrapod cover layers.

Damage or Safety-Factor Tests.—Because storm-wave trains contain waves higher than the significant height ($H_{1/3}$), it is important that rubble-mound breakwaters be designed so that they will not fail when subjected to waves with heights moderately larger than the selected design-wave height. Thus, quarry-stone armor units were subjected to tests in which wave heights were greater than the previously selected design-wave heights for the no-damage and no-overtopping criteria to obtain information concerning safety factors for rubble-mound breakwaters designed on the basis of Eq. 18. Results of these tests for quarry-stone armor units are presented in the form of a log-log plot in Fig. 5, with the stability number (N_s) as the ordinate, cot α as the abscissa, and the percentage of damage to the cover layer (D) as the parameter. The damage tests were conducted using 0.10-lb and 0.30-lb armor stones and relative depths of 0.10 and 0.25. The solid line AB in Fig. 5 is the same as line AB in Fig. 4; that is, it is the approximate best-fit line through the data points for the no-damage and no-overtopping criteria. The dashed lines in Fig. 5 were drawn parallel to line AB through data points delineating approximate ranges of percentages of damage to the cover layer. Although the dashed lines represent only rough approximations of the amounts of damage obtained for the different wave heights, it is believed that, considering the nature of the tests and the significance of the damage parameter, they reflect the test results with sufficient accuracy for the immediate needs of the design engineer.

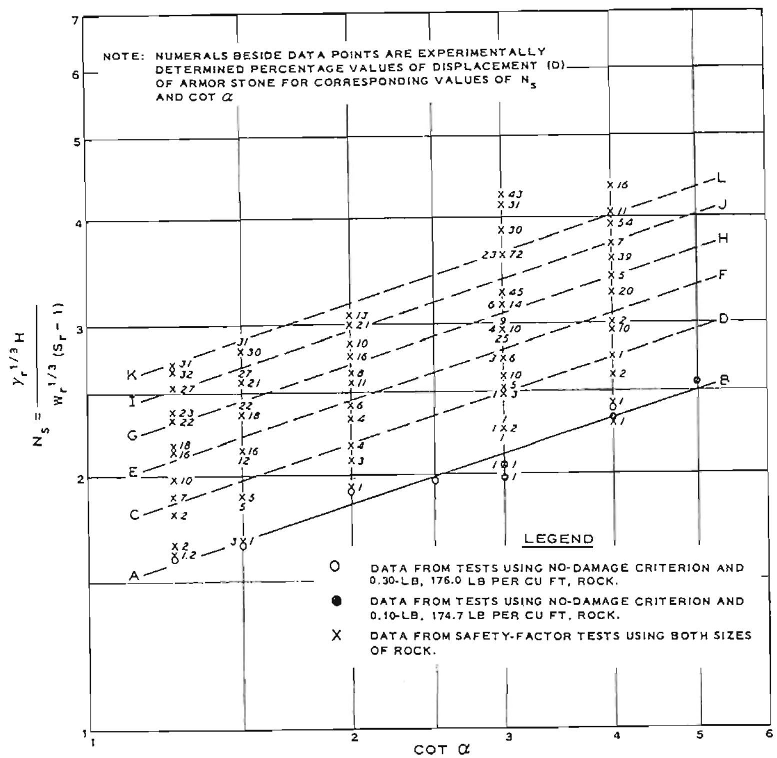

FIG. 5.—STABILITY OF QUARRY-STONE COVER LAYERS: N_s AS A FUNCTION OF α AND D

The form of the equation for the dashed lines in Fig. 5 is the same as that of lines AB and MN of Fig. 4. Therefore, the general formula for stability of quarry-stone armor units, for $H \geqq H_{D=0}$, is

$$W_r = \frac{\gamma_r \; H^3}{K_D \left(S_r - 1\right)^3 \cot \alpha} \quad \ldots\ldots\ldots\ldots (19)$$

in which K_D is the experimentally determined damage coefficient, and H is the corresponding wave height. Table 3 shows values of D, $H/H_{D=0}$, and K_D corresponding to the various lines in Fig. 5.

In this tabulation, the amounts of damage to the test sections are given in terms of percentages of the armor units removed from the cover layer. In the damage tests the breakwater sections were of the type shown in Fig. 3(a), with both the crown height above still-water level and the maximum distance below still-water level to which the armor units extended being equal in magnitude to the previously determined design-wave height. Thus, the percentages of damage for these tests are considerably smaller than the corresponding percentages of damage that would obtain for breakwaters of the type shown in Fig. 3(b), other conditions being equal. In the Fig. 3(b) breakwater section, the volume of the cover layer is smaller than that shown in Fig. 3(a), consequently, for equal amounts of damage to the cover layer, the percentage of damage is proportionally larger for the cover layer of smaller volume.

Comprehensive tests to determine the amount of damage to tetrapod cover layers as a function of $H/H_{D=0}$ have not been conducted. However, preliminary tests of tetrapods in which waves larger than $H_{D=0}$ were used indicate

TABLE 3.—EXPERIMENTALLY DETERMINED DAMAGE COEFFICIENTS FOR QUARRY-STONE ARMOR UNITS

Line (1)	Range of D, in percentage (2)	$H/H_{D=0}$ (3)	K_D (4)
AB	0-1	1.00	3.2
CD	1-5	1.18	5.1
EF	5-15	1.33	7.2
GH	10-20	1.45	9.5
IJ	15-40	1.60	12.8
KL	30-60	1.72	15.9

that the limit of stability of tetrapod armor units, with n = 2, is reached when the ratio $H/H_{D=0}$ becomes equal to approximately 1.2. For values of $H/H_{D=0}$ slightly larger than 1.3, failure of the tetrapod cover layer occurs. It is believed, therefore, that a value of K_Δ of 8.3, which corresponds approximately to the lower envelope of data points for tetrapods in Fig. 4, line M'N', should be used for design of tetrapod cover layers until more quantitative information is available concerning safety factors for tetrapod armor units.

It is emphasized that the wave heights in Eqs. 18 and 19 are the selected significant waves that occur at the position of a proposed breakwater before the breakwater is constructed, and not the heights of waves moving up, or breaking on, a breakwater slope. Also, it is pointed out that the angle (α) in these equations is the angle of the breakwater slope as first constructed, and not the angle of the breakwater slope after the breakwater has been stabilized by waves of height H.

SUPPLEMENTARY DESIGN DATA

Wave Run-Up.—The primary function of breakwaters is to provide adequate protection from wave action in selected harbor areas. Consequently, overtopping usually can be tolerated only if it is negligible or does not exceed allowable limits as determined by the type of harbor and the use for which different areas in the harbor are designed. There is considerable experimental data in the literature concerning wave run-up on paved slopes, beach slopes, and shore-line structures such as seawalls (11), (12), (13), (14), and a theoretical method of computing run-up on smooth, impervious slopes by Miche (15), has been noted by Bruun (16). However, comparatively little run-up data are available for structures with slopes as rough and porous as rubble-mound breakwaters.

Although limited in scope, the small-scale tests of wave run-up on sloping structures conducted by Granthem (17) provide some information on this subject. Granthem's tests were conducted in a manner that approximated the action of waves on rubble-mound breakwaters. Although derivation of a theoretical basis for interpretation and correlation of test data was not attempted, it is believed that the important parameters suggested by Granthem's tests can be used to correlate data obtained in the present testing program. Granthem concluded from the results of his tests that the primary variables affecting wave run-up are the wave steepness (H/λ), the relative depth (d/λ), the angle of the seaside slope (α), and the porosity of the structure (P).

Hydraulic roughness of the slope surface and the angle of obliquity of wave attack (β) are also believed to affect wave run-up. The hydraulic roughness of a breakwater slope is difficult to define quantitatively, however, for the quarry-stone armor units placed pell-mell, such as those used in this investigation, the average thickness of one layer of armor units should provide an approximate measure of this variable. Thus, correlation of the run-up data for rubble-mound breakwaters may be accomplished by the functional relationship

$$R/H = f(\alpha, H/\lambda, P, d/\lambda, r, \beta) \quad \ldots\ldots\ldots\ldots\ldots (20)$$

The percentage of voids in the quarry-stone cover layers of the breakwaters tested was essentially constant, and the angle of wave obliquity was 0 deg. Therefore, for the tests completed to date, Eq. 20 reduces to

$$R/H = f(\alpha, H/\lambda, d/\lambda, r) \quad \ldots\ldots\ldots\ldots\ldots (21)$$

Wave run-up data were obtained by visual observation. The average of five individual readings was recorded for each size wave used in the testing of each section. Each of the five individual readings represented the average run-up for a wave train consisting of from 10 waves to 15 waves.

Results of the run-up observations are presented graphically in Figs. 6 through 8. These data show that the wave run-up factor (R/H) is a function of breakwater slope, wave steepness and, to some extent, the hydraulic roughness of the breakwater surface. The effects of relative depth are obscured by the wide range of scatter in the observed values of run-up, that is attributed to difficulties in defining and observing the extent of run-up on a rough, porous, sloping surface, and the complexity of the phenomenon of wave motion on rubble-mound slopes. The range of scatter should be even larger for wave run-up measurements on full-scale structures. Therefore, it is believed that the upper limits of the envelopes of data points, indicated by the solid lines in Figs. 6 through 8, should be used in selecting design crown elevations when overtopping of a proposed rubble-mound breakwater cannot be tolerated.

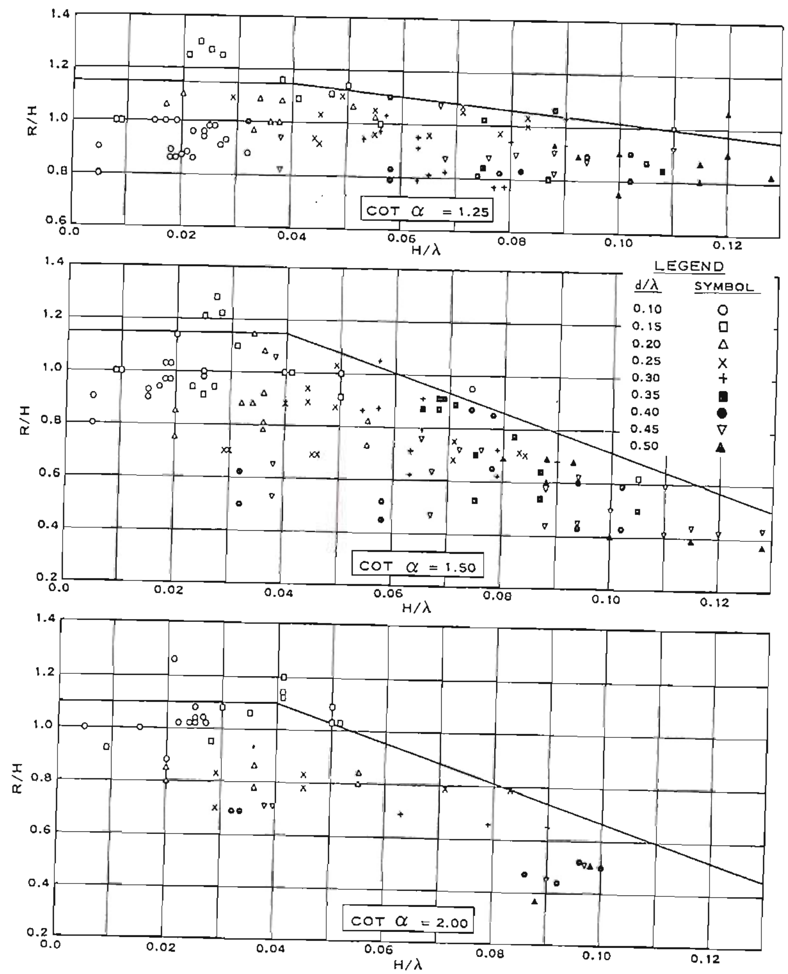

FIG. 6.—WAVE RUN-UP ON RUBBLE-MOUND BREAKWATERS: $R/H = f(H/\lambda, d/\lambda)$

The test data show that breakwater slope and wave steepness are primary variables affecting wave run-up on porous rubble-mound breakwaters of the type tested. Within the range of test conditions used to date, R/H decreases when either cot α or H/λ is increased.

