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**Labour Supply, Commodity Demand
and Marginal Tax Reform**

by

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Labour Supply, Commodity Demand and Marginal Tax
Reform

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Abstract: This paper examines the implications of extending the Ahmad-Stern model of indirect tax reform to include labour supply. The inclusion of labour supply alters the basic measure of marginal revenue cost (MRC) of indirect taxation and introduces the possibility of calculating a MRC for direct taxation. The paper derives the expressions for these revised MRCs and provides estimates from Irish data. It then examines the sensitivity of the results to assumptions regarding functional form and, in particular, goods/leisure separability.

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Labour Supply, Commodity Demand and Marginal Tax Reform¹

1. Introduction

Optimal taxation theory dates back to the seminal paper by Ramsey (1927) where he examined the question of how to set commodity tax rates to collect a given revenue, with the minimum loss of utility. Ramsey's original analysis dealt with the case of a one-consumer economy and did not discuss issues of equity or distribution. Papers by Diamond and Mirrlees (1971) and later by Diamond (1975) and Mirrlees (1975) extended this analysis to a many-person economy, where issues of equity and efficiency and the trade-off between the two could be examined.

These papers were concerned with the design of optimal tax systems. Optimal tax design poses considerable practical difficulties for the policy-maker, who needs to make explicit assumptions regarding the underlying preferences of the consumer (i.e. choose a functional form for the utility and demand functions) and also make some assumption about how expenditures are distributed. Derived optimal tax rates will obviously be sensitive to assumptions regarding underlying preferences. This is discussed by Deaton (1981) and is empirically examined by Ray (1986). Policy-makers also face the problem that even when they

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do provide explicit forms for demand functions they face difficulties regarding the reliability of estimates and are also evaluating demand responses at a point possibly quite far away from the optimum.

The difficulties outlined above led to the development of the marginal tax reform literature, with the seminal paper in this area by Ahmad and Stern (1984). This approach has the considerable advantage of not requiring the choice of explicit utility functions, nor of distributions of expenditure, but instead merely requiring information on the actual position of the economy at a single point in time, using actual consumptions and distributions of expenditure. It must be noted that tax reform does require information on demand responses, so that results in this area can still be sensitive to the specification of consumer preferences and demand systems. However, tax reform results appear to be less sensitive than do tax design results to this specification (see Decoster and Schokkaert (1990) and Madden (1993b)).

Ahmad and Stern (henceforth AS, 1984) examined indirect tax reform for India.² This model addressed tax reform using a measure which they called the marginal social cost (MSC) of raising revenue via an increase in the tax on a specific good. Each good has a particular MSC and optimality requires that the MSC be equal for all goods. If the MSC are not equal then

²This model is usually referred to as the Ahmad-Stern model although calculations of the central parameter first appear in a paper by Christiansen and Jansen (1978) for the Norwegian economy.

directions of tax reform at the margin can be identified. The tax on the good with a higher MSC is lowered while that on the good with the lower MSC is raised. The actual expression for MSC is the ratio of a welfare effect and a revenue effect and its calculation requires information on household demands for goods, tax rates, welfare weights and price responses. AS then present calculations of MSC for the Indian economy and identify directions of indirect tax reform at the margin. Similar calculations to those of AS have been carried out for Norway (Christiansen and Jansen (1978)), Belgium (Decoster and Schokkaert (1990)), Canada (Craggs (1990)), Germany (Kaiser and Spahn (1989)), Italy (Brugiavini and Weber (1988)), Pakistan (Ahmad and Stern (1991)) and Ireland (Madden (1989, 1993c)).

In recent work (Madden (1993b, 1993c)) the author has demonstrated a problem that may arise in the use of the original MSC measure used by AS. This term includes a revenue effect in the denominator. However, should this revenue effect be zero then MSC is undefined, while should it be negative then this introduces difficulties into the ranking of goods by MSC. To overcome this problem the inverse of MSC, what we term the marginal revenue cost (MRC), can be used. This shows the marginal cost in revenue foregone when tax is lowered so as to raise welfare by one unit.

A feature of the AS approach is that it examines indirect

taxes only.³ In the case of optimal tax design, the introduction of direct taxation may, under certain conditions, lead to optimally uniform commodity taxes. The conditions depend upon whether a linear or non-linear direct tax is available. If an optimal non-linear direct tax is available, then weak separability of goods and leisure is sufficient for optimality of uniform commodity taxes, provided individuals have identical preferences and differ only in the wage (see Atkinson and Stiglitz (1976) and Deaton (1979)).⁴ Effectively, the optimal direct tax can address both the issues of equity and efficiency, without any recourse to differential commodity taxation. If the government is constrained to using a linear direct tax then commodity taxes should be uniform in the presence of an optimum poll tax or grant, where once again everyone has identical preferences, leisure is separable from goods and goods demand functions have linear Engel curves in terms of income (Deaton (1979)). This result has been generalised by Deaton and Stern (1986) to an economy with different demographic groups, provided that households in each demographic group receive an optimally chosen payment which is uniform within each group but differs between groups. In this case, uniform commodity taxation is still optimal provided preferences are weakly separable between goods and leisure. Ebrahimi and Heady (1988) provide calculations for the UK showing the sensitivity of this result

³ This may well reflect the fact that the original Ahmad-Stern exercise was carried out in the explicit context of tax reform in developing countries where direct tax systems are typically not well developed.

