# Lane Detection Using B-Snake 

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#### Abstract

In this paper, we proposed a new B-snake based lane detection algorithm. Compared with other lane models, the B-snake based lane model is able to describe a wider range of lane structures, since $B$-spline can form any arbitrary shape by a set of control points. The problems of detecting both sides of lane markings (or boundaries) have been formulated here as the problem of detecting the mid-line of the lane, by using the knowledge of the perspective parallel lines. A robust algorithm called CHEVP is presented for providing a good initial position for the $B$-snake. Furthermore, a minimum energy method by MMSE (Minimum Mean Square Energy) is suggested to determine the control points of the $B$-snake model by the overall image forces on two sides of lane. Experimental results show that the proposed method is robust against noise, shadows, and illumination variations in the captured road images, and also applicable to both the marked and the unmarked roads, and the dash and the solid paint line roads.


## 1. Introduction

Autonomous Guided Vehicles (AGV) have found many applications in the industries. In most applications, these AGVs have to navigate in the unstructured environments. Path findings and navigational control under these situations are usually accomplished from the images captured by camera mounted on the vehicles. These images are also interpreted to extract meaningful information such as positions, road markings, road boundaries, and direction of vehicle's heading. Among many extraction methods, the lane marking (or road boundary) detection from the road images had received great interest. As the captured images are usually corrupted by noises, lots of boundary-detection algorithms have been developed to achieve robustness
against these noises.
The main properties that the lane marking (or boundary) detection techniques should possess are:

- The quality of lane detection should not be affected by shadows, which can be cast by trees, buildings, etc.
- It should be capable of processing the painted and the unpainted roads.
- It should handle the curved roads rather than assuming that the roads are straight.
- It should use the parallel constraint as a guidance to improve the detection of both sides of lane markings (or boundaries) in the face of noises in the images.
- It should produce an explicit measurement of the reliability of the results obtained.
In Section 2, reviews on existing lane-detection techniques are presented. Section 3 introduces a novel Bspline lane model with dual external forces. In Section 4, an algorithm, called CHEVP, is described both for vanishing line detection and $B$-snake lane model initialization. Section 5 presents a minimum energy method, called MMSE (Minimum Mean Square Energy), to determine the parameters for lane detection. This paper concludes in Section 6.


## 2. RELATED WORKS

Up to present, various vision-based lane detection algorithms have been developed. They usually utilized different road models (2D or 3D, straight or curve) and different techniques (Hough, template matching, neural networks, etc.).

The approach based on morphological filtering has been suggested [1][2]. This technique used the morphological "watershed" transformation to locate the lane edges in the intensity gradient magnitude image. Although this technique has the advantage of not requiring any thresholding for the gradient magnitudes, it
has the disadvantage of not imposing any global constraints on the lane edge shapes.

A curve road model was proposed by [3][4]. It was supposed that the lane boundaries could be presented by a parabolic curve on a flat ground. Although it can approximate normal road structures, it still cannot describe some cases, i.e. a "T" turn. A deformable template method was proposed by optimizing a likelihood function based on this model. However, this algorithm cannot guarantee a global optimum and the accuracy, without requiring huge computational resources.

An edge-based road detection algorithm was presented by [5]-[8], it could work nicely in well-painted roads even under shadowy condition, but it will fail for the unpainted roads.

An approach by combining the Hough transform and the Line-Snake model was presented by [9], it divided an image into a few sub-regions along the vertical direction. The Hough transform was then performed for each subregion to obtain an initial position estimation of the lane boundaries. Afterwards, the Line-Snake improved the initial approximation to an accurate configuration of the lane boundaries. This approach suffers from two problems. One is, in the case of broken lane markings, it may not extend all the ways to the upper of the image. Another is, the contrast of one (or both) of the lane edges may not be high enough to detect near the bottom of the image.

In [10][11], an approach of detecting lane boundary, especially for the country roads, by artificial vision was described. It used statistical criteria, i.e. energy, homogeneity, contrast, etc., to distinguish between the roads and the non-roads. It combined the random searching with the chi-square fitting to obtain the best set of parameters of a deformable template. However, they used the same road model as [3][4].