The tests were not designed to study the effects of the hydraulic roughness of the breakwater surface on wave run-up. However, two sizes of armor stones

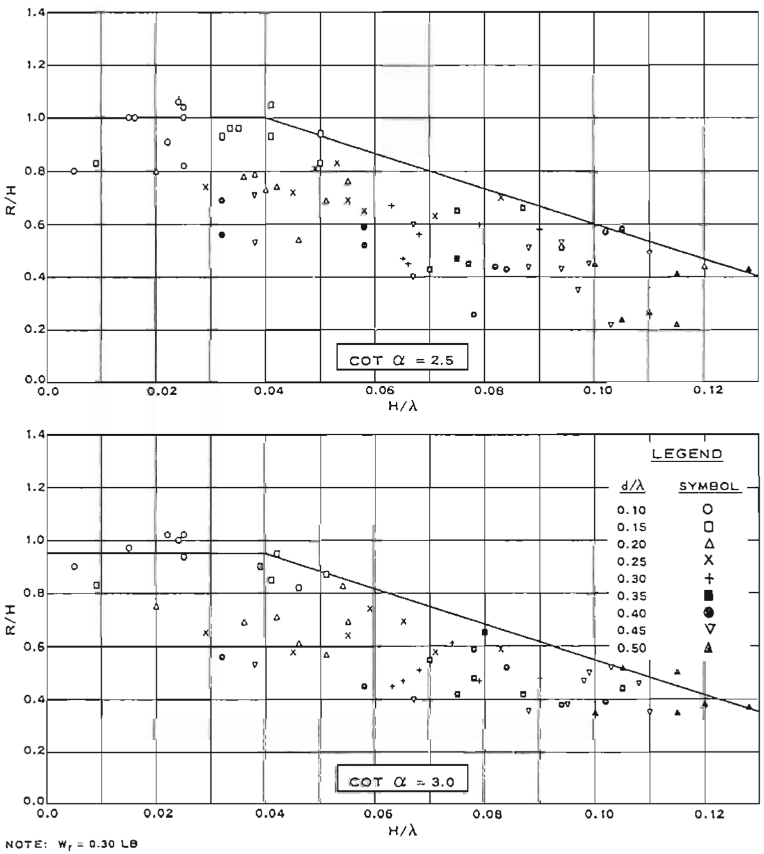

FIG. 7.—WAVE RUN-UP ON RUBBLE-MOUND BREAKWATERS: R/H = f (H/λ, d/λ)

were used in tests of sections with side slopes of 1-on-4 and 1-on-5. Results show that size of stones does not affect wave run-up on a slope of 1-on-4. For the 1-on-5 slope, however, the run-up factors for the smoother surface, that is, the slope composed of 0.10-lb armor stones, averaged approximately 20% greater than the corresponding run-up factors for the slope composed of

0.30-lb stones. This can probably be explained by the fact that waves tend to break more readily on flatter slopes, and as the breaking waves rush up the slope, the depth of flow decreases, resulting in a greater percentage of energy loss for the rougher surface. Also, the flatter slopes provide a greater distance over which the losses of energy may occur. However, these tests are

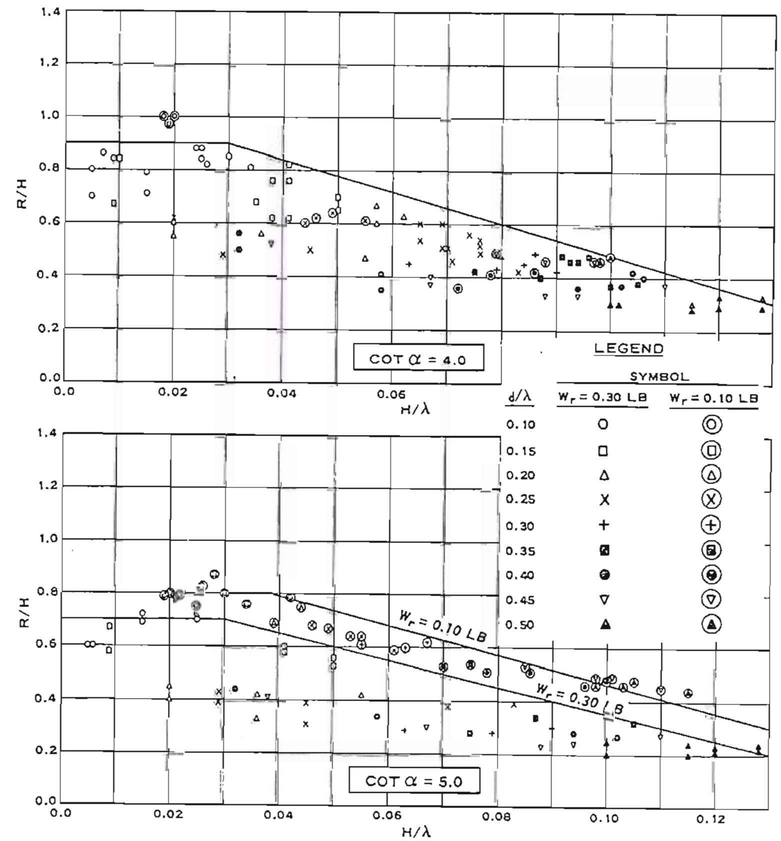

FIG. 8.—WAVE RUN-UP ON RUBBLE-MOUND BREAKWATERS: R/H = f (H/λ, d/λ, r)

not sufficient to determine fully and accurately the effects of hydraulic roughness on wave run-up, and additional tests are therefore necessary.

A qualitative measure of the effects of porosity can be obtained by comparing the results of the present tests with results of those conducted at the Waterways Experiment Station during 1954 and 1955 in an investigation of wave run-up on Lake Okeechobee levee slopes (18). The porosity of the armor-stone cover layers used in the present tests averages about 41%. Levee slopes used in the Lake Okeechobee tests were smooth and impervious. The comparison of the results of these two sets of tests showed that the run-up factor for the smooth impervious slopes averages about twice that obtained for the comparatively rough, porous slopes used in the tests of rubble-mound breakwaters.

Thickness and Porosity of Cover Layers.—Breakwater design requires, in addition to quantitative data to insure stability of armor units and prevent excessive overtopping, accurate information concerning the thickness and porosity of the cover layer as functions of shape, weight, and specific weight of the individual armor units. The thickness of a layered pile of quarry stones or other type of armor units may be computed by the equation

$$r = n\, k_\Delta \left(\frac{W_r}{\gamma_r}\right)^{1/3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

in which r is the thickness of n layers of armor units of weight W_r and specific weight γ_r. The experimental thickness coefficient k_Δ is a function of armor-unit shape and, to some extent, the manner of placing armor units. The porosity of a given number of layers of armor units of given shape Δ, weight W_r, and specific weight γ_r can be determined by the equation

$$P = \left(1 - \frac{N_r\, W_r}{A\, \gamma_r\, r}\right) 100 \quad \ldots\ldots\ldots\ldots\ldots (23)$$

in which P is the porosity in percentage, and N_r is the experimentally determined number of armor units for a given surface area, A. Eqs. 22 and 23 may also be used to estimate the thickness and porosity of underlayers.

The preparation of cost estimates and the necessary planning for construction of breakwaters are facilitated if the number of armor units required for breakwater sections of different types, and for different shapes of units, is known. The required number of armor units for a full-scale breakwater can be determined from the equation

$$N_r = A\, n\, k_\Delta \left(1 - \frac{P}{100}\right)\left(\frac{\gamma_r}{W_r}\right)^{2/3} \quad \ldots\ldots\ldots\ldots (24)$$

Tests to determine k_Δ and P as functions of armor-unit shape have been conducted using tetrapods and quarry stones of seven different shapes (designated A through G) varying from nearly round to flat. The shapes of the rocks were determined by measuring their average dimensions in three mutually perpendicular planes. The rocks were placed pell-mell, by layers, in a square box 2 ft wide and 1 ft high. The surface of each layer was sounded to determine its average thickness, and the number of rocks required to form each layer was counted. The thickness coefficient (k_Δ) and the porosity of the rock layers (P) were then computed by means of Eqs. 22 and 23.

Thickness and porosity data were obtained for one, two, three, and four layers of each shape of rock. Individual stones of each type having approximately the same weights and shapes were selected. The rocks varied in weight from 0.12 lb to 0.46 lb, and had an average specific weight of 176.0 pcf. The manner of placing the rounder rocks (shapes A, B, and C) corresponds to pell-mell construction. For the more elongated rocks (shapes D to G), the manner of placement corresponded roughly to masonry-type construction,

TABLE 4.—SHAPE AND POROSITY CHARACTERISTICS OF QUARRY-STONE ARMOR UNITS

Number of Layers, n	Characteristics of Stone			
	x/z	y/z	k_Δ	P, %
(1)	(2)	(3)	(4)	(5)
		Stone Shape A		
1	1.5	1.2	0.95	38
2	1.5	1.2	0.95	40
3	1.5	1.2	0.93	41
4	1.5	1.2	0.91	38
			Avg 0.94	39
		Stone Shape B		
1	1.6	1.3	0.95	44
2	1.6	1.3	0.93	41
3	1.6	1.3	0.92	40
4	1.6	1.3	0.93	40
			Avg 0.93	41
		Stone Shape C		
1	1.6	1.3	0.92	40
2	1.6	1.3	0.91	39
3	1.6	1.3	0.92	42
4	1.6	1.3	0.91	41
			Avg 0.92	40
		Stone Shape D		
1	1.7	1.4	0.89	43
2	1.7	1.4	0.92	45
3	1.7	1.4	0.91	43
4	1.7	1.4	0.89	42
			Avg 0.90	43
		Stone Shape E		
1	2.2	1.5	0.81	43
2	2.2	1.5	0.81	42
3	2.2	1.5	0.81	42
4	2.2	1.5	0.80	41
			Avg 0.81	42
		Stone Shape F		
1	2.6	1.5	0.76	46
2	2.6	1.5	0.77	47
3	2.6	1.5	0.75	45
4	2.6	1.5	0.75	45
			Avg 0.76	46
		Stone Shape G		
1	3.3	2.5	0.62	49
2	3.3	2.5	0.65	45
3	3.3	2.5	0.66	48
4	3.3	2.5	0.64	46
			Avg 0.64	47

with the largest dimension of the rock parallel to the breakwater slope. This manner of placing elongated stones was used to determine the effects of shape factor on the coefficients k_Δ and P, and is not recommended for full-scale breakwater construction.

