

Large MIMO Systems

Large MIMO systems, with tens to hundreds of antennas, are a promising emerging communication technology. This book provides a unique overview of this technology, covering the opportunities, engineering challenges, solutions, and state-of-the-art of large MIMO test beds. There is in-depth coverage of algorithms for large MIMO signal processing, based on metaheuristics, belief propagation, and Monte Carlo sampling techniques, and suited for large MIMO signal detection, precoding, and LDPC code designs. The book also covers the training requirement and channel estimation approaches in large-scale point-to-point and multiuser MIMO systems; spatial modulation is also included. Issues like pilot contamination and base station cooperation in multicell operation are addressed. A detailed exposition of MIMO channel models, large MIMO channel sounding measurements in the past and present, and large MIMO test beds is also presented. An ideal resource for academic researchers, next generation wireless system designers and developers, and practitioners in wireless communications.

A. CHOCKALINGAM is a Professor in the Department of Electrical Communication Engineering, Indian Institute of Science (IISc), Bangalore, India. He has made pioneering contributions in the area of low complexity near-optimal signal detection in large MIMO systems. He is a recipient of the Swarnajayanti Fellowship from the Department of Science and Technology, Government of India, and a Fellow of the Indian National Academy of Engineering (INAE), the National Academy of Sciences, India (NASI), and the Indian National Science Academy (INSA).

B. SUNDAR RAJAN is a Professor in the Department of Electrical Communication Engineering, Indian Institute of Science (IISc), Bangalore, India. He is a well-known authority in the area of space-time coding for MIMO channels and distributed space-time coding, and a leading expert in the design of space-time codes based on algebraic techniques. He is a recipient of the Professor Rustum Choksi Award from IISc for excellence in research in engineering, and a Fellow of the Indian National Academy of Engineering (INAE), the National Academy of Sciences, India (NASI), the Indian National Science Academy (INSA), and the Indian Academy of Sciences (IASc).

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“This cutting-edge portrayal of large-scale MIMO systems provides a shrewd long-term outlook on this salient wireless subject.”

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University of Southampton

“This is a very timely and useful book written by authors who are pioneers in the area of large MIMO systems.”

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The University of British Columbia

“Large MIMO will power our wireless networks before this decade is out and the race is just starting. Chockalingam and Sundar Rajan have compiled an excellent companion for this journey.”

Arogyaswami Paulraj
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A. CHOCKALINGAM AND B. SUNDAR RAJAN

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To our teachers and students

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Preface

The physical layer capabilities in wireless transmissions are growing. In particular, the growth trajectory of the achieved data transmission rates on wireless channels has followed Moore's law in the past decade and a half. Over a span of 15 years starting mid-1990s, the achieved wireless data transmission rates in several operational scenarios have increased over 1000 times. The data transmission rate in WiFi which was a mere 1 Mbps in 1996 (IEEE 802.11b) had reached 1 Gbps by 2011 (IEEE 802.11ac). During the same span of time, the data rate in cellular communication increased from about 10 kbps in 2G to more than 10 Mbps in 4G (LTE). One of the promising technologies behind such a sustained rate increase is multiantenna technology – more popularly referred to as the multiple-input multiple-output (MIMO) technology, whose beginnings date back to the late 1990s.

The interest shown in the study and implementation of MIMO systems stems from the promise of achieving high data rates as a result of exploiting independent spatial dimensions, without compromising on the bandwidth. Theory has predicted that the greater the number of antennas, the greater the rate increase without increasing bandwidth (in rich scattering environments). This is particularly attractive given that the wireless spectrum is a limited and expensive resource.

More than a decade of sustained research, implementation, and deployment efforts has given MIMO technology the much needed maturity to become commercially viable. More and more wireless products and standards have started adopting MIMO techniques, mainly in the small number of antennas regime (2–8 antennas). However, the promise of achieving very high spectral efficiencies using a much larger number of antennas still remains open to research and subsequent commercial exploitation. We call MIMO systems which achieve spectral efficiencies of tens to hundreds of bps/Hz using tens to hundreds of antennas “large MIMO systems.” This book is exclusively about large MIMO systems.

