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Large Hierarchies from Approximate *R* Symmetries

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We show that hierarchically small vacuum expectation values of the superpotential in supersymmetric theories can be a consequence of an approximate R symmetry. We briefly discuss the role of such small constants in moduli stabilization and understanding the huge hierarchy between the Planck and electroweak scales.

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Introduction.—One of the major puzzles in contemporary physics is the existence of large hierarchies in nature, such as the ratio between the Planck and electroweak scales $M_P/m_W \sim 10^{17}$. Some of the most promising explanations of such hierarchies rely on dimensional transmutation. Here the dynamical scale $\Lambda = M_P e^{-a/g^2}$ (with g and a denoting the gauge coupling and a constant, respectively) can be naturally much smaller than the fundamental scale. However, if one is to embed this mechanism in a more fundamental framework, one often encounters the problem that there has to be a hierarchically small quantity right from the start. Concretely, if one is to make use of the dynamical scale in string theory, one has first to fix the modulus that determines the coupling strength. This in turn often requires the introduction of a small constant. One faces then the well-known "chicken-or-egg problem."

Motivated by results obtained in the framework of string theory model building, we present here a potential resolution of the problem. We shall show that, if the superpotential in a supersymmetric theory exhibits an approximate $\mathrm{U}(1)_R$ symmetry, it generically acquires a suppressed vacuum expectation value (VEV). Such accidental $\mathrm{U}(1)_R$ symmetries, which get broken at higher orders, are naturally present in string compactifications. They arise as remnants from exact, discrete R symmetries. Such symmetries allow us to control the VEV of the (perturbative) superpotential and, in particular, to avoid deep anti–de Sitter vacua. We will discuss the role of the resulting hierarchically small superpotential VEVs in the context of moduli stabilization in string theory, for giving a plausible explanation of the huge hierarchy between M_P and m_W, and for providing, in the context of a class of promising string models [1], a solution to the μ problem of the minimal supersymmetric standard model (MSSM).

Supersymmetric Minkowski vacua as a consequence of a $\mathrm{U}(1)_R$ *symmetry.*—Consider a superpotential of the form

$$\mathcal{W} = \sum c_{n_1\cdots n_M}\phi_1^{n_1}\cdots\phi_M^{n_M}. \tag{1}$$

Here and in the following we work in Planck units; i.e., we set $M_P = 1$ unless stated differently. Assume that $\mathcal{W}$ has an exact R symmetry, under which $\mathcal{W}$ has R charge 2,

$$\mathcal{W} \to e^{2i\alpha}\mathcal{W}, \tag{2}$$

and the fields transform as

$$\phi_j \to \phi_j' = e^{ir_j\alpha}\phi_j \tag{3}$$

such that each monomial in (1) has total R charge 2.

Let $\langle\phi_i\rangle$ denote a field configuration which solves the F-term equations,

$$F_i = \frac{\partial\mathcal{W}}{\partial\phi_i} = 0 \quad \text{at } \phi_j = \langle\phi_j\rangle \quad \forall\ i, j. \tag{4}$$

Consider now an infinitesimal $\mathrm{U}(1)_R$ transformation,

$$\mathcal{W}(\phi_i) \to \mathcal{W}(\phi_i') = \mathcal{W}(\phi_i) + \sum_j \frac{\partial\mathcal{W}}{\partial\phi_j}\Delta\phi_j. \tag{5}$$

At $\phi_j = \langle\phi_j\rangle$ the superpotential goes into itself, which can only be consistent with (2) if $\mathcal{W} = 0$ at $\phi_j = \langle\phi_j\rangle$. This proves that, if the F equations are satisfied, $\mathcal{W}$ vanishes.