⁴ Deaton (1981) shows how these results are sensitive to seemingly innocuous changes in the particular type of separability imposed.

to the various underlying assumptions. They find that the result shows greatest sensitivity to weak separability and the optimal setting of the chosen payment (in their example a child benefit). They find that the result shows little sensitivity to linearity of Engel curves.

The above discussion highlights the sensitivity of optimal tax design results to assumptions regarding separability between goods and leisure and raises the issue of how these features might affect tax reform. The AS model assumes that incomes are fixed, thus imposing weak separability between goods and leisure. This is not satisfactory in two respects. Firstly, depending on whether an optimal linear or non-linear direct tax is available and subject to the conditions outlined above, optimality may require uniform commodity taxes, thus rendering the tax reform exercise superfluous. Secondly, if weak separability does not hold then price responses estimated from a system which imposes weak separability will be biased and inconsistent. Most tests for separability tend to reject it (e.g. the paper by Barnett (1979) for the US, Blundell and Walker (1982) for the UK and Murphy and Thom (1986) for Ireland) which raises the question of how marginal tax reform recommendations will be affected when weak separability is not imposed, as well as raising the possibility of including leisure (and hence labour supply and direct taxation) in our derived tax reforms.⁵

⁵ Tests for separability can also be carried out via the estimation of consumer demand systems conditional on the quantity of labour supplied. These provide for more exact tests of separability and permit more general functional forms. However they do not permit the estimation of a full commodity demand-labour supply system. For an example of this approach, which also rejects separability, see Browning and Meghir (1991).

Intuitively, the formal inclusion of leisure will alter indirect tax reform in two ways: firstly, there will be the interaction between the demand for commodities and the demand for leisure, which will affect the calculated MRCs for all goods; and secondly, there will be the associated MRC for leisure (and equivalently for labour) which raises the possibility of comparing direct and indirect taxation at the margin. Atkinson and Stern (1980) discuss issues of sensitivity of tax reforms to the specification of preferences and separability. They analyse a switch from direct to indirect taxation, using a Stone-Geary utility function. However, they use an activity model, so that consumer preferences are a function of activities requiring both goods and time. This allows for richer interaction between goods and leisure and the derived commodity demands are not as restricted as they would be under the usual Linear Expenditure System (LES). In particular, this model does not impose weak separability as the marginal rate of substitution (MRS) between two goods may be affected by leisure, owing to the means whereby time is used with commodities in the production of activities. They estimate commodity demand-labour supply responses using cross-sectional data, thus restricting price effects to work via the price of time (i.e. they have only one observation on the price of each commodity, but they have a distribution of observations on the price of time). Their results broadly show that a switch from direct to indirect taxation will benefit the comparatively well-off at the expense of the less well-off. However, they do not explicitly weight the utilities of different households, so they do not arrive at any overall conclusions

concerning the welfare effect on society as a whole.

In this paper we show how the inclusion of labour supply alters the expression for MRC and we derive the expression for the MRC for direct taxation. We then present calculations of these measures based on estimates from a commodity demand-labour supply model for Ireland. We estimate a number of different versions of this model thus providing us with a variety of estimates of MRCs for indirect and direct taxation. Thus we can assess the sensitivity of these estimated MRCs to assumptions regarding separability etc.

In this paper we will be assuming that the labour market clears and that there is no rationing. In later work we hope to also examine the sensitivity of tax reforms to different assumptions regarding the labour market and in particular to the imposition of rationing in this market. Once again, separability will have a crucial role to play, since the presence of separability implies that rationing in the labour market affects commodity demands via an income effect only.

The layout of the paper is as follows: section 2 shows how leisure and labour supply can be formally included in the AS model and derives the expression for the "revised" MRCs which take account of leisure. Section 3 discusses the data and estimation issues involved in calculating revised MRCs. Section 4 presents calculations of these revised MRCs for Ireland, while section 5 offers some concluding remarks.