Tests of tetrapod armor units were made using the 2-ft-sq box as described previously, with the units placed pell-mell, two layers thick. Tests of tetrapods were also made in which the two layers were placed in the cover layer of a breakwater test section in a more dense and geometrical pattern.

The results of tests to determine the thickness coefficient (k_Δ) and the porosity (P) for the different shapes of quarry-stone armor units are shown in Table 4. Both k_Δ and P vary with shape of the stones; neither, however, vary with the number of stone layers (n). The thickness coefficient has an average value of 0.94 for the shape A (nearly round) armor stones, and decreases as the shape of the stones becomes flatter and more elongated, to an average value of 0.64 for the shape G stones. Porosity increases as the shape of stones becomes flatter and more elongated. The average values of P for the shape A and shape G stones are 39% and 47%, respectively.

TABLE 5.—SHAPE AND POROSITY CHARACTERISTICS FOR TETRAPOD ARMOR UNITS

n	k_Δ	P, %	Placement	Reference
2	1.02	49	Pell-mell	Danel
2	1.06	43	Geometrical	WES data - 30 tests
2	1.00	52	Pell-mell	WES data - 5 tests
2	1.13	46	Pell-mell below swl, geometrical above swl	Hudson and Jackson
3	1.02	46		
4	0.96	46		

The results of tests to determine values of k_Δ and P for tetrapod armor units are shown in Table 5. For tetrapods placed geometrically, two layers thick, average values of k_Δ and P are 1.06 and 43%, respectively. For tetrapods placed pell-mell, two layers thick, the respective values are 1.00 and 52%. In addition, results of Danel (7) and Hudson and Jackson (9) are shown for tetrapods placed pell-mell and placed semi pell-mell, respectively. It is believed that values of k_Δ and P of 1.0 and 50% are representative of the conditions that would obtain when placing tetrapods in two layers to form cover layers of full-scale breakwaters.

CONCLUSIONS

The following is concluded from the results of tests completed (as of 1959) on small-scale rubble-mound breakwaters with quarry-stone and tetrapod-shaped armor units:

1. Iribarren's formula is not sufficiently accurate to be used in designing rubble-mound breakwaters unless it is used in conjunction with values of the experimentally determined coefficient K', as a function of breakwater slope, shape of armor unit, and the other important variables described herein.

2. Use of the Iribarren formula in correlating the stability-test data for rubble-mound breakwaters is not feasible, because the experimental coefficient K' varies appreciably with the coefficient of friction μ, and accurate values of the friction coefficient for the different types of armor units are very difficult to obtain.

3. The assumptions on which the analysis of the phenomenon of waves attacking a rubble-mound breakwater was based are sufficiently accurate for purposes of this investigation.

4. Results of the stability tests conducted for the no-damage and no-overtopping criteria are represented with sufficient accuracy by Eq. 18.

5. The amount of damage that will be done to a quarry-stone cover layer of the type tested by waves larger than the selected design wave can be estimated from the results of damage tests presented in this paper.

6. The safety factor for rubble-mound breakwaters with quarry-stone armor units and $n > 2$, designed in accordance with Eq. 18 using $K_\Delta = 3.2$, is adequate. However, in view of the fact that nature wave trains contain waves of heights as large as $1.6\ H_{1/3}$ approximately 1% of the time, compared with a corresponding value of $1.1\ H_{1/3}$ for the small-scale test waves, there is some doubt as to which of the various wave heights in natural wave trains should be selected as the design wave.

7. Eq. 18, with a value of 8.3 for K_Δ, can be used to design tetrapod cover layers for rubble-mound breakwaters. However, because preliminary tests have indicated that tetrapod cover layers with n = 2 are damaged appreciably by waves slightly larger than $1.3\ H_{D=0}$, it is recommended that design-wave heights for breakwaters having this type of cover layer be selected with caution.

8. For the conditions tested, in which the H/d ratio was comparatively small, the stability of rubble-mound breakwaters is not appreciably affected by variations in the d/λ and H/λ ratios. However, special stability tests concerning a breakwater at Nawiliwili Harbor, Kauai, T. H. (19), where the H/d ratio is critical and waves break directly on the breakwater slope, showed that the ratios H/λ and d/λ are important variables for these conditions.

9. Two layers of armor units are optimum for tetrapod cover layers.

10. Breakwater slope (tan α) and wave steepness (H/λ) are the primary variables affecting wave run-up on rubble-mound breakwaters where the H/d ratio is sufficiently large so that breaking waves do not occur on or seaward of the breakwater slope. Wave run-up decreases when values of either H/λ and cot α are increased.

11. The thickness of cover layers and the number of armor units required to cover exposed slopes of rubble-mound breakwaters can be determined by the Eqs. 22 and 24. Conservative values of k_Δ and P for selected quarry-stone armor units placed pell-mell are 1.0 and 40%, respectively. Corresponding values of k_Δ and P for tetrapods are 1.0 and 50%, respectively.

ADDITIONAL TESTS

Tests being conducted (April, 1959) at the Waterways Experiment Station to determine the relative efficiencies of quarry-stone, tetrapod, tribar, tetrahedron, and other special-shape armor units. Tribars (20),(21) were developed by R. Q. Palmer of the U. S. Army Engineer District, Honolulu, T. H.

As of this date (April, 1959), test results obtained at the Waterways Experiment Station indicate that, with n = 2; (a) tetrahedrons are inferior to both tetrapods and tribars with respect to stability, (b) tribars are slightly better than tetrapods with respect to stability, (c) a tribar cover layer has a slightly higher porosity than a tetrapod cover layer, and (d) a smaller number of tribars are required for a two-layer cover for rubble-mound breakwaters. Also, it has been determined that tribars can be placed as a one-layer unit above still-water level in such a way that the stability provided is considerably greater than the stability provided by two layers of either tetrapods or tribars.

ACKNOWLEDGMENTS

The experiments reported herein were performed in the Wave Action Section, Hydrodynamics Branch, of the Hydraulics Division, U. S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi, by Mr. R. A. Jackson, Hydraulic Engineer, with the aid of various assistants. Their work was under the immediate supervision of the writer. The testing program is sponsored by the Office, Chief of Engineers, U. S. Army, Washington, D. C., under Civil Works Investigation 815, "Stability of Rubble-Mound Breakwaters." The aid of Mr. J. G. Housley, M. ASCE, Hydraulic Engineer, Waterways Experiment Station, in the preparation of material and review of this paper is greatly appreciated.

APPENDIX I.—BIBLIOGRAPHY FOR RUBBLE-MOUND BREAKWATERS

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APPENDIX II.—NOTATION

A = Surface area, square feet;

a = coefficient in Eqs. 17, 18, and 19;

b = exponent in Eq. 17;

C = coefficient;

D = damage parameter, percent;

d = water depth, feet;

F = force, pounds;

f = reads "function of";

g = acceleration due to gravity, feet per second squared;

H = wave height, i.e., the vertical distance from trough to crest, measured at the location of a proposed breakwater, feet;

h = height of breakwater crown above swl, feet;

K = coefficient in new breakwater stability formula for conditions of no damage and no overtopping, varies with shape of armor unit;

K' = coefficient in modified Iribarren formula;

K_D = coefficient in new breakwater stability formula, varies with percentage of damage to cover layer for a given shape armor unit;

k = coefficient;

l = characteristic linear dimension of armor unit, feet;

m = width of breakwater crown, feet;

N = number;

n = number of layers of armor units;

P = porosity of cover layer, percent;

R = wave run-up, vertically above swl, feet;

R = Reynolds number;

r = thickness of cover layer, measured perpendicular to slope of breakwater face, feet;

S = specific gravity, for example, $S_r = \gamma_r/\gamma_w$;

T = wave period, seconds;

t = time, seconds;

V = velocity, feet per second;

W = weight, pounds;

W_r' = buoyant weight of armor unit, pounds;

x = abscissa;

y = ordinate;

z = vertical distance, measured positively upward from swl, feet;

α = angle of breakwater slope, measured from horizontal degrees;

β = angle of obliquity of wave attack, degrees, for example, when wave crest is parallel to breakwater alignment, $\beta = 0$;

γ = specific weight, pounds per cubic foot;

Δ = shape of armor unit;

λ = wave length, feet;

μ = coefficient of friction;

σ = angle of beach slope, measured from horizontal, degrees;

ϕ = angle of repose of armor units, degrees;

$\cot \alpha$ = reciprocal of breakwater slope;

$\tan \alpha$ = breakwater slope;

∂ = partial differential symbol;

R/H = wave run-up factor;

$d/$ = relative depth;

$H/$ = wave steepness;

H/d = relative height;

$\frac{1}{V^2}\frac{\partial V}{\partial t}$ = form of Iversen's modulus; and

swl = still-water level.

SUBSCRIPTS

a = refers to area;

b = refers to break-wave condition;

D = refers to damage of cover layer;

d = refers to drag;

m = refers to inertia;

q = refers to total;

r = refers to armor unit;

s = refers to stability;

v = refers to volume;

w = refers to water; and

Δ = refers to slope factor.

DISCUSSION

JOSÉ REIS DE CARVALHO[3] and DANIEL VERA-CRUZ,[4] A.M. ASCE.—The writers have followed the extensive and valuable work done in the Waterways Experiment Station on breakwater stability, of which Mr. Hudson's paper is a part. The paper shows the new trend in the study program of that station due to the difficulty in obtaining the exact value of the friction coefficient in Iribarren's formula, the accurate determination of which was the purpose of the former program. In spite of the considerable work already carried out, much remains to be done before the design engineer will be able to achieve an optimum design. For that purpose, the design engineer must not only know how to design the armor cover layer, but also, especially when dealing with great depths, to design the underwater slopes below the lower level of the cover layer. He must also know how to solve the problems involved in the singular points of breakwaters, such as curves and heads. When dealing with shallow depths, the situation is very different from that prevailing at great depths, especially on sandy bottoms when the waves break just before or on the breakwater toe. In this case, even when measures are taken against the setting down of the toe, the increased specific gravity of the water resulting from the great amounts of sand stirred up, must be taken into account. Believing that this is an extremely important point, the writers suggest that, although the experimental conditions during the tests are presented in the text, it would be preferable to stress in the figures of the paper summing up the experimental results that these, due to the test conditions, have a restricted field of application.

