# Lagrange–Chebyshev Interpolation for image resizing

Donatella Occorsio<sup>a,1</sup>, Giuliana Ramella<sup>b</sup>, Woula Themistoclakis<sup>b,\*</sup>

<sup>a</sup>Department of Mathematics and Computer Science, University of Basilicata, Viale dell'Ateneo Lucano 10, 85100 Potenza, Italy

<sup>b</sup>C.N.R. National Research Council of Italy, Institute for Applied Computing "Mauro Picone", Via P. Castellino, 111, 80131 Naples, Italy

### Abstract

Image resizing is a basic tool in image processing and in literature we have many methods, based on different approaches, which are often specialized in only upscaling or downscaling. In this paper, independently of the (reduced or enhanced) size we aim to get, we approach the problem at a continuous scale where the underlying continuous image is globally approximated by the tensor product Lagrange polynomial interpolating at a suitable grid of first kind Chebyshev zeros. This is a well-known approximation tool that is widely used in many applicative fields, due to the optimal behavior of the related Lebesgue constants. Here we show how Lagrange–Chebyshev interpolation can be fruitfully applied also for resizing an arbitrary digital image in both downscaling and upscaling. The performance of the proposed method has been tested in terms of the standard SSIM and PSNR metrics. The results indicate that, in upscaling, it is almost comparable with the classical Bicubic resizing method with slightly better metrics, but in downscaling a much higher performance has been observed in comparison with Bicubic and other recent methods too. Moreover, in downscaling cases with an odd scale factor, we give an estimate of the mean squared error produced by our method and prove it is theoretically null (hence PSNR equals to infinite and SSIM equals to one) in absence of noise or initial artifacts on the input image.

*Keywords:* Image resizing, Image downscaling, Image upscaling, Lagrange interpolation, Chebyshev nodes *2000 MSC:* 94A08, 68U10, 41A05, 62H35

### 1. Introduction

In this paper, we deal with the problem of image resizing. This has been widely investigated over the past decades and is still an active research area characterized by many applications in different domains, including image transmission, satellite image

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<sup>\*</sup>Corresponding author

Email addresses: donatella.occorsio@unibas.it (Donatella Occorsio),

giuliana.ramella@cnr.it (Giuliana Ramella), woula.themistoclakis@cnr.it (Woula Themistoclakis)

analysis, gaming, remote sensing, etc. (see e.g. [8, 22, 25, 28, 50, 51]) In literature, downscaling and upscaling are often considered separate problems (see e.g. [7, 30, 50]), and most of the existing methods are specialized in only one direction, sometimes for a limited range of scaling factors (see e.g. [13, 46]). The method we are going to introduce works in both down and up scaling directions and for "large" scaling factors as well. It falls into the class of the interpolation methods and it is based on a non-standard modeling of the image resizing problem.

In order to introduce the adopted model we premise that, from the mathematical viewpoint, a continuous image  $\mathcal{I}$  is a function f(x, y) of the spatial coordinates which, without losing the generality, we can assume belonging to the open square  $A = ]-1, 1[^2$ . Hence, its digital version I of  $n \times m$  pixels is supposed to be made of the values that f takes on a discrete grid of nodes  $X_{n \times m} \subset A$ . Regarding such grid, it is generally supposed that  $X_{n \times m} = X_n \times X_m$  where we set

$$X_{\mu} := \{ x_{k}^{\mu} : k = 1, \dots, \mu \} \subset ] - 1, 1[, \quad \forall \mu \in \mathbb{N}.$$
 (1)

A typical and natural choice of the (univariate) system of nodes  $X_{\mu}$  is to divide [-1, 1]in  $(\mu + 1)$  equal parts and to take the  $\mu$  internal equidistant nodes, i.e.

$$X_{\mu}^{equ} = \left\{ -1 + \frac{2k}{\mu+1} : k = 1, \dots, \mu \right\}.$$
 (2)

However, it is well known that equally spaced nodes are not the best choice for Lagrange interpolation since they lead to exponentially growing Lebesgue constants, whereas optimal Lebesgue constants (growing at the minimal projection rate) are provided by the Chebyshev nodes of the first kind

$$x_k^{\mu} = \cos\left[\frac{(2k-1)\pi}{2\mu}\right], \qquad k = 1, \dots, \mu, \qquad \forall \mu \in \mathbb{N},$$
(3)

(see e.g. [27, 45] for a short excursion on the topic).

Discrete polynomial approximation based on Chebyshev zeros is a pillar in approximation theory and practices. It has been widely studied and applied in several fields and also recently, Chebyshev–like grids such as Xu points and Padua points have been introduced to get optimal interpolation processes on the square (see e.g. [48, 49, 9, 11, 33]). Nevertheless, to our knowledge, its usage in image processing has been mainly limited to particular cases, such as Magnetic Particle Imaging that is strictly related to Lissajous curves generating the Padua points (see e.g. [21, 18, 14]).

Indeed, at a first look, Chebyshev zeros may seem unsuitable for sampling an arbitrary image since they are not equidistant on the segment ]-1,1[ of each spatial coordinate, but they are *arc sine* distributed, becoming denser at the endpoints  $\pm 1$  (see Figure 1). Nevertheless, in our model, we transfer the sampling question from the segment ]-1,1[ to the semicircle of radius 1 centered at the axes origin. Thus, using the standard setting  $t = \arccos x$ , instead of the usual equidistant nodes on the segment, our method takes the following equidistant nodes on the semicircle

$$t_k^{\mu} := \frac{(2k-1)\pi}{2\mu}, \qquad k = 1, \dots, \mu, \qquad \mu \in \mathbb{N},$$
 (4)

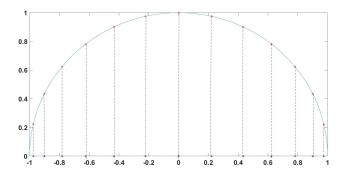


Figure 1: The first kind Chebyshev zeros of order  $\mu = 15$  (black stars) and their accosine (red stars)

These nodes define the Chebyshev zeros  $x_k^{\mu} = \cos t_k^{\mu}$  in (3) and the grid

$$X_{n \times m} = \{ (x_i^n, x_j^m) : \quad i = 1 : n, \ j = 1 : m \}, \qquad n, m \in \mathbb{N}$$
 (5)

which we actually choose to sample arbitrary (continuous) images  $\mathcal{I}$  and to obtain digital images of  $n \times m$  pixels, for arbitrary  $n, m \in \mathbb{N}$ .

According to this model, let  $I = f(X_{n \times m})$  be the digital image of  $n \times m$  pixels, i.e.

$$I_{i,j} := f(x_i^n, x_j^m), \qquad i = 1: n, \ j = 1: m,$$
(6)

and let  $I^{in}$  be its more or less corrupted version really available as input data, say

$$I_{i,j}^{in} := \tilde{f}(x_i^n, x_j^m), \qquad i = 1: n, \ j = 1: m,$$
(7)

where  $\tilde{f}$  denotes a corrupted version of the continuous image. Starting from the data  $I^{in}$ , the resizing problem aims to find a good approximation of the resampled image  $I^{res}$  consisting of the samples of f at the different (more or less dense) grid  $X_{N\times M}$ , i.e.

$$I_{k,h}^{res} := f(x_k^N, x_h^M), \qquad k = 1: N, \ h = 1: M.$$
(8)

Hence, under our modeling, the resizing problem raises the following approximation question: starting from the approximate values of f at the Chebyshev grid  $X_{n\times m}$ , how to approximate f at another (coarser in downscaling or finer in upscaling) Chebyshev grid  $X_{N\times M}$ ? The solution we propose relies on the global approximation of a function by its bivariate Lagrange polynomial interpolating at the Chebyshev grid  $X_{n\times m}$ .