2. Integrating Labour Supply into Indirect Tax Reform

The major change to the usual derivation of MRC in the AS model is the abandonment of the assumption of weak separability between goods and leisure and thus the introduction of endogenous labour supply. We achieve this by assuming that household utility is defined over n goods x_1, \dots, x_n and leisure $x_0 = T - l_0$, where T is the fixed time endowment and l_0 is labour supply. Thus household h has a utility function

$$u^h = u(x_0^h, x_1^h, \dots, x_n^h) \quad (1)$$

"Full" expenditure per period is given by

$$M^h = w^h x_0^h + \sum_i q_i x_i^h \quad (2)$$

where $w = w^* - t_0$ is the after tax wage i.e. the opportunity cost of leisure, with t_0 the specific amount of income tax paid (we assume that income tax is levied on wage income only) and $q_i = p_i + t_i$ is the tax-inclusive price of good i , with p_i the producer price and t_i the specific indirect tax on good i . We constrain total commodity expenditure per period to equal the sum of labour and non-labour incomes so that $q'x^h = w^h l_0 + y^h$, where y^h is non-labour income of household h . Substitution into (2) gives the full income budget constraint

$$M^h = w^h T + y^h \quad (3)$$

The solution of the maximisation of (1) subject to (3) gives us the indirect utility function of household h defined over consumer prices, after-tax wages and non-labour incomes $v^h(q, w^h, y^h)$. There are n goods and H households indexed by $h=1,2,\dots,H$.

Before proceeding further, it is necessary to specify how households differ. We assume that households differ in their pre-tax wage rate, w^h , which is fixed, and where wages are dependent on underlying ability (which is unobservable to the government and thus cannot be taxed; otherwise it would be optimal for the government to simply tax underlying ability). Governments can observe w^h and l^h and thus also pre-tax labour income $w^h l^h$. Note that because this is a tax reform rather than a tax design exercise, we do not have to provide explicit distributions of pre or post-tax wages, nor of expenditures. We can simply use the actual distribution of whichever variable is chosen to rank households. In previous work on indirect tax reform we have ranked households according to expenditure per equivalent adult and we will continue that practice in this paper.

We assume the government maximises a Bergson-Samuelson indirect social welfare function which we can write as

(4)

$$V(q, \bar{w}, \bar{y}) = W(v^1(q, w^1, y^1), v^2(q, w^2, y^2), \dots, v^H(q, w^H, y^H))$$

We also have an aggregate demand vector given by

$$X(q, \bar{w}, \bar{y}) = \sum^h x^h(q, w^h, y^h) \quad (5)$$

where w and y are some average or representative value of w^h and y^h . The form which this representative wage and non-labour income takes involves issues of aggregation which we will return to later. Aggregate labour supply is given by

$$L = \sum^h l^h(w^h, q_1, y^h) \quad (6)$$

Government tax revenue is given by

$$R = \sum_i t_i X_i(q_1, \bar{w}, \bar{y}) + \sum^h t_0 l^h(q_1, w^h, y^h) \quad (7)$$

The crucial parameter in this model is the MRC of raising the tax on each good, including leisure (i.e. lowering the tax on labour). An increase in the tax on good i raises revenue by an amount $\partial R / \partial t_i$. It also changes welfare by an amount $\partial V / \partial t_i$. The ratio of these two measures gives the MRC of raising one unit of revenue from increasing the tax on good i . Thus we define the MRC of good i , often referred to as ρ_i as

$$\rho_i = - \frac{\partial R / \partial t_i}{\partial V / \partial t_i} \quad (8)$$

where we insert the minus sign to denote marginal cost. It is intuitively obvious that at an optimum all the ρ_i should be equal, since otherwise we could lower the tax on a good with a low ρ_i and raise the tax on a good with a high ρ_j , thus increasing welfare for no change in revenue.

Our next task is to find an expression for ρ_i that is

readily calculable. From Roy's Identity we know that

$$\frac{\partial v^h}{\partial q_i} = -\alpha^h x_i^h \quad (9)$$

where α^h is the private marginal utility of income of household h. We can also apply Roy's Identity to the price of leisure (labour) to derive

$$\partial v^h / \partial w^h = \alpha^h l^h \quad (10)$$

where l^h is the quantity of labour supplied by household h. Similarly the change in utility following an increase in non-labour income, y^h , is simply given by α^h .

We can then say that

$$\partial v^h / \partial t_i = -\sum^h \beta^h x_i^h \quad (11)$$

and

$$\partial v^h / \partial t_0 = \sum_h \beta^h l^h \quad (12)$$

where β^h is the social marginal utility of income of household h i.e. its welfare weight.

