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# Laboratory Investigations of the Mechanism of Cavitation



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**The paper describes some experimental investigations of the formation and collapse of cavitation bubbles. The experiments were carried on in the high-speed water tunnel of the Hydrodynamics Laboratory of the California Institute of Technology under the sponsorship of the Research and Development Division of the Bureau of Ordnance of the U. S. Navy and the Fluid Mechanics Section of the Office of Naval Research. A detailed study of the formation and collapse of the individual bubbles has been carried on by the use of high-speed motion pictures taken at rates up to 20,000 per sec. From these records calculations have been made of rate of formation and collapse of the bubbles. Deductions have been drawn from these results concerning the physical mechanism of the cavitation phenomenon.**

## INHERENT DIFFICULTIES OF OBSERVATION OF CAVITATION PROCESS

THERE is little doubt but that most workers in the field of cavitation would agree that there is considerably more conjecture than knowledge on the physical events that take place during cavitation. Much of this lack of knowledge is due to the fact that it is inherently difficult to observe and record the details of the phenomenon. The individual bubbles or voids form and collapse with great rapidity. Furthermore, cavitation is generally caused by fast-moving bodies in liquid, either with a free surface (propeller, torpedo), or in closed conduits (pump or turbine impeller), so that even the study of simpler cases with a stationary object and fast-moving liquid to attain the same relative speed is difficult. The result is that most of the experimental observations in the past have been restricted either to the study of the effect of cavitation, i.e., cavitation damage, or to the recording of the over-all or instantaneous pictures of some stage of the cavitation process. As a consequence of the lack of such detailed information, no quantitative description has been developed of the actual physical processes which take place during cavitation. Thus although many attempts have been made to develop analytical interpretations they have been based upon widely different physical assumptions, many of which have little background of experimental fact. The objective of the present study has been to attempt to furnish a more quantitative physical knowledge concerning the mechanism of cavitation and to formulate some elementary analytical descriptions of the phenomenon on the basis of these physical observations. This paper, in turn, is only a preliminary report for the purpose of presenting some of the first experimental observations, together with tentative analyses of their significance and implications.

## EXPERIMENTAL METHODS AND EQUIPMENT

The experimental approach to the problem may be divided naturally as follows:

1 The production of the desired degree of cavitation under measurable and reproducible conditions which are suitable for observation.

2 The photographic recording of the details of the cavitation process.

The equipment and technique required for each part will be described separately.

*Production of Cavitation in High-Speeed Water Tunnel.* The high-speed water tunnel was chosen as the major piece of equipment for use with this project because the pressure, velocity, and temperature of the liquid in the working section could be controlled accurately at any desired set of values within the range necessary to produce or eliminate cavitation on a wide variety of experimental shapes. A detailed description of the construction and operation of this tunnel has been given in another paper.[3]

In the series of experiments now under consideration, measurements have been made at velocities of from 30 to 70 fps, with absolute pressures at the wall of the working section ranging from about $1^1/_2$ to 50 psi above vapor pressure. Temperature range has been held to within a few degrees of room temperature. Nearly all the observations have been made on flow around bodies of revolution which have been mounted with their axes either parallel or within a few degrees of parallel with the direction of flow. Wide ranges of forebody or nose shapes and afterbody shapes have been studied. Pressure-distribution measurements have been made on some selected shapes of these series. All of the bodies studied have had a uniform maximum diameter of 2 in. The observations under consideration at this time have all been made on cavitation occurring on or adjacent to a series of ogive noses. The ogive nose is a very simple shape, as may be seen from Fig. 1. It can be defined as being generated by revolving a circular arc about the axis of revolution of the cylinder. One end of the arc is tangent to an element of the cylinder; the

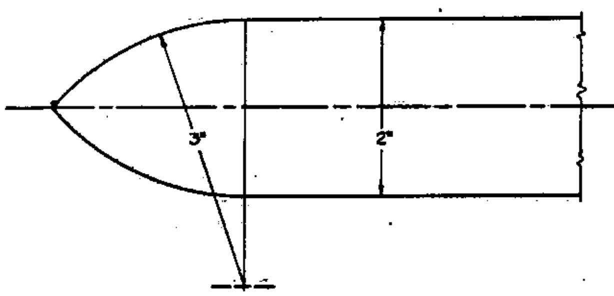


FIG. 1 1.5-CALIBER OGIVE NOSE

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society. Paper No. 47—A-150.

[3] "The Hydrodynamics Laboratory at the California Institute of Technology," by R. T. Knapp, Joseph Levy, F. Barton Brown, and J. Pat O'Neill, Trans. ASME, this issue, pp. 437-457.

other end intersects the axis. It is convenient to express the radius of this arc in terms of the diameter of the cylinder, i.e., the generatrix of a $1^1/_2\, d$ ogive is an arc whose radius is $1^1/_2$ times the diameter of the cylinder.

*High-Speed Motion-Picture Photography.* The tool selected to record the physical details of the cavitation phenomenon is high-speed motion-picture photography. Motion pictures taken at one speed and projected at another can be thought of as performing the function of a time telescope or microscope. With this conception, the ratio of magnification will be measured by the ratio of the picture-taking speed to the projecting speed of the picture. For example, if pictures are taken of a given phenomenon at relatively long intervals and then projected at the normal speed necessary for viewing movies, the time scale of the phenomenon is changed in a manner similar to the way the distance scale of an object is changed when observed through a telescope. The telescope brings the distant object close enough to the observer, so that details of its structure can be observed; the speeded-up projection of the pictures brings the time details of the phenomenon close enough together, so that they can be observed. Conversely, motion pictures taken at a high rate of speed and projected at a much lower rate of speed serve as a time microscope, since the process resolves the details in time in the same manner as the microscope resolves the details in space.

In the present study, pictures of cavitation have been taken at varying rates from 64 per sec to 20,000 per sec. When these are projected at the normal viewing speed of 16 per sec, time magnifications covering ratios of 4:1 to 1250:1 are secured. Equipment such as this is needed to change the time scale for exactly the same reason that telescopes and microscopes are needed to change the length scale. The human senses and brain have a limited range in which they can get an undistorted concept of what is occurring. Therefore it is necessary to transform the actual times and distances involved in a given phenomenon until they fall within these limited ranges.

*Description of Photographic Equipment.* Photographic equipment used in this study is of the multiflash type. The pioneer development in this field was carried on by Prof. Harold E. Edgerton and his associates at the Massachusetts Institute of Technology. It consists of a simple camera in which the recording film moves constantly past the focal plane at a high speed. The camera has no shutter. Illumination required to take the picture is provided by one or more synchronized flash lamps, which also act as the camera shutter. This requires that the flash duration be so short that neither the image of the object on the film nor the film itself move an appreciable distance while the light is on. As the number of the pictures taken per second increases, the film motion becomes the controlling factor in most cases. Up to the present time satisfactory pictures have been taken at rates up to 30,000 exposures per sec. The lamp equipment has been operated up to 50,000 flashes per sec, but as yet the obtainable film speeds have not been high enough to give a satisfactory frame height for use at this rate.

(*a*) *Camera.* The camera itself is the standard General Radio type instrument as shown in Fig. 2. A series of lenses of varying focal lengths have been fitted to it to increase its flexibility. The commutator provided on the film drum is not used; instead, the pulsing of the flash lamps is controlled by an oscillator.

(*b*) *Flash Lamps.* Considerable development work has been carried on to increase the rate at which the flash lamps can be operated. The original equipment, as developed by Edgerton, operated satisfactorily at the rate of 3000 flashes per sec. Investigation showed that this limitation was in the control circuits and not in the lamp itself. Consequently, the laboratory has undertaken the development of a system which utilizes several control circuits synchronized through a common multiphase oscillator circuit, but discharging in rotation through a single lamp.

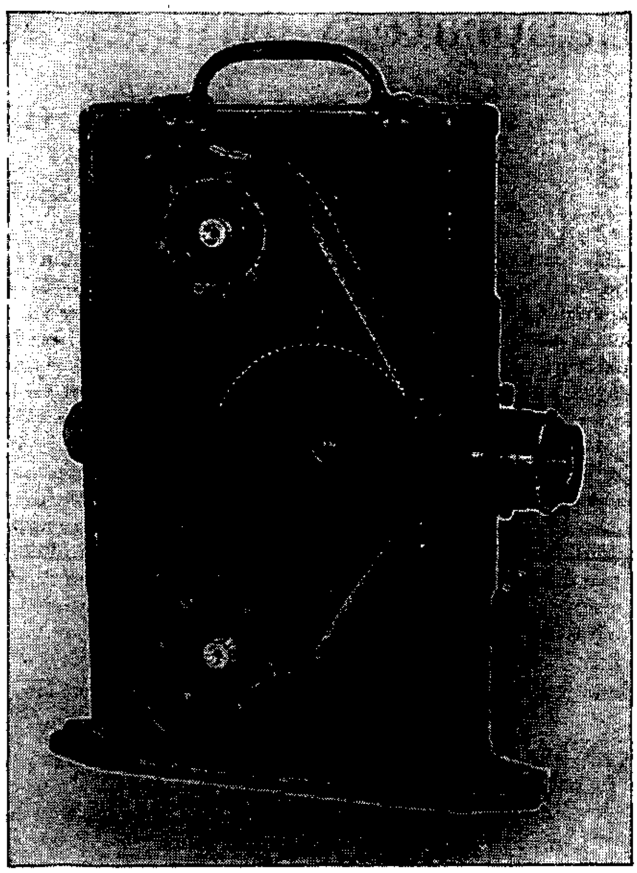


Fig. 2 High-Speed Motion-Picture Camera

By the use of this system the flash rate becomes equal to the maximum rate at which a single control circuit can be operated, multiplied by the number of circuits involved. At the present time six circuits have been used simultaneously in 6-phase array with proper electronic switching devices to permit all of the circuits to discharge through a common lamp.

In the design and development of a combination camera and flash-lamp system of this type, it is necessary to bear in mind the extreme importance of the relationship between the camera and the lights because the lights function as the camera shutter. In fact, the characteristics of the flash lamps exert a controlling influence upon the work that can be done with the combination. The most important characteristic of the flash lamp is the effective duration of the flash. The minimum available flash duration limits the maximum usable film speed.

In this system of photography, the film moves continuously. Therefore the flash duration must be short enough to stop the motion of the film; otherwise, the record will be blurred. For critically sharp results, the maximum usable film speed can be calculated from the criterion that the allowable film motion during one flash should not be greater than the diameter of the circle of confusion of the lens system. For extremely high-speed work it may be necessary to lower this requirement somewhat. The permissible deviation will depend upon the accuracy of measurements required from the record. At first sight this criterion may seem incomplete, since no consideration is given to the speed of the object being photographed.

A simple example will show that, at least for the present use, this is not the case. Blurring is caused by a relative movement between the image and the film during exposure. While making cavitation photographs in the laboratory, the flow is at right angles to the motion of the film. The motion with respect to the film will be the vector sum of the motion of the image of the bubble with respect to the camera frame and the motion of the film with respect to the camera frame. The maximum flow velocity in the tunnel that has been photographed is about 75 fps. The smallest reduction ratio used in the photograph is about 6:1, i.e., the image on the film, and hence the image velocity on the film is not over $^1/_6$ of the bubble velocity in the tunnel. Thus the maximum velocity of the image with respect to the camera frame is $12^1/_2$ fps.