This well-known interpolation polynomial can be easily deduced via tensor product from the univariate case (see for instance [31, 32, 34, 35]) and fast algorithms can be implemented for its computation (see e.g. [33]). Moreover, we recall that wavelets techniques can be applied to such kind of interpolation [20, 15, 16].

Hence, in order to get the resampled image, instead of f, we propose to sample a suitable Lagrange interpolation polynomial at the new grid. More precisely, we use the pixels of the input image  $I^{in}$  to build the Lagrange-Chebyshev polynomial  $L_{n,m}\tilde{f}$  interpolating  $\tilde{f}$  at  $X_{n\times m}$  and we get the output image of the desired  $N \times M$  size (here denoted by  $I^{out}$ ) by sampling  $L_{n,m}\tilde{f}$  at the mesh  $X_{N\times M}$ .

To test the performance of such Lagrange–Chebyshev Interpolation (LCI) method we have considered the usual SSIM (Structured Similarity Index Measurement) and PSNR

(Peak Signal to Noise Ratio) metrics as computed by MatLab, and we have carried out the experimentation on several kinds of images collected in 5 datasets, having different characteristics on the size and the quality of the images, the variety of subjects, etc.

For an initial comparison, we focused on interpolation methods working as well in both down and up scaling directions. In general, these methods find a resized image by local or global interpolation of the continuous image f, using in some way the initial  $n \times m$  pixels, namely the more or less corrupted values of f at the grid of nodes  $X_{n \times m}$ . Typically, for such grid it is assumed the equidistant nodes model (2) and most of the interpolation methods act locally on each pixel of the input image. This is the case of the classical bicubic interpolation method BIC [19] implemented by the Matlab built-in function **imresize** which we have used either in upscaling and downscaling to get a first comparison.

By the extensive experimentation we carried out, we assess that the performance of the LCI method is superior in downscaling (d-LCI method), while in upscaling (u-LCI method) there are only slight differences and the two methods seem almost comparable.

Thus it seemed relevant to us to further investigate only the downscaling case by testing d-LCI method in comparison with two other recent downscaling methods, here denoted by DPID [47] and  $L_0$  [23], both of them not belonging to the family of interpolation methods for downscaling. The effective better performance in downscaling has been confirmed also in these cases, with an increasing gap as the downscaling factor increases. Moreover, by running the public available Matlab code, we have observed that DPID and  $L_0$  methods work only for integer scale factors n/N and m/M and they are not always implementable for images with large sizes due to too long running times or too much memory. On the contrary, LCI method keeps the runtimes contained and offers high flexibility since it works in both downscaling and upscaling with non integer scale factors too, being possible to specify the desired final size or the scale factor as input parameters. We remark that similar nice features are shared by the Matlab function imresize, but with lower performances in downscaling. Moreover, in downscaling, for all odd scale factors, we state a theoretical estimate of the Mean Squared Error (MSE) obtained by our d-LCI method arriving to prove that it is null if  $I^{in} = I$ . Such a theoretical result has been confirmed by our experimentation that also shows it does not hold for imresize applied to the same input images.

The paper is organized as follows. For a simpler exposition of the idea behind the LCI method, we first introduce the method for a monochrome image in Section 2 and then, in Section 3, we expose it for RGB color images. Both monochrome and color cases are considered in Section 4 where some advantages of our approach are discussed. Finally, Section 5 consists of all the numerical experiments.

# 2. Re-sampling gray-level images

In the case of a monochrome image,  $\mathcal{I}$  is represented at a continuous scale by a scalar function f(x, y) whose values are the grey levels at the spatial coordinates  $(x, y) \in A$ . Moreover, at a discrete scale, digital images of  $n \times m$  pixels are matrices  $I \in \mathbb{R}^{n \times m}$ whose elements are the samples of f at the discrete nodes set  $X_{n \times m}$  defined in our model by (5) and (3). Then, according to the notation introduced in the previous section, the resizing problem of  $\mathcal{I}$  (in upscaling or in downscaling, respectively) consists in finding a good approximation of the matrix  $I^{res} \in \mathbb{R}^{N \times M}$  given by (8) starting from the matrix (having a reduced or enlarged size, respectively)  $I^{in} \in \mathbb{R}^{n \times m}$  defined as in (7) being  $\tilde{f}$  a more or less corrupted version of f.

We approach such an approximation problem by using the following bivariate (tensor product) Lagrange–Chebyshev polynomial based on the available data  $I^{in}$ 

$$L_{n,m}\tilde{f}(x,y) := \sum_{i=1}^{n} \sum_{j=1}^{m} \tilde{f}(x_i^n, x_j^m) \ell_i^n(x) \ell_j^m(y), \qquad (x,y) \in A,$$
(9)

where, for all  $\mu \in \mathbb{N}$ ,  $\ell_k^{\mu}$  denotes the k-th fundamental Lagrange polynomial related to the nodes system  $X_{\mu}$ , namely

$$\ell_k^{\mu}(\xi) := \prod_{\substack{s = 1\\s \neq k}}^{\mu} \frac{\xi - x_s^{\mu}}{x_k^{\mu} - x_s^{\mu}}, \qquad \xi \in [-1, 1], \qquad , \qquad k = 1 : \mu.$$
(10)

It is well-known that the Lagrange–Chebyshev polynomial in (9) interpolates  $\tilde{f}$  at the grid  $X_{n \times m}$ , i.e.

$$L_{n,m}\tilde{f}(x_i^n, x_j^m) = \tilde{f}(x_i^n, x_j^m) = I_{i,j}^{in}, \qquad i = 1:n, \ j = 1:m.$$
(11)

The samples of such polynomial at the re-scaled grid  $X_{N \times M}$  constitute the approximate resized image provided by the LCI method. Hence, the LCI method computes the matrix  $I^{out} \in \mathbb{R}^{N \times M}$  whose entries are the following

$$I_{k,h}^{out} := L_{n,m} \tilde{f}(x_k^N, x_h^M), \qquad k = 1 : N, \quad h = 1 : M.$$
(12)

Introducing the Vandermonde-like matrices

$$V_1 := \left[\ell_i^n(x_k^N)\right]_{i,k} \in \mathbb{R}^{n \times N}, \qquad V_2 := \left[\ell_j^m(x_h^M)\right]_{j,h} \in \mathbb{R}^{m \times M},\tag{13}$$

by (9) and (12), the output matrix  $I^{out}$  can be computed from the input matrix  $I^{in}$  according to the following matrices identity

$$I^{out} = V_1^T I^{in} V_2, (14)$$

Note that in case we have to resize a lot of images for the same fixed sizes, formula (14) also allows working in parallel with pre-computed matrices  $V_i$ .