Here, we present a novel B-snake lane model, its initialization by CHEVP, and its iteration by a Minimum Mean Square Energy (MMSE). Details were given in the following sections.

## 3. ROAD MODEL

### 3.1. The Modeling of Lane Boundaries

In this paper, we focus on constructing the 2-D lane model, by assuming that the two sides of the road boundaries are parallel on the ground plane.

In addition, let's assume that the right side of road is the shifted version of the left side of road at a horizontal distance, $D=\left(x_{r}-x_{l}\right)$, along the $x$ (horizontal) axis in the ground plane. Here, $x_{r}$ and $x_{l}$ are the $x$ coordinates
of the two correspondence points, $P_{l}\left(x_{l}, y\right)$ and $P_{r}\left(x_{r}, y\right)$, in the ground plane. After projection from the ground plane to the image plane, the horizontal distance $d=\left(c_{r}-c_{l}\right)$ between the corresponding points $p_{l}\left(c_{l}, r\right)$ and $p_{r}\left(c_{r}, r\right)$ in the image plane, which are the projected points of $P_{l}\left(x_{l}, y\right)$ and $P_{r}\left(x_{r}, y\right)$, is:

$$
d=k(r-h z)
$$

where $k=\frac{\lambda^{2} D}{H\left(\lambda^{2}+h z^{2}\right)}, \lambda$ is the focal length of the lens, $H$ is the height of the camera location, $h z$ is the position of vanish line in the image pane, and $r$ is the vertical coordinate used in the image plane.

Let's define the mid-line of the road in the image plane as

$$
L_{m i d}=\left(c_{m}, r_{m}\right)
$$

Thus the left side of the modeled road is

$$
L_{l e f t}=\left(c_{1}, r_{l}\right)
$$

where

$$
c_{l}=c_{m}-\frac{1}{2} d=c_{m}-\frac{1}{2} k\left(r_{l}-h z\right) \text { and } r_{l}=r_{m}
$$

Similarly, the right side of the modeled road is

$$
L_{r i g h t}=\left(c_{r}, r_{r}\right)
$$

where

$$
c_{r}=c_{m}+\frac{1}{2} d=c_{m}+\frac{1}{2} k\left(r_{r}-h z\right) \text { and } r_{r}=r_{m}
$$

From the above modeling, it is easy to observe that the problem of detecting two sides of road can be merged as the problem of detecting the mid-line of road.

### 3.2. B-spline Snake

Snakes [12], or active contours, are curves defined within an image domain which can move under the influence of internal forces from the curve itself and external forces from the image data. Once internal and external forces have been defined, the snake can detect the desired object boundaries (or other object features) within an image.

A more economical realization of snake can be reached by using far fewer state variables by cubic Bsplines. B-splines can represent curves by four or more state variables (control points). As required, the represented curves may be open or closed. The flexibility of the curve increases as more control points are added. Each additional control point either allows one more inflection in the curve or, when multiple knots are used [13], reduces continuity at one point.

### 3.2.1. Uniform Cubic B-Splines

The B-splines are piecewise polynomial functions that provide local approximations to contours using a small
number of parameters (control points).
In this paper, we deal with the open curves that are $C^{2}$ continuous, have both their continuous slopes and curvatures, and are modeled by cubic B -splines. Figure 1 shows a cubic B-spline.


Figure 1. Cubic B-spline curve.
An open cubic B-spline, with $n+1$ control points $\left\{Q_{0}, Q_{1}, \cdots, Q_{n}\right\}$, consists of $(n-2)$ connected curve segments, $g_{i}(s)=\left(r_{i}(s), c_{i}(s)\right), \quad i=1,2, \ldots,(n-2)$. Each curve segment is a linear combination of four cubic polynomials by the parameter $S$, where $S$ is normalized between 0 and $1(0 \leq s \leq 1)$, it can be expressed as:

$$
\begin{align*}
s_{i}(s) & =X_{0}(s) Q_{i-1}+X_{1}(s) Q_{i}+X_{2}(s) Q_{i+1}+X_{3}(s) Q_{i+2} \\
& =\left[\begin{array}{llll}
s^{3} & s^{2} & s & 1
\end{array}\right]\left[\begin{array}{cccc}
-\frac{1}{6} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{6} \\
\frac{1}{2} & -1 & \frac{1}{2} & 0 \\
\frac{1}{2} & -1 & \frac{1}{2} & 0 \\
\frac{1}{6} & \frac{2}{3} & \frac{1}{6} & 0
\end{array}\right]\left[\begin{array}{c}
Q_{i-1} \\
Q_{i} \\
Q_{i+1} \\
Q_{i+2}
\end{array}\right]  \tag{1}\\
& =M_{R}(s)\left[\begin{array}{c}
Q_{i-1} \\
Q_{i} \\
Q_{i+1} \\
Q_{i+2}
\end{array}\right] . \quad i=1,2, \cdots, n-2
\end{align*}
$$

where

$$
M_{R}(s)=\left[\begin{array}{llll}
s^{3} & s^{2} & s & 1
\end{array}\left[\begin{array}{cccc}
-\frac{1}{6} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{6}  \tag{2}\\
\frac{1}{2} & -1 & \frac{1}{2} & 0 \\
-\frac{1}{2} & -1 & \frac{1}{2} & 0 \\
\frac{1}{6} & \frac{2}{3} & \frac{1}{6} & 0
\end{array}\right] .\right.
$$

### 3.2.2. Relationship Between Knot Points and Control Points

The points connecting the neighboring segments (Figure 1) are called the knot points $P_{i}(i=1.2, \ldots, n-1)$, where the B -spline bases are tied together. Given the set of knot points $P=\left(R_{1}, P_{2} \ldots P_{n-1}\right)$ of a uniform cubic B-spline curve (Figure 1), we can uniquely determine control points $Q=\left(Q_{0}, Q_{1}, \ldots, Q_{n}\right)$, by substituting $s=0$ into equation (1). The relationship between the control points and the connection points is given as:

$$
\begin{equation*}
P_{i}=\frac{1}{6} Q_{i-1}+\frac{2}{3} Q_{i}+\frac{1}{6} Q_{i+1}, \quad i=1,2, \ldots, n-1 \tag{3}
\end{equation*}
$$

### 3.3. Using B-Snake to Describe Lane Markings (or Boundaries)

We use a set of control points to describe the mid-line of the road by B -spline, and a additional parameter $k$ (as described in section 3.1) to determine the left and the right sides of road model. In order to make B-splines pass through the first and the last control points, we set the first three control points equal and the last three control points equal. The mid-line of road model can be expressed by a $B$-spline as

$$
L_{m i d}=\left(c_{m}, r_{m}\right)
$$

$$
=M_{R}(s)\left[\begin{array}{c}
Q_{i-1} \\
Q_{i} \\
Q_{i+1} \\
Q_{i+2}
\end{array}\right], \quad i=-1,0,1,2, \cdots, n .
$$

The mid-line of lane model can be deformed by the external forces $E_{\mathrm{M} \text { _sum }}(s)$, which is the sum of the dual external forces calculated from the left and the right sides of lane model, $E_{L}(s)$ and $E_{R}(s)$.

$$
E_{\mathrm{M}_{-} \text {sum }}(s)=E_{L}(s)+E_{R}(s) .
$$

Also, the difference of horizontal components of $E_{L}(s)$ and $E_{R}(s)$, denoted as $E_{\mathrm{M}_{-} \text {dif }}^{c}(s)$, would lead to adjustment of the parameter $k$.

$$
E_{\mathrm{M} \text { dif }}^{c}(s)=E_{L}^{c}(s)-E_{R}^{c}(s)
$$

There are two advantages for using dual external forces to deform the B-snake model: First, the processing time will be reduced since two deformation problems have been formulated to one deformation problem; Second, the B -snake model would be robust against shadows, noises, etc., since the knowledge of parallel lines on the ground plane has been used.