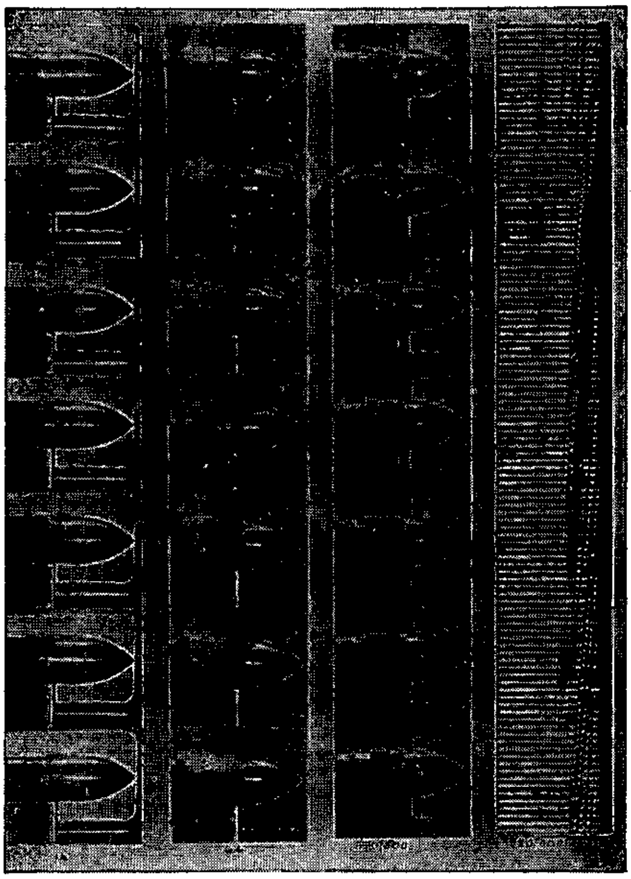
Most of the pictures were taken at a rate of 20,000 frames per sec. In order to keep the film speed to a minimum, very small frame heights were employed. Thus, on the average, each individual picture was about 1 in. wide × $^1/_{16}$ in. high. This frame height of $^1/_{16}$ in. requires a film speed of over 100 fps when the taking rate is 20,000 per sec and, even at this speed, all spacing between frames has to be eliminated. The vector sum of 100 and $12^1/_2$ is less than 101, i.e., the speed of the object had something less than 1 per cent effect upon the relative speed between the image and the film. When the camera is further from the tunnel, or if a shorter-focal-length lens is used, the effect is even smaller. It is safe to conclude, therefore, that in the design of photographic equipment of this kind the speed of the object can be ignored safely.

Another very important characteristic of the flash lamp when used in this system of photography is the intensity of the light. This intensity must be very high to produce an image of reasonable density in the very short exposure time available. The importance of illumination intensity can be seen clearly if the operation of this type of equipment is compared to that of a hypothetical motion-picture camera of the standard type, using a normal shutter but operating at 20,000 exposures per sec. The normal type of shutter has an opening of about 180 deg, which means that the effective exposure time is one half of the elapsed time between successive pictures. In this case the exposure would be $^1/_{40,000}$ sec. If this is compared to the exposure time of $^1/_{30}$ to $^1/_{50}$ sec for a normal camera operating at a conventional speed, it will be seen that an extremely intense illumination would be required if an adequate exposure were to be secured. However, $^1/_{40,000}$ sec is 25 microseconds. This is 25 times as long as the flash duration, which is 1 microsecond. Hence the flash intensity must be at least 25 times as great ast hat required for this hypothetical conventional-type camera. The energy input to the lamp is at a rate corresponding to a continuous flow of 20 kw; however, as the lamp is burning only $^1/_{50}$ of the total time, the energy input during the exposure is at the rate of 1000 kw.

An example of the difference in the information obtainable with different taking rates is seen in Figs. 3(*a* to *d*). The film strips are all taken under the same conditions in the tunnel for the same degree of cavitation on the same model. In comparing strips 3(*a*) and 3(*d*), it should be remembered that there are 1250 individual exposures on the *d* strip between each one on the *a* strip. The *a* strip was taken at the normal motion-picture rate.

### Experimental Observations

An examination of some of the records shows that if all stages of cavitation are considered the phenomenon is very complex. For example, Fig. 4 presents a series of pictures showing increasing degrees of cavitation from the incipient point to the formation of a cavity large enough to contain the entire body. In this case, the shape is a hemispherical nose with a straight cylindrical afterbody. This entire series was obtained while the tunnel was operating at constant velocity with gradually decreasing pressure in the working section. It will be observed in each picture that many complex bubble groups are formed and if the life history of such a group is examined it will be seen that the individual bubbles interact and often combine in either the formation or the collapse stage. At the present time no attempt will be made to investigate these complicated interactions. Instead, consideration will be restricted to the simplest appearances that can be found on the records. For that reason a shape was chosen which, at least for low degrees of cavitation, tends to produce individual bubbles spaced far enough apart so that occasional ones can be found which throughout their entire life history of formation, collapse, and rebound are not seriously affected by interference from other bubbles.



(*a*) (*b*) (*c*) (*d*)
(*a*) 16 per sec (*b*) 64 per sec (*c*) 1500 per sec (*d*) 20,000 per sec
Fig. 3 Comparison of Picture-Taking Rates

An example of the effect of surface curvature of the body on the appearance of the cavitation is shown in Figs. 5(*a*, *b*, and *c*). These pictures were taken for approximately the same degree of cavitation. However, the body noses are different. Fig. 5(*a*) is a hemispherical nose, Fig. 5(*b*) an 0.875-caliber ogive, and Fig. 5(*c*) a 1.5-caliber ogive. The appearance of the cavitation in Fig. 5(*a*) is typical of that found on the blunter nose forms; whereas that in Fig. 5(*c*) is characteristic of the finer shapes. Fig. 5(*b*) is a transition shape, showing some of the characteristics of both. The experimental material used in the rest of this presentation has all been obtained from records taken with a 1.5-caliber ogive nose mounted on a long cylindrical afterbody.

Fig. 6 shows a record of the complete life history of a cavitation bubble. Strip (*b*) is a direct continuation of strip (*a*). These photographs were obtained at a tunnel velocity of 40 fps and a picture-taking rate of 20,000 frames per sec. It will be

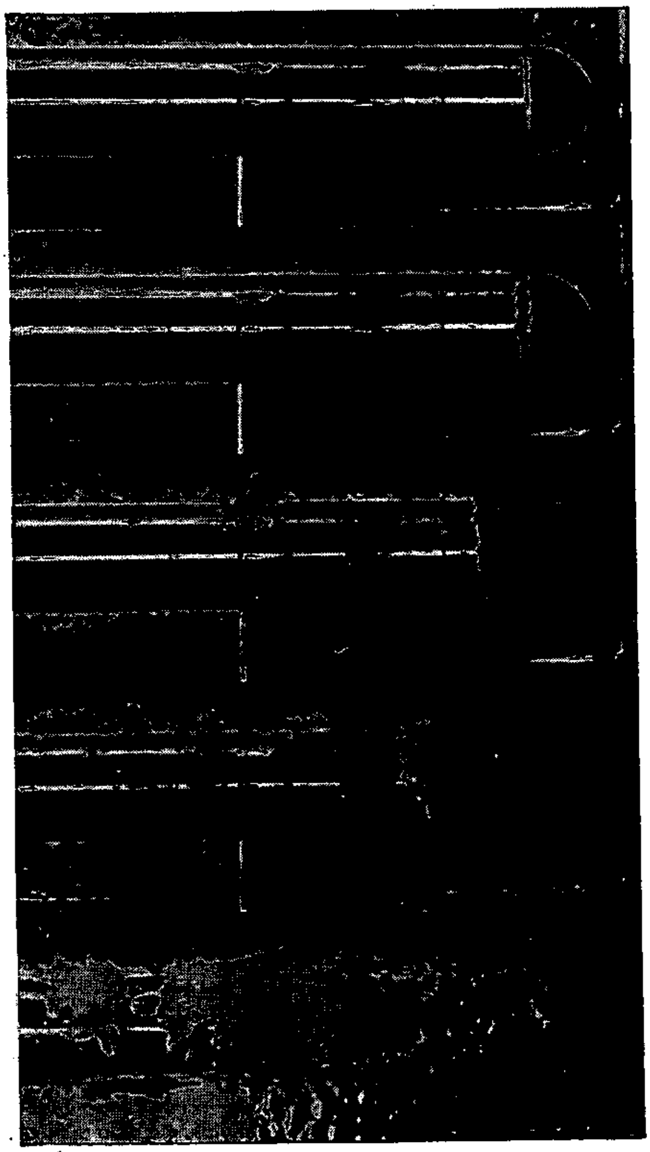


FIG. 4 CAVITATION DEVELOPMENT ON BODY WITH HEMISPHERICAL NOSE

[Views from top down: (a) $K = 0.62$; (b) $K = 0.55$; (c) $K = 0.45$; (d) $K = 0.40$; (e) $K = 0.31$, respectively.]

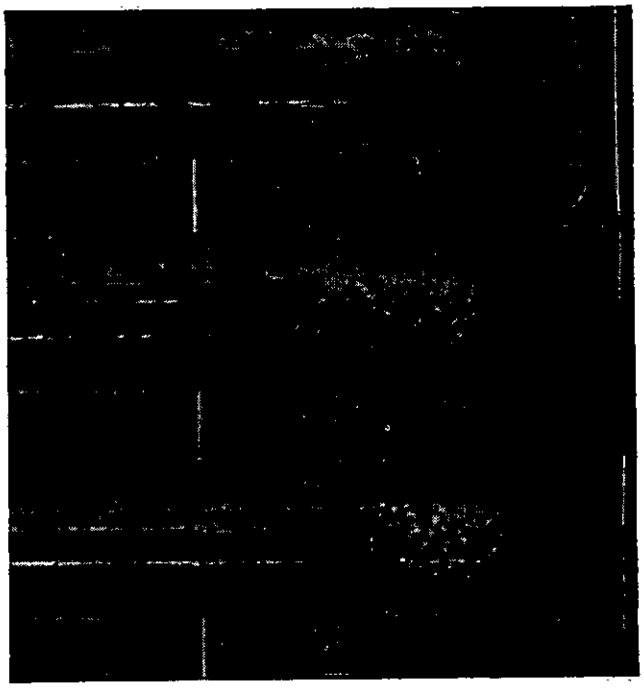


FIG. 5 EFFECT OF SURFACE CURVATURE ON APPEARANCE OF CAVITATION

[Views from top down: (a), (b), (c)]

seen that the life cycle of a bubble can be divided into a series of natural stages, as follows:

1 Formation and growth, from first appearance to maximum diameter.

2 First collapse, from maximum diameter to first disappearance.

3 First rebound, from first disappearance to second maximum.

4 Second collapse, from second maximum to second disappearance.

5 Second rebound.

6 Third collapse.

7 Final rebound, collapse, and disappearance.

A large share of the existing literature on cavitation has considered only the second stage. The growth, rebound, and recollapse phases have been ignored, in general, either because their existence was unknown or because they were considered an unwarranted complication. This can be understood on the basis discussed previously, i.e., most of the investigators have been concerned either with the investigation of methods for preventing the occurrence of cavitation, or with the determination of cavitation damage and the relative resistance of different materials to such damage.

One of the assumptions commonly made is that the pressure in the bubble is approximately equal to the vapor pressure of the liquid at the mean temperature of the flow. There is much indirect evidence to support the belief that this is the right order of magnitude for the pressure. For example, Fig. 4 shows a series of pictures of the development of cavitation on the hemispherical nose. It will be noted that in the initial stages, Figs. 4 (*a* to *c*), the cavitation area is not symmetrical around the nose, but in each case it is wider at the top than at the bottom of the model. The only significant difference in the flow conditions from top to bottom is a change in the hydrostatic pressure which has an over-all magnitude of 2 in. of water. Thus the degree of cavitation is sensitive to a fraction of this very slight change of pressure. This furnishes a strong inference that the pressure within the bubble must likewise be small, that is, of the same order as the vapor pressure at the existing temperature, or it would not be affected by this small change in pressure. Similar evidence is given by the difference betwen the successive pictures. The change of the measured tunnel pressure between the pictures is very small in contrast to the great change in the cavitation areas. More direct evidence is given by the agreement between pressure-distribution measurements made under noncavitating conditions on a specific shape, with the pressure at which cavitation first appears on that shape.

