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ABSTRACT

We report a numerical study of the large-scale structure of turbulent Rayleigh–Bénard convection (RBC) in a slim-box using direct numerical simulations. The simulations are performed in a rectangular cell of 1/6 depth-to-width ratio with the Rayleigh number from $Ra = 1 \times 10^7$ to 5×10^9 and Prandtl number equal to 0.7. It turns out that the large-scale circulation is driven by the jet flows, which consist of thermal plume clusters emitted from the conducting plate. The oblique impinging jet presents similar behavior for Ra. Moreover, the Reynolds number defined by the jet speed is approximated as a power law $Re_m \sim Ra^{0.50}$. The oblique jet impinges onto the horizontal plate and develops into a wall jet. The similar flow patterns over the plate indicate the coherent motion of the wall jet. The wall jet presents a three-layer structure including the viscous sublayer, the mixing layer, and the bulk. The velocity in each layer has its characteristic parameters. We analyzed the turbulent kinetic energy and dissipation and obtained the scaling laws of the horizontal and vertical velocity fluctuations and the heights of their peaks. The thermal boundary layer and the heat transfer on the plate are investigated. The self-similarity of the thermal boundary layer solution is verified by boundary layer theory. Further analysis reveals that the Nusselt number on the conducting plate is possessed by an exponential law of the horizontal location, $Nu_x = Nu_m \exp(-\tilde{x})$, where Nu_m is the maximum Nusselt number on the plate and $\tilde{x}$ is the normalized horizontal distance to the stagnation point. We derived a power law of the maximum heat transfer on the plate by scaling analysis, $Nu_m \sim Ra^{0.2925}$, in agreement with the simulations. All results indicate that both the oblique impinging jet and the wall jet characterize the near-wall flow and the global heat transfer of turbulent RBC at moderate Rayleigh numbers.

I. INTRODUCTION

Thermal convection plays an important role in many natural and engineering systems. Enormous phenomena in nature such as convection in the earth's mantle, cloud formation, and large-scale circulation (LSC) in the ocean are mainly driven by convection.[1–6] Of the many different forms of thermal convection, Rayleigh–Bénard convection (RBC) is by far the most studied. RBC occurs when a fluid in a confined area is heated from the bottom at high temperature and cooled from the top.[7] For a certain range of temperature differences, special features such as thermal plumes, LSC, and flow reversals have been observed.[8–13] Turbulent RBC is conventionally divided into a boundary layer (BL) and a bulk flow region.[14,15]

The heat transfer in a confined area filled with a fluid of width L_x, depth L_y, and height L_z is determined by a temperature difference ΔT. The conditions are usually described by three dimensionless parameters, namely the Rayleigh number $Ra = g\beta\Delta T L_z^3/(\nu\kappa)$, the Prandtl

number $Pr = \nu/\kappa$, and the aspect ratio L_x/L_z. Here, g is the acceleration due to gravity, β the thermal expansion coefficient, ν the kinematic viscosity, and κ the thermal diffusivity, respectively. Two global output parameters are dimensionless heat transfer $Nu = q/(\kappa\Delta T/L_z)$ and flow intensity $Re = UL/\nu$, where q is heat flux and U and L are characteristic velocity and length, respectively, chosen appropriately for specific research. In order to control the flow and enhance the heat transfer for engineering purposes, huge efforts have been paid to distinguish the optimal boundary conditions and fluid properties by adopting the geometrical confinement,[16–21] adding roughness elements,[22–24] applying harmonic vibrations,[25,26] and inducing phase change.[27] Despite the simple configuration, many outstanding questions regarding the nature of turbulent RBC remain unanswered.

In a slim rectangular container of aspect ratio one ($L_x/L_z = 1$) with relatively small depth ($L_y/L_x = 1/6$), the flow is usually characterized by a dominant roll (also LSC) lying in the vertical square

section parallel to the front-end walls.[10,28] The LSC confined in between the walls may induce other large-scale structures. Those structures include the corner roll (CR),[29] the wind-shearing boundary layer and intensively ejecting plumes.[10,30–32] They jointly dominate heat transfer in an RB cell. The fluid driven by LSC and CR near the horizontal plate determines the local heat flux. It has been found that the local Nu (defined as $Nu_x = -[(\partial T/\partial z)/(\Delta T/L_z)]_{z=0}$) reaches its maximum at the stagnation point and decays along the streamwise direction.[10,33–35] Additionally, thermal structures and the second peak of streamwise turbulent kinetic energy (TKE) are present in this region.[10,36] These phenomena are attributed to the flow between LSC and CR impinging on the plate. Additionally, the wall jet in RBC (RB-WJ) develops along the plate in the presence of the oblique jet impingement. These observations urge to pay more attention to the heat transfer induced by the jet.

The development of flow ejected by a nozzle parallel to an infinite plane is called the plane wall jet (PWJ) here, to distinguish it from the jet-like flow in the RBC. Both the oblique impinging jet (OIJ) and the PWJ have been studied for decades due to its enormous applications for engineering purpose. Goldstein and Franchett[37] found that the local Nu of the oblique plane jet exhibits a local maximum, along with an exponential decay. Narayan and Narasimha first suggested that the wall-bounded flow possesses a unique scaling law while being normalized with the momentum rate and the height of the outer region of PWJ.[83] The streamwise similarity of PWJ is characterized by the momentum rate $M = \int_0^\infty U^2 dz$, the maximum velocity at a given section U_m, the streamwise location x, and viscosity ν

$$\frac{U_m \nu}{M} \sim \left(\frac{xM}{\nu^2}\right)^{n_u}, \qquad (1)$$

$$\frac{z_t M}{\nu^2} \sim \left(\frac{xM}{\nu^2}\right)^{n_{zt}}, \qquad (2)$$

where z_t is the height of the mean velocity equal to one half of the maximum velocity U_m. The normalized velocity in Eq. (1) and the height in Eq. (2) are the parameters of the jet flow expressed as functions of streamwise position.

The scaling laws of the momentum rate have been asserted in a wide range of jets without heat transfer.[38–41] For the PWJ, the power exponent $n_u = -1/2$ was first deduced by Glauert[39] by introducing similarity solution of plane laminar and turbulent wall jet (WJ) without wall stress. Myers et al.[42] and Gersten[43] also obtained $n_u = -1/2$ with an integral method. By experimentally measuring n_{zt}, Gupta et al.[41] found the n_{zt} is close to 1.00, whereas Wygnanski et al.[44] suggested $n_{zt} = 0.88$, and Narasimha et al.[40] gave $n_{zt} = 0.91$. Launder[45] proposed that z_t demonstrates a linear growth with regard to the case of $n_{zt} = 1.00$. Besides the scaling law of the streamwise similarity, the TKE was investigated,[46,47] and twin peaks of the vertical TKE profile were found to be a feature of turbulent PWJ. The results showed that, when production is low, more energy would be transported to the region in between the two peaks. Fruitful knowledge of the turbulent PWJ flow is helpful for understanding the energy transportation of large-scale motions in RBC.