Moreover, it is well known that the fast computation of the matrices  $V_i$  can be achieved by using, instead of (10), the following more convenient form of the fundamental Lagrange polynomial

$$\ell_k^{\mu}(\cos t) = \frac{2}{\mu} \sum_{r=0}^{\mu-1} \cos\left[\frac{(2k-1)r\pi}{2\mu}\right] \cos\left[rt\right], \qquad k = 1:\mu, \quad t \in [0,\pi], \tag{15}$$

where, as usual, the prime on the summation symbol means that the first addendum is halved. Hence, by (15) we get that the matrices  $V_1$  and  $V_2$  can be computed by using

fast cosine transform algorithms (see e.g. [37]) being

$$(V_1)_{i,k} = \ell_i^n(x_k^N) = \frac{2}{n} \sum_{r=0}^{n-1} \cos\left[\frac{(2i-1)r\pi}{2n}\right] \cos\left[\frac{(2k-1)r\pi}{2N}\right], \quad (16)$$
$$i = 1:n, \ k = 1:N,$$

$$(V_2)_{j,h} = \ell_j^m(x_h^M) = \frac{2}{m} \sum_{s=0}^{m-1} \cos\left[\frac{(2j-1)s\pi}{2m}\right] \cos\left[\frac{(2h-1)s\pi}{2M}\right], \qquad (17)$$
$$j = 1:m, \ h = 1:M.$$

### 3. Re-sampling RGB color images

Now we are going to introduce the method for a general color image  $\mathcal{I}$ . In this case, behind  $\mathcal{I}$  there is a vector function  $\mathbf{f} : A \to \mathbb{R}^3$  whose components  $\mathbf{f} = (f_R, f_G, f_B)$ represent  $\mathcal{I}$  in the RGB color space. According to our model, the input image  $\mathbf{I}^{in}$  and the target resized image  $\mathbf{I}^{res}$  are represented in the RGB space as the follows

$$\begin{aligned} \mathbf{I}^{in} \in \mathbb{R}^{n \times m \times 3}, & \mathbf{I}^{in} \equiv (I_R^{in}, I_G^{in}, I_B^{in}) \\ \mathbf{I}^{res} \in \mathbb{R}^{N \times M \times 3}, & \mathbf{I}^{res} \equiv (I_R^{res}, I_G^{res}, I_B^{res}), \end{aligned}$$

where we assume that

$$(I_R^{in})_{i,j} = \tilde{f}_R(x_i^n, x_j^m), \qquad (I_R^{res})_{k,h} = f_R(x_k^N, x_h^M), 
(I_G^{in})_{i,j} = \tilde{f}_G(x_i^n, x_j^m), \qquad (I_G^{res})_{k,h} = f_G(x_k^N, x_h^M), 
(I_B^{in})_{i,j} = \tilde{f}_B(x_i^n, x_j^m), \qquad (I_B^{in})_{k,h} = f_B(x_k^N, x_h^M).$$
(18)

holds for all [i, j] = [1 : n, 1 : m] and [k, h] = [1 : N, 1 : M].

Thus, similarly to the monochrome case and with obvious meaning of the notation, we start from

$$\mathbf{I}_{i,j}^{in} = \tilde{\mathbf{f}}(x_i^n, x_j^m) \qquad i = 1: n, \quad j = 1: m,$$
(19)

and approximate  $\mathbf{I}^{res}$  by

$$\mathbf{I}_{k,h}^{out} = L_{n,m} \tilde{\mathbf{f}}(x_k^N, x_h^M) \qquad k = 1: N, \qquad h = 1: M$$
(20)

i.e., for k = 1 : N and h = 1 : M, we define

$$\mathbf{I}_{k,h}^{out} = \left( L_{n,m} \tilde{f}_R(x_k^N, x_h^M), \ L_{n,m} \tilde{f}_G(x_k^N, x_h^M), \ L_{n,m} \tilde{f}_B(x_k^N, x_h^M) \right).$$
(21)

Hence, by applying the same argument of the monochrome case, we get that the RGB components of the output image  $\mathbf{I}^{out} \equiv (I_R^{out}, I_G^{out}, I_B^{out})$  are given by

$$\mathbf{I}^{out} = (V_1^T I_R^{in} V_2, \ V_1^T I_G^{in} V_2, \ V_1^T I_B^{in} V_2),$$

with  $V_1, V_2$  defined in (16)–(17).

### 4. Model analysis

For a general analysis of the error we get in approximating the target resized image  $I^{res} = [\mathbf{f}(x_i^n, x_j^m)]_{i,j}$  with the output image  $I^{out} = [\mathbf{L}_{n,m} \mathbf{\tilde{f}}(x_h^N, x_k^M)]_{h,k}$  produced by LCI method, we refer the reader to the wide existing literature on the Lagrange interpolation error estimates (see e.g. [27] and the references therein). In this paper, we are interested to measure the performance of LCI method through some standard metrics usually used in Image Processing (see e.g. [38] and the references therein). In particular, we focus on the Mean Squared Error (MSE) defined as follows

$$MSE(I^{res}, I^{out}) = \begin{cases} \frac{1}{MN} \|I^{res} - I^{out}\|_{F}^{2}, & \text{gray-level images,} \\ \\ \frac{1}{3NM} \left\{ \|I_{R}^{res} - I_{R}^{out}\|_{F}^{2} + \|I_{G}^{res} - I_{G}^{out}\|_{F}^{2} + \|I_{B}^{res} - I_{B}^{out}\|_{F}^{2} \right\}, \\ & \text{RGB color images,} \end{cases}$$

being  $\|\cdot\|_F$  the Frobenius norm

$$||A||_F := \left(\sum_{k=1}^N \sum_{h=1}^M A_{k,h}^2\right)^{\frac{1}{2}}, \qquad A = (A_{k,h}) \in \mathbb{R}^{N \times M}.$$

Moreover, we consider the Peak Signal to Noise Ratio (PSNR) defined by the previous MSE as follows

$$PSNR(I^{res}, I^{out}) = 20 \log_{10} \left( \frac{max_f}{\sqrt{MSE(I^{res}, I^{out})}} \right),$$
(22)

with  $\max_{f} = 255$ , since we use the representation by 8 bits per sample.

Finally, in our experiments we evaluate the Structured Similarity Index Measurement (SSIM). For grey-level images it is defined as

$$SSIM(I^{res}, I^{out}) = \frac{[2\mu(I^{res})\mu(I^{out}) + c_1] [2cov(I^{res}, I^{out}) + c_2]}{[\mu^2(I^{res}) + \mu^2(I^{out}) + c_1] [\sigma^2(I^{res}) + \sigma^2(I^{out}) + c_2]},$$
(23)

where  $\mu(A), \sigma(A)$  and  $\operatorname{cov}(A, B)$  indicate the average, variance and covariance, respectively, of the matrices A, B, and  $c_1, c_2$  are constants usually fixed as  $c_1 = (0.01 \times L), c_2 = (0.03 \times L)$  with the dynamic range of the pixel values L = 255 for 8-bit images.

In the case of RGB color images, making the conversion to the color space YCbCr, the SSIM is computed by the above definition applied to the intensity Y (luma) channel.

Based on the previous metrics, the results of wide experimentation will be reported in the next section. In the sequel, we are going to underly two particular features of LCI method.

A first aspect is related to the model choice which, for odd downscaling factors s := n/N = m/M > 1, leads to state the following

**Proposition 4.1.** Under the previously introduced notation, if there exists  $\ell \in \mathbb{N}$  s.t.  $n = (2\ell - 1)N$  and  $m = (2\ell - 1)M$  hold, then we have

$$MSE(I^{res}, I^{out}) \le s^2 MSE(I, I^{in}), \qquad s := (2\ell - 1).$$

$$(24)$$

Moreover, if  $I = I^{in}$  then we get the best results

$$MSE(I^{res}, I^{out}) = 0, \qquad and \qquad SSIM(I^{res}, I^{out}) = 1.$$
(25)

*Proof.* Let's give the proof in the case of a gray-level image since for RGB color images the statement will easily follow by considering the single RGB components.