In the analysis of cavitation phenomena, the cavitation parameter has been found very useful. This is defined as follows

$$K = \frac{p_L - p_B}{\rho \frac{V^2}{2}} = \frac{h_L - h_B}{\frac{V^2}{2g}}$$

in which

$K$ = cavitation parameter
$p_L$ = absolute pressure in the undisturbed liquid, psf
$h_L$ = same in feet of liquid, $h_L = \frac{p_L}{\gamma}$
$p_B$ = vapor pressure corresponding to water temperature, psf
$h_B$ = same as $p_B$ in feet of liquid, $h_B = \frac{p_B}{\gamma}$
$V$ = relative velocity between body and liquid, fps
$\rho$ = mass density of liquid, slugs per cu ft = $\gamma/g$
$\gamma$ = specific weight of liquid, lb per cu ft
$g$ = acceleration of gravity, fps per sec

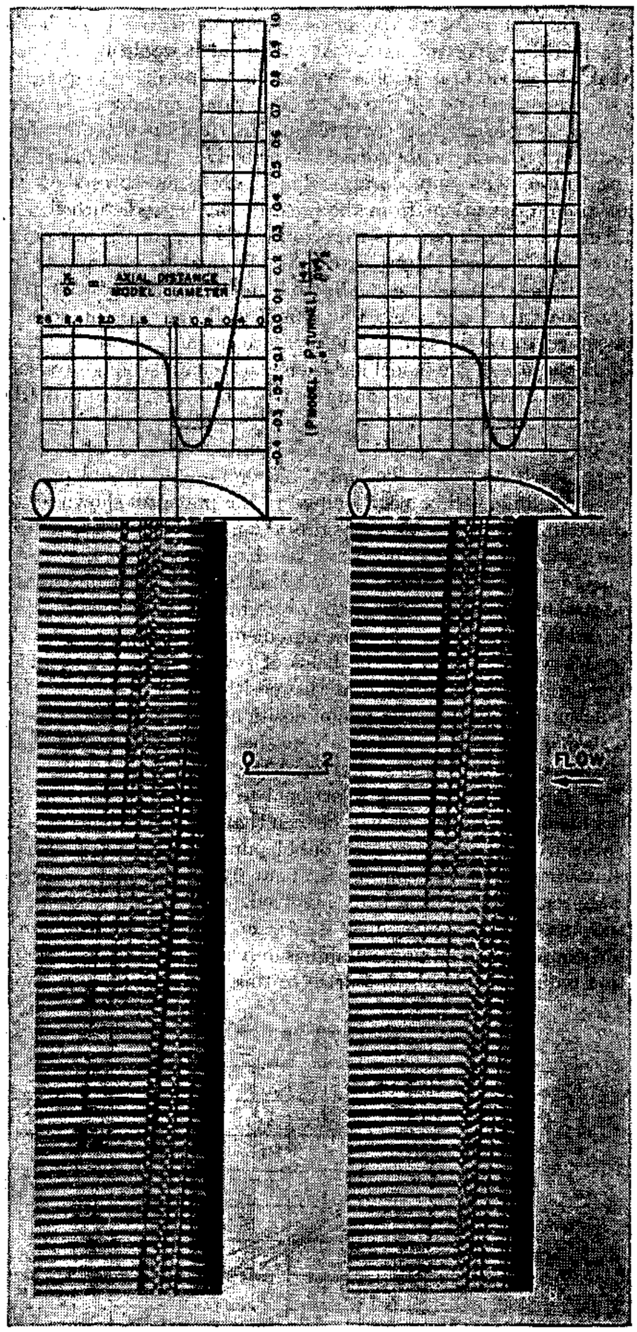
It will be seen that the numerator of both expressions is simply the net pressure or head acting to collapse the cavity or bubble. The denominator is the velocity pressure or head. Since the entire variation in pressure around the moving body is a result of the velocity, the velocity head may be considered a measure of the pressure available to open up a cavitation void. From this point of view, the cavitation parameter is simply the ratio of the pressure available for keeping the stream in contact with the body to the pressure available for opening the stream and permitting bubble formation. If the $K$ for incipient cavitation is considered, (designated $K_i$), it can be interpreted to mean the maximum reduction in pressure on the surface of the body measured in terms of the velocity head. Thus if a body starts to cavitate at the cavitation parameter of 1, it means that the lowest pressure at any point on the surface is one velocity head below that of the undisturbed fluid.

It was found that for greater degrees of cavitation, measured by the extension of the bubble-covered section to $^1/_4$, $^1/_2$, $^3/_4$, or full length of the body, the parameter $K$ is equally significant, i.e., it signifies similar extensions for the same $K$ values independently of the velocity.

## Analysis of Observations

The high-speed water tunnel is a piece of equipment which can be operated under accurately known and controlled conditions. The associated instruments and apparatus, including the photographic equipment used in making the records of the formation and collapse of the cavitation bubble are quantitative instruments. Therefore it is possible to evaluate the records of the cavitation bubbles with reasonable accuracy. Time measurements are based upon the interval between the individual exposures on the high-speed motion pictures. This interval is determined by the flash rate of the lamps. This rate is controlled by an oscillator whose frequency is known with great accuracy. Thus the flashes are spaced at very uniform known intervals. The time measurement is completely unaffected by the film speed in this system of measurements. Motion is determined by measuring the position of the bubble on the individual pictures on the film. The light path from the camera to the bubble traverses air, lucite, and water, which produces some optical distortion. This distortion is comparatively small because the outside surfaces of the lucite windows are planes. Therefore the cylindrical lens effect of the water-filled circular working section is largely eliminated. The amount of distortion which does exist is eliminated by applying correction factors that have been determined by photographing horizontal and vertical test scales mounted in the tunnel area in the position normally occupied by the model. Thus the actual dimensions of the bubbles and the amount of their movement can be determined with a good degree of approximation.

Fig. 6 is a suitable record for this purpose. It shows the life cycle of an isolated bubble which happens to be far enough removed from other similar bubbles to make it reasonable to




(a) (b)
Fig. 6. Life History of a Cavitation Bubble

assume that it is relatively unaffected by other elements of the cavitation. The diagram at the top of the figure shows the pressure distribution on the surface of the body. This distribution was measured on the model in the tunnel. The full line is for noncavitating conditions, and the dotted line is for the degree of cavitation shown in the photographs. For these measurements the tunnel was operated at a cavitation parameter of $K$ = 0.33. The tunnel velocity was 40 fps. This corresponds to a dynamic head of about 24.8 ft or 10.7 psi. The vapor pressure of the water at the temperature of the measurements was approximately 0.4 psi. The absolute pressure in the undisturbed flow, corresponding to these conditions, is about 4 psi. (See also Fig. 8)

If we examine the pressure-distribution diagram it will be seen that at point $A$ the pressure has decreased until it has reached the vapor pressure. At this point it would be expected that the cavitation bubble would first appear. At point $B$ the pressure starts to rise above the vapor pressure. It must not be forgotten that the pressure-distribution diagram gives only the pressures on the surface of the body. Therefore, neglecting transient pressure during collapse, these pressures show the maximum deviations from the pressure in the undisturbed flow. If pressure measurements were to be taken at one given tunnel cross section for points between the body and the tunnel, this deviation would become smaller and smaller as the distance from the body increased. At the tunnel wall the pressure can be assumed to be the true static pressure in the undisturbed flow, since the size of the model has been chosen small enough (ratio of sectional areas of model to tunnel is $^1/_{80}$) to cause very little disturbance at the wall.

The photographs show that the cavitation bubbles follow paths very nearly touching the body. Therefore the liquid pressure on the bubble will be nearly equal to the pressures shown on the diagram. Probably the pressure along the line of the bubble path will be slightly higher than the diagram, but for the present purposes the values on the diagram may be considered as a reasonable approximation of the lower limit of possible pressures.

*Bubble Formation.* The measurements made from these records have been used as the basis of several different graphical presentations. Fig. 7 shows the position of the bubble as a function of time, with the zero of position at the point of tangency of the ogive to the cylinder. Note that the three lines show the leading edge, the trailing edge, and the mid-point of the bubble. The slope of the line is proportional to the axial velocity of the bubble in the tunnel. It will be seen that this is not constant but varies with the position of the bubble along the body. The maximum diameter of the bubble is about 0.3 in. This is relatively large for the size of the body involved. Nevertheless, the life of the bubble from the instant it is large enough to be detected until the completion of its first collapse is only about 0.003 sec. Formation requires about three fourths of this time, leaving one fourth for the collapse. An interesting point to observe in passing is that during the final stages of the first collapse the leading edge of the bubble is moving radially inward so rapidly that it is actually moving upstream in the tunnel. Fig. 8 gives the measured radius and volume of the bubble, plotted on the pressure-distribution diagram from Fig. 6. Fig. 9 shows the bubble radius and volume as functions of time. For this diagram the bubbles have been assumed to be spheres having a radius equal to the average of the horizontal and vertical dimensions measured in the photographs.

In the analysis of these diagrams it is necessary to consider some of the physical factors which must influence the growth and collapse of the bubble.

Any fluid particle may be considered as a free body moving in accordance with the forces acting upon it. In such an analysis the inertia of the particle plays a very important role. If particles of liquid on the bubble surface are studied, it may be assumed as a first approximation that they move symmetrically, i.e., that the bubble remains spherical. Thus spheres are equal pressure surfaces, hence only radial forces and velocities are involved.

A consideration of the shape of the body and the pressure-distribution diagram on its surface leads to the explanation of why the cavitation bubble forms. Imagine a particle of liquid in the flow impinging on the nose of the body and following along the surface. First it is forced radially outward and the pressure-distribution diagram shows the amount of force required to make it conform to this portion of the body shape. Outward acceleration continues for a short distance but decreases rapidly in magnitude as shown by the rapid fall in the pressure on the surface. At the point where the pressure on the body has fallen until it is equal to the static pressure in the undisturbed flow, outward acceleration ceases, i.e., the particle is moving out fast enough to keep out of the way of the body. Downstream from this point it is necessary to apply a force acting toward the body to keep the particle in contact with it, because now the surface is curving away from the path of the particle. Since the pressure in the