The PWJ is usually scaled by two reference heights and velocities corresponding to the inner and outer layers, but this scheme seems oversimplified for turbulent shear flows.[48,49] The turbulent boundary layer (TBL), for example, presents a multilayer structure as being,

respectively, dominated by its own balance in each layer. Recently, the wind-shearing and plume-ejecting of turbulent RBC have been investigated and described by the theory of structure ensemble dynamics (SED),[10] which provides a method to illuminate the multilayer structure of wall-bounded turbulent thermal convection.

In this paper, we investigate OIJ and WJ by analyzing the momentum and heat transfer in RBC. The present work is organized as follows. In Sec. II, we introduce the mathematical equations and numerical methods of simulating the turbulent Rayleigh–Bénard convection. The features of the impinging jet flow including the similarity of integration variables and local mean profiles of WJ are discussed in Sec. III. The energy flux carried by the OIJ and the scaling of heat transfer on the plate are presented in Sec. IV. Finally, the summary is given in Sec. V.

II. COMPUTATIONAL SETUP

Direct numerical simulation (DNS) of turbulent Rayleigh–Bénard convection has been performed using a second-order finite-difference code in a slim-box domain as described in detail by Verzicco and Orlandi[50] and Verzicco and Camussi.[51] The dimensionless conservation laws of momentum, energy, and mass in the Oberbeck–Boussinesq approximation with gravitational acceleration in the z-direction read

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \theta \boldsymbol{e}_z + \sqrt{\frac{Pr}{Ra}}\nabla^2 \boldsymbol{u}, \qquad (3)$$

$$\frac{\partial \theta}{\partial t} + (\boldsymbol{u} \cdot \nabla)\theta = \sqrt{\frac{1}{PrRa}}\nabla^2 \theta, \qquad (4)$$

$$\nabla \cdot \boldsymbol{u} = 0. \qquad (5)$$

Here $\boldsymbol{u}$, θ, p, and $\boldsymbol{e}_z$ denote the dimensionless velocity, temperature, dynamic pressure, and unit vector in z-direction, respectively. We have performed DNS in a slim-box of aspect ratio $L_x : L_y : L_z = 1 : 1/6 : 1$.

The boundary conditions are set to close the equations as follows. Fixed temperatures are set on top and bottom plates, and adiabatic condition on side-walls. Periodic boundary is applied in the depth direction and nonslip condition for all solid walls. The simulations are initialized by a fluid at rest and an even temperature. The box height L_z and the free fall velocity $U_f = (\beta L_z g \Delta T)^{1/2}$ are used as references for distance and velocity, and the temperature is normalized by $\theta = (T - T_c)/\Delta T$, where ΔT denotes the difference between the temperatures of the top and bottom plates and T_c is their mean value. Therefore, the Pr number $Pr = \nu/\kappa$ and the aspect ratio present as the parameters fixing the set of equations. The variables are normalized using the free fall velocity $U_f = \sqrt{RaPr}(\kappa/L_z)$, the pressure $P_0 = RaPr(\rho\kappa^2)/L_z^2$, and the timescale $T_0 = (L_z^2/\kappa)\sqrt{RaPr}$. These parameters are determined according to Zou et al.[10] to fulfill the resolution requirements for a DNS. The Ra number varies between 1×10^7 and 5×10^9, while the Pr number is kept constant at $Pr = 0.7$.

Kolmogorov and Batchelor scales were resolved to ensure appropriate number of nodes. The globally averaged Kolmogorov length scale η_k was calculated by the relation $\eta_k = Pr^{1/2}[Ra(Nu - 1)]^{-1/4}$, and the Batchelor length scale η_{Ba} was estimated through $\eta_{Ba} = \eta_k Pr^{-1/2}$. Since the Pr number in the present work is less than unit, resolving Kolmogorov scale is necessary. The BL region requires finer spatial resolution than the bulk region to resolve the near-wall flow. Neglecting this might lead to unrealistic plumes, and thus, Nu

TABLE I. Simulation parameters for $Pr = 0.7$, $L_x : L_y : L_z = 1 : 1/6 : 1$. The Reynolds number Re_m as defined in Eq. (7); the number of mesh nodes in the i-direction N_i, $i = x, y, z$; the number of nodes in the thermal/viscous BL as used in DNS; the Nusselt number Nu_j, where $j = 1, 2, 3$ denote the Nu, respectively, defined by volume integration, momentum dissipation, and thermal dissipation; the Kolmogorov scale η_k; the Batchelor scale η_{Ba}; the mesh nodes for the viscous and thermal BL $N_{u,bl}$, $N_{\theta,bl}$; the number of dimensionless time units τ used for averaging; and the minimum and maximum heights in z-direction Δ_{min} and Δ_{max}.

Ra	Pr	Re_m	$N_x \times N_y \times N_z$	Nu_1	Nu_2	Nu_3	Δ_{min}/L_z	Δ_{max}/L_z	η_k/L_z	η_{Ba}/L_z	$N_{u,bl}$	$N_{\theta,bl}$	t_{avg}/t_f
1×10^7	0.7	715	$768 \times 128 \times 512$	18.2	18.2	18.2	0.23×10^{-3}	4.2×10^{-3}	7.30×10^{-3}	8.73×10^{-3}	$72 > 4$	$52 > 4$	480
5×10^7	0.7	1500	$768 \times 128 \times 512$	29.1	29.1	29.0	0.23×10^{-3}	4.2×10^{-3}	4.32×10^{-3}	5.16×10^{-3}	$53 > 4$	$71 > 5$	320
1×10^8	0.7	2069	$768 \times 128 \times 512$	34.7	34.2	34.3	0.23×10^{-3}	4.2×10^{-3}	3.47×10^{-3}	4.15×10^{-3}	$72 > 5$	$29 > 5$	300
5×10^8	0.7	4446	$768 \times 128 \times 512$	55.8	54.8	55.3	0.23×10^{-3}	4.2×10^{-3}	2.06×10^{-3}	2.46×10^{-3}	$62 > 6$	$19 > 7$	400
1×10^9	0.7	5189	$1024 \times 128 \times 800$	68.9	69.3	68.5	0.08×10^{-3}	1.8×10^{-3}	1.64×10^{-3}	1.96×10^{-3}	$83 > 7$	$38 > 7$	300
5×10^9	0.7	13862	$1024 \times 128 \times 800$	110.8	109.1	109.7	0.10×10^{-3}	1.8×10^{-3}	0.97×10^{-3}	1.16×10^{-3}	$86 > 8$	$22 > 9$	400

will be higher than the real solution.[52] The essential number of nodes in the BL has been estimated in the simulations of Zhou and Chen[29] and displayed in Table I. The spatially averaged Kolmogorov scale is used as the required resolution for the bulk region in each direction. In order to examine the Nu number, we compared three types of Nu, which are listed in Table I. They are the spatiotemporal average of the heat flux in the box $Nu_1 = 1 + \sqrt{RaPr}\langle w\theta \rangle_{V,t}$, the heat flux of global thermal dissipation $Nu_2 = \langle |\nabla\theta|^2 \rangle_{V,t}$, and the global momentum dissipation $Nu_3 = 1 + Pr\langle |\nabla \boldsymbol{u}|^2 \rangle_{V,t}$. The $N_{u,bl}$ and $N_{\theta,bl}$ are the least number of grid nodes required inside the viscous/thermal BL estimated by Shishkina et al.[53] The simulated data[29] showed that the Ra dependence of the geometry of flow structures is obvious. The size of flow structures usually decreases with increasing Ra.