As a keynote of the proof, we observe that for any  $\ell \in \mathbb{N}$ , we have

$$n = (2\ell - 1)N \implies X_N \subset X_n.$$

More precisely, setting  $s := (2\ell - 1)$ , it is easy to check that

$$n = sN \qquad \Longrightarrow \qquad x_k^N = x_i^n \quad \text{with} \quad i = \frac{s(2k-1)+1}{2}, \quad k = 1:N,$$
 (26)

where we remark that for all k = 1 : N, the numerator s(2k - 1) + 1 is certainly even since s is odd.

Consequently, we deduce (24) from (26) as follows

$$MSE(I^{res}, I^{out}) = \frac{1}{NM} \sum_{k=1}^{N} \sum_{h=1}^{M} \left[ f\left(x_{k}^{N}, x_{h}^{M}\right) - L_{n,m} \tilde{f}\left(x_{k}^{N}, x_{h}^{M}\right) \right]^{2}$$
$$= \frac{1}{NM} \sum_{k=1}^{N} \sum_{h=1}^{M} \left[ f\left(x_{\frac{s(2k-1)+1}{2}}^{n}, x_{\frac{s(2h-1)+1}{2}}^{m}\right) - \tilde{f}\left(x_{\frac{s(2k-1)+1}{2}}^{n}, x_{\frac{s(2h-1)+1}{2}}^{m}\right) \right]^{2}$$
$$\leq \frac{1}{NM} \sum_{i=1}^{n} \sum_{j=1}^{m} \left[ f\left(x_{i}^{n}, x_{j}^{m}\right) - \tilde{f}\left(x_{i}^{n}, x_{j}^{m}\right) \right]^{2} = s^{2}MSE(I, I^{in}).$$

Moreover, in the case that  $I = I^{in}$ , we have that  $L_{n,m}f = L_{n,m}\tilde{f}$  and, due to the nesting property  $X_{N \times M} \subset X_{n \times m}$ , by (11), for any h = 1 : N and k = 1 : M, we get

$$I_{h,k}^{out} = L_{n,m}\tilde{f}(x_h^N, x_k^M) = L_{n,m}f(x_h^N, x_k^M) = f(x_h^N, x_k^M) = I_{h,k}^{res},$$
(27)

 $\diamond$ 

that implies the first equation in (25). Finally, (27) also yields

$$2\operatorname{cov}(\mathbf{I}^{\operatorname{res}}, \mathbf{I}^{\operatorname{out}}) = \sigma(\mathbf{I}^{\operatorname{res}})^2 + \sigma(\mathbf{I}^{\operatorname{out}})^2,$$
$$2\mu(I^{\operatorname{res}})\mu(I^{\operatorname{out}}) = \mu(I^{\operatorname{res}})^2 + \mu(I^{\operatorname{out}})^2$$

which conclude the proof of (25).

We remark that the previous proposition is a direct consequence of our choice of the sampling model based on Chebyshev zeros (3), instead of the usual equally spaced nodes (2).

Such choice (and more generally the choice of "good" interpolation knots) allows to use global interpolation processes that are not possible in the case of equally spaced nodes. In fact, as second aspect, we aim to highlight the relevance of the choice of good interpolation knots by showing the effects we have in the performance of global Lagrange interpolation methods associated with the univariate nodes set  $X^{equ}_{\mu}$  in (2). The disastrous effects of the exponential growth of Lebesgue constants are visible in the next experiment concerning an image zooming  $\times 2$  from the size  $n \times n$  with n = 256 to the size  $N \times N$  with N = 512. Figure 2 displays the images obtained starting from the same input image (Fig.2, left) and using the Lagrange interpolation polynomial based on equally spaced nodes  $X_n^{equ} \times X_n^{equ}$  (Fig.2, right) and on the Chebyshev grid  $X_{n \times n}$  (Fig.2, middle). The images we see are the samples of the previous Lagrange polynomials at the respective  $(N \times N)$ -grid and we can check how the equally-spaced sampling model (2) yields wrong results outside of a small region of the output image.



Figure 2: The well–known image *Flowers* taken from USC–SIPI [1] (left) upsampled at the double scale by Lagrange interpolation at Chebyshev nodes (middle) and at equally spaced nodes (right)

# 5. Experimental results

In this section, we propose a selection of the extensive experimentation we carried on to test the benefits of our procedure in downscaling (d-LCI method) and in upscaling (u-LCI method), comparing our results with other resizing techniques.

### 5.1. Comparison methods

As already mentioned, we compare LCI with BIC provided by MatLab imresize with 'bicubic' option where we recall that a new pixel is determined from a weighted average of the 16 closest pixels using an interpolating cubic convolution function satisfying prescribed assumptions of smoothness on f to gain at least a cubic order of convergence [19]. The comparison BIC-LCI regards either down and up resizing cases, giving as input either the scale factor or the final size of the desired image. Note that we have also tested the other different options of imresize (namely 'linear' and 'nearest neighborhood') but for brevity, we do not report the results since they give no new insight.

Moreover, to further investigate the performance in downscaling case, we compare d-LCI with other two recent downscaling methods not belonging to the interpolation method class. These methods are briefly indicated as DPID method (described in [47] with the code available at [2]) and  $L_0$  method (described in [23], with the code available at [3]). To be fair, we remark that we forced DPID and  $L_0$  also in upscaling direction for several scaling factors (the choice of the desired size is not allowed by DPID and  $L_0$ ) but the results were very poor w.r.t. BIC and LCI and they have been not reported here.

Finally, we point out that all the previous methods have run on the same PC with Intel Core i7 3770K CPU @350GHz configuration.

# $5.2. \ Datasets$

Since the results of any method depend on the type of input image, we have conducted the experimentation on different kinds of 8–bits images collected in five publicly available datasets, whose characteristics (acronym, number of images, range of their sizes) are synthesized in Table 1.

Table 1: Datasets list									
Dataset	n. of Images	Sizes							
BSDS500	500	$481 \times 321 \text{ or } 321 \times 481$							
NASA	17	from $500 \times 334$ to $6394 \times 3456$							
YAHOO	96	from $500 \times 334$ to $6394 \times 3456$							
13US	13	from $241 \times 400$ to $400 \times 310$							
URBAN100	100	from $1024 \times 564$ to $1024 \times 1024$							

We point out that we have chosen the datasets in Table 1 since they are the same considered in [47, 23] for DPID and  $L_0$  methods. In particular:

- BSDS500 dataset [26], available at [4], has been used for testing  $L_0$  in [23]. It is sufficiently general and provides a large variety of images often employed for testing many other methods with different image analysis tasks such as image segmentation [29, 39, 40, 42]), color quantization [12, 41, 10, 43], etc.
- NASA Image Gallery [6] and YFCC100M (Yahoo Flickr Creative Commons 100 Million) [44] datasets (here briefly denoted by NASA and YAHOO, respectively) are used by DPID in [47].
- 13US [36] contains natural images, available at [5] and originally taken from the MSRA Salient Object Database [24]. It is another dataset used in [47].
- URBAN100 dataset [17], concerning urban scenes with images having one dimension equal to 1024, is commonly used to evaluate the performance of superresolution models. It is used as test images for  $L_0$  in [23].

All the previous datasets have been considered for either up and down scaling.

We remark that in all the performed experiments the input images are not available from the datasets. Hence, following a typical approach in image quantitative evaluation, we fix all the images from the datasets as target images (i.e.  $I^{res}$  in our notation) and we apply BIC to generate the input image (namely  $I^{in}$ ) common to all methods. More precisely, to run  $[\times s]$  upscaling methods ([: s] downscaling methods, resp.) we generate the input image  $I^{in}$  by applying BIC to  $I^{res}$  in the opposite [: s] ([×s], resp.) scaling direction.