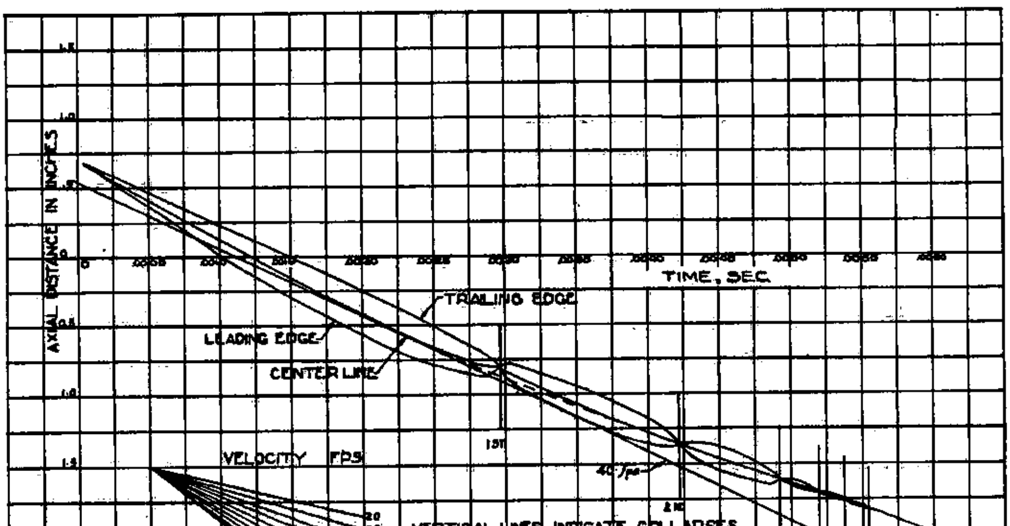



Fig. 7 Bubble Movement During Formation and Collapse

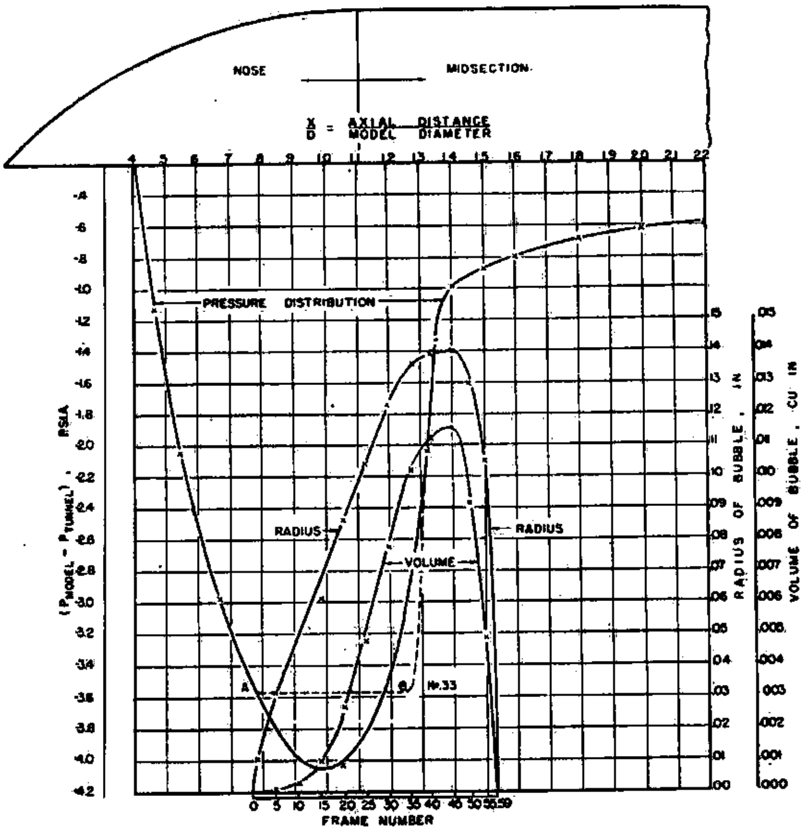



Fig. 8 Relation of Bubble Growth and Collapse to Pressure Field

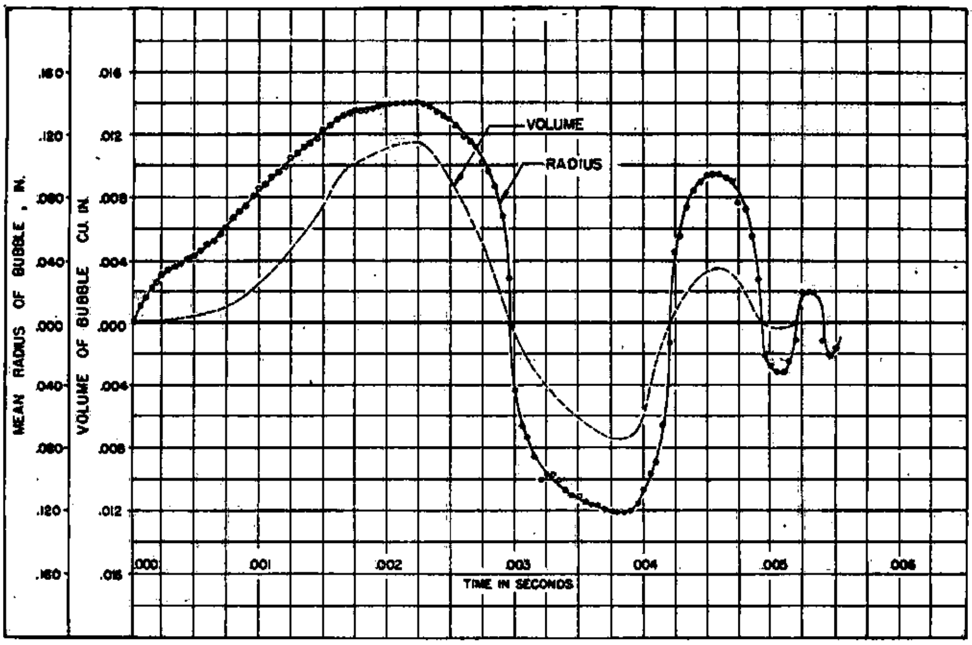



Fig. 9 Time History of Life of Bubble

undisturbed flow is the upper limit of the available pressure, the pressure differences required to keep the flow in contact with the body must be produced by a reduction in pressure on the body surface. As the body curves more rapidly away from the flow, a greater and greater inwardly acting pressure difference is required, i.e., a lower and lower absolute pressure on the body as shown on the pressure-distribution diagram. Note that this pressure difference is utilized to reduce the axially outward velocity that was set up during the flow around the forward part of the nose. The maximum pressure difference available is reached when the pressure on the surface falls to the vapor pressure of the liquid. This occurs at point $A$ on the pressure-distribution diagram, Fig. 8. However, the outward radial velocity of the liquid particle under consideration has not been reduced to zero; hence when it moves downstream from point $A$, there is no longer enough pressure difference acting to cause the curvature of its path to match that of the body. Therefore it separates from the body, which is just a way of saying that a cavitation bubble is formed. Putting it another way, a cavitation bubble appears when there is no longer a large enough pressure gradient acting toward the body to hold the flow against it.

Attention is shifted now to the bubble itself. It will be seen that surface tension forces are acting, which from their physical nature always tend to decrease the size of the bubble. Hence as the bubble expands, work must be done against these forces if growth is to take place.

The record shows that the bubble expands very rapidly. The question arises concerning the pressure and composition of the gas inside the bubble. In this discussion it has been assumed tacitly that the bubble is full of water vapor, and, at least at inception, the pressure is the vapor pressure at the temperature of the liquid. Several other possibilities must be considered. The bubble might contain air, which was previously dissolved in the water, since the present experiments were made with water saturated with air at atmospheric pressure. As the total time from formation to collapse is very small, it would be impossible for air molecules to migrate through the liquid any appreciable distance. Therefore the only air which might come out of solution would be that dissolved in a thin layer of liquid adjoining the bubble surface. This amount is so small that the pressure in the bubble would of necessity be less than a millimeter of mercury during the most of its life. The pressure can be estimated roughly by dividing the volume of air at atmospheric pressure dissolved in the liquid layer $\Delta R_A$ thick adjacent to the bubble, by the volume of the bubble itself. If it is assumed that air-saturated water at atmospheric pressure contains 2 per cent of air by volume, the pressure in the bubble of radius $R$ is

$$p_A = 0.06\,\frac{\Delta R_A}{R}$$

Since the individual air bubbles must be very small because of the low concentration of the dissolved air, it is difficult to imagine their migrating any appreciable distance through the liquid in the 0.0022 sec available for growth to the maximum bubble diameter. If $\Delta R$ is estimated to be 0.001 in., the pressure becomes

$$p_A = 0.0004 \text{ atm} = 0.3 \text{ mm Hg}$$

Another possibility is that the bubble might contain water vapor but at a pressure much lower than the equilibrium pressure corresponding to the average temperature of the flow (0.39 psi at 72 F). However, physical measurements obtained from pressure-distribution models show that when the cavitation voids touch the body and are large enough to cover some of the piezometer openings, the pressure in these voids is approximately equal to the vapor pressure of the liquid. If this is actually the case in all cavitation bubbles, such as the one under consideration, then liquid must be evaporated during the growth period. If a portion of the surrounding liquid evaporates into the bubble, it must secure the necessary heat of vaporization to do so. The only available source of heat is in the heat of the liquid layer immediately surrounding the bubble. Because of the extremely short time available for heat transfer, the effective thickness of this layer must be very small. On the other hand, if the temperature of the surface layer of liquid falls very far, the pressure in the bubble must decrease appreciably, since it is difficult to imagine how the vapor pressure in an expanding bubble can be greater than that corresponding to the temperature of the surface layer of the liquid.

The thermal considerations just outlined suggest the desirability of some rough calculations to determine whether enough vaporization is physically possible to maintain the pressure in the bubble near the vapor pressure of the liquid. Therefore computations have been made on the following assumptions:

(*a*) The vapor pressure in the bubble is in equilibrium with the temperature of the surface layer of liquid.

(*b*) The vapor to fill the bubble is produced by the evaporation of a uniform thin layer over the surface.

(*c*) The necessary heat for this evaporation comes from the heat of the liquid of a shell of uniform thickness surrounding the bubble.

The temperature of the inside surface of this shell is assumed to be the temperature corresponding to the vapor pressure in the bubble, and the temperature of the outside of the shell is the average tunnel temperature. The thickness of the shell which must be evaporated to fill the bubble with vapor is equal to the volume of the bubble divided by the product of the bubble surface and the ratio of the specific volumes of the vapor to the liquid. The ratio of the thickness of the outer shell which furnishes the heat to evaporate this liquid to the thickness of the evaporated layer is equal to the heat of vaporization divided by the average temperature drop of this outer shell. Thus

$$\Delta R_v = \frac{\frac{4}{3}\pi R^3}{4\pi R^2 \dfrac{V_v}{V_L}} = \frac{RV_L}{3V_v}$$

or

$$\frac{\Delta R_v}{R} = \frac{V_L}{3V_v}$$

and

$$\frac{\Delta R_H}{R} = \frac{\Delta R_v}{R}\;\frac{H_v}{1/2(T_L - T_B)}$$

where

$R$ = radius of bubble
$R_v$ = thickness of evaporated shell
$R_H$ = thickness of shell furnishing heat
$V_v$ = specific volume of vapor
$V_L$ = specific volume of liquid
$H_v$ = heat of vaporization
$T_L$ = temperature of undisturbed liquid
$T_B$ = equilibrium temperature of liquid corresponding to pressure in bubble

For the bubble shown in Fig. 6, $T_L$ was 72 F. If the vapor in the bubble is assumed to be 10 F below this, i.e., 62 F, $\dfrac{V_v}{V_L}$ = 70,000, $H_v$ = 1050. Therefore

$$\frac{\Delta R_v}{R} = \frac{1}{210{,}000}$$

$$\frac{\Delta R_H}{R} = \frac{1}{210{,}000} \times \frac{1050}{5} = \frac{1}{1000}$$

When the bubble has grown to maximum size, $R_0 = 0.15$ in., hence

$$\Delta R_v = 7 \times 10^{-7} \text{ in.}$$

$$\Delta R_H = 1.5 \times 10^{-4} \text{ in.}$$

These thicknesses are so small that the evaporative process appears very plausible even in face of the short time of growth and much more plausible than the evolution and migration of minute air bubbles through a layer of liquid 7 times as thick as the heating shell. Furthermore the vapor pressure corresponding to 62 F is 0.019 atmosphere, which is 47 times greater than the 0.004 atmosphere calculated for the air migration from the 0.001-in-thick shell.

A further inspection of the pressure-distribution diagram yields some additional facts. As previously stated, point $A$ should be the point at which the bubble first appears. Point $B$ should be the point of the maximum rate of bubble growth. Up to this point the outward radial velocity of the bubble surface should have increased. Here the acceleration should reverse, i.e., the rate of growth should slow down. Note, however, that the growth should continue until the radial kinetic energy is expended in working against the pressure difference. Thus, the point of maximum bubble diameter should be downstream from point $B$. At the point of the maximum bubble diameter, the liquid no longer has any kinetic energy with respect to the center of the bubble. However, the kinetic energy has been expended in a conservative manner, i.e., it has done work against the pressure difference and against the surface tension. The point of maximum diameter is not an equilibrium condition, the bubble starts to collapse immediately. If in Fig. 8 the bubble size is compared with the pressure-distribution diagram, it will be seen that at least qualitatively the foregoing deductions agree with the observations.

*Bubble Collapse.* All of the factors investigated during bubble growth must be considered during the collapse period. In examining this collapse, it might be well to state explicitly an assumption that is implicit in the previous discussion. The fluid system is thought of not as a purely mechanical one, but as a thermodynamic one as well. Two sources of energy have been considered during the bubble growth, the mechanical energy present, as a result of the motion of the fluid, and the thermal energy made available by a change in temperature of the liquid. It was assumed that none of the mechanical energy was transferred from the liquid to the gas. During the collapse period it may not be possible to avoid considering energy interchange between the liquid and the gas and vapor in the bubble. The collapse period begins at the maximum diameter of the bubble. At this point the vapor may be assumed to be in thermal equilibrium with the inner surface of the liquid which is at a lower temperature than that of the surrounding liquid. The progress of the collapse furnishes the mechanism for compressing the vapor in the bubble, thus raising its temperature above the surface of the liquid and reversing the temperature gradient, which provides a means for carrying away the heat of condensation.

It will be seen from Fig. 9 that the rate of collapse is considerably higher than the growth; consequently the rate of condensation must be similarly increased. Furthermore, as the bubble gets smaller, the thickness of the shell of surrounding liquid, whose temperature has been raised by the heat of condensation, increases appreciably. Both factors require corresponding increases in the temperature difference between the vapor and the average temperature of the liquid. This is easily available when the bubble has grown small because the necessary energy for compressing and raising the temperature of the vapor can be taken from the kinetic energy of the surrounding liquid.