The Formula Proposed by the Author.—The author considers that the armor units acted on by the wave are subjected to two principal forces: a drag force and an inertia force. It is the opinion of the writers that the inertia force is very important and that, if the influence of the term $\frac{\partial V}{\partial t}$ could be investigated, a better understanding of the phenomena occurring in a breakwater would result. However, it being difficult to determine this influence, the author includes this term in the same coefficient that involves the drag coefficient, the virtual-mass coefficient, and the shape of the units.

To establish the incipient instability condition, Hudson makes the resultant of the two forces developed by the waves equal to the buoyant weight of the armor unit. He, however, takes only the magnitude of those forces into consideration, neglecting their directions. It is the opinion of the writers

[3] Ass't., Research Laboratório Nacional de Engenharia Civil, Lisbon, Portugal.
[4] Civ. Engrg. Ass't., Research Laboratório Nacional de Engenharia Civil, Lisbon, Portugal.

that an effort must be made to express the vectorial character of the forces that is supposed to be all the more important the steeper the slopes. With these simplifications, the proposed formula is:

$$\frac{\gamma_r^{1/3}}{\left(S_r-1\right)W_r^{1/3}} = f\left(\alpha, \frac{H}{\lambda}, \frac{d}{\lambda}, C_d, C_n, K_a, K_v, \ \ldots\right) \ldots\ldots (25)$$

in which the first member is a dimensionless parameter and the symbols are those defined in the paper.

For the test conditions, the author found,

$$\frac{\gamma_r^{1/3} H}{W_r^{1/3}\left(S_r-1\right)} = a\,(\cot\alpha)^{1/3} = \left(K_\Delta \cot\alpha\right)^{1/3} \ldots\ldots\ldots (26)$$

in which K_Δ is the shape coefficient, numerical values of which are presented for quarry-stone and tetrapod-shaped units.

Criteria of Stability.—One of the chief advantages of Hudson's paper lies, no doubt, in the adoption of numerical, consequently well defined, stability criteria. In all the experimental studies so far carried out, both at European laboratories and in those previously described by the Waterways Experiment Station, the criterion left an important part of the experimenter's personal judgment in regard its definition and above all its appreciation. It has thus been impossible to make an accurate comparison of the very numerous results already obtained by the different laboratories in which the subject was studied.

The writers believe, nevertheless, that a real effort should be attempted to improve the stability criterion established by the author, for the no-damage condition at least, so that the results can be generalized for use in the design of similar structures.

In fact, the geometrical characteristics of the breakwater sections used in the tests on the quarry-stone armor units and the tetrapod armor units being considerably dissimilar, the results obtained are not comparable, because, as Hudson states, the percentage of damage is proportionally higher for the cover layer of smaller volume. It seems that the most adequate criterion for the no-damage condition should be one in which only the active portion of the breakwater and one or two layers of the protective cover were taken into account.

As regards the tests for determination of the damage or safety factor, it seems that the results obtained apply only to the types of breakwater sections tested. It suffices to point out that, according to Hudson, failure of the tetrapod cover layer occurs for values of $H/H_{D=0}$ slightly exceeding 1.3, whereas for identical values of $H/H_{D=0}$ in the quarry-stone armor unit tests, only 5% to 15% damages were observed. Consequently, we believe that safety factors should be determined for each particular case, as it is very difficult, if not impossible, to find a criterion of stability in which the multiple causes conducive to damage in breakwater subjected to waves with a $H > H_{D=0}$ could be considered.

Iribarren's Formula.—In the deduction of his formula, Iribarren considers as the principal resistant force the one resulting from friction between blocks. His formula is Hudson's Eq. 1.

With regard to μ, Iribarren says that it will be very close to the tangent of the natural slope of the layer-units. According to Iribarren, μ is greater than 1, but he recommends that one use $\mu = 1$, that would introduce a certain safety factor. This point must be explained, and the writers need to present here the conditions near the quarry-stone breakwater, the observation of which permitted Iribarren to determine the coefficient to be used with quarry-stone breakwaters. Iribarren used the non-homogeneous expression of Eq. 1,

$$W_r = \frac{N H^3 S_r \mu^3}{(\mu \cos \alpha - \sin \alpha)^3 \left(S_r - 1\right)^3} \quad \ldots\ldots\ldots\ldots (27)$$

in which

$$N = K' \gamma_w \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

The breakwater observed by Iribarren was of quarry stone with a sandy bottom, its water depth in high water was 4.5 m and the slope toe lay at the low-water level. The stones had a weight of 3 metric tons, $S_r = 2.65$, and the equilibrium slope was $\cot \alpha = 3.1$. Iribarren assumed that the height of the maximum wave was $H = 4.5$ m (water depth in high water) and $\mu = 1$. A value of N results equal to 0.015. If, according to Iribarren, one takes $\mu > 1$, a new value would result for N. Let us call this new value N'. It is easy to see that $N' > N$ and that, with such a value N' and the corresponding μ, no safety factor would have been introduced by taking $\mu = 1$ and the corresponding value of N. If now, according to Iribarren, we assume that N and N' are not variable with the slope, it can be seen by a simple computation, that $\mu = 1$ is an unsafe assumption when $\cot \alpha < 3.1$, and a safe one when $\cot \alpha > 3.1$. However, the safety factor introduced in these slopes varies with the slope, and this variation can be very important (for example, if the actual value is $\mu = 1.09$, the safety factor with $\mu = 1$ is about 1.1 if $\cot \alpha = 2$ and 2.2 if $\cot \alpha = 1.25$). The difficulty in defining with exactitude what is the natural slope, makes the use of Iribarren's formula too uncertain.

Besides, it is the opinion of the writers that, owing to the particular conditions near the quarry-stone breakwater observed by Iribarren (very small depths and sandy bottom), the generalization of the coefficient determined is very problematical.

Comparison Between Hudson's and Iribarren's Formulas.—Let us consider Eq. 26. We can give a similar expression to Iribarren's Eq. 1:

$$\frac{\gamma_r^{1/3} H}{W_r^{1/3}\left(S_r - 1\right)} = \frac{(\mu \cos \alpha - \sin \alpha)}{K'^{1/3} \mu} \quad \ldots\ldots\ldots\ldots (29)$$

Let $\left(\frac{1}{K'}\right)^{1/3} = K$, equating the two second members of Eqs. 26 and 29:

$$K \cos \alpha - \frac{K}{\mu} \sin \alpha = K_\Delta^{1/3} (\cot \alpha)^{1/3}$$

$$K \left(\cos \alpha - \frac{1}{\mu} \sin \alpha\right) = a (\cot \alpha)^{1/3}$$

$$K = \frac{a (\cot \alpha)^{1/3}}{\cos \alpha - \frac{1}{\mu} \sin \alpha} = a \frac{(\cot \alpha)^{1/3}}{\cos \alpha - \frac{1}{\mu} \sin \alpha}$$

$$K'^{1/3} = \frac{\cos \alpha - \frac{\sin \alpha}{\mu}}{a (\cot \alpha)^{1/3}}$$

$$K' = \frac{\left(\cos \alpha - \frac{\sin \alpha}{\mu}\right)^3}{K_\Delta \cot \alpha} \quad \ldots\ldots\ldots\ldots\ldots\ldots (30)$$

This expression relates Iribarren's coefficient K' with Hudson's coefficient K_Δ. And, K_Δ being a constant for the test conditions, the preceding expression shows that Iribarren's coefficient is a function of the slope α.

Using Eq. 30 we can always suppose $\mu = 1$ when $\cot \alpha$ is greater than one. By this means, the writers think that Iribarren's formula is susceptible of experimental verification, provided that its coefficient is not considered a constant.

Summing up, the writers think that the great merit of Hudson's coefficient lies in the important advantage of being constant, whatever the slope. It should not, however, be called a shape coefficient, as it necessarily involves the friction as well and is also dependent on the physical nature of the surface of the blocks.

WILLIAM H. BOOTH, F. ASCE.[5]—Due to the peculiar character of rubble structures and the phenomena of wave action thereon, it has been difficult, in the past, to arrive at designs that are both safe, from a structural standpoint, as well as being economical to construct. The many variables present in the phenomena of waves attacking a rubble structure greatly complicate the problem of analysis. The science of rubble breakwater structures is still in an early stage of development. Hudson has done a great service to the engineering profession in presenting the model data developed to date. However, much work remains to be done.

[5] Engr., Corps of Engrs., Gravelly Point, Washington, D. C.

Until recent years, the design of rubble-mound breakwaters was usually based on observed performance of like structures without any regard to the wave characteristics at the proposed location. As a result, several rubble-mound structures have been damaged and maintenance costs have been high.

The original purpose of the laboratory investigation was to provide data from which an efficient design of rubble-mound breakwaters could be selected for different conditions of use and wave attack. It was believed that the results of the investigation would allow the formulation of equations for more scientific designs. The original investigation has been expanded and the results to date are contained in the paper.

The new stability formula presented has been used to determine the weight of armour units for rubble-mound structures on several projects constructed by the Corps of Engineers. However, the values of the experimentally determined coefficient (K_{Δ}) should not be considered as final at this stage of the testing program. Additional model studies of rubble-mound breakwaters are being conducted in a wave tank capable of producing 6-ft waves. Information from these studies will provide data on scale effects. The wave tank is located at the Beach Erosion Board, Washington, D. C.

The weight of the tetrapods described by John E. Deignan,[6] F. ASCE, was determined from results of the small scale model study. His paper presents some of the problems encountered in the design and construction of projects of this nature. From a structural and hydraulic standpoint, the tetrapod is considered to have a desirable shape for good stability when subjected to wave forces. However, an indicated royalty cost in the United States of $3.00 per cu yd of tetrapod volume may limit its use to a small number of projects due to economy.

The tribar has been patented in the United States by R. Q. Palmer, M. ASCE. The patent provides for the manufacture and use by or for the Government for its purposes without royalty payment. Patent applications are pending in several countries. This shape of armour unit has been manufactured and used for the repair of the breakwater at Nawiliwili Harbor, Island of Kauai. The project is under the jurisdiction of the U. S. Army Engineer District, Honolulu. The weight of the units was determined under the same testing program. This breakwater is a rubble-mound structure about 2,150 ft in length that had been damaged by storm waves. Quarry rock of sufficient weight to insure stability of the breakwater, without excessively flat slopes, was not available. Because preliminary model studies indicated that the tribar was slightly better than the tetrapod, it was decided to prepare three alternate plans for reconstruction of the breakwater. One plan was designed to use only stone. Two other plans were designed using precast concrete shapes on the seaward slope. One of these called for the use of 20-ton tetrapods and the other called for the use of 17.8-ton tribars. The tribar armour facing as well as the tetrapod armour facing, requires a solid backing to present ravelling when subjected to design wave overtop the structure. Therefore, both plans provided for concrete caps. Bids for the reconstruction were opened by the Honolulu District Engineer on April 30, 1958. The low bid was on the tribar plan. Since completion of the project, the breakwater has been subjected to heavy wave action. Preliminary information available at this time (December, 1959) indicates

[6] "Breakwater at Crescent City, California," by John E. Deignan, Proceedings, ASCE, Vol. 85, No. WW3, September, 1959, p. 167.

minor shifting of the tribars down the slope. The maximum measured distance was about 9 in. This movement was expected due to the consolidation of the rubble-mound and the tribar cover layer.