III. SIMILARITY OF THE JET IN RB CELL

A. The oblique impinging jet and its Reynolds number scaling

Figure 1(a) presents the speed field, that is, the magnitude of time-averaged velocity, $U_S(x, z) = \sqrt{U^2 + W^2}$. All data presented below have been averaged in the depth (y) direction. The downwelling stream, called the "ejecting stream" near the left sidewall, is deflected by the CR and leads to an oblique jet impinging on the horizontal plate. Therefore, the motion of LSC in combination with the CR forms an OIJ analogous to the OIJ impinging on an infinite plane[37] along with a PWJ.[40,41,43,48] Note that this scenario occurs at the diagonal (down-left/up-right) corners of the RB cell.

The speed profiles normal to the slip line at five locations named $loc1$ to $loc5$ [as marked in Fig. 1(a)] are plotted together in Fig. 1(b). The line at $loc3$ connects the centers of the CR and LSC. Other lines of speed profiles are parallel to this line. The inclination angle between the line and the bottom plate is called θ_0. The speed profiles present a parabolic function. The similarity of the profiles along the stagnation streamline indicates that the fluid motion along the stagnation streamline sustains a free-jet-like flow between CR and LSC. Moreover, the deviation of the profile at $loc5$ is caused by the stagnation of the jet on the bottom plate.

To investigate the effect of Ra, the speed profiles at $loc3$ for different Ras are scaled by the correspondent maxima, and the length scales are scaled by the distance between LSC and CR

$$\tilde{U}_j = \frac{U_S}{U_{max}} \quad \text{and} \quad \tilde{L} = \frac{L_{cr}}{L_{lc}}, \quad (6)$$

where the size of CR L_{cr} is the distance to the center of CR on $loc3$, $L_{lc} = \sqrt{(x_{lsc} - x_{cr})^2 + (z_{lsc} - z_{cr})^2}$, (x_{lsc}, z_{lsc}), and (x_{cr}, z_{cr}) are the coordinates of the centers of LSC and CR, respectively. Figure 2(a) shows a similarity of OIJ speed profile for different Ras, which can be empirically described in form of beta function $\tilde{U}_j = 4.0\tilde{L}^{0.80}(1 - \tilde{L})^{1.30}$. The slip line is laid at $\tilde{L} = 0.126$, closer to CR than LSC, indicating that the OIJ contributes more energy to LSC than CR.

The similarity of the speed profiles of U_S indicates that the normalized jet speed U_{max} and L_{lc} can be used as the parameters to scale the OIJ speed profile. Figure 2(b) shows Re_m, and the Reynolds number defined by the above two parameters in combination with viscosity ν fulfills an Ra power law

$$Re_m \equiv \frac{U_{max}L_{lc}}{\nu} = 0.20Ra^{0.50}. \quad (7)$$

The scaling is consistent with the previous scaling of the Reynolds number defined by large-scale motion.[54–56] The Reynolds number defined by the velocity of LSC in the RBC (also called the most probable velocity in Ref. 56) has the same scaling, $Re \sim Ra^{0.50}$. The coincidence of the scalings indicates that the OIJ is the large-scale structure driving the motions of LSC and CR.

B. Similarity of RB-WJ

The fluid of OIJ convecting along the shear layer between LSC and CR eventually impinges onto the horizontal plate. Since the impingement causes an intensive pressure gradient near the stagnation point, the WJ flow on the plate no longer conforms to the conventional theory of zero pressure gradient TBL. We here apply the knowledge of PWJ to describe the velocity and heat transfer of the WJ in turbulent RB system.

Kinematic momentum rate $M = U^2L$ is often used as a scale for the PWJ and the boundary layer.[41,43,48,57] Two definitions of the momentum rate are usually used to scale WJs.[40] They are the nozzle momentum rate $M_{noz} = U_{noz}^2 b$ and the local momentum rate $M(x) = \int_0^L U_j^2(x, z)dz$, where U_j is the nozzle exit velocity and b is the nozzle height, $U_j(x, z)$ is the local velocity of the jet flow, and L is a reference height.

If the WJ is independent of the initial conditions of the nozzle, the flow would be scaled by local momentum rate $M(x)$, allowing for self-similar development with regard to the local conditions.[41] Since
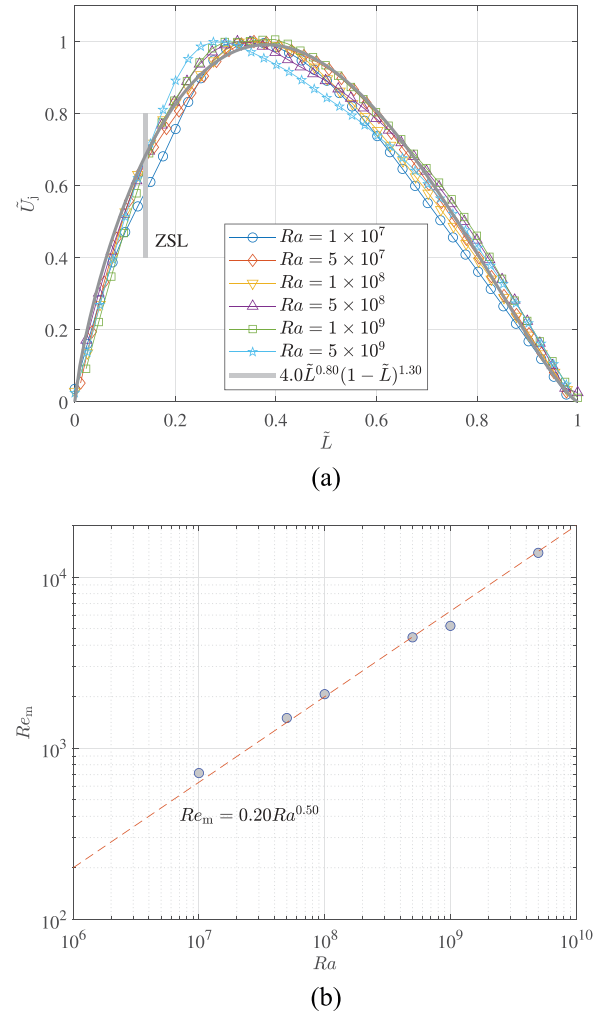
(a)