Fig. 10 is a plot of the radial velocity of the bubble surface during the collapse period. It is seen that this velocity increases very rapidly as the bubble becomes small. The accuracy of the calculation is limited by the experimental measurements. The points show the consecutive frames of the photographic record. In the final collapse and initial rebound phases, the readings are too far apart even though they are separated by only $^1/_{20,000}$ sec. For this reason, an attempt is being made to increase the photographic rates to at least 50,000 per sec.

In analyzing the mechanism of collapse and rebound, it is necessary to explain what happens to the kinetic energy of the liquid.

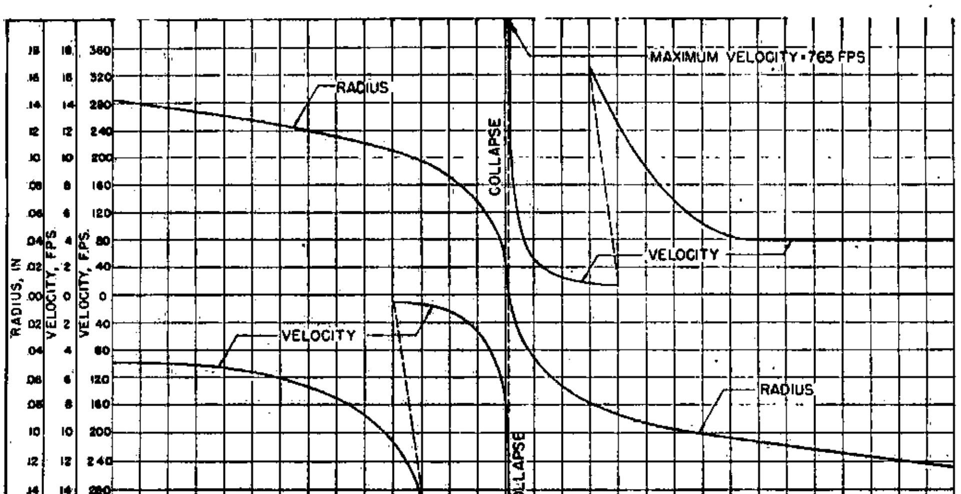



Fig. 10 Velocity of Bubble Surface During Collapse and Rebound

Since there is no apparent way of dissipating this energy, it must be assumed to be stored in some other form when the bubble is completely collapsed. The very fact that rebound occurs and the cavity reopens to nearly the same size as the original maximum radius is strong evidence that the kinetic energy was stored and given back essentially undiminished. Several possible methods of storing the energy suggest themselves. The most probable energy storage is in the compression of the liquid itself. Other possible ways are in the energy of compression of the noncondensable gas or the vapor in the bubble. However, as will be shown later, storage of a major part of the energy in the gas or vapor leads to impossible values of pressure and volume. The storage of the energy in the liquid is accomplished by the common "water-hammer" phenomenon. This method of energy storage permits the development of extremely high localized pressures. If particles of the liquid from opposite sides of the bubble are assumed to hit each other and come to rest, the resultant pressure may be estimated by the normal water-hammer calculations provided that the velocity of the liquid ($V$), at the time of impact is known. The resulting pressure is given by the simple water-hammer equation

$$P = \frac{\rho c V}{144}$$

where

$P$ = pressure, psi
$c$ = velocity of sound in liquid, fps

If values for $c$ and $\rho$ for cold water are substituted in the equation, this becomes

$$P = 65\,V$$

It must be remembered that this water-hammer equation is derived on the basic concept that the kinetic energy of a given element of moving liquid is stored within that same element in elastic compression when the element is brought to rest. This concept explains the "rebound" or re-formation of the bubble after collapse. Since there is no way to hold the liquid in a compressed condition after the inward radial velocity has been reduced to zero, the stored elastic energy goes into producing outward radial velocity. Since there has been nothing to cause appreciable energy loss, the bubble should grow to its original size at the same rate it collapsed, provided that the surrounding pressure remained constant. This cycle of growth and collapse should continue indefinitely. Actually, losses through fluid friction or possibly heat conduction, damp this oscillation. The photographic records show clearly that many bubbles go through 4 or 5 cycles before final decay. The pressure diagram shows that, except for the original formation period, the rest of the life of the bubble occurs in a relatively constant pressure field.

It is interesting to compare this measured history of an actual bubble with the analysis presented by Rayleigh in his classical paper[4] of 1917. He considered the collapse of an empty spherical bubble in an incompressible fluid having a constant pressure at infinity. He equated the kinetic energy of the resulting motion of the fluid to the work done at infinity by the constant pressure acting through a change of volume equal to the change of bubble volume. He obtained expressions for the velocity of the bubble surface as a function of the radius, for the time of collapse, for the acceleration of a point on the surface, and for the pressure distribution in the surrounding fluid. He also calculated the behavior of the bubble if it were filled with a gas at an arbitrary pressure at the beginning of collapse, on the assumption of isothermal compression. This included an expression for the ratio of initial to final volume of the bubble if all of the kinetic energy of the incompressible fluid was stored in compressing the gas. Finally, he calculated the pressure produced if an empty bubble collapsed on an absolutely rigid sphere of arbitrary radius. Here he abandoned the assumption of an incompressible fluid, but only after contact with the rigid sphere. He found the resulting pressure to be given by the water-hammer equation.

Fig. 11 shows a comparison of the Rayleigh prediction for the empty bubble with the measured radius versus time for the collapse of the bubble in Fig. 6. It is felt that the agreement is quite remarkable. In the calculation, the pressure acting is assumed to be the pressure at the tunnel wall minus the vapor pressure of the water. Rayleigh's derivation permits of such a constant bubble pressure. The curve in Fig. 12 is Rayleigh's calculated velocity of the surface as a function of the time measured from the beginning of collapse. The points shown are the slope of the measured curve in Fig. 11. The deviations of the measured from the predicted curves are in the right direction to agree with physical conditions. Rayleigh assumed no energy storage up to the instant of complete collapse, because up to that instant he assumed an incompressible fluid. Actually, there is some energy storage, especially in the last stages, in the liquid and also in the gas or vapor in the bubble. All of this energy storage reduces the work available for increasing the velocity; hence the collapse time must be longer.

In a previous section of this paper it was estimated that the amount of air available would fill the bubble to a pressure of 0.0004 atm at maximum diameter. If all of the kinetic energy of the liquid were to be stored in this air by isothermal compression, Rayleigh's calculations indicate a required compression ratio of $4 \times 10^{26}$, or a radius ratio of $7.3 \times 10^{8}$. The initial radius of the actual bubble was 0.140 in. This means the compressed air bubble would be $2 \times 10^{-10}$ in. diam and would have a pressure of $1.6 \times 10^{23}$ atm. Obviously, this is impossible since the energy could all be stored in the liquid at a much lower pressure. It seems most probable that the energy is stored in compression in all three fluids, i.e., liquid, vapor, and gas, and that the compression processes of the vapor and the gas lie between the adiabatic and the isothermal.

*Cavitation Damage.*[5] At the beginning of this discussion it was stated that the objective of this investigation is the study of the mechanism of cavitation and not of cavitation damage. However, there are a few tentative conclusions which can be formulated concerning certain phases of cavitation damage on the basis of the results obtained to date. It has been seen that the maximum collapse velocity of the cavitation bubble is controlled by (1) the maximum bubble size, and (2) the pressure difference existing between the surrounding fluid and the bubble. Factors which affect the maximum bubble size are the length of the zone in which bubble growth occurs, the average velocity of flow, and the velocity component normal to the guiding surface. The length of the zone is determined by the size and the shape of the guiding surface that is responsible for the cavitation. This alone would indicate that there should be a scale effect in cavitation damage. Consider, for example, two shapes geometrically similar but differing in size. If the velocity of flow past these two shapes is the same, the pressure and also the cavi-

[4] "On the Pressure Developed in a Liquid During the Collapse of a Spherical Cavity," by Lord Rayleigh, *Philosophical Magazine*, vol. 34, 1917, pp. 94–98 (see Appendix).

[5] No bibliography of cavitation literature is included in this paper because many exhaustive lists have already been published. One of the most recent of these will be found in the paper by A. J. Stepanoff, "Cavitation in Centrifugal Pumps," Trans. ASME, vol. 67, 1945, p. 539. Another which refers particularly to cavitation damage, but includes much work on cavitation, is contained in the book on "Werkstoffzerstörung durch Kavitation" by Nowotny, published by V.D.I. Verlag, 1942, and reprinted by J. W. Edwards, Ann Arbor, Mich., 1946.

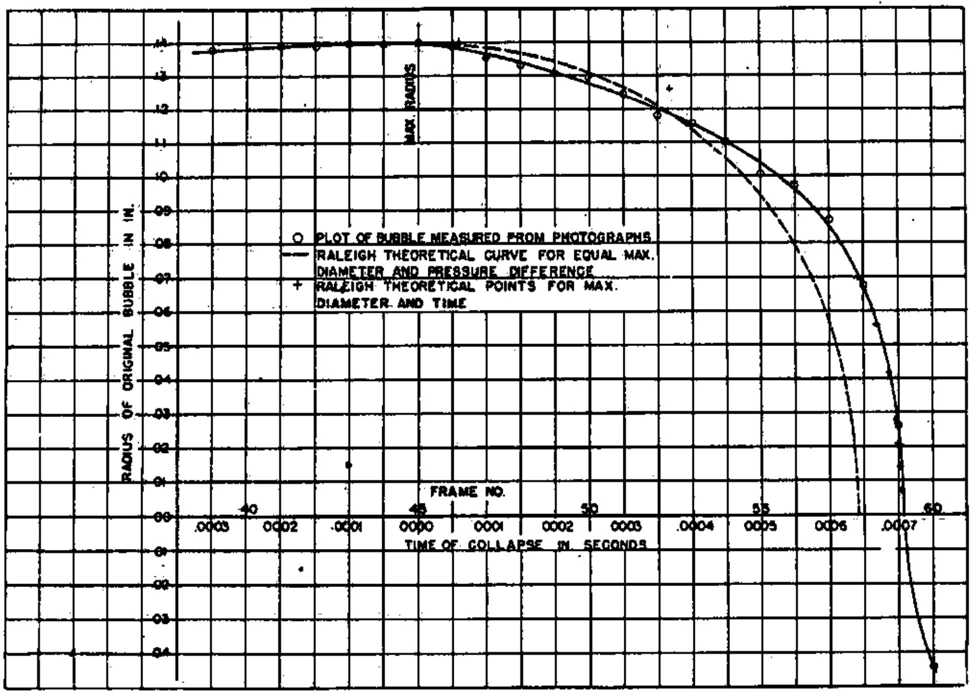



Fig. 11 Comparison of Measured Bubble Size With Rayleigh Prediction

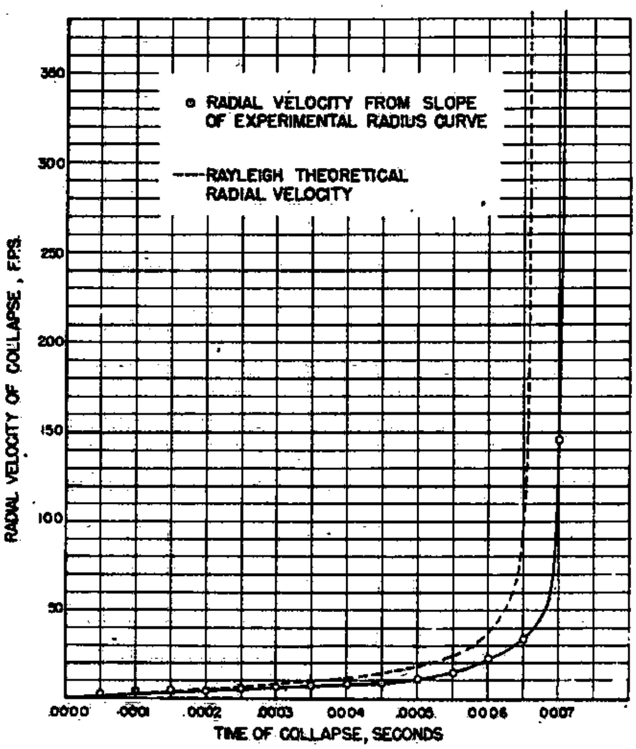



Fig. 12 Comparison of Measured Surface Velocity During Collapse With Rayleigh Prediction

tation parameter, $K$, at corresponding points will be identical. If one shape is twice as large as the other, the length of the formation zone will be twice as long, which means that since the velocities are the same, the bubble should grow under the same pressure difference for twice as long a time; hence it should be larger. The two bubbles will collapse under identical pressure conditions; hence the final collapsing velocity will be higher for the larger bubble. Thus cavitation damage may be expected to increase in severity with increase in size. A similar line of reasoning leads to the conclusion that if the flow velocity is increased over a given surface while the flow pressure is adjusted to keep the cavitation parameter $K$ constant, the collapsing velocity, and hence the cavitation damage, will increase. Both of these conclusions are contrary to the concept that the cavitation parameter alone determines the severity of cavitation damage.