A few comments are in order. First, this statement holds regardless of whether the configuration $\langle\phi_i\rangle$ preserves $\mathrm{U}(1)_R$ or breaks it spontaneously. Second, in the context of supergravity, the statements above imply that the $D_i\mathcal{W}$ vanish for $\phi_i = \langle\phi_i\rangle$; i.e., also the supergravity F terms vanish and one obtains a supersymmetric Minkowski vacuum. Third, our findings are related to an observation by Nelson and Seiberg made in [2], where it is stated that, in order to have a theory without supersymmetric ground state, the superpotential has to exhibit a continuous R symmetry. The statements do, however, not tell us whether or not a theory with a superpotential exhibiting a continuous R symmetry has a supersymmetric ground state or not. Our findings and [2] imply that, if there is a continuous R symmetry, there are two options: (i) there is a supersymmetric ground state with $\mathcal{W} = 0$ [with $\mathrm{U}(1)_R$ spontaneously broken or unbroken]; (ii) there is no supersymmetric ground state, and in the ground state $U(1)_R$ is spontaneously broken [2].

In this Letter we focus on case (i). If the U(1) that acts on the scalar components of the superfields gets spontaneously broken at $\phi_i = \langle\phi_i\rangle$ (which is the case if, for instance, all $\langle\phi_i\rangle$ are nontrivial), it follows then from Goldstone's theorem that there is a massless mode, the so-called R axion.

Small constants from approximate $U(1)_R$ *symmetries.*—Let us now study what happens if the R symmetry is "slightly" broken, i.e., by higher order terms. We can write the superpotential as

$$\mathcal{W}(\phi_i) = \mathcal{W}_0(\phi_i) + \sum_j \mathcal{W}_j(\phi_i), \tag{6}$$

where $\mathcal{W}_0(\phi_i)$ consists of monomials up to order $N-1$ which preserve the R symmetry while the $\mathcal{W}_j(\phi_i)$ are monomials of order $\geq N$ which break the R symmetry. This means that the superpotential transforms under $U(1)_R$ as

$$\mathcal{W}(\phi_i) \to e^{2i\alpha}\mathcal{W}_0(\phi_i) + \sum_j e^{i\alpha R_j}\mathcal{W}_j(\phi_i) \simeq \mathcal{W}(\phi_i) + i\alpha\Big(2\mathcal{W}_0(\phi_i) + \sum_j R_j\mathcal{W}_j(\phi_i)\Big) \tag{7}$$

with $R_j \neq 2$, and

$$\mathcal{W}(\phi_i) \to \mathcal{W}(e^{i\alpha r_i}\phi_i) \simeq \mathcal{W}(\phi_i) + i\alpha\sum_j \frac{\partial\mathcal{W}}{\partial\phi_j} r_j\phi_j. \tag{8}$$

Combining these two expressions and assuming that the F terms vanish in our vacuum, $\frac{\partial\mathcal{W}}{\partial\phi_i} = 0$, we see that

$$\langle\mathcal{W}(\phi_i)\rangle = -\frac{1}{2}\sum_j (R_j - 2)\langle\mathcal{W}_j(\phi_i)\rangle. \tag{9}$$

This means that in the case of an approximate $U(1)_R$ symmetry one obtains suppressed superpotential VEVs, written symbolically as

$$\langle\mathcal{W}\rangle \sim \langle\phi\rangle^{\geq N}. \tag{10}$$

In many situations there is a mild hierarchy between the fundamental scale and a typical VEV, $\langle\phi\rangle/M_P < 1$. This is, for instance, the case in string models where a U(1) factor appears "anomalous", and where the one-loop Fayet-Iliopoulos term forces some VEVs to be roughly 1 order of magnitude smaller than M_P [3]. According to the above discussion, the suppression of $\langle\mathcal{W}\rangle$ gets then enhanced by the Nth power of this mild hierarchy, similarly to the Froggatt-Nielsen picture [4].

Further, we have seen that there might be a Goldstone mode η. With explicit $U(1)_R$ breaking, it will generically receive a mass, $m_\eta \sim \langle\phi\rangle^{\geq N-2}$. (The "$-2$" comes from the second derivative.) In supergravity theories, $\langle\mathcal{W}\rangle$ sets the gravitino mass,

$$m_{3/2} \simeq \langle\mathcal{W}\rangle. \tag{11}$$

This leads then to the expectation that there is a mode whose (supersymmetric) mass scales like $m_{3/2}$,

$$m_\eta \sim \frac{m_{3/2}}{\langle\phi\rangle^2}. \tag{12}$$

Let us comment that, if one is to include supergravity effects, $\mathcal{W} \neq 0$ does not necessarily imply anti–de Sitter solutions (see, e.g., the discussion in [5], section 4).