Large MIMO systems, by their very nature, merit special attention and treatment. For example, algorithms and techniques which are known to work well with a small number of antennas may not scale well for a large number of antennas. Therefore, newer and alternative approaches are needed. Also, in addition to increased rate and diversity gains, large dimensionality brings other advantages (e.g., channel hardening, which can be exploited to achieve low complexity signal

processing) which do not come with smaller systems. Bringing out such large MIMO centric opportunities, issues, and solution approaches and techniques is one of the key objectives in this book.

A few words about what motivated us to write this book are in order. Our teaching and research interest in space-time coding and multiuser detection in the early- to mid-2000s brought us together to collaborate on MIMO wireless research. Being in the same department and having offices in the same building helped – we could discuss ideas over casual chats during coffee/tea breaks and evening walks. Our first set of results on large MIMO systems were published in mid-2008. Since then, we have continued our research on various signal processing aspects in large MIMO systems, which has led to several of our subsequent publications on large MIMO. The large MIMO idea seems to have caught on, as we can see in the chapter on large MIMO testbeds (Chapter 12). Over these years, we have given tutorial talks on this topic to conferences and industry. We felt that, in the process, we had generated a critical mass of material, enough to write a book on large MIMO systems. Also, we found that a book written exclusively on large MIMO systems was yet to appear at the time of proposing this book to the publisher. We thank the publisher for having accepted our proposal for writing this book, and here we are with our intended book on large MIMO systems.

It is heartening to see that large MIMO systems have become more popular now compared to the days when we first started publishing on this topic in 2008. Large MIMO systems seem to have started to flourish under several names; large-scale MIMO, massive MIMO, hyper-MIMO, higher-order MIMO, to name a few. It is even more heartening to realize that large MIMO technology is one of the key technologies being considered for standardization in 5G and beyond.

We hope that this book will be of interest and use to researchers, graduate students, and wireless system designers and implementers, and will create the interest needed to take large MIMO research, development, and standardization activities to the next level.

Acknowledgments

We would first like to thank our graduate students for their valuable contributions to our large MIMO research. At a time when people started thinking that there is not much of interest left in MIMO research, they took on the challenges of exploring the uncharted area of MIMO systems with tens to hundreds of antennas. Thanks to their dedicated and sustained efforts, we were able to make some of the early contributions to the field of large MIMO systems. This book to a large extent draws on these contributions, and we thank all our students for their commitment, hard work, and help. Our many thanks are due to: K. Vishnu Vardhan, Saif K. Mohammed (currently an Assistant Professor at the Indian Institute of Technology, Delhi), Ahmed Zaki, N. Srinidhi, Suneel Madhekar, P. J. Thomas Sojan, Pritam Som, Tanumay Datta, N. Ashok Kumar, Suresh Chandrasekaran, Yogendra Umesh Itankar, P. M. Chandrakanth, M. Raghavendra Nath Reddy, Harsha Eshwaraiah, T. Lakshmi Narasimhan, Kamal Agarwal Singhal, Manish Mandloi, and Shovik Biswas.

Our research and teaching in multiuser detection and space-time coding had a positive influence on our understanding of and contribution to large MIMO research. We thank all the students who attended our courses on CDMA and multiuser detection, and space-time signal processing and coding. We also thank the students who contributed to our research in these areas. Parts of early drafts of this book were used in the CDMA and multiuser detection course. We thank the students for the valuable feedback on these drafts.

N. Srinidhi, Tanumay Datta, and T. Lakshmi Narasimhan were helpful in the development of the manuscript in many ways (generating figures, proof-reading, LaTeX help, offering general feedback and comments on the structure and contents of the book). Our special thanks are due to them. We also thank Ms. G. Nithya, our project associate, for her help in the preparation of the manuscript. We appreciate her technical support to our laboratory activities and large MIMO related activities.

We thank Emanuele Viterbo and Yi Hong for their fruitful research collaboration on MIMO precoding and sampling based lattice decoding. We also thank Onkar Dabeer for his collaboration on AdaBoost for MIMO signal detection. We are grateful to Rajesh Sundaesan and Vivek Borkar for useful discussions on MCMC techniques.

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We thank the academic institutions and industries who hosted our talks and discussion meetings on large MIMO systems, on one occasion or another. These interactions stimulated us to continuously engage in and broaden our views and scope of our research in large MIMO.