These two cases deal with similar geometric shapes. As a third case, consider two different shapes which, however, have the same incipient cavitation parameter. If these are operated at the same velocity but at a lower cavitation parameter identical for both shapes, the cavitation damage may be quite different. If the velocity component of the flow normal to and away from the surface of one shape is lower than that of the other, the maximum bubble size for the shape having the lower normal velocity component should be smaller and the damage less.

## Conclusion

In conclusion, the authors wish to emphasize that the foregoing interpretation of the experimental measurements of the life history of a cavitation bubble is only a tentative presentation of the simplest possible case, i.e., a bubble which forms and collapses without interference from other bubbles. An examination of the photographic record shows that this is a relatively rare occurrence; more often clusters of bubbles form and collapse very close together. In many of the records it is obvious that the collapse of one bubble has a major effect on the collapse of its neighbor. Furthermore, as the severity of the cavitation is increased, the bubble concentration builds up very rapidly, so that rarely if ever can a single bubble be seen to form and collapse without interference. An inspection of the records indicates that the presence of many bubbles offers complicating factors. Thus the most this discussion can represent is the first short step in the correlation of this new supply of experimental facts with the analysis of the physical mechanism of the cavitation phenomenon. It is regretted that time has not permitted the presentation of a more complete comparison of the laboratory results with the various analytical descriptions of the cavitation process which are to be found in the literature. Such comparison will be included in the next step in the development of the cavitation program of the laboratory.

ACKNOWLEDGMENT

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# Appendix

In 1917 Lord Rayleigh presented his classical paper[4] on the pressure development in a liquid during the collapse of a spherical cavity. Since this paper is not regularly accessible to the engineer, a brief summary of it will be presented here.

Rayleigh quotes Besant's formulation of the problem: "An infinite mass of homogeneous incompressible fluid acted upon by no forces is at rest, and a spherical portion of the fluid is suddenly annihilated; it is required to find the instantaneous alteration of pressure at any point of the mass, and the time in which the cavity will be filled up, the pressure at an infinite distance being supposed to remain constant." Rayleigh first sets up an expression for the velocity $u$, at any distance $r$, which is greater than $R$, the radius of the cavity wall, that has a velocity $U$, at time $t$. It is

$$u/U = R^2/r^2 \qquad [1]$$

Next, the expression for the kinetic energy of the entire body of fluid at time $t$, is developed by integrating the kinetic energy of a concentric fluid shell of thickness $dr$, and density $\rho$. The result is

$$\frac{\rho}{2}\int_R^\infty u^2\, 4\pi r^2 dr = 2\pi\rho U^2 R^3 \qquad [2]$$

The work done on the entire body of fluid as the cavity is collapsing from the initial radius, $R_0$ to $R$ is a product of the pressure, $P$ at infinity and the change in volume of the cavity, i.e.

$$\frac{4\pi P}{3}(R_0^3 - R^3) \qquad [3]$$

Since the fluid is incompressible, all of the work done must appear as kinetic energy. Therefore Equation [2] can be equated to Equation [3], which gives

$$U^2 = \frac{2P}{3\rho}\left(\frac{R_0^3}{R^3} - 1\right) \qquad [4]$$

An expression for the time $t$, required for the cavity to collapse from $R_0$ to $R$ can be obtained from Equation [4] by substituting for the velocity $U$, of the boundary, its equivalent $dR/dt$, and performing the necessary integration. This gives

$$t = \sqrt{\frac{3\rho}{2P}}\int_R^{R_0}\frac{R^{3/2}dR}{(R_0^3 - R^3)^{1/2}} = R_0\sqrt{\frac{3\rho}{2P}}\int_\beta^1\frac{\beta^{3/2}d\beta}{(1-\beta^3)^{1/2}} \qquad [5]$$

The new symbol $\beta$, is $R/R_0$. The time $\tau$, of complete collapse is obtained if Equation [5] is evaluated for $\beta = 0$. For this special case the integration may be performed by means of $\Gamma$ functions with the result that $\tau$ becomes

$$\tau = R_0\sqrt{\frac{\rho}{6P}}\;\frac{\Gamma\left(\frac{5}{6}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{4}{3}\right)} = 0.91468\, R_0\sqrt{\frac{\rho}{P}} \qquad [6]$$

Rayleigh does not integrate Equation [5] for any other value of $\beta$. In the detailed study of the time history of the collapse of a cavitation bubble, it is convenient to have a solution of all values between 0 and 1. Fig. 13 and Table 1 give the value of this integral over this range.

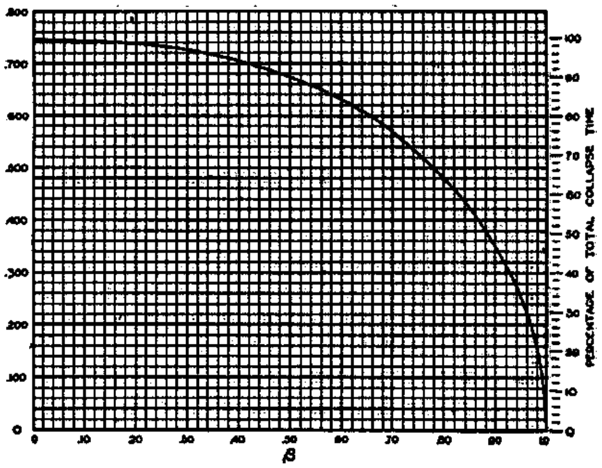


FIG. 13 INTEGRAL FOR DETERMINING TIME OF PARTIAL COLLAPSE AS FUNCTION OF RADIUS RATIO

TABLE 1 VALUES OF THE INTEGRAL OF EQUATION [5]

| $\beta$ | $\int_\beta^1 \frac{\beta^{3/2}d\beta}{(1-\beta^3)^{1/2}}$ | $\beta$ | $\int_\beta^1 \frac{\beta^{3/2}d\beta}{(1-\beta^3)^{1/2}}$ |
|---|---|---|---|
| 0.000 | 0.74684 | 0.600 | 0.62915 |
| 0.040 | 0.74671 | 0.640 | 0.60676 |
| 0.080 | 0.74611 | 0.680 | 0.58133 |
| 0.120 | 0.74484 | 0.720 | 0.55240 |
| 0.160 | 0.74274 | 0.760 | 0.51935 |
| 0.200 | 0.73967 | 0.800 | 0.48128 |
| 0.240 | 0.73552 | 0.840 | 0.43684 |
| 0.280 | 0.73016 | 0.860 | 0.41159 |
| 0.320 | 0.72349 | 0.880 | 0.38380 |
| 0.360 | 0.71539 | 0.900 | 0.35285 |
| 0.400 | 0.70575 | 0.920 | 0.31782 |
| 0.440 | 0.69443 | 0.940 | 0.27716 |
| 0.480 | 0.68129 | 0.960 | 0.22785 |
| 0.520 | 0.66616 | 0.980 | 0.16220 |
| 0.560 | 0.64886 | 1.000 | 0.00000 |

Equation [4] shows that as $R$ decreases to 0, the velocity $U$ increases to infinity. In order to avoid this, Rayleigh calculates what would happen if, instead of having zero or constant pressure within the cavity, the cavity is filled with a gas which is compressed isothermally. In such a case, the external work done on the system as given by Equation [3] is equated to the sum of the kinetic energy of the liquid given by Equation [2], and the work of compression of the gas, which is $4QR_0^3\log_e(R_0/R)$, where $Q$ is the initial pressure of the gas. Thus Equation [4] is replaced by

$$U^2 = \frac{2P}{3\rho}\left(\frac{R_0^3}{R^3} - 1\right) - \frac{2Q}{\rho}\frac{R_0^3}{R^3}\log_e\frac{R_0}{R} \qquad [7]$$

For any real (i.e., positive) value of $Q$, the cavity will not collapse completely, but $U$ will come to 0 for a finite value of $R$. If $Q$ is greater than $P$, the first movement of the boundary is outward. The limiting size of the cavity can be obtained by setting $U = 0$ in Equation [7], which gives

$$P\left(\frac{z-1}{z}\right) - Q \log_e z = 0 \quad [8]$$

in which $z$ denotes the ratios of the volumes $R_0^3/R^3$. Equation [8] indicates that the radius oscillates between the initial value $R_0$ and another which is determined by the ratio $P/Q$ from this equation. Although Rayleigh presents this example only for isothermal compression, it is obvious that any other thermodynamic process may be assumed for the case in the cavity and equations analogous to Equation [7] may be formulated.

As another interesting aspect of the bubble collapse, Rayleigh calculates the pressure field in the liquid surrounding the bubble, reverting to the empty cavity of zero pressure. He sets up the radial acceleration as the total differential of the liquid velocity $u$, at radius $r$, with respect to time, equates this to the radial pressure gradient divided by the density, and integrates to get the pressure at any point in the liquid. Hence

$$a_r = -\frac{du}{dt} = \frac{\delta u}{\delta t} - u\frac{\delta u}{\delta r} = \frac{1}{\rho}\frac{\delta p}{\delta r} \quad [9]$$

Expression for $\frac{\delta u}{\delta t}$ and $u\frac{\delta u}{\delta r}$ as functions of $R$ and $r$ are obtained from Equations [1] and [4], the partial differentials of Equation [1], with respect to $r$ and $t$, and the partial differential of Equation [4], with respect to $t$. Substituting these expressions in Equation [9] yields

$$\frac{1}{P}\frac{\delta p}{\delta r} = \frac{R}{3r^2}\left[\frac{(4z-4)R^3}{r^3} - (z-4)\right] \quad [10]$$

By integration this becomes

$$\frac{1}{P}\int_P^p \delta p = \frac{R}{3}\left[(4z-4)R^3\int_\infty^r \frac{\delta r}{r^5} - (z-4)\int_\infty^r \frac{\delta r}{r^2}\right] \quad [11]$$

which gives

$$\frac{p}{P} - 1 = \frac{R}{3r}(z-4) - \frac{R^4}{3r^4}(z-1) \quad [12]$$

The pressure distribution in the liquid at the instant of release is obtained by substituting $R = R_0$ in Equation [12], which gives

$$p = P\left(1 - \frac{R_0}{r}\right) \quad [13]$$

The point of maximum pressure may be found by setting $dp/dr$ equal to zero in Equation [10]. This gives a maximum value for $p$ when

$$\frac{r_m^3}{R^3} = \frac{4z-4}{z-4} \quad [14]$$

If this value for $r_m$ is substituted back into Equation [12], the maximum value of $p$ is obtained

$$\frac{p_m}{P} = 1 + \frac{(z-4)R}{4r_m} = 1 + \frac{(z-4)^{4/3}}{4^{4/3}(z-1)^{1/3}} \quad [15]$$

If this equation is inspected it will be seen that so long as $z$ is less than 4, the second term of the equation is negative; hence $p_{max}$ and therefore all other pressures in the liquid are less than $P$ at infinity; but when $z$ exceeds 4, then $p_{max}$ becomes greater than $P$. The radial location of $p_{max}$ is given by Equation [14]. As the cavity approaches complete collapse, $z$ becomes great and Equations [14] and [15] may be approximated by

$$r_m = 4^{1/3}R = 1.587\,R \quad [16]$$

and

$$\frac{p_m}{P} = \frac{z}{4^{4/3}} = \frac{R_0^3}{4^{4/3}R^3} \quad [17]$$

Equation [16] shows that as the cavity becomes very small, the pressure in the liquid near the boundary becomes very great in spite of the fact that the pressure at the boundary is always zero. Although Rayleigh does not mention it, this would suggest the possibility that some energy can be stored in compressing the liquid which would add an additional term to Equation [7]. Of course this would mean that the assumption of incompressibility would have to be abandoned for the entire calculation. This, however, would not change the physical concepts involved. Rayleigh himself abandons the assumption of the incompressible fluid to consider what happens if the cavity collapses on an absolutely rigid sphere of radius $R$, as proposed by his correspondence to Mr. Cook. However, he abandons the supposition of incompressibility only at the instant that the cavity wall comes in contact with the rigid sphere. From that instant on he makes the assumption common to all water-hammer calculations that the kinetic energy of each particle of fluid is changed to elastic energy of deformation of the same particle as determined by the volume modulus of elasticity of the fluid. On this basis he obtained

$$\frac{(P')^2}{2E} = \frac{1}{2}\rho U^2 = \frac{P}{3}\left(\frac{R_0^3}{R^3} - 1\right) = \frac{P}{3}(z-1) \quad [18]$$

where $P'$ is the instantaneous pressure on the surface of the rigid sphere and $E$ is the volume modulus of elasticity. Both must be expressed in the same units.