Figure 2. B-snake based lane model.
For most lanes, we found that using 3 control points is
efficient to describe their shapes. Therefore, we select 3 control points in this paper for constructing the lane model. Figure 2 shows a lane model formed by a set of 3 control points, $Q_{0}, Q_{1}$ and $Q_{2}$.

## 4. Initialization of B-Snake Lane Model: CHEVP Algorithm

The CHEVP (Canny/Hough Estimation of Vanishing Points) algorithm has been developed to initialize the Bsnake. The road is assumed to have two parallel boundaries on the ground, and in the short horizontal band of image, the road is approximately straight. As a result of the perspective projection, the road boundaries in the image plane should intersect at a shared vanishing point on the horizon. There are following five processing stages.
(1) Edge pixel extraction by Canny edge detection.
(2) Straight Lines Detection by Hough Transform.
(3) Horizon and Vanishing Points Detection.
(4) Estimate the mid-line of road and the parameter $k$ by the detected road lines.
(5) Initial the control points of the lane model to approach the mid-line detected by last step.
The CHEVP algorithm has been applied to more than 50 images. Some results are shown in Figure 3.

## 5. B-Snake Parameters Updated from Image Data

Based on the initial location of the control points that are determined either by CHEVP algorithm or lane detection result of previous frame, the B-snake would further approach to road edge accurately in the current frame. This section deals with this problem.

### 5.1. Minimum Mean Square Energy Approach

The advantage of using B -snake is that internal forces are not required, since the $B$-snake representation maintains smoothness via hard constraints in the representation.

B-snake should be updated to minimize (1) the sum of the external forces from the both sides of the road model for achieving accurate position of B-snake, and (2) the difference of the external forces from the both sides of the road model for achieving suitable parameter $k$. In addition, external forces should be transmitted to each control point when updating B-snake. Details are described as follows.

When the B-snake approaches the road boundaries, its external force should satisfy the equation.

$$
\begin{equation*}
E_{e x t}=0 \tag{4}
\end{equation*}
$$



Figure 3. Results of CHEVP Algorithm.
where

$$
E_{e x t}=E_{\mathrm{M}_{-} \text {sum }}(s)=E_{L}(s)+E_{R}(s)
$$

If external force of the B-snake is zero, then there is no change in both the position and the shape of the midline of road. So we can define the following equation for solving the requirement of external force being zero.

$$
\begin{align*}
E_{\text {ext }} & =\gamma\left(L_{\text {mid }}(t)-L_{\text {mid }}(t-1)\right) \\
& =\gamma M_{R}(s)(Q(t)-Q(t-1))  \tag{5}\\
& =M_{R}(s) \Delta Q(t)
\end{align*}
$$

where $\gamma$ is a step-size and $\Delta Q(t)$ is defined as the adjustment of the control points $Q$ in each iteration step.

$$
\begin{equation*}
Q(t)=Q(t-1)+\Delta Q(t) \tag{6}
\end{equation*}
$$

External force can be sampled along the B-spline of B-snake at a certain distance. Then equation (5) can be solved digitally. Here, the Minimum Mean Square Energy solution for the digital version of the equation (5) is given as a matrix form.

$$
\begin{equation*}
\Delta Q(t)=\gamma^{-1}\left[M^{T} M\right]^{-1} M^{T} E_{e x t} \tag{7}
\end{equation*}
$$

where

$$
\begin{gathered}
M=\left[\begin{array}{ccccc}
M_{-1} & 0 & \cdots & \cdots & 0 \\
0 & M_{0} & 0 & \cdots & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
0 & \cdots & 0 & M_{n-1} & 0 \\
0 & \cdots & \cdots & 0 & M_{n}
\end{array}\right], \\
M_{i}=\left[\begin{array}{cccc}
s_{i, 1}^{3} & s_{i, 1}^{2} & s_{i, 1} & 1 \\
s_{i, 2}^{3} & s_{i, 2}^{2} & s_{i, 2} & 1 \\
\cdot & \cdot & \cdot & \cdot \\
s_{i, m}^{3} & s_{i, m}^{2} & s_{i, m} & 1
\end{array}\right]\left[\begin{array}{cccc}
-\frac{1}{6} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{6} \\
\frac{1}{2} & -1 & \frac{1}{2} & 0 \\
-\frac{1}{2} & -1 & \frac{1}{2} & 0 \\
\frac{1}{6} & \frac{2}{3} & \frac{1}{6} & 0
\end{array}\right],
\end{gathered}
$$