(b)
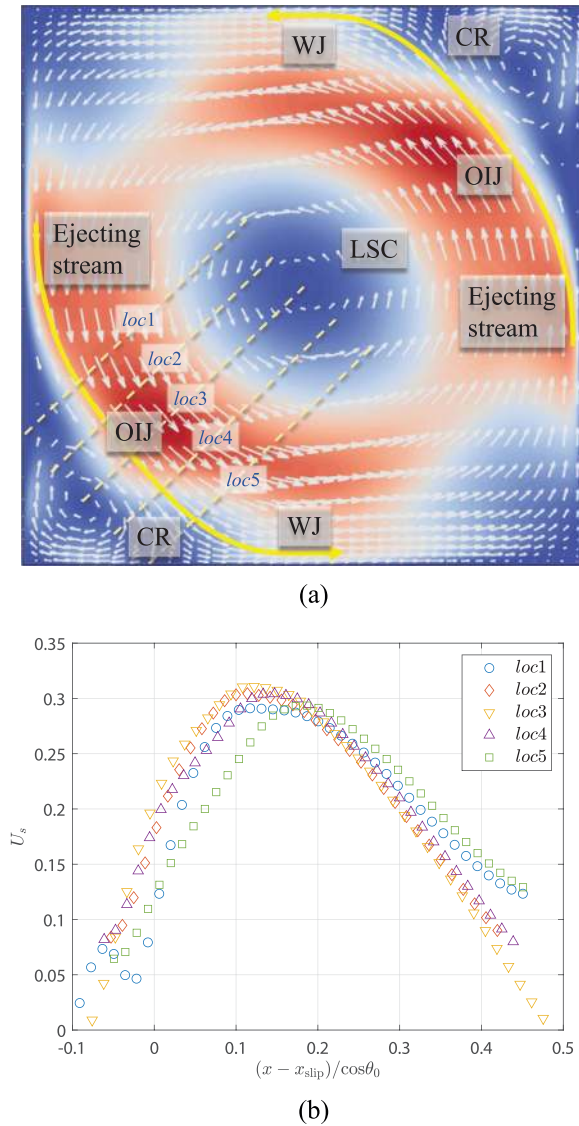
FIG. 1. Spatial distribution of the mean speed at $Ra = 1 \times 10^8$: (a) the time-averaged speed on xz-plane. Blue denotes low speed, and red denotes high speed. (b) Speed profiles on the five lines normal to the slip line, and x_{slip} is the abscissa of slip line.

(a)

(b)

FIG. 2. The normalized speed of OIJ and the Reynolds number defined by it: (a) the speed profiles of OIJ $\tilde{U}_j$ on the line connecting the cores of LSC and CR with Ra from 1×10^7 to 5×10^9 and (b) the Reynolds number of OIJ Re_m as a power function of Ra.

the convection flow in response to RB-OIJ does not have a definite initial condition like the flow being ejected from a nozzle, the local momentum rate $M(x)$ is then employed to scale WJ here.

The RB-WJ consists of two flow regions—the near-wall region and the bulk region, as schematically shown in Fig. 3. The flow in the near-wall region is intensively confined by the plate and dominated by viscosity. The near-wall region includes the viscous sublayer and the mixing layer. The bulk region, on the other hand, is dominated by inertia. The mixing layer is formed by the interaction between the viscosity of the sublayer and the inertia of the bulk. The characteristic velocity and the height of the bulk are the maximum velocity $U_{m,b}$ and

half-height $z_{t,b}$, where the mean velocity is equal to $U_{m,b}/2$, respectively.[40] The inner peak velocity $U_{m,w}$ and the corresponding height $z_{t,w}$ are the characteristic scales for the near-wall region. For the viscous sublayer, the flow is dominated by the law of wall and correspondingly scaled by the friction velocity $u_\tau = \sqrt{(\nu \frac{\partial U}{\partial z})_{z=0}}$ along with the viscous scale ν/u_τ.

The inner peak velocity in the near-wall region $U_{m,w}$ along the bottom plate (the direction of x) is shown in Fig. 4(a), which takes with the Ra of 1×10^7 to 5×10^9 over a range of the distance to the left sidewall, $x = 0.40–0.60$. To eliminate the geometrical and bulk flow effects, the distance to the stagnation point is used as the horizontal distance, $\Delta x = x - x_a$, where x_a is the horizontal distance of the stagnation point to the left sidewall. Here, both $U_{m,w}$ and Δx are scaled with the local momentum rate $M_w(x) = \int_0^{z_{min}} U^2(x,z)dz$ (per unit width) and viscosity ν, where z_{min} is the height of the minimum
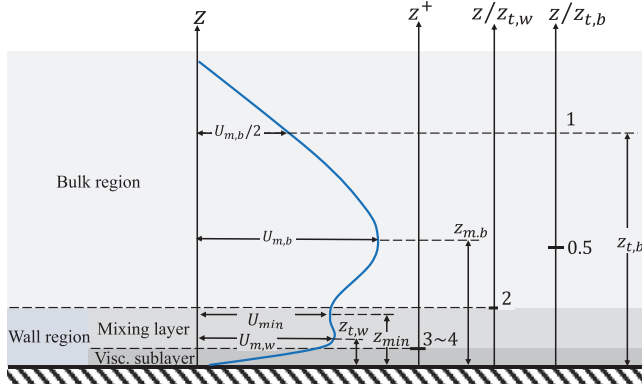
FIG. 3. Schematic plot of the velocity profile of turbulent RBC including the viscous sublayer, the mixing layer and the bulk region. The thickness of each layer is determined by corresponding normalized height—$z^+ = 3$–4 for the viscous sublayer, $z/z_{t,w} = 2$, for the mixing layer, and $z/z_{t,b} = 0.5$ for the bulk region, as discussed in Sec. III D.

velocity in between two velocity peaks, $U_{m,b}$ and $U_{m,w}$ (see Fig. 3). For the "single peak" of velocity, z_{min} is set to equal $2z_{t,w}$. Figure 4(a) shows that the normalized maximum velocity $U_{m,w}\nu/M_w$ varying as a power function of $\Delta x M_w/\nu^2$ with an exponent of $n_u = -0.50$

$$\frac{U_{m,w}\nu}{M_w} = A_u \left(\frac{\Delta x M_w}{\nu^2}\right)^{n_u}, \qquad (8)$$

where $A_u = 2.4$ for all Ra numbers of the present study. The results of turbulent PWJs are also presented in Fig. 4(a) for comparison, and their power exponent and the factor are correspondingly $n_u = -0.50$ and $A_u = 4.1$. Equation (8) can be rewritten as $M_w = \int_0^h U^2 dz = U_{m,w}^2 \Delta x/A_u^2$ for $n_u = -0.50$. Considering the different outer regions of the two systems, the integration scales for the near-wall region are taken as $h = z_{min}$ for the RBC and $h = \infty$ for the PWJ, respectively.