Note that, according to such procedure, in testing  $[\times s]$  upscaling methods we may find that  $I^{res}$  does not have both the dimensions N and M that are divisible for s. In this case, in order to generate  $I^{in}$  we use **imresize** with the size option, requiring  $n = \lfloor \frac{N}{s} \rfloor$  and  $m = \lfloor \frac{M}{s} \rfloor$ . Moreover, once obtained  $I^{in}$ , both BIC and u-LCI run by specifying again the size  $N \times M$  instead of the scaling factor.

## 5.3. Visual comparison

As first experiment, we focus on the visual comparison of some original images chosen in the previous datasets with the output images produced by the methods in Subsection 5.1. Such images are given in Figure 3 for upscaling and in Figure 4 for downscaling.

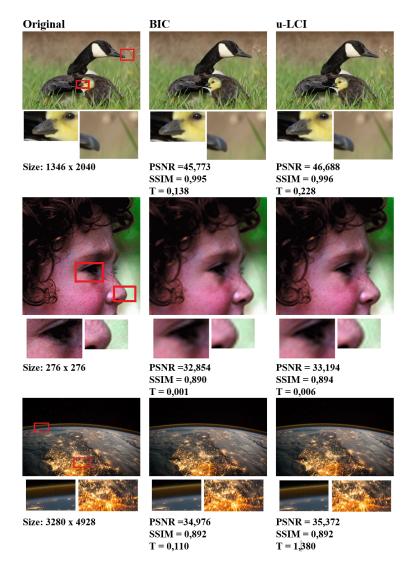


Figure 3: Examples of upscaling performance results at the scale factor 2 (top), at the scale factor 3 (middle), at the scale factor 4 (bottom).

In both the figures at the first column we show the target images and at the successive columns we display the resized images obtained as the output of the considered methods. Moreover, the first, second and third row of both the figures correspond to the scaling factors 2, 3 and 4, respectively. In all the cases the images have been reported with evidence of some magnified regions of interest (ROI).

From the qualitative point of view, by inspecting Figures 3 and 4, we can deduce that in terms of visual quality LCI has a good performance, also with respect to the chosen comparing methods. It preserves the visual structure of the object without losing image details, the local contrast, and the luminance of the input image, by generating resized images close to the target ones. Ringing and over smoothing artifacts are limited and

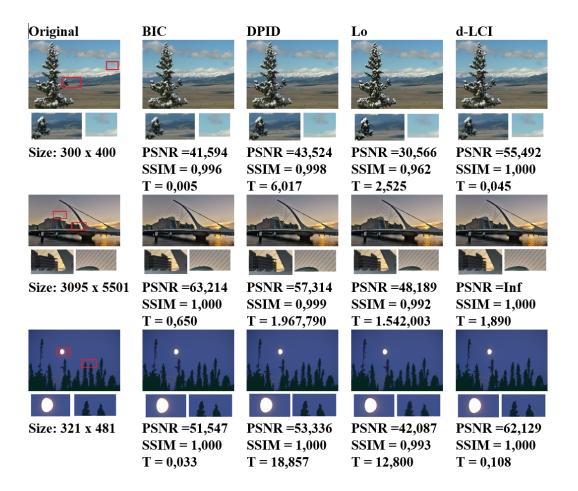


Figure 4: Examples of downscaling performance results at the scale factor 2 (top), at the scale factor 3 (middle), at the scale factor 4 (bottom).

the resized images seem fair, sufficiently not blurred, and with well-balanced colors.

# 5.4. Quantitative comparison: average results

For each dataset selected in Subsection 5.2, we computed the averages of the PSNR and SSIM values achieved by LCI and by the comparison methods described in Subsection 5.1, for the scaling factors 2, 3, 4. The results are displayed in Table 2 for upscaling and in Table 3 for downscaling, reporting in both the tables also the averages of the required CPU times.

As announced in the introduction, from Table 2 we see that the values attained by u-LCI are only slightly better than those achieved by BIC, and the execution time of u-LCI is a little bit greater than BIC. Of course, as the scale factor increases, both PSNR and SSIM decrease in all the upscaling methods. Moreover, the gaps between BIC and u-LCI in both the metrics are maintained also for large size images.

On the contrary, in downscaling, by inspecting Table 3 we observe that the performance provided by d-LCI is much better than those achieved by the other methods for both the metrics. In particular, when the downscaling factor is 3 the theoretical results claimed in Proposition 4.1 are confirmed. Moreover, looking at the even downscaling factors 2, 4 the improvement by d-LCI seems to increase as the scale factor increases

 $\mathbf{x}\mathbf{2}$  $\mathbf{x4}$ x3PSNR Т SSIM PSNR SSIM Т PSNR SSIM Т BSDS500 BIC 26,3410,886 0,003 24,821 0,837 0,002 22,251 0,701 0,003 u-LCI 26,3810,888 0,014 24,868 0,839 0,010 22,446 0,769 0,008 NASA 0,071 0,091 31,205 0,9240,074 BIC 34,806 0,95829,808 0,907 u-LCI 35,4120,960 1,35731,5420,924 0,839 30,183 0,908 0,634YAHOO 0,055 28,741BIC 33,391 0,9530,056 29,7540,913 0,891 0,051 33,900 29,956 u-LCI 0,955 0,812 0,914 0,56728,986 0,891 0,459URBAN100 21,703 0,8820,009 21,3060,7550,008 0.008 BIC 25,4330,741u-LCI 25,896 0,886 0,064 21,3250,7540,05121,919 0,793 0,059 13US 24.1160.8610.002 20.7540.002 0.002 BIC 0.73420.5450,710u-LCI 24,4860,868 0,012 20,7800,738 0,010 20,656 0,713 0,009

Table 2: Average results in upscaling.

Table 3: Average results in downscaling

		:2			:3			:4	
	PSNR	SSIM	Т	PSNR	SSIM	Т	PSNR	SSIM	Т
BSDS500									
BIC	38,872	0,993	0,006	39,253	0,993	0,009	39,183	0,993	0,017
DPID	41,745	0,996	7,696	41,827	0,996	12,246	41,206	0,996	$18,\!615$
$L_0$	29,317	0,961	$3,\!647$	32,873	0,971	8,020	34,174	0,971	14,295
d-LCI	53,732	1,000	0,057	Inf	1,000	0,091	$55,\!889$	1,000	0,137
NASA									
BIC	45,969	0,995	0,229	47,114	0,996	0,325	46,973	0,996	$0,\!639$
DPID	$47,\!675$	0,998	448,023	$47,\!657$	0,998	731,498	47,112	0,997	1.098, 113
$L_0$	34,754	0,972	208,574	37,285	0,979	$617,\!386$	oom	oom	oom
d-LCI	54,265	0,999	$6,\!614$	Inf	1,000	$13,\!317$	$57,\!491$	1,000	22,859
YAHOO									
BIC	44,757	0,996	$0,\!155$	45,858	0,997	0,219	$45,\!682$	0,996	$0,\!422$
DPID	46,743	0,998	$291,\!685$	46,948	0,998	$479,\!638$	46,421	0,998	$714,\!908$
$L_0$	33,913	0,974	$133,\!190$	oom	oom	oom	oom	oom	oom
d-LCI	54,067	0,999	4,698	Inf	1,000	$^{8,279}$	$57,\!223$	1,000	$13,\!898$
URBAN00									
BIC	35,661	0,989	0,027	36,010	0,990	0,041	35,940	0,989	0,068
DPID	39,178	0,996	37,592	39,178	0,996	60,834	38,702	0,995	$93,\!996$
$L_0$	26,718	0,951	13,267	31,061	0,969	28,845	33,571	0,973	50,312
d-LCI	$52,\!613$	0,999	0,234	Inf	1,000	0,416	$55,\!452$	1,000	$0,\!618$
13 US									
BIC	35,129	0,990	0,005	35,469	0,991	0,009	35,397	0,991	0,013
DPID	38,061	0,996	5,593	38,326	0,996	8,905	37,707	0,995	$13,\!819$
$L_0$	25,521	0,949	2,572	30,231	0,972	5,706	32,929	0,979	9,939
d-LCI	$52,\!813$	1,000	0,042	Inf	1,000	0,073	$55,\!374$	1,000	0,108

while BIC seems to be affect by saturation, with an increasing gap between d-LCI and the remaining methods.