It is desired to emphasize the point that weight of armour units, determined in accordance with experimental data presented, will not necessarily apply to the seaward end, or head, of breakwaters. The ends are subjected to severe wave action and must be strengthened accordingly. In 1958, storm waves moved tetrapods weighing approximately 33 tons each off the heads of the breakwaters at Kahului Harbor, Territory of Hawaii. The tetrapods on the head were of the same weight as those designed and placed on the seaward slope of the breakwater. The expanded laboratory investigation program includes tests for the treatment of the head of breakwaters. It is hoped the investigation can be continued in order to provide sufficient data to insure safe and economical designs of rubble structures.

FRANCIS B. SLICHTER, F. ASCE.[7]—Hudson's development of hydraulic model testing procedures and formulas for evaluating test results are significant contributions toward rationalizing the variables involved in breakwater design.

To be effective, a breakwater must accomplish its purpose of reducing the waves generated by severe storms to tolerable levels in the designated harbor area. Often, several alternative breakwater layouts can be developed that, although not equal in performance, may give acceptable results. Consequently, decision on the layout to be selected will depend on an economic analysis considering first cost, maintenance costs, and relative performance in protecting the harbor area. In fact, economic limitations can force the acceptance of a breakwater design that allows severe harbor disturbance or breakwater damage during critical storm periods.

Breakwater design is far removed from status as a science in which application of formulas will lead to an exact solution. In addition to gaps in knowledge of wave phenomena wherefrom we derive the "design wave," we lack finite information on the reaction of the breakwater armour units and of its core material and foundation under the complex hydraulic impact of the waves. When we adopt a design using armour units of pre-cast concrete, we must accept unknown limitations imposed by structural strength of fabricated units and durability of concrete (and reinforcing steel, if used) in seawater. Therefore, although the guide lines contributed by Hudson's work are valuable aids, the design of breakwaters must remain largely in an area dominated by sound judgment based on observations of past experience.

In an effort to augment its experience record on fabricated armour units, the Corps of Engineers has constructed a tetrapod armoured breakwater at Crescent City, Calif., and has used the tribar shape in reconstruction of the breakwater at Nawaliwili Harbor, Island of Kauai, Hawaii. The weights of individual armour units are 25 tons, and 17.8 tons, respectively. Observations over several years are anticipated before conclusive data can be assembled as design criteria.

The value of experience is demonstrated by that of the Corps of Engineers at Kahului Harbor, Island of Maui, Hawaii. At this harbor two breakwaters converge to form the harbor entrance. The design wave height that is in the

[7] Chf., Engrg. Div., Civ. Works, Office, Chf. of Engrs., Washington, D. C.

general magnitude of 28 ft actually will overtop the breakwater. The breakwaters were repaired in 1952, with armour stone of 12-ton weight. In the storm of March 1954, the ends of both breakwaters were damaged to the extent that restoration was essential to continued harbor use. Past experience with breakwater heads subjected to wave attack from divergent sources has demonstrated a requirement for monolithic cast-in-place concrete blocks with weight of several hundred tons. However, funds in the amount needed for such head construction were not available, and it was decided to rebuild the heads with the same weight armour unit selected for the breakwater proper. Consequently, the 33-ton units of the tetrapod shape used on the breakwater were also placed around each head. In December 1958, the breakwater was subjected to a storm delivering waves of design magnitude. Examination, subsequent to the storm, revealed that the tetrapods had been stripped from the face of each breakwater head. Some of the 33-ton tetrapods had been moved over 100 ft into the interior of the harbor. In fairness to Neyrpic (Grenoble, France), inventors of the tetrapod, their representative had advised that tetrapods used in armouring the head should be of greater weight than those used on the breakwater proper.

In view of the magnitude of the weight scale ratio between the author's model armour unit and the prototype (over 300,000 for the Kahului tetrapod), it is doubted that there is any practical design significance in abandoning the Iribarren formula with its troublesome friction coefficient in favor of Hudson's new formula. On the other hand, the derivation of Hudson's formula results in a valuable tool for his use in evaluation of his model test data.

In developing his formula for stability, he has considered that friction between armour units can be neglected. This assumption is questioned, in particular for fabricated shapes such as the tetrapod and tribar. It would appear that part of the large differences between stone and fabricated shapes in test values for K could be attributed to reactions caused by shear forces between units. Further research to explore the relative effects of unit-shape, porosity and reaction between units should contribute information useful in breakwater design.

Further work also is needed in developing criteria for armour shapes and in placement procedures for aid in design of the seaward end (head) of a breakwater.

LELAND B. JONES,[8] F. ASCE.—The advantages of the formula developed by Hudson are evident when one considers the uncertainties inherent in breakwater design. Use of the formula with the shape factor K_Δ, if selected with reasonable care, should lead to consistently better design than use of Iribarren's formula in which the accurate selection of the friction factor is both critical to accurate design and difficult to do, especially when manufactured shapes are used.

Because close control can be maintained in sizes and shapes of materials used for breakwater construction, testing of uniform materials leads to practicable design and reduces the number of variables that would otherwise require testing. Also, breakwaters are usually constructed in relatively shallow water, and it has not been necessary to investigate reduction in stability requirements very far below the water surface. For these reasons, rubble-mound breakwater investigations have been confined primarily to testing the stability of uniform size quarry stone and manufactured units at elevations within the range of wave action.

[8] Civ. Engr., Chf. of Soils Sect., Corps of Engrs., Walla Walla, Wash.

When railroads, highways, and other structures are constructed along lakes and reservoirs it is often necessary to protect them against wave action, but it has not been considered economical to require uniform size materials or manufactured units for this purpose. Instead, riprap consisting of quarry-run rock having a gradation range is used. The selection of rock sizes and other riprap features has not always been very rational, and often they have been based on the engineer's judgment, supplemented by experience and the limited amount of available data. In the past some effort has been made to utilize breakwater design criteria for determining required rock sizes needed for graded riprap, but there has always been an uncertainty as to the effects of gradation and how to determine the significant rock sizes.

Development of flood control, navigation, and water power along the lower Columbia and Snake Rivers, in Oregon and Washington, makes a more precise knowledge of riprap requirements necessary. The Corps of Engineers has completed three dams on the lower Columbia River: Bonneville, The Dalles and McNary. Construction is just commencing (1960) on John Day Dam on the Columbia River, and Ice Harbor Dam on the lower Snake River is nearing completion. Three more dams are authorized for construction on the lower Snake River downstream of Lewiston, Idaho. All of these completed, under construction, and authorized projects have these things in common: all are in deep, narrow, steep-walled canyons; all have railroad and highway relocations that are or will be located along the edges of the reservoirs with high fills extending well below the water surface; and all are subject to relatively severe wave action. Riprap design for each of the completed projects was based primarily on Corps of Engineer's experience criteria, but required rock sizes were compared with W_r in Iribarren's formula, in which the assumed friction factor was 1.05. There was some question as to the validity of this assumption because W_r might represent a stone-size smaller than maximum size, water pressures inside the riprap might assume greater importance than in open, one-size stone, and the friction factor might be in error. It was found by measurement that dumped rock fill generally assumed outside slopes of 1 vertical on 1.3 or 1.4 horizontal, that could mean that a friction factor of 0.75 or 0.8 might be more reasonable. If the friction value were 0.75 the stone weight would be increased four times for a 1-on-2 slope.

Completion of Bonneville, The Dalles, and McNary Dams has given the Corps of Engineers, highway departments, and railroad companies with relocations along the reservoirs opportunity to observe the effects of wave action. It is obvious now that additional study is required for adequate and economical riprap design. More than 100 miles of riprap have been provided along the completed projects, and several hundred miles of riprap will be required along projects under construction and authorized. Inadequacies and overdesign can result in substantial extra cost here, where they would not be particularly important on smaller jobs.

As a result, the Waterways Experiment Station is investigating the following features of riprap design and other protection against wave action. These tests are being made for the Walla Walla district, Corps of Engineers, under the direct supervision of the writer.

1. Effects of wave heights and wave periods on graded riprap at the water surface and at variable depths below water surface;

2. necessary depth of riprap toe below water surface when embankment is rock fill;
3. necessary depth of riprap or rock fill toe below water surface when embankment is gravel;
4. height of wave ride-up and depth of wave rundown for graded riprap;
5. limited tests for variable riprap slopes; and
6. comparative effects of wave action on riprap for angles of attack of 30°, 60°, and 90°.

While the testing program has not progressed to the point at which results can be published, it has produced some interesting trends, as suspected, and these trends may become important factors in riprap design. For example:

a. The value of W_r for quarry stone, as determined from the author's formula, was compared with results of model tests for graded riprap. Preliminary data indicate the riprap stone size comparable to W_r may be in the order of 20% by weight or 50% by size of the maximum size stone. This was based on tests for well graded riprap where the minimum size stone was about 6% by weight of the maximum size. On this basis, it might be necessary to require more uniform stone sizes where wave action is severe and available stone sizes tend to be small, or where the volume of material required for graded riprap would make the cost excessive.

b. Tests on river gravel having 2-in maximum size and underwater slope of 1 vertical on 2 horizontal indicate that gravel fill may require riprap protection for 10 ft below water surface for wave heights in the order of 4 ft or 5 ft, and for about 30 ft below for wave heights in the order of 8 ft. The wave period appears to have considerable effect on stability of the gravel. It is interesting to note that the gravel, when tested for damage by wave action at water surface, resisted wave heights of about 0.5 ft.

Because tests are incomplete, it is not possible to present firm conclusions. However, it is believed that the work being performed on riprap is of general interest. These studies are, of necessity, geared to problems and materials typical of the Columbia and Snake River areas, but it is expected that significant data will be obtained to be of value on other related work. It will also be a starting point for future expansion of riprap testing.

OMAR J. LILLEVANG,[9] M. ASCE.—The engineer who struggles to keep abreast of the growth of knowledge in the field of harbor and coastal engineering can find valuable assistance in Hudson's paper. The bibliography refers to information that is found in only a few libraries and not many of the references include discussions by reviewers or supplementary contributions. To the reader who finds too little time for study, the discussion by others of the work of technical authors is a great aid to quick evaluation. Wider distribution of information on new work and ideas is needed; it is appreciated that work such as the present paper, originally given limited distribution by the Waterways Experiment Station, is published in the journals of the Society.

Design waves for rubble-mound structures may not be the same for stone size determination as for concerns with uprush of the seas. Where wave statistics are available for a problem site the record normally presents frequency of occurrence of the "Significant" waves and periods, the term being a precise one relating to the average of the highest one-third of the waves in a sample. Other averages within the sample are related by Hudson, and, in 1952, R. R. Putz[10] published an excerpt from an actual wave record that graphically shows characteristic variations. Fig. 9 is Putz' illustration, and from it one can clearly appreciate the uncertainty of what may come next in any wave series.