From eq. (5) we have

$$\frac{\partial R}{\partial t_i} = x_i + \sum_j t_j \cdot \frac{\partial x_j}{\partial t_i} + \sum_h t_0 \cdot \frac{\partial l^h}{\partial t_i} \quad (13)$$

and

$$\frac{\partial R}{\partial t_0} = \sum_j t_j \frac{\partial x_j}{\partial t_0} + \sum_h l^h + \sum_h t_0 \frac{\partial l^h}{\partial t_0} \quad (14)$$

The expression for $\partial R/\partial t_i$ differs from the expression in the original AS paper by the inclusion of the ${}_h t_0 \partial l^h/\partial t_i$ term. This term takes account of the effect on labour supply (and hence direct tax revenue) of a change in the price of good i .

There is a further important point to be made concerning equations (12) and (14). In both cases we have assumed that a change in direct taxation has no effect on non-labour income. As Hausman (1984) points out, this is only true when the direct tax system is directly proportional. If the direct tax system is progressive (even to the extent that tax free allowances are admitted) or if non-labour income is subject to taxation, then what is known as "virtual income" will be affected by a change in the after-tax wage. While we may not feel too uncomfortable with the assumption that non-labour income is not taxed, the same cannot be said for the assumption that the direct tax system is directly proportional. The incorporation of virtual income would alter the expressions for $\partial V/\partial t_0$ and $\partial R/\partial t_0$ as follows:

$$\frac{\partial V}{\partial t_0} = \sum_h \beta^h l^h + \sum_h \beta^h \frac{\partial y^h}{\partial t_0} \quad (15)$$

and

$$\frac{\partial R}{\partial t_0} = \sum_h l^h + \sum_j t_j \left(\frac{\partial x_j}{\partial t_0} + \sum_h \frac{\partial x_j^h}{\partial y^h} \frac{\partial y^h}{\partial t_0} \right) + \sum_h t_0 \left(\frac{\partial l^h}{\partial t_0} + \frac{\partial l^h}{\partial y^h} \frac{\partial y^h}{\partial t_0} \right) \quad (16)$$

However, for the moment we will adopt the simplifying assumptions regarding the effect of t_0 on y (i.e. $\partial y/\partial t_0=0$).

Thus the expressions for ρ_i and ρ_0 are:

$$\rho_i = \frac{X_i + \sum_j t_j \frac{\partial X_j}{\partial t_i} + \sum_h t_0 \frac{\partial l^h}{\partial t_i}}{\sum_h \beta^h X_i^h} \quad (17)$$

and

$$\rho_0 = \frac{\sum_j t_j \frac{\partial X_j}{\partial t_0} + \sum_h l^h + \sum_h t_0 \frac{\partial l^h}{\partial t_0}}{\sum_h \beta^h l^h} \quad (18)$$

We need to express (17) and (18) in terms that are easily calculated. To do this we multiply (14) above and below by q_i and (15) by w , the net wage. From (1) and (2) we know that $\partial X_i / \partial t_i = \partial X_i / \partial q_i$ (likewise with $\partial l^h / \partial t_i$) and $\partial l^h / \partial t_0 = -\partial l^h / \partial w$ (likewise with $\partial X_i / \partial t_0$). After some manipulation to re-express in elasticity terms, this gives us

$$\rho_i = \frac{q_i X_i + \sum_j \tau_j \epsilon_{ji} q_j X_j + \sum_h \tau_0 w^h l^h \epsilon_{li}}{\sum_h \beta^h q_i X_i^h} \quad (19)$$

and

$$\rho_0 = \frac{\sum_h l^h w^h - \sum_h \tau_0 w^h l^h \epsilon_{lw} - \sum_j \tau_j q_j X_j \epsilon_{jw}}{\sum_h \beta^h l^h w^h} \quad (20)$$

where τ_j refers to the tax as a proportion of consumer price for good j , τ_0 is tax as a proportion of the net wage, ϵ_{ji} is the uncompensated cross-elasticity of demand between goods j and i , ϵ_{li} is the uncompensated elasticity of supply of labour with respect to commodity i , ϵ_{lw} is the uncompensated elasticity of labour supply to the after-tax wage and ϵ_{jw} is the uncompensated elasticity of demand for good j with respect to the after-tax wage. Note also that we place a minus sign before the second and third terms in the numerator, since we are capturing the effect on labour supply and commodity demand of an increase in direct



taxation, which, of course, implies a decrease in the after-
 (11) wage.

Equations (17) and (18) permit an attractive decomposition which allows us to give an intuitive interpretation to the terms in these expressions. We can rewrite eqn. 17 as

$$p_i = \frac{x_i}{\sum_h \beta^h x_i^h} + \frac{\sum_j t_j \frac{\partial x_j}{\partial t_i}}{\sum_h \beta^h x_i^h} + \frac{\sum_h t_0 \frac{\partial l^h}{\partial t_i}}{\sum_h \beta^h x_i^h}$$

The first of these terms takes account of distribution considerations. It is the reciprocal of the "distribution characteristic" of the good introduced by Feldstein (1972) and its importance in the ranking of goods by MRC will depend upon the degree of inequality aversion of the government. If there is zero inequality aversion, then $\beta^h=1 \forall h$ and this term will be unity for all goods and will play no part in the ranking of goods.