Our thanks are due to Dheeraj Sreedhar, N. Srinidhi, Tanumay Datta, Sanjay Vishwakarma, T. Lakshami Narasimhan, S. N. Padmanabhan, and S. V. R. Anand for useful discussions on large MIMO system development. We also thank our colleagues K. J. Vinoy, Gaurab Banerjee, and Bharadwaj Amrutur for discussions on RF and hardware related issues. Working with the scientists of DEAL, Dehradun, and the engineers of Tata Elxsi, Bangalore, on large MIMO system development was an exciting experience. We thank them for their zeal and commitment to the development effort.

We thank Cambridge University Press for accepting our proposal to write this book. It was a pleasure working with Philip Meyler and Mia Balashova, Cambridge University Press, on this project. Special thanks are due to Mia for the excellent support she rendered throughout the various stages of writing this book. Her response to our queries and concerns at every stage was always prompt and clear. She made our book writing experience a smooth one.

Finally, we express our sincere gratitude to our families – our indulgence in several of our academic pursuits, including writing of this book, would not have been possible without their patience, understanding, and support.

Abbreviations

2G	Second generation
3G	Third generation
3GPP	Third generation partnership project
4G	Fourth generation
5G	Fifth generation
ADC	Analog-to-digital conversion
AGC	Automatic gain control
AoA	Angle of arrival
AoD	Angle of departure
AP	Access point
APP	A posteriori probability
AS	Angular spread
ASIC	Application specific integrated circuit
AWGN	Additive white Gaussian noise
BC	Broadcast channel
BCJR	Bahl–Cocke–Jelinek–Raviv
BER	Bit error rate
BP	Belief propagation
bpcu	Bits per channel use
BPSK	Binary phase shift keying
BQP	Binary quadratic program
BS	Base station
CCDF	Complementary cumulative distribution function
CDA	Cyclic division algebra
CDF	Cumulative distribution function
CDMA	Code division multiple access
CN	Check node
COMP	Coordinated multipoint
COST	Cooperation in science and technology
CP	Cyclic prefix
CPSC	Cyclic prefixed single-carrier
CRB	Cramer–Rao bound
CRLB	Cramer–Rao lower bound
CSI	Channel state information

CSIR	Channel state information at receiver
CSIT	Channel state information at transmitter
DAC	Digital-to-analog conversion
dB	Decibel
DFT	Discrete Fourier transform
DoA	Direction of arrival
DoD	Direction of departure
DPC	Dirty paper coding
EPA	Extended pedestrian A model
ETU	Extended typical urban model
EVA	Extended vehicular A model
EXIT	Extrinsic information transfer
FDD	Frequency division duplex
FDMA	Frequency division multiple access
FIR	Finite impulse response
FFT	Fast Fourier transform
FGBP	Factor graph belief propagation
FPGA	Field-programmable gate array
GA	Gaussian approximation
GAI	Gaussian approximation of interference
GDL	Generalized distributive law
GPDA	Generalized PDA
GPS	Global positioning system
GSM	Generalized spatial modulation
GTA	Gaussian tree approximation
HDTV	High-definition television
IC	Integrated circuit
ICI	Inter-carrier interference
IDFT	Inverse DFT
IF	Intermediate frequency
IFA	Inverted F antenna
IFFT	Inverse FFT
iid	independent and identically distributed
ILS	Integer least-squares
ISDIC	Iterative soft decision interference cancelation
ISI	Inter-symbol interference
IUI	Inter-user interface
KL	Kullback–Leibler
LAN	Local area network
LAS	Likelihood ascent search
LD	Linear dispersion
LDPC	Low-density parity-check
LHS	Left hand side
LLL	Lenstra–Lenstra–Lovasz