If the major waves were always tightly grouped, any heavy overtopping by uprush on seawalls would be an extremely burdensome problem of water disposal. Some risk can prudently be taken in most instances that the highest waves do not ordinarily group themselves within the series, and the structure design may, therefore, often be related to some lower average wave. Overtopping by the wave uprush doesn't necessarily mean the design is a failure. Accepting overtopping and providing for it in the design may be the best economic choice in many instances. Fig. 10 shows three basic variations of a reference cross section that may be considered in treating with the wave run-up question. Hudson's work provides excellent tools for raising the crest of the structure or for flattening the seaward slope to minimize or eliminate overtopping. It is apparent that if one would consider making the crest wider than the minimum width dictated by construction procedures, and drain off the cresting water through the armor voids, then data on the volume of water at any elevation in an uprushing wave would be most helpful. This need for knowledge of volume versus elevation becomes particularly acute if the structure is a seawall of normal crest width, and gutters or storm drains must be proportioned for the area landward of the seawall. If such information exists, its presentation should be sought; if not, it is an area of investigation which could very beneficially be undertaken in one of the research centers active in this field.

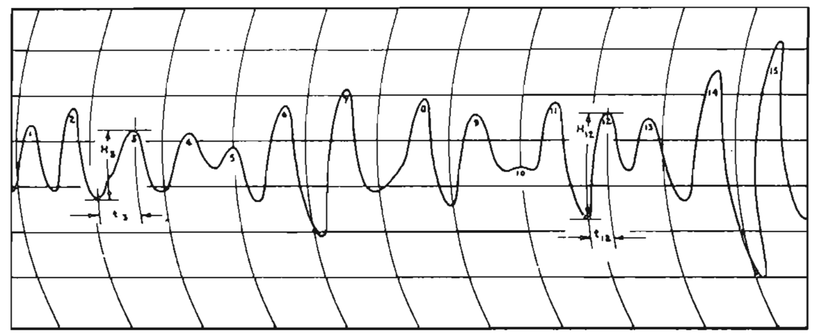

FIG. 9.—TYPICAL SECTION OF A WAVE RECORD

9 Asst. Engr., Leeds, Hill and Jewett, Los Angeles, Calif.

10 "Statistical Distribution for Ocean Waves," by R. R. Putz, Transactions, AGU, Vol. 33, No. 5, p. 685.

Armor stone might be selected of a size to completely resist the maximum wave in a train of waves, but something less is usually acceptable. Armor stone in well-built and successfully maintained breakwaters and seawalls are not often as large as would be indicated to be necessary from application of formulas that neglect interaction between stones (other than friction), or consider the interaction to be a safety factor. "Well-built," in this writer's view, requires that armor stone be placed with care, not dumped, and that above the low water line the individual pieces be selected for shape of fit to other stones and laid in juxtaposition with them and in an attitude of optimum stability. Wedging may then be anticipated as the whole structure accepts wave attack,

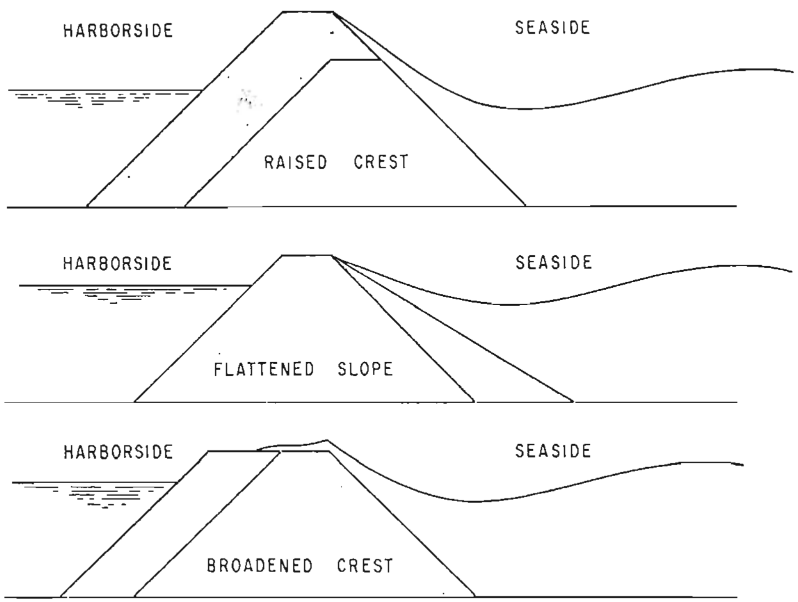

FIG. 10.—ALTERNATIVE TREATMENTS FOR WAVE RUNUP

and settles into a coherent unit. Lacking such careful selection and placing of armor stones, the wave-affected slope should be flattened so only friction is relied on, if maintenance is not to be excessively costly soon after the first heavy storms. Obviously, the variability of dimensions of stones of a given weight, their angularity or blockiness, will affect the degree of stability achieved by careful placement. In essence this is recognized in Hudson's use of a "Shape Factor." It would be useful if in the author's closing discussion some added data to illustrate the Shape Factor were included.

The writer is intrigued by Hudson's statement that the effect of non-breaking wave forces is comparable with that of the forces of breaking waves, though the wave motions are different in characteristics. An elaboration of this statement would be most interesting. Would this view lead one to rational design procedures in determining underwater slopes and stone sizes? For that unseen portion of rubble structures little has been found.

Presumably the tests, under way in the Beach Erosion Board's giant wave flume, on stability of very much larger stones in rubble-mound structures will evaluate the significance of scale effect in Hudson's experiments. As a follow-up of Hudson's paper, it is hoped the results of the large-scale work and its relationship to the small-scale will be the subject of a paper. The subject is worthy of the wider readership.

THORNDIKE SAVILLE, JR.,[11] M. ASCE.—The author has ably presented a quantity of useful data leading to continued progress toward a sounder basis for stability and economic design of rubble-mound structures. In the course of the extensive testing carried out, Hudson has also obtained a great deal of valuable data pertaining to wave run-up on rubble slopes. These data are presented in Figs. 8 through 10 in which relative run-up (R/H) has been plotted as a function of the wave steepness (H/λ) determined for the water depth at the structure toe. As the wave steepness for any particular wave train critically depends on the depth of water for which it is determined (varying by a factor of as much as 3 or 4, for ordinary waves), the plotted points have also been segregated according to relative depth (d/λ). However, the wave steepness range covered for each relative depth tested is generally small, and, as the author notes, the scatter induced by measurement difficulties and general complexity of action is large, so that the true effect of relative depth is obscured. Hudson has, accordingly, drawn a near-envelope curve, and indicates that he feels it should be used for determining design values.

Unfortunately, because values of both height (H) and length (λ) depend on the relative depth in which they are measured, use of such a curve under the assumption that it is independent of relative depth produces different values of run-up for any particular wave train, depending on where the wave characteristics are measured. An illustrative example is given in Table 6, in which values of wave run-up (R) as determined from Fig. 7 for a 1 on 2.5 slope are tabulated for a single wave train moving from deep water into shallow. The tabulations are made for arbitrarily selected water depths for which the wave characteristics are determined. The particular wave train chosen has a height and period of 8 ft and 6 sec in a 10-ft water depth. In making the computations, Fig. 7 was used as though it were completely independent of relative depth.

It will be noted that, in this case, predicted run-up of 5.9 ft is obtained if the wave characteristics, as measured in 10 ft of water, are used. However, if the designer had chosen to determine his wave characteristics at a depth of 25 ft instead, he would have obtained a predicted run-up of 6.8 ft. And if he were working with deep water characteristics, he would have determined a 7.8 ft predicted run-up. As the table shows, a different value of run-up will be determined for each relative depth value for which the wave characteristics may be measured. Actually, however, we know that a wave incident on a given structure must have only one value of run-up for a particular shore condition.

[11] Asst. Chf., Research Div., Beach Erosion Bd., U. S. Army Corps of Engrs., Washington, D. C.

It would seem, therefore, that run-up curves such as these should be referred to a specific relative depth rather than being considered as independent of relative depth. Fortunately, there are readily available methods of determining heights and lengths for any depth, or relative depth, if the height and length are known at some particular depth. Wiegel, for example, has tabulated[12] ratios of height and length to their deep water values as a function of relative depth. Using these tables, Hudson's data for each relative depth may be transformed into equivalent data for a single selected reference relative depth. The particular relative depth selected as reference is immaterial, and one is probably as good as another. However, a deep-water-reference relative depth has previously been used[13,14] and has some advantages because deep water values are frequently the actual known values, and the tabulated ratios relate to deep water values.

Accordingly, the author's run-up data for the 1 on 2.5 slope have been referred to a deep water basis (d/λ = 1.0), and are replotted in Fig. 11. The scales of the graph have been changed to logarithmic (rather than to arithmetic, as the author used) to stretch out the points at the lower steepness values.

TABLE 6.—RUN-UP COMPUTATIONS, 1 ON 2.5 SLOPE

Depth (d) (ft) (1)	Height (H) (ft) (2)	Period (T)(Sec) (3)	Length (λ) (ft) (4)	$\frac{d}{\lambda}$ (5)	$\frac{H}{\lambda}$ (6)	$\frac{R^a}{H}$ (7)	R^a (ft) (8)	$\frac{R^b}{H}$ (9)	R^b (ft) (10)
10	8.00	6.0	101.5	.0986	.0788	.74	5.9	.80	6.4
15	7.56		120.6	.124	.0627	.84	6.4	.84	6.4
18.45	7.40		130.9	.141	.0565	.89	6.6	.86	6.4
25	7.26		145.8	.172	.0497	.93	6.8	.88	6.4
36.9	7.28		163.9	.225	.0444	.96	7.0	.89	6.5
184.5	7.93		184.5	1.00	.0430	.98	7.8	.81	6.4

[a] From Hudson, Fig. 9.
[b] From present discussion, Fig. 3c.

Such stretching seems to indicate a possible tendency for relative run-up to decrease with decreasing steepness below a certain critical steepness value—although this conclusion is largely dependent on the location of but a single point.

All plotted points in this figure now refer to a single relative depth, and a mean curve could be drawn through them. However, the scatter is still large, and a curve somewhat above the mean has been drawn. (This curve is also plotted in Fig. 11 and 13(c), on arithmetic scales.)

Similar curves (also referred to a deep water relative depth, (d/λ = 1.0) have been drawn for the other slopes for which the author gives data. These

[12] "Gravity Waves, Tables of Functions," by R. L. Wiegel, Council on Wave Research, Engrg. Foundation, 1954.
[13] "Wave Run-up on Shore Structures," by Thorndike Saville, Jr., Transactions, ASCE, Vol. 123, 1958, p. 139.
[14] "Wave Run-up on Roughened and Permeable Slopes," by R. P. Savage, Transactions, ASCE, Vol. 124, 1959, p. 852.

are shown as a family of curves in Fig. 12. Also shown in this figure, as a matter of comparison, are similar curves for smooth slopes as derived from previously published reports.[13,14,15,16] The roughness and permeability of the rubble had an obvious reducing effect! These curves are shown in terms of H_o'/T^2 (in which T is the wave period) rather than H_o'/L_o purely as a matter of personnel convenience, because the deep water wave length, L_o, equals 5.12 T^2, the two ratios bear a constant relationship, with one being essentially five times the other.