The magnitudes of (n_u, A_u) for turbulent PWJ have been extensively studied in the literature. Schwarz and Cosart suggested $(-0.555, 5.395)$,[58] Narasimha et al. $(-0.506, 4.6)$,[40] and Wygnanski et al. $(-0.472, 1.473)$.[44] Note that the above parameter sets of (n_u, A_u) were scaled with M_j rather than $M(x)$. As for the difference of M_{noz} and $M(x)$, Narasimha et al. claimed that the correlation is the same either for M_j or $M(x)$,[40] whereas Gupta et al. suggested that the local $M(x)$ is a better scale for the PWJ.[41] As shown in Fig. 4, the unique power exponent of the maximum velocity while being scaled by $M(x)$–ν indicates the same behavior of the near-wall flows for the PWJ and the RB-WJ.

It is noteworthy that $\Delta x M_w/\nu^2$ of PWJ is several order of magnitude greater than that of RB-WJ. In order to understand the derivation of the two systems, we take the nozzle height b as an extra parameter in the WJ flow and will get three dimensionless independent variables $\Pi_1 = z/b$, $\Pi_2 = x/b$, and $\Pi_3 = \sqrt{M_j b/\nu^2}$. The nozzle height b as estimated by the half width of speed profile of OIJ $b = L_{lc}/2$ along with the maximum velocity of the OIJ leads to Re_j for the RBC on the order of 10^2 to 10^3. On the other hand, Re_j for PWJ is usually on the order of 10^3 to 10^4. The Reynolds numbers of the two systems are close. Considering the fact that the RB-WJ is driven by circulation of the thermal plumes rather than by momentum fluid from a nozzle, the absence of an exit of fluid makes it hard to accurately determine b for
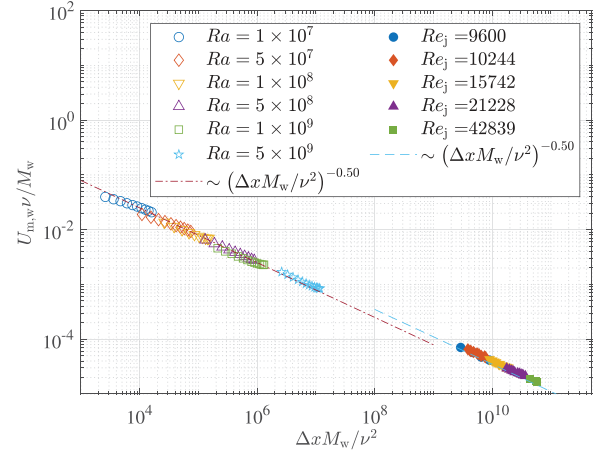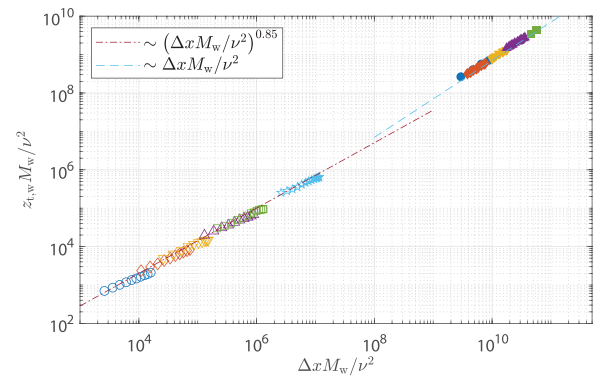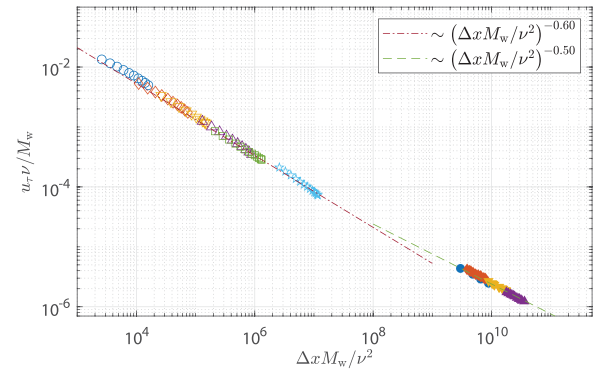
FIG. 4. The scalings of the normalized (a) $U_{m,w}$, (b) $z_{t,w}$, and (c) u_τ against the normalized x. The data are obtained from the experiments of Schwarz et al.[58] $Re_j = 42\,839$, Eriksson et al.[59] $Re_j = 9600$, and Gupta et al.[41] $Re_j = 10\,244$, $15\,742$, and $21\,228$, where $Re_j = U_j b/\nu$, U_j is the nozzle exit velocity, and b is the nozzle height.

RBC. The flow state in the near-wall region in RBC indicates the similar behavior of the two systems. Then, the magnitude of $\sqrt{M_j b/\nu^2}$ can be measured by the nozzle exit Reynolds number Re_j. The estimated b/L_z for RBC is about 0.3, and the maximum of x/b is about 3.

On the other hand, x/b for the PWJ (presented in Fig. 4) is usually on the order of 10^3. Considering $\Delta x M_w / \nu^2 = \Pi_2 \Pi_3^2$, one then obtains that the normalized Δx is on the order of 10^4 to 10^6 for RBC, and 10^9 to 10^{11} for PWJ. This fact is consistent with the results in Fig. 4. The data of the two systems collapse well with a similar scaling law that strongly implies the similarity of the scales, which are independent of the nozzle exit Reynolds number Re_j and the nozzle height b.

The thickness of the near-wall region $z_{t,w}$ scaled by M_w-ν, as shown in Fig. 4(b), has a scaling law

$$\frac{z_{t,w} M_w}{\nu^2} = A_{zt} \left(\frac{\Delta x M_w}{\nu^2} \right)^{n_{zt}}, \tag{9}$$

where $A_{zt} = 0.73$ and $n_{zt} = 0.85$ for the simulated RB system. Here, the near-wall region thickness $z_{t,w}$ is defined as the height of the maximum velocity in the inner layer, corresponding to the half-height of the PWJ. As George et al. pointed out, the maximum velocity height and the half-height have the similar behavior along the streamwise direction,[48] but the half-height is more stable for experimental results of the PWJ.[41] Narasimha et al.[40] also found that the ratio of half-height to the maximum velocity height is about 6.2, independent of $\Delta x M_w / \nu^2$. Hence, the two scales are comparable.

The thickness of the near-wall flow of the PWJ has been extensively investigated. George et al. suggested that $n_{zt} = 0.915$ by semiempirical analysis (normalized by $M_j - \nu$) and also derived that the streamwise gradient of $z_{t,w}$, $dz_{t,w}/dx$ is proportional to the shear stress.[48] For asymptotic condition, $dz_{t,w}/dx \rightarrow 0$, $n_{z_{t,w}}$ is less than unit. Here, one can see that the flow in RBC satisfies the scenario of George et al., but the power exponent is less than that in the WJ.[40,48,49] This indicates that the turbulence is suppressed by the downward flow of LSC in vertical direction, and therefore, the growth of wall-bounded turbulence of RBC is weaker than that in turbulent PWJ.