The second term takes account of demand responses, for good only. The third term is the new term introduced by the inclusion of labour supply. It takes account of the change in direct tax revenue following a change in the price of good i . If good i and labour are substitutes (e.g. the often cited case of a set of golf clubs, which presumably ignores professional golfers!), then $\delta l^h / \delta t_i > 0$, thus increasing the MRC of the good in question. The converse holds when the good in question and labour are complements.

An alternative way of looking at this is to say that our "new" measure of MRC incorporating labour supply, which we could call ρ^*_i , is equal to the sum of two terms: ρ_i , the measure calculated when labour supply is not taken account of, plus ρ_{0i} , the extra effect on direct tax revenue arising from any change in labour supply following an increase in the tax on good i . When examining whether the inclusion of labour supply alters our measure of MRC we are effectively examining whether this term is zero or non-zero. It should also be remembered that this term could be non-zero, yet still not affect the ranking of goods by MRC.

Below equation (18) is similarly decomposed into three terms, the first of which, once again, takes account of distributional issues. The second term shows the change in indirect tax revenue, arising from changes in the consumption of goods following a change in the rate of direct taxation, while the third term incorporates any changes in direct taxation owing to changes in labour supplied.

$$\rho_0 = \frac{\sum_h l^h}{\sum_h \beta^{h1^h}} + \frac{\sum_j t_j \frac{\partial x_j}{\partial t_0}}{\sum_h \beta^{h1^h}} + \frac{\sum_h t_0 \frac{\partial l^h}{\partial t_0}}{\sum_h \beta^{h1^h}}$$

3. Data Requirements and Estimation.

Calculation of the expressions in (19) and (20) requires a variety of information. Much of this information is identical to the data requirements for the usual AS calculations viz. household budget data, indirect tax rates, welfare weights and consumer demand responses. The inclusion of labour supply requires additional information viz. data on hours worked and wage rates, direct tax rates and elasticity estimates from a jointly estimated consumer demand-labour supply model.

Madden (1993c) gives details of sources of data for the case where labour supply is not included. For the data on hours worked and wage rates, note that in our expressions (16) and (17) we do not have separate terms for hours worked and wage rates. Thus we could use the aggregate gross of tax wage bill as our measure for $\sum_h w^h l^h$ and the net of tax bill for $\sum_h w l^h$. However, we also require estimates of labour supply elasticities and labour supply-commodity demand elasticities. Thus, for these terms we will need separate wage and hours worked data. As discussed below, the estimation of a commodity demand-labour supply model from aggregate time-series data will also require that conditions of aggregation will have to be met. For exact linear aggregation to hold, Muellbauer (1981) showed that preferences must be of the Gorman Polar form. With time-series data, this rules out such flexible functional forms as Deaton and Muellbauer's Almost Ideal Demand System (AIDS, Deaton and Muellbauer (1980)). Nevertheless, it still permits functional

forms which allow for a reasonable degree of substitutability and which can nest varying degrees of separability.

Welfare weights are introduced exogenously and generated from a utility of income function of Atkinson's (1970):

$$U^h(I^h) = \frac{kI^{1-e}}{1-e}, e \geq 0, e \neq 1; -k \log(I), e = 1. \quad (21)$$

where I^h is expenditure per equivalent adult of the h^{th} household, with the equivalence scales of Conniffe and Keogh (1988) used. Given the expression for $U^h(I)$ above, we have $\beta^h = U'(I^h)$ and we normalise β^h so that the welfare weight of the poorest household is unity. Thus $\beta^h = (I^1/I^h)^e$ and e can be regarded as an inequality aversion parameter. When $e=0$, we have zero inequality aversion, while the case of $e=5$ would approach the extreme Rawlsian case of only considering the welfare of the poorest. We now proceed to describe the model used for estimation.

Recall equation 1 where household preferences are described by a utility function defined over leisure x_0^h and a vector of goods x^h . The household maximises this function subject to the constraint in equation 2. Dropping the superscripts, we can also view the household as choosing x_0 and x so as to minimise the full cost of achieving the utility level given by the solution to the primary problem. We can define the consumer cost or expenditure function for this problem as

$$c(w, q, u) = \min(w x_0 + q'x : s. t. u(x_0, x) = u) \quad (22)$$

From Shephard's Lemma we can derive the compensated or Hicksian labour supply and commodity demands as

$$I - T - c_0(w, q, u) \quad (23)$$

$$x_i = c_i(w, q, u), \quad i=1, \dots, n. \quad (24)$$

where c_0, c_i are the partial derivatives of the cost function with respect to w and q_i respectively. If we invert (22) we obtain the indirect utility function $v(q, w, M)$ and substituting for u in (23) and (24) we obtain the uncompensated or Marshallian labour supply and commodity demands