About the CPU time, except for BIC, d-LCI is faster than the other methods that, in case of very large images, need very long computational time (see the results for NASA and YAHOO datasets that include target images of size  $6394 \times 3456$ , i.e. input images with size  $25576 \times 13824$  in case of downscaling factor equals to 4) and in some cases, wherever we see oom (which means out of memory), the available code of L<sub>0</sub> does not arrive to produce any result.

#### 5.5. Quantitative comparison: some pointwise results

Here we focus on each single image from the smaller datasets of Table 1 , namely we consider 13US dataset in downscaling, with target images of small size displayed in

Figure 5, and NASA dataset in upscaling, with target images of large size displayed in Figure 6.



Figure 5: 13US dataset image: [U1 -U13] from left to right and from top to bottom .

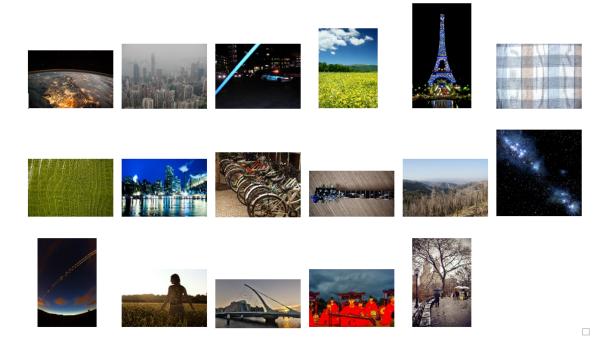


Figure 6: NASA dataset images: [N1, N2, N3, N5, N6, N7, N8, N9, N10, N11, N12, N13, N14, N15, N16, N17, N19] from left to right and from top to bottom .

The PSNR values achieved for every single image have been plotted in Figure 7 for NASA dataset with upscaling factor 2, 4, 8, 16, and in Figure 8 for 13US dataset with downscaling factor 6, 18, 30.

On the same datasets, more detailed results are given in Tables 4–6 reporting in the first columns the name and the size of all the images from 13US and NASA.

In particular, Table 4 contains the detailed upscaling results on the NASA dataset, for the scaling factors  $s \in \{2, 4, 8, 16\}$  while Tables 5-6 concern the downscaling results

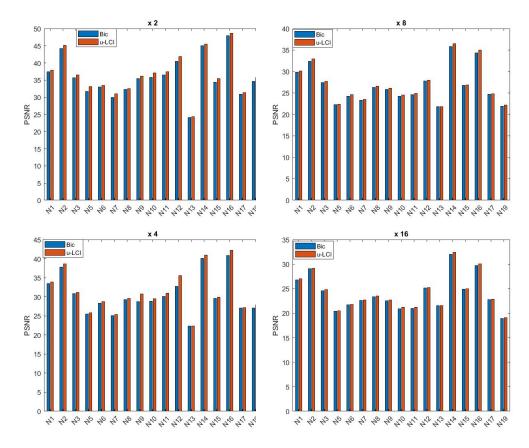


Figure 7: Pointwise PSNR for the NASA set for scale factors s=2,4,8,16

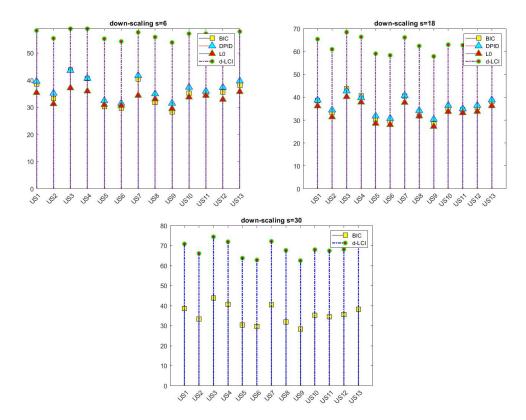


Figure 8: Pointwise PSNR values for the set 13US, with scale factors s=6 (top), s=18 (middle), s=30 (down)

rable 4: Pointwise results on NASA dataset in upscaling           x2         x4         x8         x16										
	PSNR X	SSIM	PSNR X4	<sup>1</sup> SSIM	PSNR	SSIM	$\frac{x16}{PSNR}$ SSIM			
<b>N1</b> 4928×3280	PSIN	551M	PSING	551M	FSING	5511/1	PSING	SSIM		
BIC	37,472	0,945	33,453	0,892	29,828	0,858	26,795	0,833		
u-LCI	37,903	0,945 0,950	33,856	$0,892 \\ 0,892$	<b>30,133</b>	0,855	<b>26,795</b> <b>26,999</b>	0,833		
<b>N2</b> 3072×2304	31,303	0,900	33,830	0,092	30,133	0,855	20,999	0,001		
BIC	44,235	0,986	37,820	0,952	32,417	0,888	29,065	0,840		
u-LCI	45,203	0,980 0,988	<b>38,621</b>	0,952 0,957	32,943	0,888 0,894	<b>29,005</b> <b>29,183</b>	0,840 0,840		
<b>N3</b> 2048×1536	40,200	0,300	50,021	0,301	52,345	0,034	23,105	0,040		
BIC	35,708	0,963	30,845	0,919	27,474	0,874	24,593	0,830		
u-LCI	36,552	0,965	31,136	0,916	27,731	0,865	24,850	0,808		
N5 2701×3665	00,002	0,000	01,100	0,010		0,000	_1,000			
BIC	31,743	0,987	25,444	0,960	22,262	0,932	20,388	0,913		
u-LCI	33,077	0,989	25,792	0,962	22,426	0,933	20,487	0,913		
<b>N6</b> 1836×3264		- )	- /	- /	, -	- /	- /	- )		
BIC	33,011	0,986	28,291	0,965	24,250	0,927	21,677	0,886		
u-LCI	33,443	0,986	28,779	0,961	24,614	0,908	21,784	0,850		
<b>N7</b> 1600×1200		,		,		,		,		
BIC	29,939	0,886	25,087	$0,\!670$	23,364	0,567	22,627	0,545		
u-LCI	30,981	0,905	25,409	$0,\!684$	23,542	0,569	22,683	0,546		
<b>N8</b> 3008×2000										
BIC	32,298	0,987	29,295	0,974	26,346	0,956	23,345	0,932		
u-LCI	32,577	0,988	29,594	0,976	$26,\!619$	0,958	23,508	0,933		
N9 5430×3520										
BIC	35,440	0,985	28,707	0.945	24,336	0,883	22,494	0,846		
u-LCI	36,116	0,986	30,735	0,955	26,121	0,899	22,712	0,845		
<b>N10</b> 3264×2448										
BIC	35,758	0,973	28,875	0,916	24,213	0,837	20,914	0,767		
u-LCI	37,047	0,977	29,531	0,920	$24,\!529$	0,838	$21,\!187$	0,768		
<b>N11</b> 4368×2326										
BIC	36,525	0,982	30,149	0,955	24,649	0,906	20,966	0,849		
u-LCI	37,461	0,981	30,986	0,951	24,918	0,897	21,193	0,846		
<b>N12</b> 3504×2336										
BIC	40,471	0,989	32,786	0,951	27,777	0,888	25,116	$0,\!850$		
u-LCI	41,889	0,991	35,581	0,956	27,982	0,890	25,191	0,851		
<b>N13</b> 1200×1600										
BIC	24,145	0,746	22,370	0,603	21,838	0,530	21,555	0,483		
u-LCI	24,316	0,730	22,390	0,592	21,852	0,529	21,579	0,486		
<b>N14</b> 3456×5184	45.041	0.000	40.105	0.000	ar	0.070	91.000	0.02		
BIC	45,041	0,992	40,137	0,986	35,808	0,978	31,982	0,967		
u-LCI	45,539	0,992	40,907	0,986	36,531	0,978	32,407	0,966		
N15 3072×2048	94 457	0.000	20 574	0.007	0.0 791	0.041	04.000	0.000		
BIC	34,457	0,990	29,574	0,967	26,731	0,941	24,886	0,920		
u-LCI	35,427	0,991	29,872	0,968	26,883	0,940	24,976	0,919		
<b>N16</b> 5501×3095 BIC	47.000	0.002	10.946	0.000	24 204	0.061	20.760	0.040		
u-LCI	47,929	0,993	40,846 <b>42,202</b>	0,982	34,304	0,961	29,760 30,027	<b>0,940</b>		
<b>N17</b> 2048×1363	48,691	0,994	42,202	0,983	35,029	0,961	30,027	0,939		
BIC	30,892	0 000	97 090	0.054	21 600	0.097	22 80K	0,899		
u-LCI	<b>30,892</b> <b>31,380</b>	0,980 <b>0,981</b>	27,028 27,219	$0,954 \\ 0,954$	24,688 <b>24,834</b>	$0,927 \\ 0,927$	<b>22,895</b> 21,044	0,899		
<b>N19</b> 3039×4559	51,300	0,901	41,419	0,904	44,004	0,921	21,044	0,011		
BIC	34,581	0,977	27,048	0,884	21,873	0,718	18,910	0,615		
u-LCI	35,844	0,981	<b>27,040</b> <b>27,881</b>	0,884 0,896	<b>21</b> ,875 <b>22,178</b>	<b>0</b> ,718 <b>0</b> ,723	<b>19,077</b>	<b>0,617</b>		
u-101	00,044	0,301	21,001	0,000	22,110	0,140	10,011	0,017		