Two Reynolds numbers, Re_{Mx} and Re_{Mz}, are defined by the normalized physical parameters, $U_{m,w}\nu / M_w$, $\Delta x M_w / \nu^2$, and $z_{t,w} M_w / \nu^2$, to characterize streamwise and vertical development of the near-wall flow, respectively. The streamwise Re number $Re_{Mx} = \sqrt{\Delta x M_w}/\nu$ and vertical Re number $Re_{Mz} = U_{m,w} z_{t,w}/\nu$. The correlation of Re_{Mx} and Re_{Mz} is obtained by combining Eqs. (8) and (9): $Re_{Mz} = \frac{U_{m,w} z_{t,w}}{\nu} = (U_{m,w}\nu/M_w)(z_{t,w}M_w/\nu^2) = A_u A_{zt} Re_{Mx}^{2(n_u + n_{zt})} \sim Re_{Mx}^{0.70}$. Thus, $Re_{Mx} \approx 2300$ and $Re_{Mz} \approx 360$ for $Ra = 5 \times 10^9$ at $x = 0.5$.

The friction velocity u_τ scaled by M_w-ν is shown in Fig. 4(c), and its scaling is shown as follows:

$$\frac{u_\tau \nu}{M_w} = A_{u_\tau} \left(\frac{\Delta x M_w}{\nu^2} \right)^{n_{u_\tau}}, \tag{10}$$

where $A_{u_\tau} = 1.40$ and $n_{u_\tau} = -0.60$ for the present RBC. The turbulent PWJ gives $A_{u_\tau} = 0.25$ and $n_{u_\tau} = -0.50$. In comparison, Wygnanski et al. suggested $A_{u_\tau} = 0.382$ and $n_{u_\tau} = -0.535$ as scaled by $M_j - \nu$ in turbulent PWJ.[44]

The results of turbulent PWJ show that $u_\tau/U_{m,w}$ weakly depends on $\frac{xM_w}{\nu^2}$. George et al.[48] suggested $u_\tau/U_{m,w} = 0.023(z_T^+)^{-0.0362}$ $\times \exp\left(\frac{2.9}{(ln z_T^+)^{0.46}} \right)$, where $z_T^+ = z_T u_\tau/\nu = \frac{z_T M_w}{\nu^2} \frac{u_\tau \nu}{M_w} \sim \left(\frac{xM_w}{\nu^2} \right)^{0.25}$. In the present study, combining Eq. (8) with (10) and taking the power exponents of the RBC, we have $u_\tau/U_{m,w} \sim \left(\frac{xM_w}{\nu^2} \right)^{-0.10}$. It is seen that the

impingement angle and confinement of the RB cell have finite influence on the similarity of the near-wall flow of RB system.

C. Correlation of scalings

The scaling of $U_{m,w}$ in RBC agrees well with the turbulent WJ as seen in Fig. 4. The scalings of $z_{t,w}$ and u_τ show that the LSC and the confinement of the flow have finite effects on the near-wall flow. The correlation can be formularized by the horizontal equation of Eq. (3)[48,60]

$$\frac{\partial U^2}{\partial x} + \frac{\partial UW}{\partial z} = \frac{\partial}{\partial z} \left[\nu \frac{\partial U}{\partial z} - \overline{u'w'} \right] - \frac{\partial}{\partial x} [\overline{u'u'} - \overline{w'w'}]. \tag{11}$$

Equation (11) has eliminated the pressure by integrating the vertical momentum equation of Eq. (3) and ignored the vertical integration of $\frac{\partial \theta}{\partial x}$. By integrating in z direction and considering the boundary conditions of wall, we obtain

$$\frac{d}{dx} \int_0^{z_o} U^2 + \overline{u'u'} - \overline{w'w'} dz + \frac{dz_o}{dx} [U^2 + \overline{u'u'} - \overline{w'w'}]|_{z_o} + UW|_{z_o}$$
$$= -u_\tau^2 + \nu \frac{\partial U}{\partial z}\Big|_{z_o} - \overline{u'w'}|_{z_o}, \tag{12}$$

where z_o is an appropriate height, corresponding to the infinite height in PWJ. As mentioned before, we have $z_o = z_{min}$, which means a position far away from the wall but not reaching out into the bulk. Finally, we have

$$UW|_{z_o} \approx 0, \quad \nu \frac{\partial U}{\partial z}\Big|_{z_o} \approx 0, \quad \overline{u'w'}|_{z_o} \approx 0, \quad \frac{dz_o}{dx} \approx 0 \tag{13}$$

and

$$\frac{d}{dx} \int_0^{z_o} U^2 + \overline{u'u'} - \overline{w'w'} dz = -u_\tau^2. \tag{14}$$

Since $U^2 \gg \overline{u'u'} - \overline{w'w'}$, Eq. (14) is as follows:

$$\frac{dM_w}{dx} = \frac{d}{dx} \int_0^{z_o} U^2 dz = -u_\tau^2. \tag{15}$$

Equation (15) is similar to result of Ref. 48. It demonstrates that the wall friction is attributed to the momentum loss.

A nondimensional momentum rate k_m is obtained when M_w is scaled by $U_{max}^2 z_T$

$$k_m = \frac{M_w}{U_{max}^2 z_T}. \tag{16}$$

Wygnanski et al.[44] suggested $k_m = 4.93$ for turbulent WJ.[61] The normalized momentum rate k_m varies in x direction with different Ras as seen in Fig. 5. With x and Ra increasing, k_m slowly grows till saturated, indicating an incomplete similarity. Similar behavior in PWJ is referred to Ref. 62. Renormalizing k_m with its value at $x = x_a + 0.25$ gives that $k_m/k_{m,s}$ is in range of 0.8–1.2. The study in turbulent PWJ by Wygnanski et al.[44] suggested that with increasing Ra, k_m reaches a constant for $Ra > 5.8 \times 10^{12}$. In contrast, k_m at the location $x = x_a + 0.25$ fulfills a power law $k_m = 0.145 Ra^{0.12}$ (see the inset of Fig. 5). The finite Re effect on the flow is considerable at moderate Ra.
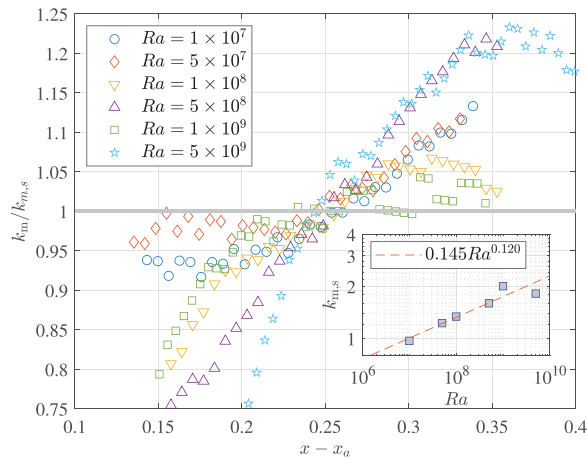
FIG. 5. The renormalized momentum rate $k_m/k_{m,s}$ against x. The value of $k_{m,s}$ is k_m at $x = x_a + 0.25$. The inset is the Ra dependence of $k_{m,s}$.