$$I - T - g_0(w, q, M) \quad (25)$$

$$x_i = g_i(w, q, M), \quad i=1, \dots, n. \quad (26)$$

We will be estimating this system with aggregate time-series data and so it is necessary to choose a functional form for (22) which permits exact linear aggregation over w^h and y^h . Muellbauer (1981) shows that exact linear aggregation over h consumers requires that the "micro" and "macro" cost functions be of the Gorman polar form

$$c_h(w_h, q, u_h) = a_h(q) + w_h d(q) + w_h^k b(q)^{1-k} u_h \quad (27)$$

$$c(w, q, u) = a(q) + wd(q) + w^k b(q)^{1-k} u \quad (28)$$

where we have indexed households by subscripts rather than superscripts to avoid confusion with exponents, and $a(q) = (1/H) \sum a_h(q)$, $w = (1/H) \sum w_h$, $u = (1/H) \sum u_h$; $a(q)$ and $b(q)$ are homogeneous of degree one and $d(q)$ is homogeneous of degree zero. The cost function (28) will then yield the following

uncompensated labour supply and commodity demands

$$wl = w(T - d(q)) - kF \quad (29)$$

$$q_i x_i - q_i a_i(q) + w q_i d_i(q) + q_i (1-k) b_i^*(q) F \quad (30)$$

where $F = wT + y - a(q) - wd(q)$ and $b_i^*(q) = b_i(q)/b(q)$. The labour hours supply function (29) does not permit linear aggregation over households with differing wage rates. Thus we use a labour income function and express commodity demands in expenditure terms.⁶

The cost function (28) also permits a simple test for separability. Goldman and Uzawa (1964) show that goods and leisure are weakly separable if $c_{i0} = z c_{i1}$ where z is constant for all $i=1, \dots, n$. For expression (28) this reduces to $d_i(q) = 0$ for all i .

The actual estimation of the system (29)-(30) requires specific functional forms for $a(q)$, $b(q)$ and $d(q)$. Following Murphy and Thom, the most general specification we can choose is

$$a(q) = \sum_i \sum_j a_{ij} (q_i q_j)^{1/2}; \quad a_{ij} = a_{ji} \quad (31)$$

$$d(q) = d_0 \prod_i q^{d_i}; \quad \sum d_i = 0 \quad (32)$$

$$b(q) = b_0 \prod_i q^{b_i}; \quad \sum b_i = 1 \quad (33)$$

As with the LES the terms $a(q)$ and $d(q)$ can be regarded as minimum or subsistence expenditures on goods and leisure

⁶ Deaton and Muellbauer (1981) show that if the cost function is specified so that l is linear in w , then the resulting commodity demands are non-linear in w .

respectively, while F can be regarded as supernumerary income. Thus the minimum quantity of good i purchased is

$$x_i = \sum_j a_{ij} (q_j/q_i)^{1/2} + w d_0 d_i \prod q^{d_i}/q_i \quad (34)$$

Thus minimum quantities are general functions of prices rather than constants as in the LES. Our cost function can be viewed as a generalised form of the LES and reduces to the LES when the off-diagonal terms in $a(q)$ and the d_i terms ($i=1, \dots, n$) are zero. In this case preferences are separable and additive. Note however, that unlike a typical LES encountered in consumer demand studies, the marginal budget shares i.e. the b_i are defined over full income as opposed to total expenditure.

Using (31)-(33) we have the following labour supply and commodity demand equations for estimation

$$w l = w(T - d_0 \prod q^{d_i}) - k F \quad (35)$$

$$q_i x_i = \sum_j a_{ij} (q_i q_j)^{1/2} + w d_0 d_i \prod q^{d_i} + (1-k) b_i F; \quad i=1, \dots, n. \quad (36)$$

$$F = w T + y - \sum_i \sum_j a_{ij} (q_i q_j)^{1/2} - w d_0 \prod q^{d_i} \quad (37)$$

In order to provide us with a reasonable range of estimates of elasticities and consequently a reasonable range of calculated p_i s, we estimated five versions of the above system. Firstly, we estimated equations (35)-(37), which we call LES01. Secondly, we estimate the system in equations 35-37, except that we impose separability i.e. we constrain the d_i to be zero, but retain the specification for the subsistence quantities. This system we call LES02. Thirdly we estimated what we might regard as an

augmented LES, with subsistence quantities specified as constants rather than as general functions of prices. We estimated this system unrestricted and with separability imposed, thus giving us LES03 and LES04 respectively.⁷ Finally, we estimated what we may regard as a "traditional" LES, defined over total expenditure rather than "full income". This we label LES05.