and $m$ is the sampling points number in $i$ th segment of the B-spline. $E_{e x t}$ is the force vector digitized on the Bspline. Here, $n=2$ is for the case of using three control points.

The difference of the external forces from the left and the right side of lane model would lead to changing the parameter $k$ (as given in section 3.1). Estimation of the parameter $k$ can be similarly given as follows.

$$
\begin{gather*}
E_{k}=E_{\mathrm{M}_{-} \mathrm{dif}}^{c}=\left(E_{L}^{c}(s)-E_{R}^{c}(s)\right)  \tag{8}\\
E_{k}=\tau(k(t)-k(t-1))=\tau \Delta k(t)  \tag{9}\\
k(t)=k(t-1)+\Delta k(t) \tag{10}
\end{gather*}
$$

where $\tau$ is a step-size for $k$. Thus,

$$
\begin{equation*}
\Delta k(t)=E_{k} / \tau \tag{11}
\end{equation*}
$$

Here we choose Gradient Vector Flow (GVF) [14] as the external force for B -snake to perform the lane detection, since GVF has a larger capture range.

### 5.2. Application in Lane Detection

In order to achieve the solutions in equations (6) and (10), an iterative procedure is adopted. The steps contained in this iterative minimization process are as follows:

1. Initialization Step. Initialize the control point parameters by CHEVP algorithm introduced in section 4.
2. Calculate the GVF of the edge road image as the external force of B-snake.
3. Calculate MMSE in equations (7) and (11) for obtaining $\Delta Q(t)$ and $\Delta k(t)$, respectively.
4. Obtain $Q(t)$ and $k(t)$.
5. If $\|\Delta Q(t)\|>$ threshold $^{1}$ and $\|\Delta k(t)\|>$ threshold $^{2}$, then set $Q(t)$ to $Q(t-1)$ and $k(t)$ to $k(t-1)$, and go to step 3; Otherwise, go to step 6.
6. Stop. The last estimations of $Q(t)$ and $k(t)$ are regarded as the solutions of MMSE.

Application of the MMSE approach to real image is shown as follows. We take Figure 4 as an example. Although CHEVP algorithm can provide a quite good initialization for B-snake lane model (as shown in Figure 4(b)), in order to show the robustness of our algorithm, the initialization for B-snake is set far from the CHEVP result, as shown in Figure 4 (c). The convergence procedure of the B-snake lane takes 55 iterations. The final result of MMSE approach is shown in Figure 4(d). It can be seen that the $B$-snake lane model approaches to the lane boundaries accurately.

gure 4. An Example of MMSE approach.

### 5.3. Lane Detection Results

This lane detection algorithm has been simulated by Matlab codes and tested on more than 50 images grabbed by a camera at different locations and at different times. These lane images include curve and straight road, with or without shadows and lane marks. Some of these results are shown in Figure 5.


Figure 5. Lane detection results.

## 6. Conclusion

A novel B-snake based lane model, that describes the perspective effect of parallel lines, is established for generic lane boundaries (or markings). It is able to describe a wider range of lane structures than other lane models, such as straight and parabolic models.

A robust algorithm, called CHEVP, is presented for initializing the B-snake lane model. This algorithm is robust against noise, shadows, and illumination variations in the captured road images, and is also applicable to both the marked and the unmarked, and dash and solid paint line roads.

A minimum energy method, MMSE (Minimum Mean Square Energy), that measures the matching degree between the model and the real edge map is presented to determine the control points of road model for lane detection by iteration. The obtained results are quite good and accurate even under shadow conditions.

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