With Eq. (16), Eq. (15) can be rewritten as

$$\frac{dk_m U_{max}^2 z_T}{dx} = -u_\tau^2. \tag{17}$$

Thus, by ignoring small variation of k_m the power exponents, n_u, n_{zt} and n_{u_τ}, follow a relation as

$$2n_u + n_{zt} - 1 = 2n_{u_\tau}. \tag{18}$$

With this equation, we have $n_{u_\tau} = -0.575$ consistent with DNS result. The correlation of power exponents n_u, n_{zt}, and n_{u_τ} reflects the fact that RB-WJ is featured by developing PWJ, but distinct from well-developed and wall-bounded turbulence.

D. Multilayer structure of velocity profiles

The velocity profiles of RBC are compared with those of turbulent PWJ. The mean velocity profile normal to the plate is schematically shown in Fig. 3, which includes three layers. They can be scaled by the inner scale, the mixing layer scale, and the bulk scale, respectively.

Figure 6 shows the vertical velocity profiles at various horizontal locations and Ras being scaled by the friction velocity u_τ and viscous scale ν/u_τ, that is, $U^+ = U/u_\tau$ and $z^+ = zu_\tau/\nu$. The linear viscous sublayer conforms to the law of wall, $U^+ = z^+$. The thickness of the sublayer is about $z^+ = 3$–4 in agreement with the measurements of a two-dimensional PWJ.[59,63] Note that this sublayer is thinner than that in TBL, which is about five to 10 wall units.[64] The different thicknesses are attributed to finite Re effect under the influence of buoyancy and pressure gradient.

The TBL at a relatively high Reynolds number with zero pressure gradient yields a well-developed viscous sublayer. However, the Re number of simulated RBCs is on the order of hundreds as estimated by Re_{Mz}. The small Re suppresses the momentum transport in the near-wall region and reduces the sublayer thickness. It is noted that with increasing Ra, the Re number increases, and the thickness of the sublayer increases thereafter [see Fig. 6(b)]. Moreover, the linearity region may reach up to the viscous sublayer of TBL at high Ra.

The mixing layer is scaled by $U_{m,w}$, and the height is scaled by $z_{t,w}$ as shown in Fig. 7 in the range of $0.6 < z/z_{t,w} < 2$ within the WJ
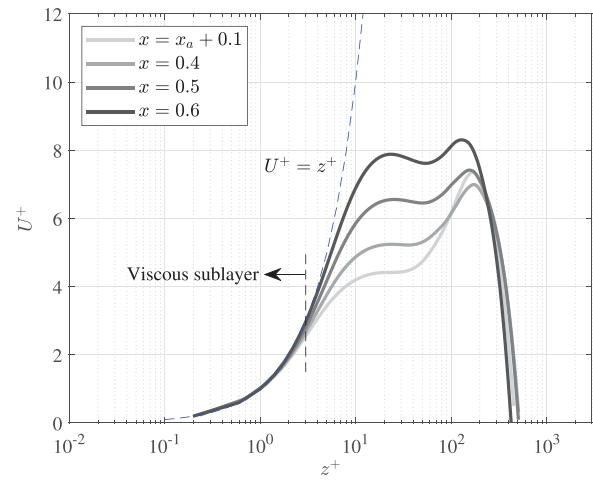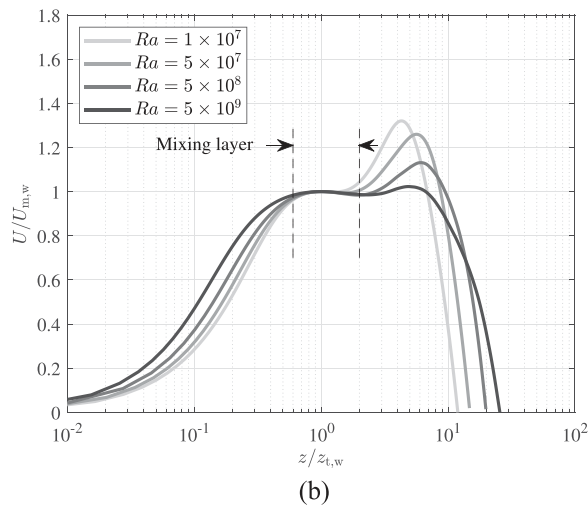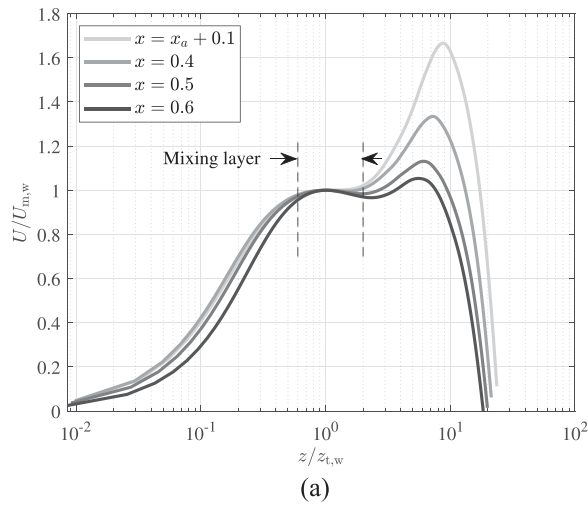

(a)
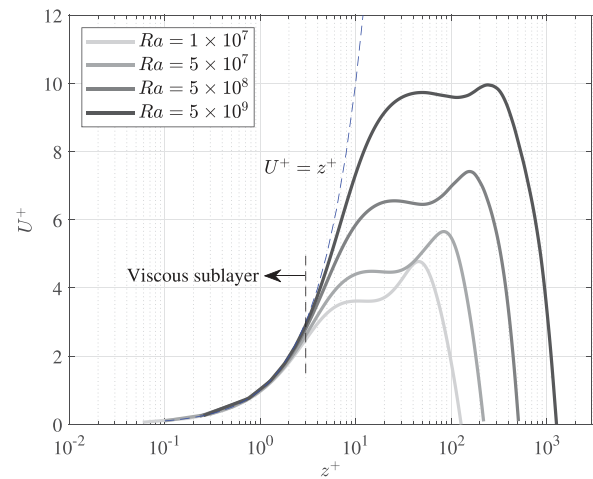
(b)

FIG. 6. The velocity profiles scaled by the inner velocity and length as a function of the height at (a) $x = x_a + 0.1$, 0.4, 0.5, 0.6 with $Ra = 5 \times 10^8$, and (b) $Ra = 1 \times 10^7$, $Ra = 5 \times 10^7$, 5×10^8, 5×10^9 at $x = 0.5$.

region and the simulated Ras. In the mixing layer, the effect of momentum diffusion is remarkable, while the pressure gradient, the Reynolds stress, and the mean advection jointly dominate the flow. This region can be considered as the overlap region of turbulent PWJ,[41,43,48] which is laid below the maximum velocity. The Re_τ dependence of the mixing layer can be expressed as the log-law[43] or the power law.[41] However, it has a larger extent in RBC, and its velocity profile is quite flat. Note that the flow further away from the plate no longer sustains its similarity due to the circulation of the bulk.