Explicit string theory realization.—One of the central themes of string theory is the issue of moduli stabilization, which is closely connected to the question of supersymmetry breaking. In the traditional approach, supersymmetry is broken by dimensional transmutation [6], e.g., by gaugino condensation [7]. However, for this elegant mechanism to work, one needs first to fix the gauge coupling, whose strength is given by the VEV of the dilaton S or another modulus in string theory. This can be achieved in various ways: for instance, in the race-track scheme [8] one has two competing nonperturbative superpotentials which provide a nontrivial minimum of the dilaton potential. The drawback of this mechanism is that it only works if one has two rather large "hidden" gauge groups with rather special matter contents. A somewhat more economic scheme is that of Kähler stabilization [9,10] where one needs only one hidden sector. However, in the relevant regime where dilaton stabilization may be achieved, the theory is not calculable. More recently, an alternative has been studied (with the most prominent example being that of KKLT [11]) where the superpotential is of the form

$$\mathcal{W} = c + Ae^{-aS}. \tag{13}$$

The first term c is a constant and the second term reflects hidden sector strong dynamics; i.e., S is related to the gauge coupling, $\mathrm{Re}S \propto 1/g^2$, and a is related to the β-function of the hidden gauge group. In the KKLT setup, the constant comes from fluxes. The minimum of the scalar potential for S occurs at a point where

$$|aSAe^{-aS}| \sim |c|. \tag{14}$$

The VEV of $\mathcal{W}$, i.e., the gravitino mass, is of the same order. In order to have MSSM superpartner masses at the TeV scale, the gravitino mass cannot exceed $\mathcal{O}(100)$ TeV, hence

$$|c| \lesssim 10^{-12} \tag{15}$$

in Planck units. The small scale in this setting is therefore *not* explained by dimensional transmutation but originates from the smallness of the constant c. KKLT and others argue that, due to the large number of vacua, some of them might have such c by accident.

In what follows, we will exploit the observation of the previous section that small VEVs of the (perturbative) superpotential can be explained by an approximate $U(1)_R$ symmetry. We will use this in order to discuss moduli stabilization in the context of the heterotic string. We focus on orbifold compactifications [12] since they possess many (and well-understood) discrete symmetries, which, as it turns out, imply approximate $U(1)_R$ symmetries of the superpotentials describing the effective field theories derived from these constructions. As we shall see, superpotential VEVs of the order $10^{-\mathcal{O}(10)}$ can naturally be obtained. Orbifold compactifications allow us to embed the MSSM into string theory [1,13,14].

In our calculations we focus on the models of the "heterotic MiniLandscape" [1,15]. These models exhibit the standard model gauge group and the chiral matter content of the MSSM. They are based on the $\mathbb{Z}_6$-II orbifold with three factorizable tori (see [13,16] for details). The discrete symmetry of the geometry leads to a large number of discrete symmetries governing the couplings of the effective field theory [17,18] (cf. also [13,16,19]). Apart from various bosonic discrete symmetries, one has a

$$[\mathbb{Z}_6 \times \mathbb{Z}_3 \times \mathbb{Z}_2]_R \tag{16}$$

symmetry; other orbifolds have similar discrete symmetries. Further, in almost all of the MiniLandscape models there is, at one-loop, a Fayet-Iliopoulos (FI) D term,