LLR	Log-likelihood ratio
LOS	Line-of-sight
LR	Lattice reduction
LS	Local search
LTE	Long-term evolution
LTE-A	Long-term evolution advanced
LTS	Layered tabu search
MAC	Media access control
MAP	Maximum a posteriori probability
MCMC	Markov chain Monte Carlo
MF	Matched filter
MGS	Mixed-Gibbs sampling
MGS-MR	Mixed-Gibbs sampling with multiple restarts
MIMO	Multiple-input multiple-output
ML	Maximum likelihood
MMSE	Minimum mean square error
MMSE-ISDIC	MMSE based iterative soft-decision interference cancelation
MMSE-SIC	MMSE successive interference cancelation
MRF	Markov random field
MSE	Mean square error
MUBF	Multiuser beamforming
MUD	Multiuser detection
NDS	Norm descent search
NLOS	Non-line-of-sight
NO-STBC	Non-orthogonal space-time block code
OFDM	Orthogonal frequency division multiplexing
OFDMA	Orthogonal frequency division multiple access
OLA	Overlap-and-add
OSTBC	Orthogonal space-time block code
PAM	Pulse amplitude modulation
PAPR	Peak-to-average power ratio
PAS	Power angular spectrum
PC	Personal computer
PDA	Probabilistic data association
pdf	Probability density function
PDP	Power delay profile
PIC	Parallel interference cancelation
PIFA	Planar inverted F antenna
pmf	Probability mass function
PSK	Phase shift keying
QAM	Quadrature amplitude modulation
QPSK	Quadrature phase shift keying
R3TS	Random-restart reactive tabu search
RF	Radio frequency

RFID	Radio-frequency identification
rms	Root mean square
RS	Randomized search
RTS	Reactive tabu search
SA	Seysen's algorithm
SAGE	Space-alternating generalized expectation-maximization
SC-FDMA	Single-carrier frequency division multiple access
SCM	Spatial channel model
SCME	Spatial channel model – extended
SD	Sphere decoder
SDMA	Space division multiple access
SDR	Semi-definite relaxation
SFBC	Space-frequency block code
SGA	Scalar Gaussian approximation
SIC	Successive interference cancelation
SIMO	Single-input multiple-output
SINR	Signal-to-interference plus noise ratio
SISO	Single-input single-output
SM	Spatial modulation
SMSE	Sum mean square error
SNR	Signal-to-noise ratio
spcu	Symbols per channel use
SSK	Space shift keying
STBC	Space-time block code
STTC	Space-time trellis codes
SVD	Singular value decomposition
TCM	Trellis coded modulation
TDD	Time division duplex
TDL	Tapped delay line
TDMA	Time division multiple access
TGn	Task group IEEE 802.11n
THP	Tomlinson–Harashima precoding
TOA	Time of arrival
TS	Tabu search
TV	Television
UCA	Uniform circular array
UE	User equipment
UHF	Ultra high frequency
ULA	Uniform linear array
USB	Universal serial bus
UT	User terminal
UWB	Ultra wideband
V-BLAST	Vertical Bell laboratories layered space-time architecture
VGA	Vector Gaussian approximation

VHF	Very high frequency
VLAN	Virtual local area network
VP	Vector perturbation
VP-SE	Vector perturbation with sphere encoding
WINNER	Wireless world initiative new radio
WiFi	Wireless fidelity
WLAN	Wireless local area network
ZF	Zero forcing
ZF-SIC	Zero forcing successive interference cancelation
ZP	Zero padding
ZPSC	Zero padded single-carrier

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Notation

$(\cdot)^*$	Complex conjugation
$(\cdot)^H$	Hermitian transposition
$(\cdot)^T$	Transposition
$ \cdot $	Absolute value of a complex number (or cardinality of a set)
$\ \cdot\ $	Euclidean norm of a vector
$\lfloor \cdot \rfloor$	Rounding operation to the nearest integer
$\lfloor c \rfloor$	Largest integer less than c
\odot	Element-wise multiplication operation
\otimes	Kronecker product
$\mathcal{CN}(\mu, \sigma^2)$	Circularly symmetric complex Gaussian distribution with mean μ and variance σ^2
n_t	Number of transmit antennas
n_r	Number of receive antennas
$\text{vec}(\cdot)$	Stack columns of the input matrix into one column vector
$\det(\mathbf{X})$	Determinant of matrix \mathbf{X}
$\text{tr}\{\mathbf{X}\}$	Trace of matrix \mathbf{X}
\mathbf{I}_n	$n \times n$ identity matrix
\mathbf{x}	Vector \mathbf{x}
\mathbf{X}	Matrix \mathbf{X}
\mathbb{C}	Field of complex numbers
$\mathbb{E}[\cdot]$	Expectation operation
\mathbb{R}	Field of real numbers
\mathbb{R}^+	Non-negative real numbers
\mathbb{Z}	Set of all integers
$\Re(\cdot)$	Real part of the complex argument
$\Im(\cdot)$	Imaginary part of the complex argument