As these curves are all referred to a deep water relative depth, the designer in using them must compute the deep water wave steepness and height from whatever design wave information he has, and use these values in computing the run-up. Similar curves referred to any other relative depth could, of course, be drawn, and would be equally usable. However, in using them, one

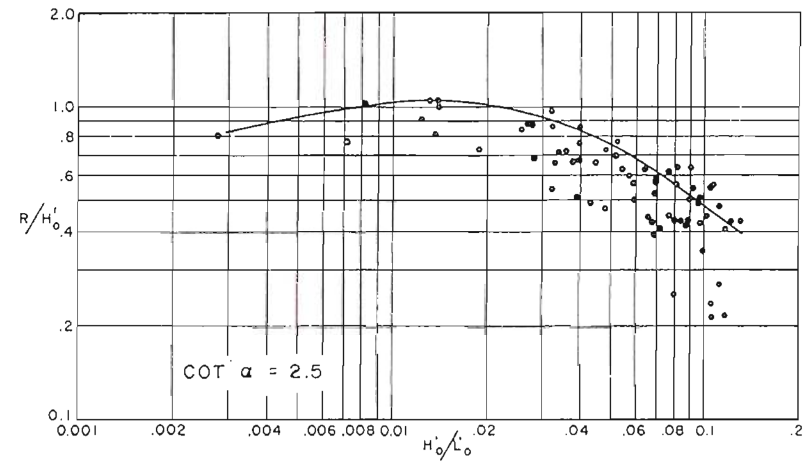

FIG. 11.—WAVE RUN-UP ON RUBBLE SLOPE, 1 ON 2.5 DEEP WATER REFERENCE RELATIVE DEPTH

must know the relative depth of reference, and be careful to use wave characteristics for that particular reference relative depth. This is true even though this reference depth may not physically exist in the actual field location (as "deep water" does not for many lakes, for example). In this way, one always gets the same run-up computed for a given wave and structure. For the case tabulated previously, and using the curve shown in Fig. 12, this run-up value would be 6.4 ft.

[15] "Wave Run-up on Composite Slopes," by Thorndike Saville Jr., Proceedings, 6th Internatl. Conf. on Coastal Engrg., Council on Wave Research, Engrg. Foundation, 1958.
[16] "Shore Protection Planning and Design," Beach Erosion Bd., Tech. Report No. 4, U. S. Army Engr. Beach Erosion Bd., Washington, D. C.

FIG. 12.—WAVE RUN-UP ON RUBBLE MOUND AND SMOOTH SLOPES (FOR VALUES OF $d/H_0 > 3$)

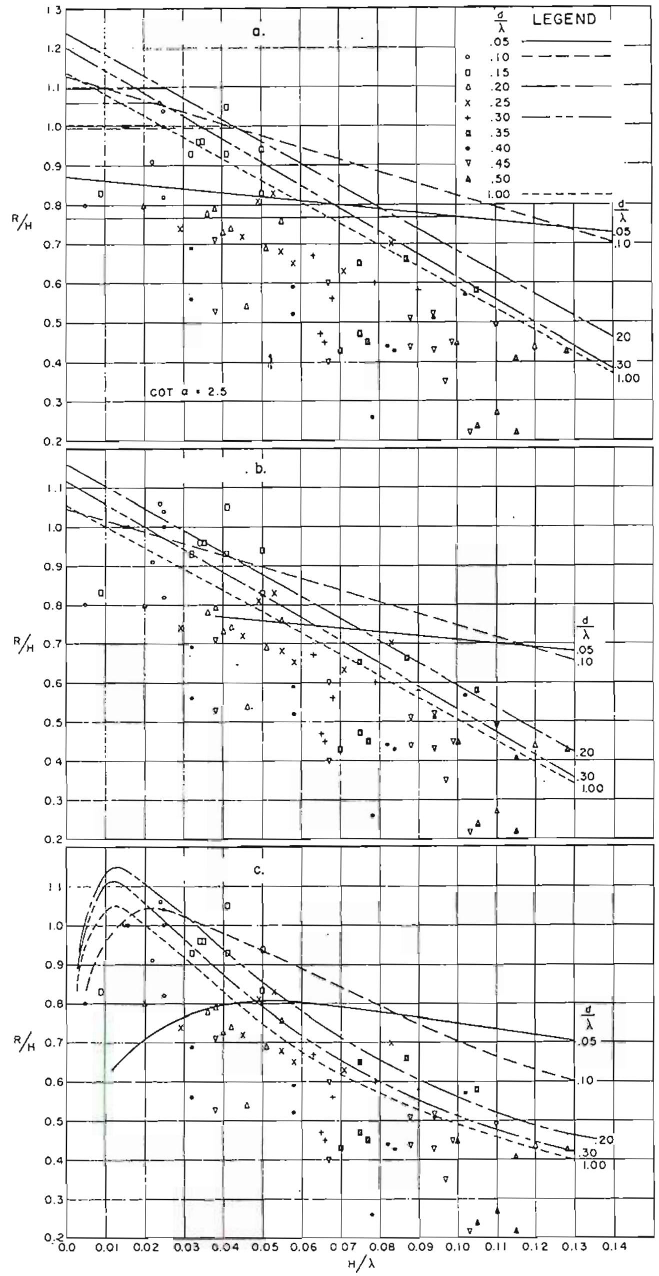

FIG. 13.—WAVE RUN-UP ON RUBBLE SLOPE, 1 ON 2.5. RELATIONSHIP TO RELATIVE DEPTH FOR SEVERAL POSSIBLE RUN-UP CURVES

If, because of the large quantities of data to be handled, one prefers not to have to make the many and repeated computations involved in obtaining wave characteristics for the referenced relative depth, curves for other relative depths may be determined from the first curve. These curves, if plotted as a family of curves, enable interpolation directly for values referred to any other reference relative depth. This has been done in Fig. 13 for the 1 on 2.5 slope data. In so doing, three different basic deep water reference curves have been drawn. The first of these, in Fig. 13(a), is essentially an envelope curve; the second, in Fig. 13(b), is somewhat less conservative but still retains the straight line character used by the author; the third is the (logarithmic) curve from Fig. 11. In these figures, the abscissa (wave steepness, H/λ) values for each of the curves shown are then those applicable to the particular relative depth (d/λ) for the curve. The points on the figures are those shown in Fig. 7 for the 1 on 2.5 slope, and are segregated according to relative depth by the same symbols that he used. Unfortunately, the degree of scatter still prevents a good estimation of the accuracy of the various curves shown.

Values of run-up factor (R/H) have been determined from the curves in Fig. 13(c) for the 8-ft, 6-sec wave previously used, and are also shown in Table 1. The values obtained for run-up are also tabulated and it may be seen that essentially identical values (approximately 6.4 ft) were obtained for all cases, as of course was to be expected. As a matter of comparison, the run-up values that would be obtained by use of the curves in Figs. 13(a) and 13(b), were 7.1 and 6.5 ft, respectively.

The data gathered by Hudson are certainly the most extensive and inclusive yet obtained, and probably form the best basis presently available for determining design values of wave run-up. However, these data were obtained in small scale laboratory tests using waves on the order of several inches in height. Accordingly, the possible existence of a scale effect must be borne in mind when applying them to prototype conditions. Unpublished, large scale (2 ft-5 ft waves) data, gathered at the Beach Erosion Board in connection with design of Lake Okeechobee levees, have shown the existence of such a scale effect for wave run-up on smooth slopes. On smooth slopes, this data indicates that actual run-up from prototype waves will be greater than that predicted by small scale tests by about 20% for a 1-on-3 and 10% for a 1-on-6 slope. The actual percentage increase also probably depends to some extent on the exact size of the prototype waves, being larger for larger waves. There would appear to be no reason to believe that a similar effect would not also exist for rubble slopes. Indeed, a few large-scale tests now under way at the Beach Erosion Board would seem to indicate this, although the data is not yet complete enough to permit an exact determination. This is certainly an area in which much more knowledge and understanding are needed. Consequently, it is felt that, although Hudson's run-up data certainly represent the best basis for design now available, a certain amount of conservatism in using them may be advisable.

ROBERT Y. HUDSON,[17] F. ASCE.—The extensive discussions of this paper are much appreciated. The varied backgrounds and the experience of the discussers make their contributions especially significant. Although the discussion by Carvalho and Vera-Cruz will probably be appreciated more by those engaged in laboratory tests of rubble breakwaters, design engineers in America are also indebted to these gentlemen for the information presented concerning the full-scale breakwater from which Iribarren obtained the data used to compute the coefficient (K'). The discussions by Slichter, Booth, and Lillevang are valued because they represent the ideas of practicing engineers with many years experience in the design and construction of rubble breakwaters. Jones' discussion provides excellent, but provisional, data concerning the stability of riprap cover layers for fill slopes. The stability of cover layers for rubble breakwaters and the stability of riprap cover layers for fill slopes are closely related phenomena, especially from the standpoint of laboratory testing techniques and analyses. However, there is considerable difference in the relative importance of the variables, and in the practical aspects of design.

In each discussion, the thought was expressed that the science of rubble breakwater design is still in the formative stage, and that considerable work remains to be done before stability formulas can be relied on for exact solutions. The writer concurs in this evaluation of the status. Yet, he cannot agree completely with Slichter's comment to the effect that "although the guide lines contributed by the author's work are valuable aids, the design of breakwaters must remain largely in an area dominated by sound judgment based on observations of past experience." It is believed that the observations of past experience should include observations of small-scale tests as well as observations of full-scale structures, and that the formulation of sound judgment should be based both on the relationships between variables, determined by use of small-scale models, and on lessons learned from observation of full-scale breakwaters. It is agreed, however, that the designer cannot afford to use the results of small-scale tests blindly, and that, after all possible model tests have been completed on rubble breakwaters, there will still remain a large area in design in which use of the intuition and practical experience of the designer will be necessary.

Slichter doubts that there is any practical design significance in abandoning the Iribarren formula, in view of the small size of the armor units tested relative to those of a full-scale structure. The reasons for abandoning the Iribarren formula were in no way related to the effects of model scale on the accuracy of test results. It is believed that the scale of the tests is sufficient to insure an adequate degree of dynamic similarity, model to prototype, if it can be assumed that the shape factor of the armor units, the placing of armor units, and the characteristics of incident wave trains are sufficiently similar. It is hoped that the testing programs, now in progress at the Corps of Engineers, U. S. Army Waterways Experiment Station and the Beach Erosion Board laboratory, can determine the effects of these variables.