$$V_D \supset g^2\Big(\sum_i q_i|\phi_i|^2 + \xi\Big)^2, \tag{17}$$

where the q_i denote the charges under the so-called "anomalous U(1)." It turns out that, in all models with nonvanishing FI term, ξ is of order 0.1 (see [13] for an explicit example). The first step of our analysis is to identify a set of standard model singlets ϕ_i with the following properties: giving VEVs to the ϕ_i allows us to cancel the FI term; there is no other field that is singlet under the gauge symmetries left unbroken by the ϕ_i VEVs. These properties ensure that the $\langle\phi_i\rangle$ can be consistent with a vanishing D-term potential and that the F-terms of all other massless modes vanish, implying that it is sufficient to derive the superpotential terms involving only the ϕ_i fields. A crucial property of these superpotentials is that they exhibit accidental $U(1)_R$ symmetries that get only broken at rather high orders N. As discussed, this can be regarded as a consequence of high-power discrete R symmetries [Eq. (16)]. N depends on the chosen ϕ_i configuration; as a general rule we find that the more ϕ_i fields are considered, the lower N values emerge. For instance, in a model where only seven fields are considered, we obtain $N = 26$, on the other hand, in the model 1 of [1] with 23 fields switched on, $U(1)_R$ gets broken at order 9.

Given nontrivial solutions to the F-term equations,

$$\phi_i \frac{\partial \mathcal{W}}{\partial \phi_i} = 0, \quad \text{with} \quad \phi_i \neq 0, \tag{18}$$

one can use complexified gauge transformations to ensure vanishing D terms as well [20]. Although D-term constraints do not fix the scale of the $\langle\phi_i\rangle$ in general, the requirement to cancel the FI term introduces the scale $\sqrt{\xi} \sim 0.3$ into the problem. We search for solutions of $V_D = V_F = 0$ in the regime $|\phi_i| < 1$, and find that they exist. We explicitly verify that for such solutions the superpotential is hierarchically small, $\langle\mathcal{W}\rangle \sim \langle\phi\rangle^N$, where $\langle\phi\rangle$ denotes the typical size of a VEV. A very important property of many of these configurations is that all fields acquire (supersymmetric) masses. Hereby, typically only one field has a mass of the order m_η [see Eq. (12)] while the others are much heavier. We have also checked that these features are robust under supergravity corrections.

Altogether, we find that in the models under consideration one obtains isolated supersymmetric field configurations with $|\phi_i| < 1$ where the VEV of the perturbative superpotential $\langle\mathcal{W}\rangle$ is hierarchically small.

Before discussing applications, let us compare our findings to other recent results [21]. There, using the stringy selection rules, so-called "maximal vacua" were constructed in which the superpotential vanishes term by term (and to all orders). In our approach, each superpotential term composed out of ϕ_i fields acquires a nontrivial VEV, but to the order at which the accidental $U(1)_R$ is exact, all terms cancel nontrivially. At higher orders, a nontrivial VEV of $\mathcal{W}$ gets induced.

Let us now briefly sketch how this can be used in order to stabilize the dilaton, whose VEV determines the gauge coupling. After integrating out the ϕ_i fields, one is left with a superpotential of the form (13),

$$\mathcal{W}_{\text{eff}} = c + Ae^{-aS}, \tag{19}$$

where $c = \langle\mathcal{W}\rangle = 10^{-\mathcal{O}(10)}$, and Ae^{-aS} describes some nonperturbative dynamics, such as gaugino condensation [7,22–24]. As we have discussed before in Eq. (14), this superpotential leads to a nontrivial minimum for the dilaton. In the MiniLandscape models, realistic gauge couplings are correlated with favorable sizes of the dynamical scale, $Ae^{-aS}/M_P^2 \sim$ TeV [25]. Hence, for typical expectation values $\langle\mathcal{W}\rangle = 10^{-\mathcal{O}(10)}$ one obtains reasonable gauge couplings. The fixing of the T moduli and other issues such as "uplifting" will be studied elsewhere.

Another application of our findings concerns the μ term of the MSSM. In [26] it has been proposed that in models in which the field combination $h_u h_d$ (with h_u and h_d denoting the up-type and down-type Higgs fields, respectively) is completely neutral with respect to all symmetries, there is an interesting relation between the Higgs mass coefficient μ and $\langle \mathcal{W} \rangle$,

$$\mu \sim \langle \mathcal{W} \rangle. \tag{20}$$

The heterotic MiniLandscape [15] contains many models in which the Higgs pair (and only the Higgs pair) has this property. Apart from the above property, such models exhibit "gauge-top unification"; i.e., the top Yukawa coupling is of the order of the gauge coupling, as well as many other desirable properties. In a concrete example, the benchmark model 1A of [1], it was found that solving the F-term equations for the superpotential up to order 6 always leads to $\langle \mathcal{W} \rangle = 0$. We have now obtained a better understanding of this fact: there is a $\mathrm{U}(1)_R$ symmetry that holds up to order 11, explaining this property. It is amazing to see that these models, constructed in order to reproduce the MSSM spectrum and gauge interactions, exhibit so many appealing properties automatically.