The five systems were estimated for ten goods and labour income for the period 1958-88, using the non-linear estimation procedure in SHAZAM⁸. In nearly all cases what we would regard as "reasonable" estimates were obtained. For example, own-price elasticities and the labour supply elasticities were of the expected sign and the magnitude of the own-price elasticities was consistent with estimates from more flexible functional forms, using the same data set (see Madden (1993a)). However, for the LES03 and LES04 systems the estimated subsistence quantities for two goods, transport and equipment and durables were negative, while for LES05, the estimated marginal budget share for tobacco was negative. For both LES specifications the test for separability was rejected.⁹

⁷ Note that in this case the imposition of $d_i=0$ is equivalent to imposing additive separability, since the off-diagonal terms in a_{ij} are zero.

⁸ The ten goods were food, alcohol, tobacco, clothing and footwear, fuel and power, petrol, transport and equipment, durables, other goods and services. To remove the singularity of the system, the labour supply equation was deleted. I am very grateful to John Fitzgerald and Fergal O' Brolchain for assistance with the data.

⁹ For a more detailed account of the properties of these estimates, see Madden (1993d)

4. Estimates of Revised MRCs for Ireland.

In table 1 we present values of ρ_i and ρ_0 for our five systems, with varying degrees of inequality aversion.¹⁰ Even casual inspection of the tables will reveal that the ranking of goods by MRC is quite similar for LES03, LES04 and LES05. More formally, as can be seen in table 2, the rank correlation coefficients for these three systems are around 0.8-0.9, indicating that for this data set and for these admittedly restrictive functional forms, the use of full income rather than total expenditure and, more interestingly, the imposition of additive separability between goods and leisure does not materially alter tax reform recommendations. In particular, the latter result stands in contrast to the theoretical results of Deaton (1981) and the empirical results of Ray (1986) concerning optimal tax design and supports the conjecture of Deaton (1987) that tax reform recommendations are less sensitive to assumptions regarding separability than are derived optimal tax rates.

In table 2 we have also included rank correlation coefficients for other demand systems estimated over the same data set: an Almost Ideal Demand System estimated in first differences (DAIDS03), a Rotterdam system (ROTT03) and the CBS system of Keller and Van Driel (CBS03). In all cases these systems were estimated with homogeneity and symmetry imposed. These systems show very high rank correlation coefficients

¹⁰ This is a preliminary version of the paper; MRCs for LES01 and LES02 will be presented later.

between each other (for a more detailed discussion see Madden (1993b)), and also show rank correlation coefficients ranging from 0.3 to 0.5 with the variants of the LES estimated here. These correlations also suggest that the use of the restrictive LES specification affects tax reform rankings to a greater degree than does the imposition of separability (witness the correlations of around 0.9 between the different LES systems and the lower correlations between the LES systems and the more flexible specifications). Note however, that these correlations should not be interpreted as tests of the sensitivity of rankings to the imposition of separability, since these demand systems (DAIDS03, ROTT03 and CBS03) differ from LES02 and LES03 in two crucial respects viz. a different deterministic system and the non-inclusion of leisure in preferences.

The inclusion of labour supply means that we can also calculate what we term ρ_0 , the MRC of increasing the tax on labour. Obviously since we can only calculate ρ_0 for those systems where we include labour supply, we have only three estimates. The estimated values of ρ_0 for systems LES02 and LES03, with zero inequality aversion, are 0.778 and 0.723 respectively. These results suggest that the MRC for direct taxation is broadly similar in magnitude to the MRC for indirect taxation. For both systems ρ_0 is towards the lower end of the rankings of ρ_i , suggesting that at the margin, reductions in direct tax rates would be recommended as part of a tax reform package.

Comparing rankings between LES02 and LES03 when ρ_0 is included, we can see that the imposition of separability has no effect on the relative ranking of labour by MRC. This result is difficult to interpret, but one possible explanation is as follows: the imposition of separability between goods and labour implies that a change in the wage rate will affect commodity demands via income effects only. Should separability not be imposed, then substitution effects are permitted. If substitution effects are very small, then they will be dominated by income effects, and so the calculated value of the MRC for labour will exhibit little sensitivity to the imposition of separability.

Finally, we can assess how the calculated value of ρ_0 varies with the degree of inequality aversion. We can see that at zero and low levels of inequality aversion ρ_0 is ranked about number eight or nine. However, at very high levels of inequality aversion, $e=5$, ρ_0 is ranked highest, suggesting that direct taxes should be raised as part of a tax reform package. This is in contrast to much of the debate on tax reform in Ireland e.g. the recommendations of the Commission on Taxation (1982). The result can be explained as follows: our measure of l^{hw} is direct income, which does not include state transfers (which in Ireland are untaxed). Thus relatively poor households, for whom state transfers constitute a relatively higher proportion of direct income, would suffer little utility loss from an increase in direct taxes. The greater utility loss would be borne by relatively better off households. However, when our social

welfare function exhibits a high degree of inequality aversion, these households have a very low weighting in overall social welfare. Thus ρ_0 has a high value, since we have a relatively small utility loss for a given revenue gain.