For those who favor the original Iribarren formula (Eq. 1 of the author's paper), the comparison of results shown in Table 7, obtained by substitution in the formulas of Iribarren and the author, should be informative. In Table 7 $(W_r)_I$ and $(W_r)_H$ are the weights of quarrystone armor units required for stability for the different slopes, assuming equal wave heights, obtained by substituting in the Iribarren and Hudson formulas, respectively. Values of 0.015 for K' and 3.2 for K_Δ were used. It is noted from this comparison that (a) the formulas agree for a slope of 1-on-2, (b) the weights of quarrystone armor units required for stability, as determined from the Iribarren formula, are approximately one-half those obtained from the author's formula when the slope is 1-on-3 or flatter, (c) the effects of small changes in μ are very

[17] Chf., Wave Action Sect., U. S. Army Engr. Waterways Experiment Sta., Vicksburg, Miss.

large for steep slopes, and (d) the weights of armor rock given by the Iribarren formula for slopes of 1-on-1-1/2 and 1-on-1-1/3 are exceptionally high, compared with the author's formula, especially for low values of μ.

Slichter questions the assumption, made in developing the general functional equation (Eq. 13), that friction between armor units can be neglected. This assumption is, of course, not strictly correct, but it is believed sufficiently accurate for use in developing the functional equation. This belief is based on several years of experience in testing small-scale rubble breakwaters to the point of failure. It was concluded from observing these tests that friction between armor units can be a primary resisting force for pell-mell placed units only when the entire cover layer is on the verge of sliding downslope. It was noticed that, for pell-mell placed armor units, there is considerable failure of individual units, by forces that lift and roll the units out of their nested positions, before failure occurs by sliding of the cover-layer mass. This is not true, however, for the condition in which armor units are carefully selected for shape and are positioned individually to obtain maximum wedging action. For this condition, especially for the steeper

TABLE 7.—COMPARISON OF THE IRIBARREN AND HUDSON FORMULAS

cot α	$(W_r)\,I/(W_r)\,H$		
	μ = 1.00	μ = 1.05	μ = 1.10
1-1/3	8.0	5.4	3.9
1-1/2	3.4	2.6	2.1
2	1.1	1.0	0.8
3	0.6	0.5	0.5
4	0.5	0.5	0.5
5	0.5	0.5	0.5

slopes, the cover layer usually fails by sliding along the boundary between the cover-layer bottom and the first underlayer. It is hoped that the effects of shape factor and special placing techniques can be determined in future tests.

Carvalho and Vera-Cruz indicate the following beliefs: (a) the stability of rubble breakwaters situated in relatively shallow water on sandy bottoms is affected by the increased specific gravity of the water resulting from the great amounts of sand stirred up by the waves; (b) the inertia force is important and should be investigated; (c) in the analytical basis of the stability equation, an effort should be made to express the vectorial character of the wave forces; (d) the most adequate criterion for the no-damage condition should be one in which only the active portion of the breakwater, and one or two layers of the protective cover, are taken into account; (e) safety factors should be determined for each type of breakwater section; (f) Iribarren's formula is susceptible of experimental verification, if it is assumed that μ = 1, for all slopes flatter than 1-on-1, and provided that K' is not considered constant; and (g) K_Δ should not be called a shape coefficient. The writer agrees with the ideas expressed in items (d) and (e). The work explained in

the original paper represents the first phase of a comprehensive testing program, and it is agreed that the criterion used for the no-damage tests, and the damage or safety-factor data, are applicable directly only to the type of sections used in the tests. Tests in progress (1960) compare damage to the cover layer with the volume of armor units in the cover layer itself.

The idea expressed in item (a) is new to the writer, and it should no doubt be investigated. It is predicted, however, that tests will show that the stirring-up of sand in the water by wave action does not appreciably affect the stability of rubble breakwaters. The question as to whether the forces of inertia (Eq. 3) are important, concerns a very complicated phenomena, and the writer is not qualified to argue the point. However, the success attained in correlating the test results, using the derived functional equation, in which the effects of inertia forces were included in the experimental coefficient K_Δ, indicates that the assumption made is sufficiently accurate. Also, the work of McNown and Keulegan,[18] concerning the relation of drag and inertia forces on flat plates and cylindrical bodies in periodic motion, appears to indicate that the time available for the formation and shedding of vortexes is not sufficient, in the flow around breakwater armor units caused by short-period wave action, to result in large inertia forces. It is doubted that an attempt to express the vectorial character of the wave forces in the functional equation would be successful (item c above). The direction of the wave forces vary with d/λ, H/λ, H/d, β, σ, cot α, h/H, and t/T. Thus, it was decided to omit the direction variable from the general stability equation, and let the model define the importance and the effects of its function. The success achieved to date in correlating the test results using Eq. 13 appear to substantiate the correctness of this decision.

The writer agrees that Iribarren's formula is susceptible to experimental verification if it is assumed that μ = 1. However, this is tantamount to assuming that friction is important at the time the formula is derived, then assuming it is unimportant for the purpose of experimental verification. It is believed better to let the model tests define the effects of friction. Also, Iribarren's formula includes the term involving the breakwater slope (cos α - sin α), that originated from an assumption with respect to the directions of both wave forces and friction forces. The inadequacy of this assumption is believed to be the primary reason that K' varies so considerably with breakwater slope.

The information presented by Slichter and Booth concerning the design of breakwaters in Hawaii using tribars and tetrapods should be very useful to other designers. It is good to know that the test results obtained during the model study conducted at the Waterways Experiment Station to determine the optimum design for repairing the Nawiliwili Harbor breakwater, have been partly verified by the prototype breakwater's ability to resist the action of large storm waves.

This breakwater was designed, based on the model test results, to be stable for waves as large as 24 ft in height. The storm referred to by Booth was hurricane "Dot," that passed directly over Kauai, the island on which the Nawiliwili breakwater is situated. Although an accurate estimate of the heights of waves that attacked the breakwater is not presently available, photographs

18 "Vortex Formation and Resistance in Periodic Motion," by J. S. McNown and G. H. Keulegan, Proceedings, ASCE, Vol. 85, No. EM1, January, 1959.

taken before the height of the storm show considerable overtopping of the breakwater. By comparison with similar photographs taken during the model study, it is estimated that the waves were greater than 20 ft in height.

The writer agrees with Booth's statement that values of the experimentally determined coefficient K_Δ, presented in the author's paper, should not be considered as final. However, the information presented by Booth concerning the Nawiliwili Harbor breakwater and the information presented by Deignan[19] concerning the Crescent City breakwater, indicate that the model test results can be used to predict the action of prototype structures with very good accuracy. The large-scale tests being conducted by the Beach Erosion Board for the Waterways Experiment Station, mentioned by Booth, are not complete at this time, but preliminary results indicate that the author's values of K_Δ for quarrystone are conservative.

The writer agrees with Lillevang that the placing of armor units in such a way as to effect wedging action and interlocking between adjacent units will increase the stability of rubble breakwaters, compared with the stability obtained by pell-mell placement. However, maximum benefit of especially placed armor units cannot be obtained unless all units down to an elevation equal to about -H ft, referred to swl, are so placed. The reason for this is that hydrostatic pressure of water in the cover layer is considerable, and is maximum at an elevation immediately above the trough of the wave on the structure. If the special placing of armor units is carried to the extreme, voids in the cover layer may be decreased sufficiently to cause failure due primarily to the hydrostatic pressure of water trapped in the cover layer. The test data presented in Table 8 answers Lillevang's request for information concerning the effects of shape factor on the stability of cover layers.

TABLE 8.—EFFECTS OF ARMOR UNIT SHAPE FACTOR ON STABILITY OF BREAKWATER STEM IN RELATIVELY DEEP WATER[a]

Armor Unit (1)	n (2)	Placement (3)	K_Δ (4)	k_Δ (5)	P, percentage (6)
Quarrystone	2	Pell-mell	2.6	1.0	40
Tetrapod	2	Pell-mell	8.3	1.0	50
Tribar	2	Pell-mell	12.0	1.0	54
Tribar	1	Regular	25.0	1.1	47
Tetrahedron	2	Pell-mell	3.0	1.0	40
Quadripod	2	Pell-mell	8.3	1.0	50

[a] $\cot \alpha \leq 1.5$

19 "Breakwater at Crescent City, California," by John E. Deignan, Proceedings, ASCE, Vol. 85, No. WW3, September, 1959.

These data were obtained using water of considerable depth, relative to wave height, and for the no-overtopping condition. The results of all tests to date are provisional to the extent that the effects of scale, variations in wave height (heights $> H_{1/3}$) in storm-wave trains, placing techniques, overtopping waves, waves breaking directly on the structure, and the angle of wave attack have not been determined. However, when waves do not break directly on the structure, and when there is no overtopping, it is believed that use of the values of K_Δ in the Table 8 will result in conservative designs. It is believed, also, that the relative economy of rubble breakwaters, similar except for the shape of armor unit used in the cover layer, can be determined with considerable accuracy, using the writer's formula and the above values of K_Δ, k_Δ, and P.

The information presented by Jones concerning the measured value of the friction coefficient (μ) is valuable design data. The average value of $\mu = 0.75$, determined for dumped riprap, would increase the required stone weights nearly four times for $\cot \alpha = 2$, as explained by Jones, compared with the use of $\mu = 1.0$ and $K' = 0.015$ in Iribarren's formula. It should be realized, however, that the value of $K' = 0.015$ should not be used in Iribarren's formula for riprap cover layers. A new value of K' for riprap would need to be determined, using, in this determination, the new value of $\mu = 0.75$ for the friction factor in Iribarren's formula.

Saville has correctly pointed out that both wave height (H) and wave length (λ), and thus wave steepness (H/λ), depend on depth of water in which H and λ are determined, and that different values of run-up (R) are determined from the author's run-up curves, depending on just where the wave characteristics are measured. However, this fact does not detract from the usefulness or accuracy of the author's run-up curves, as stated by Saville. In the paper, H was defined as the wave height measured at the location of the proposed breakwater. Thus, there should be no confusion as to which value of H/λ to use, because there is only one value of H/λ corresponding to a selected breakwater position and a given deep-water design wave H_o/λ_o. The dimensions of a deep-water design wave are usually selected on the basis of existing hindcasting techniques, and the corresponding value of H/λ, for depth (d) of the selected breakwater location, is determined by constructing wave-refraction diagrams for the area between deep water and the proposed structure location. It is believed that the method of plotting the run-up data selected by the author is not only correct from an analytical standpoint, but also is more convenient and less confusing than the curves proposed by Saville. Also, the tests from which the run-up data were obtained were conducted using water of considerable depth relative to wave height (H/d varied from a minimum of 0.28 to a maximum of 0.35).

Thus, it is doubtful whether the data can be used for the condition of breaking waves, as did Saville in his example (an 8-ft wave of 6-sec period will break in a water depth of about 10.5 ft). It is hoped that future tests can be conducted for the condition of gradually increasing depth seaward of the breakwater location, using both breaking and nonbreaking waves, and a larger range of values of d/λ. We are indebted to Saville for pointing out the possible effects of scale on wave run-up. The writer was not aware that scale effects for run-up on rubble breakwaters were appreciable. Until more data are available concerning scale effects on wave run-up, the crowns of breakwaters probably should be designed to withstand some over-topping, without failure, whenever the run-up curves shown in the paper are used.