# Discussion

J. M. Robertson[6] and Donald Ross.[7] This paper is an important contribution to the meager fund of knowledge of the mechanism of cavitation. The authors' development of techniques of superhigh-speed photography has made available an extremely useful tool for the study of the life of a cavitation bubble. Fig. 3 of the paper illustrates the manner in which knowledge of cavitation phenomena has increased with camera speed. The analysis of the growth and collapse of a single bubble, as presented in this paper, is in itself a major contribution to the subject. As more bubbles are analyzed and higher camera speeds are attained, more and more of the mysteries of cavitation will be dispelled.

By showing that the partial pressure of the air in the cavity is much smaller than the actual bubble pressure, the authors demonstrate that the gas in the bubble is primarily water vapor. This fact, which heretofore had only been surmised, should not be interpreted as indicating that air plays an insignificant role in the cavitation process. The cavitation studied by the authors was in water saturated with air at atmospheric pressure. In the case of the particular body studied, there was an appreciable distance preceding the cavitation region in which the pressure was below atmospheric and in which undissolved air bubbles

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[7] Assistant Professor, Ordnance Research Laboratory, Pennsylvania State College. State College, Pa.

could expand and dissolved air could come out of solution. Thus at the start of the cavitation region a number of air bubbles should have been available to act as nuclei for the formation of vapor cavities. The manner in which the bubble grows at the start of the cavitation region, as depicted in Figs. 8 and 9, would indicate that at the start of this region it may have had a radius of several thousandths of an inch.

The air content of the bubble plays an important role during the collapse period; since the air cannot be redissolved as rapidly as the water vapor, the bubble must collapse on an air nucleus. The minimum radius at the first collapse is thus primarily a function of the air content of the bubble. Osborne[8] has shown that "the abruptness and violence of the shock, the total acoustic energy produced and its frequency distribution are all highly dependent on the size of the residual air bubble around which the cavity collapses." The effectiveness of air in cushioning the cavitation shock is well known. It would be informative to know how the authors' results would be changed, if deaerated instead of saturated water were employed. The lack of air would not prevent cavitation, but its inception would be retarded. Briggs, Johnson, and Mason[9] have found that "when liquids are degassed, their natural cohesive pressure becomes effective and they will withstand a negative acoustic pressure." When air nuclei are absent, cavitation rupture is a molecular process, and the time the fluid element is in the negative pressure region is a very important factor. In tests made with model propellers at the David Taylor Model Basin,[10] and corroborated by one of the writers at the M.I.T. Propeller Tunnel, the critical pressure for the inception of cavitation was found to decrease with a reduction of air content. In an experiment in which the time in the negative pressure region was quite short, Numachi[11] found the critical pressure to be greatly affected by gas content. The critical pressure was found to be approximately vapor pressure with air-saturated water, while it was minus an atmosphere (absolute) with partially deaerated distilled water. The differences in the characteristics of the cavitation of the bodies, shown in Fig. 5 of the paper, can possibly be explained in terms of the time available for the formation of air nuclei before the fluid elements reach the cavitation region. This effect could be studied by cavitation measurements on several body shapes at different scales and with different air contents of the water.

Wilhelm Spannhake.[12] This admirable paper adds much to our present knowledge of cavitation even though the authors selected for investigation, as they state, only the simplest manifestations of the very complex phenomena they observed throughout a wide range of geometrical conditions and stages of cavitation. Specifically, they selected for study a situation which showed individual bubbles successively produced and spaced far enough apart so that from time to time some could be found which throughout their life history were not seriously affected by interference from other bubbles.

The possibility of maintaining a more or less stable stage of cavitation depends apparently upon the shape of the boundary surfaces against which cavitation occurs, or rather on the pressure distribution of the original flow pattern. This possibility will be seen from Fig. 5(*a*, *b*, and *c*) of the paper.

It is extremely interesting to notice that the same stage as in Fig. 5(*c*), showing distinctly separated bubbles, has been obtained recently at the David Taylor Model Basin in a small Venturi tube with circular cross section and a very slight longitudinal curvature in the throat, together with a small angle of divergence in the expanding passage. To a certain degree this case corresponds hydrodynamically to Fig. 5(*c*).

On the other hand, the authors mention that in other cases many complex groups of bubbles are formed in which the individual bubbles interact in either the formation or the collapse stages, as seems to be shown in Fig. 5(*a*), where the curvature of the nose of the body is sharper and consequently there are greater pressure differences along the surface.

Here there should be recalled the results obtained in 1933 and 1934 at the cavitation test stand of the Massachusetts Institute of Technology.[13]

These tests were made with a Venturi tube of rectangular cross section, and with both the longitudinal curvature in the throat and the angle of divergence in the expanding passage greater than in the circular tube mentioned as used at the Model Basin. Thus from the geometrical point of view and to a certain extent from the hydrodynamical point of view, this arrangement corresponds to the case of Fig. 5(*a*). During the tests at M.I.T., pictures were taken with a frequency of 3000 exposures per sec. No individual bubbles could be seen, but large cavitation volumes appeared separated from one another by compact volumes of water having rather definite outlines. The cavitation volumes and the intervening volumes of water were formed with a definite periodicity, and the cavitation volume collapsed as a whole when compressed between the alternate masses of water. Water completely filled the expanding passage after the collapse of the cavitation volume and was continuous with the water in the region of restored higher pressure. The writer believes that if the pictures had been taken with sufficiently higher frequency it is possible that individual bubbles and groups of bubbles would have been seen also in this case, and that the bubble groups would in turn be interacting and forming the large cavitation volumes.

The frequency of formation $f$, and collapse of the cavitation volumes turned out to be in direct proportion to the velocity $v$, in the Venturi throat and inversely in proportion to the distance $l$, over which the complete cavitation phenomenon extended. The dimensionless expression

$$\frac{f \cdot l}{v} = C$$

was the same for geometrically similar arrangements and for hydrodynamically similar stages of cavitation.

Considering the different cases described, the writer believes that cavitation is always periodical, and that the periodicity is always determined merely by the hydrodynamical boundary conditions of the original flow pattern. The formation of bubbles changes the boundary conditions and makes the flow unsteady. It likewise changes the pattern in such a way that there is a rise in pressure at the point where cavitation has started, and in this way the formation and further development of the cavity is stopped until the original conditions are restored and the cycle begins again.

It is the writer's understanding that Dr. Knapp and his asso-

[8] "The Shock Produced by a Collapsing Cavity in Water," by M. F. M. Osborne, Trans. ASME, vol. 69, 1947, pp. 253–266.

[9] "Properties of Liquids at High Sound Pressures," by H. Briggs, J. B. Johnson, and W. P. Mason, *Journal of the Acoustical Society of America*, vol. 19, 1947, pp. 664–677.

[10] "Design, Operation, and Maintenance of a Meter for Recording the Air Content of Water in the David Taylor Model Basin Water Tunnels," by A. Borden, David Taylor Model Basin Report 549, December, 1946.

[11] "Translation and Commentary on F. Numachi's Articles on 'The Effect of Air Content on the Appearance of Cavitation in Distilled Salt, and Sea Water,'" Ordnance Research Laboratory Report, Serial No. NOrd 7958–27, August 1, 1946.

[12] Technische Hochschule Karlsruhe, Baden, American Zone. Contractee U. S. Navy.

[13] "Progress Report on M.I.T. Cavitation Research," by J. C. Hunsaker, Transactions of the 4th International Congress of Applied Mechanics, Cambridge, England, June, 1934.

ciates have under way a systematic investigation of the different stages of cavitation to clear up details of this most interesting hydrodynamic problem.

As to the rebound of the bubbles which has been observed for the first time in these California tests, the writer is not sure that this would have occurred had the air content in the water been distinctly smaller. This phenomenon reminds one in a way of the periodical expansion and compression of large gas bubbles formed by underwater explosions.

With respect to the damage done by cavitation, the writer believes that this problem cannot be regarded without considering the fact that even the smoothest surface of any material is not without slight crevices. These flaws present the weak points under the impact of water following the collapse of the bubbles. There is not only the force of the blow striking into the crevices but in addition the concentration of stresses at the sharp corners, and so on. On the other hand, it will be of the greatest importance to find out whether the individual bubbles or the large cavitation volumes do the more severe damage to the boundary walls.

It seems fitting at this time to recall the memory and do honor to the name of the late Dr. Foettinger who was one of the first to envisage the nature of cavitation damage.

Dr. Knapp and his staff are to be congratulated for the outstanding work reported in this paper. All concerned with cavitation will look forward to further results with the greatest interest.

A. J. Stepanoff.[14] Although considerable progress has been made in the last two decades in understanding cavitation in hydraulic machinery, the price to avoid or alleviate cavitation is still high. Part of this cost is due to the factor of ignorance. Any further progress in cavitation study without a knowledge of the mechanism of cavitation is well-nigh impossible.

Although the authors' study was confined to the life history of a bubble, its results are important from the practical point of view. It gives size and time scale of proceedings, which permits estimation of the thermodynamic process during the birth and collapse of the bubble. This in turn may lead to an estimation of how any liquid other than water will behave under cavitation conditions. The bubble reappearance as a result of an elastic water impact, never suspected before, may serve as an explanation of the appearance of cavitation pitting in places not expected.

Lack of exact cavitation knowledge has resulted in a variety of opinions as to the importance of several factors on conditions leading to cavitation in pumps and turbines. Thus some pump engineers evaluate their cavitation data in terms of absolute velocity through the impeller eye, others use relative velocity, and some use the peripheral velocity. Still another group uses the square root of the product of the radial and peripheral velocities. Similarly, the effect of the impeller entrance angle on cavitation is frequently misunderstood.

In describing the process of appearance and collapse of the bubble, the authors emphasize the hydrodynamic conditions leading to the formation of a cavity. The writer can imagine formation and disappearance of vapor bubbles by thermodynamic means only without pressure or velocity gradients in the surrounding liquid. If the ambient temperature is brought locally to the corresponding boiling pressure, vapor bubbles will appear. This is a regular boiling process in which no external mechanical forces are required to form a bubble, heat doing the work of volume expansion. During the collapsing period, temperature-pressure equilibrium is destroyed, vapor is condensed, and the liquid rushes into an empty space. The same is true also during the rebounding process although the elastic forces of the liquid play a more conspicuous role. Perhaps the difference is just a matter of point of view. The writer prefers to think in terms of thermodynamics of the process first because he believes that heat exchange limits the extent of cavitation. When dealing with liquids other than water, the main difference will appear in thermodynamic properties of the liquid and not in hydrodynamic forces.

The effect of degree of streamlining of the body nose on cavitation, as it appears in Fig. 5 of the paper, is interesting. This is contrary to a general belief that a blunt leading edge of impeller vanes is just as good as the hatchet-shaped one. Such a misconception originated from airfoil tests on air.