Recommendations following from this result should be interpreted with caution, however. In this study we have assumed that all households pay direct tax at the same proportional rate. In practice, there are tax-free allowances and different tax rates for different income bands. The interaction with the social welfare scheme also means that there may be a very wide range of marginal tax rates, depending on such factors as demographics, housing etc. It is not possible for us to incorporate such effects in this simple model. Thus in practice, we may wish to recommend a tax package which may include tax increases for certain bands of income and tax reductions for others. Nevertheless, this approach does offer some insights into the overall balance between direct and indirect taxation at the margin.

5. Conclusion

This paper has developed the Ahmad-Stern model of indirect tax reform to include labour supply and direct taxation. It derives the augmented measure for marginal revenue cost for indirect taxes and also the expression for marginal revenue cost for direct taxation. It then presents estimates of these parameters for Ireland based on a jointly estimated commodity

demand-labour supply model.

The paper finds that the inclusion of labour supply appears to have relatively little impact on the ranking of goods by marginal revenue cost and that, in particular, the rankings do not appear to be very sensitive to assumptions regarding separability between goods and leisure. This result is contrasted with the sensitivity of derived optimal tax rates to separability assumptions. The estimated marginal revenue cost for direct taxation is found to be towards the lower range of estimated marginal revenue costs, but this finding is reversed at high levels of inequality aversion.

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APPENDIX

Table 1: MRCs for LES03

e=0		e=2		e=5	
1. C & F	0.927	1. Services	2.127	1. Services	4.770
2. Services	0.904	2. C & F	2.077	2. C & F	4.396
3. F & P	0.874	3. Petrol	1.831	3. Petrol	3.860
4. Food	0.870	4. Durables	1.751	4. Durables	3.568
5. Petrol	0.838	5. Food	1.624	5. T & E	3.381
6. Durables	0.799	6. Other G	1.603	6. Other G	3.010
7. Other G	0.792	7. T & E	1.551	7. Food	2.790
8. Tobacco	0.716	8. F & P	1.544	8. Alcohol	2.624
9. T & E	0.685	9. Alcohol	1.304	9. F & P	2.488
10. Alcohol	0.601	10. Tobacco	1.253	10. Tobacco	2.038
$\rho_0=0.778$		$\rho_0=1.313$		$\rho_0=6.279$	

MRCs for LES04

e=0		e=2		e=5	
1. Services	0.845	1. Services	1.987	1. Services	4.457
2. F & P	0.835	2. C & F	1.858	2. C & F	3.932
3. C & F	0.829	3. Durables	1.702	3. T & E	3.618
4. Durables	0.777	4. T & E	1.660	4. Durables	3.469
5. Food	0.771	5. Petrol	1.542	5. Petrol	3.251
6. Other G	0.734	6. Other G	1.484	6. Other G	2.787
7. T & E	0.733	7. F & P	1.474	7. Food	2.472
8. Petrol	0.705	8. Food	1.438	8. F & P	2.374
9. Tobacco	0.665	9. Tobacco	1.164	9. Alcohol	1.896
10. Alcohol	0.434	10. Alcohol	0.942	10. Tobacco	1.892
$\rho_0=0.723$		$\rho_0=1.220$		$\rho_0=5.840$	

MRC for LES05

e=0		e=2		e=5	
1. F & P	0.889	1. Services	2.066	1. Services	4.634
2. Services	0.878	2. C & F	1.945	2. C & F	4.117
3. Food	0.868	3. T & E	1.818	3. T & E	3.963
4. C & F	0.868	4. Durables	1.780	4. Durables	3.626
5. Tobacco	0.854	5. Petrol	1.657	5. Petrol	3.493
6. Durables	0.812	6. Food	1.620	6. Other G	3.016
7. T & E	0.803	7. Other G	1.607	7. Food	2.784
8. Other G	0.794	8. F & P	1.570	8. F & P	2.528
9. Petrol	0.758	9. Tobacco	1.495	9. Alcohol	2.475
10. Alcohol	0.567	10. Alcohol	1.230	10. Tobacco	2.430

Table 2: Rank Correlation Coefficients between Demand Systems

DAIDS	1.000					
ROTT	0.988	1.000				
CBS	0.964	0.976	1.000			
LES03	0.515	0.491	0.418	1.000		
LES04	0.503	0.454	0.479	0.842	1.000	
LES05	0.249	0.236	0.297	0.733	0.806	1.000
	DAIDS	ROTT	CBS	LES03	LES04	LES05