Conclusions.—We have shown that approximate $\mathrm{U}(1)_R$ symmetries can explain the appearance of hierarchically small constants. We find that at configurations where the F-term equations are solved, the superpotential goes like $\langle \mathcal{W} \rangle \sim \langle \phi \rangle^N$ with $\langle \phi \rangle$ denoting a typical expectation value and N being the order at which $\mathrm{U}(1)_R$ gets broken. We have analyzed various heterotic orbifold models and found that there, due to the presence of high-power discrete R symmetries, approximate $\mathrm{U}(1)_R$s are generic. We have explicitly solved the F-term equations in several models, thus obtaining points in field space in which the F- and D-term potentials vanish, and confirmed that, for $|\phi_i| < 1$, the superpotential is hierarchically small. We have argued that such suppressed superpotential expectation values can be the origin for the appearance of large hierarchies in nature: they fix the scale of the gravitino mass, which in schemes with low-energy supersymmetry sets the weak scale, and can be used to stabilize the string theory moduli at realistic values.

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[1] O. Lebedev *et al.*, Phys. Rev. D **77**, 046013 (2008).
[2] A. E. Nelson and N. Seiberg, Nucl. Phys. **B416**, 46 (1994).
[3] M. Dine, N. Seiberg, and E. Witten, Nucl. Phys. **B289**, 589 (1987).
[4] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. **B147**, 277 (1979).
[5] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).
[6] E. Witten, Nucl. Phys. **B188**, 513 (1981).
[7] H. P. Nilles, Phys. Lett. B **115**, 193 (1982).
[8] N. V. Krasnikov, Phys. Lett. B **193**, 37 (1987).
[9] J. A. Casas, Phys. Lett. B **384**, 103 (1996).
[10] P. Binétruy, M. K. Gaillard, and Y. Y. Wu, Nucl. Phys. **B481**, 109 (1996).
[11] S. Kachru, R. Kallosh, A. Linde, and S. P. Trivedi, Phys. Rev. D **68**, 046005 (2003).
[12] L. J. Dixon *et al.*, Nucl. Phys. **B261**, 678 (1985).
[13] W. Buchmüller *et al.*, Nucl. Phys. **B785**, 149 (2007).
[14] W. Buchmüller *et al.*, Phys. Rev. Lett. **96**, 121602 (2006).
[15] O. Lebedev *et al.*, Phys. Lett. B **645**, 88 (2007).
[16] T. Kobayashi, S. Raby, and R. J. Zhang, Nucl. Phys. **B704**, 3 (2005).
[17] S. Hamidi and C. Vafa, Nucl. Phys. **B279**, 465 (1987).
[18] L. J. Dixon *et al.*, Nucl. Phys. **B282**, 13 (1987).
[19] T. Kobayashi *et al.*, Nucl. Phys. **B768**, 135 (2007).
[20] B. A. Ovrut and J. Wess, Phys. Rev. D **25**, 409 (1982).
[21] W. Buchmüller and J. Schmidt, Nucl. Phys. **B807**, 265 (2009).
[22] S. Ferrara, L. Girardello, and H. P. Nilles, Phys. Lett. B **125**, 457 (1983).
[23] J. P. Derendinger, L. E. Ibáñez, and H. P. Nilles, Phys. Lett. B **155**, 65 (1985).
[24] M. Dine *et al.*, Phys. Lett. B **156**, 55 (1985).
[25] O. Lebedev *et al.*, Phys. Rev. Lett. **98**, 181602 (2007).
[26] J. A. Casas and C. Muñoz, Phys. Lett. B **306**, 288 (1993).