Under conditions of the authors' tests, it is reasonable to assume that all the kinetic energy of the bubble collapse is stored in the surrounding liquid and reappears as kinetic energy again during rebirth of the bubble. In hydraulic machinery, the minimum pressure appears at the boundary of the body, and, during collapse, a major portion of the energy may be absorbed by the body, resulting in noise and vibration. Large masses, sometimes including the supporting structure, may take part in dissipation of this energy. Thus rebounding of the bubbles may appear greatly subdued. In connection with hydraulic machinery, opinions have been expressed that, if cavitation bubbles appear and collapse in a body of water and do not contact the metal, there is no damage to the parts of the machine.[15]

The writer believes that the possibility of air in the vapor cavity is not to be excluded. This can happen not by air bubbles breaking through into the vapor space, for this they will have no energy, but by the liquid vaporizing into the air-bubble space when this reaches the low-pressure zone. Air liberation from water is quite common in hydraulic machinery when pressure drops below that of the atmosphere.

The difference in bubble grouping in the authors' tests due to a difference in elevation of 2 in. suggests that this difference alone (in the case of hydraulic machinery) may put a rotor out of balance and cause vibration. It is very unlikely that the force of buoyancy of the bubbles could have been responsible to any appreciable degree for the bubbles shifting upward due to lack of time.

In their conclusions regarding the effect of scale on cavitation in hydraulic machines of different sizes, the authors disregard the effect of Reynolds number. In larger machines, all turns will have larger radii of curvature and can be negotiated by the liquid with less velocity distortion than in a small machine. With the same linear velocities of flow, centrifugal forces, causing velocity shifting, are inversely proportional to the radius of curvature. Opinions have been expressed by several authorities on water turbines that, under the same head and submergence, the effects of cavitation are less harmful in the prototype than in the model.[16]

There is a great deal of confusion and misconception about the flow pattern in a turn or elbow. A study of cavitation under such conditions with the aid of high-speed photography would make an excellent topic for investigation. Most of the channels in hydraulic machinery where cavitation occurs are curved. There are no clear ideas as to what actually takes place under such conditions. What portion of the total flow is actually vaporized is of interest. Also, whether compound liquids, like petroleum oils, would behave similarly to water under cavitation conditions.

Research of this nature is beyond the testing facilities of the industry, and the authors and sponsors of this project deserve

[14] Development Engineer, Ingersoll-Rand Company, Phillipsburg, N. J. Mem. ASME.

[15] "Centrifugal and Axial Flow Pumps," by A. J. Stepanoff, John Wiley and Sons, New York, N. Y., 1948, p. 248.

[16] Ibid., p. 265.

credit for their undertaking and manner in which it was conducted. It is hoped that the authors will take full advantage of their equipment and recent progress in high-speed photography (5,000,000 pictures per sec) as recently reported.[17]

## Authors' Closure

Before considering the specific points raised by the individual discussers, the authors wish to point out that as more and more information becomes available on the characteristics of cavitation, it is becoming more and more apparent that it is no longer possible to talk about cavitation as a single phenomenon; instead there are certainly a number of different types of cavitation which vary quite significantly in their characteristics. The existence of these various types of cavitation probably accounts for many of the discrepancies which occur in our empirical knowledge of the phenomenon, which in turn have inspired widely different interpretations of the basic mechanism. As was pointed out in the introduction, the paper presented only the first experimental observations of a single bubble, which was selected as the simplest possible case of one particular type of cavitation. In this type, the cavity forms and collapses while moving downstream with the local velocity of the rapidly moving fluid. It is probable that many of the points brought out by the discussion apply with much more force to other types of cavitation, but since the authors have experimental knowledge concerning this case and not the others, their comments will be limited to the type presented.

Messrs. Robertson and Ross bring out clearly in their interesting discussion the empirical fact that air is known to have a definite effect on the collapse of certain types of cavitation. They then suggest that it is very probable that the cavitation bubbles discussed by the authors had for a nucleus an air bubble of appreciable size and that this air did effectively cushion the collapse of the bubble. They suggest specifically that at the beginning of cavitation when vaporization first starts the air-bubble nuclei have a radius of several thousandths of an inch. The authors feel that air bubbles of this size could not be present without having been observed because the lighting used is very intense and a bubble of this diameter would be highly reflecting, as is evidenced by the small cavitation bubbles further downstream which are so easily seen both visually and on the photographs. However, it is interesting to investigate the order of magnitude of the effect which might have been produced if such an air nucleus had served as a starting point for the bubble whose life history was presented. Assume that the bubble radius was .005 of an inch, which is the upper imaginable size that could have escaped detection in the tunnel. Assume also that the pressure within the bubble was in equilibrium with the local pressure in the flowing stream. Thus at the beginning of the cavitation zone the air will have a pressure of about $^1/_3$ psia. Assume for convenience that this is one psi. This air will remain in the bubble during expansion and collapse; thus when the bubble has collapsed to the radius of .005 in., the air will be again in the same state, assuming that during expansion and collapse to this point the process has been reversible. Up to this point all work terms on the air are negligible. The work required to compress this bubble further may be calculated easily. Rayleigh assumed an isothermal process. The authors have indicated a possible pressure due to water hammer of approximately 50,000 psi, assuming an empty bubble. The isothermal work of compressing the air to the same pressure is given by the equation

$$W = p_1 V_1 \log_e \frac{V_1}{V_2}$$

in which $p_1 = 1$ psi

$$V_1 \cong \frac{d^3}{2} \text{ and } \frac{V_1}{V_2} = \frac{p_1}{p_2} = 50{,}000$$

The solution gives

$$W = 5 \times 10^{-6} \text{ inch-pounds}$$

Now the kinetic energy of the water which must be stored at collapse in compressing the air and the water is given by the simple product of the volume of the bubble at its maximum diameter multiplied by the pressure of the liquid in the collapse zone. By measurement the pressure is about 10 psia, the diameter is .39 in. Thus the kinetic energy to be stored is .135 in-lb. Therefore, it must be concluded that the effect of this amount of air on the collapse pressure is negligible since the best it could do would be to reduce the water-hammer pressure by less than .004 per cent. If the compression were assumed to be adiabatic rather than isothermal, the energy stored could be increased by a factor of, say, about 400, but this would still be negligible and on the other hand would produce tremendously high temperatures which would be brilliantly luminous, an effect completely contrary to the observations. It might also be noted that the amount of air in this bubble nucleus is quite small as compared to the amount that the authors calculated might be possible within the bubble due to coming out of solution in the shell immediately adjacent to the bubble. Thus assuming again isothermal expansion from the nucleus to the maximum diameter of the bubble, the partial pressure of the air at maximum diameter would be $2.5 \times 10^{-6}$ atmospheres as compared to the $4 \times 10^{-4}$ calculated in the original paper. Even the latter amount has of course a negligible effect upon the collapse pressure. Messrs. Robertson and Ross state that "it would be informative to know how the authors' results would be changed if deaerated instead of saturated water were employed." The authors are in hearty agreement with this desire and wish to call attention to the fact that as will be found in the description of the new installation of the water tunnel in the companion paper[3] provision has been made for the control of air content so that just this sort of study can and will be made. However, it is the present impression of the authors that one of the most important factors is the number of air nuclei present rather than the amount of air truly dissolved in the water. By air nuclei are meant minute undissolved bubbles which are probably associated with particles of solid matter found in the water. The number of nuclei may or may not be directly proportional to the amount of total air per unit volume of water. It would appear, at least in the type of cavitation under discussion, that the presence of an air nucleus is necessary to permit the cavitation bubble to start to form, but that after formation has commenced, the role of the air is insignificant.

The authors are indebted to Dr. Spannhake for a very interesting review of some of the significant features of his classical cavitation experiments. They deplore with him that the profession can no longer have the benefit of the discussion of Dr. Foettinger, the "old master" in the field. It is indeed interesting that a definite frequency was observed by Dr. Spannhake in fully-developed cavitation in special Venturi throats. However, it would seem to the authors that this is due at least partially to the interaction between the cavitation and the flow itself, since the development of the cavitation voids effectively changes the cross section of the conduit. On the other hand, as Prof. Hunsaker pointed out in his report[18] on this same research, "The

[17] "High-Speed Camera," *Mechanical Engineering*, vol. 69, December, 1947, pp. 1045–1046.

[18] "Progress Report on Cavitation Research at M.I.T." J. C. Hunsaker. Trans. ASME, vol. 57, October 1935, p. 423.

relation $\frac{fL}{V}$ can only be determined when cavitation is well developed and $L$ is of substantial magnitude." On the other hand, as he states further in the same paper, "When cavitation is not well developed and $L$ is but one or two throat diameters, the frequency observed is highly irregular......"

An examination of the records of the authors shows little evidence of a predominant frequency; in fact, the evidence is quite to the contrary. Not only does the period between the appearance of individual bubbles seem to be variable, but the life cycle of each bubble consists not of one formation and collapse but a series of them, each of which requires a different length of time. All of this indicates a wide band of frequencies. It should be pointed out that in the authors' experiments the cross section of the cavitation zone is small as compared with that of the tunnel. Thus, little or no interaction between the cavitation and the over-all flow is to be anticipated. Hence there seems to be little reason to expect correspondence between these experiments and those of M.I.T. Dr. Spannhake expresses some doubt as to whether or not the rebound of the bubbles observed by the authors is due to the air content of the water, and suggests the possibility that had the air content been distinctly smaller, the rebound might not have occurred. It is the belief of the authors that the rebound is a necessary concomitant of the energy storage at collapse and that it would continue to occur under all conditions in which it was possible to get the original cavitation bubble to form. To expand this statement further, it might be conceived that a liquid, containing no foreign nuclei or no undissolved or dissolved gas of any kind, might reach a state in which it would support a considerable tension. If this were the case, the formation of any cavitation bubbles could be inhibited at much higher velocity than under normal conditions. However, the forces causing the first void are of much smaller intensity than those available in the rebound of the highly compressed liquid following the collapse. Thus if the cavitation void can appear at all, there is good reason to believe that rebound will occur.

With reference to Dr. Spannhake's comments on cavitation damage, the authors wish again to emphasize that their remarks on cavitation damage were in no respect a discussion of the mechanism of the damage itself, but merely some tentative predictions as to the relative behavior of cavitation in producing damage as hydrodynamic conditions are varied. The basic idea underlying the authors' comments was to raise the question of the validity of the general concept of the similarity law with regard to cavitation damage, in which $\sigma$ is usually considered as the only parameter of the hydrodynamic picture.

In reference to Dr. Stepanoff's discussion, the studies in the water tunnel demonstrated beyond any question that two of the most important variables in the production of cavitation are the shape of the body (guiding surface), and its alignment with respect to the flow. Without the knowledge and exact control of these variables it is impossible to classify cavitation in any significant and systematic form. Since this information is lacking in practically all cavitation tests of hydraulic machinery, the authors are forced to believe that the analyses of cavitation data based on any of various velocity parameters mentioned by Dr. Stepanoff as being in current use by the industry can only result at best in the determination of rules for the average design, which miss the optimum, usually by a large margin.

The authors agree that cavitation and boiling are closely related; however, they would hesitate to carry the relationship as far as Dr. Stepanoff suggests. As previously pointed out, unless the liquid can support very substantial tension, cavitation will occur with or without the benefit of heat transfer or vaporization. Fluid vaporizes into the cavity because the cavity is there; the vapor does not force the fluid away to form a cavity. The cavitation cavity is to the cavitation phenomenon as the hole is to the doughnut: It gives it its characteristic form, but not its substance.

The authors do not understand the comments regarding the effect of scale on cavitation in hydraulic machines of different sizes. If two similar passages of different size contain fluid flowing at the same absolute velocity, the pressure distribution and the flow lines will be identical if the units of length correspond to the scale of the two passages. This is one of the fundamental principles of similarity. The conditions initiating cavitation will be exactly the same for both channels.

Dr. Stepanoff calls attention to the lack of knowledge concerning the flow in curve channels even in such simple cases as stationary bends or elbows. This problem is treated in a most interesting manner in the first chapter of his new book, which includes a well-selected set of references. The authors agree that a study of this type of flow under cavitating conditions would be very valuable and are reasonably sure that the high-speed photographic technique could be adapted to this purpose.

In conclusion the authors wish to thank again the helpful discussers and to express their appreciation to many others in the field who came to them with oral and written comments and valuable suggestions.

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