Table 4: Pointwise results on NASA dataset in upscaling

					13US dataset in downscaling       :9     :18				
		3	:6		:9				
<b>TIG1</b> 200×400	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	
<b>US1</b> 300×400 BIC	38,625	0,996	38,573	0,996	38,583	0,996	38,584	0,996	
DPID	40,846	$0,990 \\ 0,997$	39,580	$0,990 \\ 0,997$	39,160	0,990 0,996	$38,384 \\ 38,764$	0,990 0,996	
L <sub>0</sub>	27,994	0,997 0,949	35,380 35,405	0,997 0,983	35,100 35,138	$0,990 \\ 0,986$	36,104 36,183	$0,990 \\ 0,991$	
d-LCI	27,994 Inf	0,949 <b>1,000</b>	58,265	1,000	55,158 Inf	1,000	<b>65,394</b>	<b>1,000</b>	
$\frac{\text{US2} 400 \times 300}{\text{US2}}$	1111	1,000	38,205	1,000	1111	1,000	05,554	1,000	
BIC	33,348	0,987	33,298	0,987	33,303	0,987	33,302	0,987	
DPID	36,428	0,991	35,200 35,217	0,992	34,823	0,992	34,450	0,991	
L <sub>0</sub>	29,184	0,963	31,289	0,962	30,647	0,968	31,394	0,971	
d-LCI	Inf	1,000	55,418	1,000	Inf	1,000	60,956	1,000	
US3 300×400		_,	,	_,		_,		_,	
BIC	43,820	0,999	43,754	0,999	43,774	0,999	43,786	0,999	
DPID	44,853	0,999	43,589	0,999	43,162	0,999	42,751	0,999	
$L_0$	34,756	0,992	37,140	0,995	38,087	0,996	40,237	0,998	
d-LCI	Inf	1,000	58,958	1,000	Inf	1,000	68,436	1,000	
<b>US4</b> 300×400			,				,	,	
BIC	40,673	0,996	40,623	0,996	40,631	0,996	40,633	0,996	
DPID	42,066	0,998	$40,\!699$	0,997	40,236	0,997	39,794	0,996	
$L_0$	32,944	0,973	$35,\!973$	0,981	36,453	0,986	$37,\!836$	0,992	
d-LCI	Inf	1,000	58,958	1,000	Inf	1,000	66, 386	1,000	
<b>US5</b> 400×300									
BIC	30,357	0,979	30,307	0,979	30,311	0,979	30,311	0,979	
DPID	$33,\!492$	0,991	32,471	0,989	32,133	0,988	31,813	0,987	
$L_0$	$26,\!807$	0,962	30,944	0,973	28,911	0,963	28,551	0,965	
d-LCI	Inf	1,000	55,257	1,000	Inf	1,000	59,065	1,000	
<b>US6</b> 300×400									
BIC	29,718	0,978	$29,\!659$	0,978	29,633	0,978	29,663	0,978	
DPID	32,578	0,990	31,486	0,988	31,111	0,987	30,752	0,985	
$L_0$	25,322	0,959	30,666	0,966	28,724	0,958	28,022	0,962	
d-LCI	Inf	1,000	54,273	1,000	Inf	1,000	58,321	1,000	
<b>US7</b> 273×400			10 510	0.00 <b>-</b>	10 501		10 500	0.00 <b>7</b>	
BIC	40,559	0,995	40,513	0,995	40,521	0,995	40,522	0,995	
DPID	43,135	0,997	41,717	0,996	41,232	0,996	40,771	0,996	
L <sub>0</sub> d-LCI	31,832	0,949	34,364	0,964	35,604	0,975	37,697	0,987	
$\frac{\text{US8 } 322 \times 400}{\text{US8 } 322 \times 400}$	Inf	1,000	57,639	1,000	Inf	1,000	66,113	1,000	
BIC	31,857	0,989	31,802	0,989	31,808	0,989	31,808	0,989	
DPID	36,382	0,989 0,996	34,979	0,989 0,994	34,601	0,983 0,994	34,239	0,903	
L <sub>0</sub>	29,562	0,930 0,974	33,051	0,934 0,977	32,378	0,980	31,828	0,985	
d-LCI	Inf	1,000	55,880	1,000	Inf	<b>1,000</b>	<b>62,394</b>	1,000	
US9 322×400		1,000		1,000		1,000	02,001	1,000	
BIC	28,407	0,984	28,350	0,984	28,536	0,984	28,354	0,984	
DPID	32,542	0,994	31,504	0,993	34,601	0,992	30,393	0,992	
L <sub>0</sub>	29,562	0,974	29,504	0,985	27,361	0,980	27,214	0,979	
d-LCI	Inf	1,000	53,879	1,000	Inf	1,000	57,877	1,000	
<b>US10</b> 400×307	1								
BIC	35,220	0,992	35,168	0,992	35,172	0,992	35,173	0,992	
DPID	$38,\!947$	0,997	37,406	0,996	36,929	0,995	36,494	0,995	
$L_0$	31,031	0,984	33,760	0,986	33,227	0,985	33,744	0,988	
d-LCI	Inf	1,000	$57,\!131$	1,000	Inf	1,000	$62,\!986$	1,000	
<b>US11</b> 400×241									
BIC	$34,\!599$	0,995	$34,\!528$	0,995	34,536	0,995	34,535	0,995	
DPID	$37,\!345$	0,998	35,950	0,997	35,432	0,997	34,937	0,996	
$L_0$	$27,\!631$	0,976	34,346	0,986	34,143	0,989	33,189	0,992	
d-LCI	Inf	1,000	$57,\!147$	1,000	Inf	1,000	62,744	61,000	
<b>US12</b> 400×266									
BIC	35,698	0,993	35,639	0,993	035,645	0,993	35,647	0,993	
DPID	38,626	0,997	37,345	0,996	36,898	0,995	36,471	0,995	
	29,883	0,972	32,902	0,978	33,091	0,982	33,776	0,987	
d-LCI	Inf	1,000	55,801	1,000	Inf	1,000	62,744	1,000	
<b>US13</b> 310×400	20.000	0.000	00 100	0.000	00.170	0.000	00 1 77	0.000	
BIC	38,220	0,996	38,166	0,996	38,173	0,996	38,177	0,996	
DPID	41,001	0,998	39,707	0,997	39,276	0,997	38,861	0,997	
	31,593	0,982	35,761	0,989	35,898	0,991	36,263	0,993	
d-LCI	Inf	1,000	57,905	1,000	Inf	1,000	$65,\!535$	1,000	

Table 5: Pointwise results on 13US dataset in downscaling

	$\begin{array}{c c c c c c c c c c c c c c c c c c c $							:33		
		-					:30		-	
	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM
<b>US1</b> 300×400										
BIC	38,583	0,996	38,585	0,996	38,586	0,996	38,585	0,996	38,584	0,996
d-LCI	$66,\!485$	1,000	Inf	1,000	Inf	1,000	$70,\!845$	1,000	Inf	1,000
<b>US2</b> 400×300										
BIC	33,302	0,987	33,302	0,987	33,302	0,987	33,303	0,987	33,303	0,987
d-LCI	66,347	1,000	Inf	1,000	Inf	66,092	1,000	1,000	Inf	1,000
<b>US3</b> 300×400										
BIC	43,786	0,999	43,787	0,999	43,786	0,999	43,788	0,999	43,784	0,999
d-LCI	69,721	1,000	Inf	1,000	Inf	1,000	$74,\!415$	1,000	Inf	1,000
<b>US4</b> 300×400										
BIC	40,633	0,996	40,634	0,996	40,636	0,996	49,636	0,996	49,635	0,996
d-LCI	68,054	1,000	Inf	1,000	Inf	1,000	$71,\!936$	1,000	Inf	1,000
<b>US5</b> 400×300										
BIC	30,311	0,979	30,311	0,979	30,311	0,979	30,311	0,979	30,311	0,979
d-LCI	59,853	1,000	Inf	1,000	Inf	1,000	63,768	1,000	Inf	1,000
<b>US6</b> 300×400										
BIC	30,311	0,979	29,663	0,978	29,664	0,978	29,663	0,978	29,663	0,978
d-LCI	59,135	1,000	Inf	1,000	Inf	1,000	62,833	1,000	Inf	1,000
<b>US7</b> 273×400										
BIC	40,524	0,995	40,521	0,995	40,524	0,995	40,536	0,995	40,520	0,995
d-LCI	67,318	1,000	Inf	1,000	Inf	1,000	$72,\!151$	1,000	Inf	1,000
<b>US8</b> 322×400										
BIC	31,808	0,989	31,807	0,989	31,807	0,989	31,807	0,995	31,806	0,989
d-LCI	$63,\!539$	1,000	Inf	1,000	Inf	1,000	$67,\!653$	1,000	Inf	1,000
<b>US9</b> 322×400										
BIC	28,354	0,984	28,355	0,984	28,354	0,984	28,355	0,984	28,355	0,984
d-LCI	$58,\!651$	1,000	Inf	1,000	Inf	1,000	$62,\!536$	1,000	Inf	1,000
<b>US10</b> 400×307										
BIC	35,174	0,992	35,172	0,992	35,174	0,992	35,175	0,992	35,175	0,992
d-LCI	63,924	1,000	Inf	1,000	Inf	1,000	$67,\!994$	1,000	Inf	1,000
<b>US11</b> 400×241										
BIC	34,535	0,995	34,535	0,995	34,534	0,995	34,534	0,995	34,535	0,995
d-LCI	63,445	1,000	Inf	1,000	Inf	1,000	$67,\!442$	1,000	Inf	1,000
<b>US12</b> 400×266										
BIC	$35,\!647$	0,993	35,646	0,993	35,645	0,993	$35,\!646$	0,993	35,648	0,993
d-LCI	63,760	1,000	Inf	1,000	Inf	1,000	$67,\!127$	1,000	Inf	1,000
<b>US13</b> 310×400										· .
BIC	38,175	0,996	38,175	0,996	38,174	0,996	38,175	0,996	38,175	0,996
d-LCI	66,347	1,000	Inf	1,000	Inf	1,000	70,960	1,000	Inf	1,000
	1 1			,		,				,

Table 6: Performance results on 13US dataset for high downscaling

obtained on 13US dataset for the scaling factors  $s \in \{3, 6, 9, 18\}$  (Table 5) and for the larger factors  $s \in \{20, 21, 27, 30, 33\}$  (Table 6).

According to the average results, the pointwise ones corroborate the global trend. In particular, Table 5 confirms that in downscaling d-LCI is the method allowing the largest PSNR and SSIM, and that the method requiring the least computation time is BIC, followed by d-LCI which is faster than DPID and L<sub>0</sub>. Also, note that the scaling factors chosen in Table 5 are all multiple of three but, according to Proposition 4.1 we can see the differnce between the odd and even downscaling factors. Moreover, we can affirm that d-LCI works fine also for large downscaling factors as reported in Table 6 where the target images in 13US have been zoomed in up to 33 times, getting input images whose size goes up to  $13200 \times 10230$ . We point out that in Table 6 the results for DPID and L<sub>0</sub> methods are missing because the publicly available MatLab codes don't work for so large scale factors. Moreover, if  $s \in \{21, 27, 33\}$  then, in accordance with Proposition 4.1, PSNR and SSIM again reach the limit values Infinite and 1, respectively, by d-LCI while the same does not hold for BIC applied to the same input images.

#### 6. Conclusions

In the context of interpolation methods for image resizing, we present the Lagrange– Chebyshev Interpolation (LCI) method. It is based on a non standard mathematical modeling that leads to the application of such an optimal Lagrange interpolation process to globally approximate the image at a continuous scale. One of the main advantages of LCI method is its high flexibility of working in both the scaling directions, either setting a scale factor or giving a particular final size. Comparisons with other resizing procedures have been reported on 5 different common datasets made of 726 images in total. The numerical experience in upscaling shows a performance comparable with the bicubic interpolation method, while in downscaling a much better performance is obtained with respect to all the considered comparison methods. Moreover, in downscaling cases with odd scale factors, we estimate the Mean Square Error (MSE) of our procedure in terms of the initial errors present in the data and we prove that it is null in absence of noise or artifacts in the input image. We wonder whether further improvements can be achieved employing wavelets technique or finer approximation polynomials. These will be the subjects of further investigations.

### Code and supplementary materials

The code and the supplementary materials are openly available (to appear at a GitHub link).

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