The turbulent PWJ is usually characterized by the momentum diffusion below $z_{m,b}$, and the outer region is filled with static fluid. In contrast, the region at the center of an RB cell is characterized by LSC. Velocity is scaled by $U_{m,b}$, and the height is scaled by $z_{t,b}$ for this region. The velocity profiles are shown in Fig. 8. The normalized velocity at different horizontal locations in the range of $z > 0.50z_{t,b}$ has the
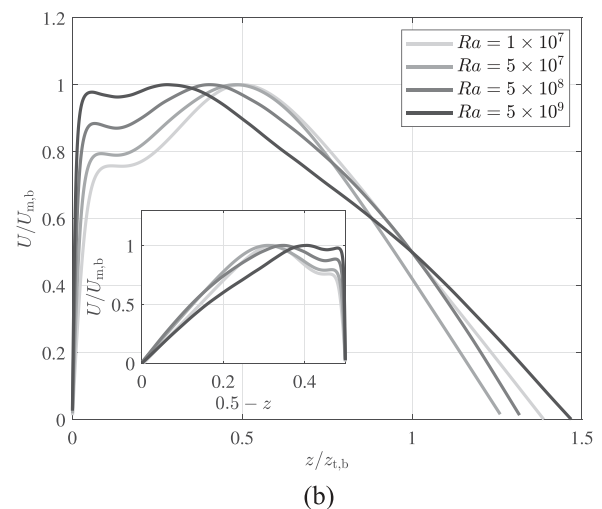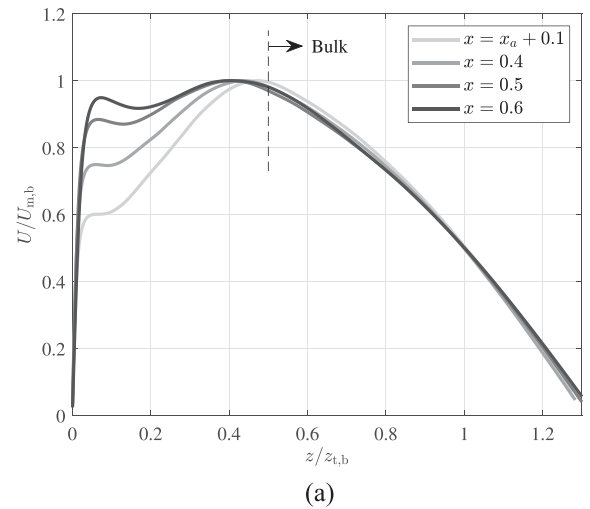
FIG. 7. The velocity profiles in scaled by the intermediate velocity and length as a function of height at (a) $x = x_a + 0.1$, 0.4, 0.5, 0.6 with $Ra = 5 \times 10^8$ and (b) $Ra = 1 \times 10^7, Ra = 5 \times 10^7, 5 \times 10^8, 5 \times 10^9$ at $x = 0.5$.

FIG. 8. The velocity profiles normalized by the outer velocity and length as a function of height at (a) $x = x_a + 0.1$, 0.4, 0.5, 0.6 at $Ra = 5 \times 10^8$ and (b) $Ra = 1 \times 10^7$, 5×10^7, 5×10^8, and 5×10^9 at $x = 0.5$.

same behavior at all simulated Ras as it in the range of $z > 0.1z_{t,b}$ for turbulent PWJ.[41] The geometry of the cell produces the circulation at the center. Therefore, the different outer flows of the two systems lead to different spatial ranges of similarity.

It is noted that the edge of the outer region, that is, $x = 0.5$ [see Fig. 8(a)], is not a universal scale. It cannot be used as the characteristic height, $z_{t,b}$, as presented in Fig. 8(b). The reason is that for a larger Ra, the LSC becomes larger and stronger, and the bulk region owns a larger volume. Moreover, based on the rotation essence, the symmetry center should lay at the center of LSC rather than the wall.

E. Similarity of the fluctuations in the near-wall region

The horizontal velocity fluctuation of RB-WJ $\sigma_u^+ = \sqrt{\overline{u'^2}}^+$ is calculated in comparison with that of turbulent PWJ. Figure 9 is the root mean square (rms) of the horizontal velocity σ_u^+ at various horizontal

locations and Ras. It shows that σ_u^+ grows rapidly in the near-wall region ($z^+ < 10$) and reaches the first peak at $z_{\sigma_u,m1}^+$ (below $z_{t,w}$), reminiscent of the results of turbulent PWJ.[46] The second peak appears at $z_{\sigma_u,m2}^+$ in bulk region. The whole WJ region is characterized by the "twin-peak" for σ_u^+ for all simulated Ras.

The scenario of the PWJ conduces to understanding the "twin peaks" of the velocity fluctuation in RB system. Twin-peak of velocity fluctuation is a common feature for turbulent PWJ even at rather low Re_{Mz}, due to the effect of the outer free-shear layer structures along some overlap of scales (cf. Ref. 46). In RB system, the "outer structure" is the well-organized LSC. Note that, for the boundary layer in RB system, the second peak would not appear unless Re is sufficiently large. Additionally, the fact that the first peak is larger than the second one is consistent with turbulent PWJs.[46]

To illustrate the similarity of the fluctuation of the horizontal velocity component, the value of σ_u^+ is scaled by the inner peak of

(a)

(a)

(b)

(b)
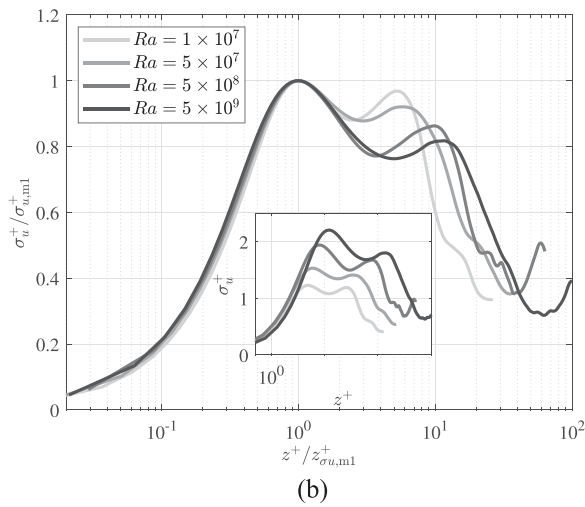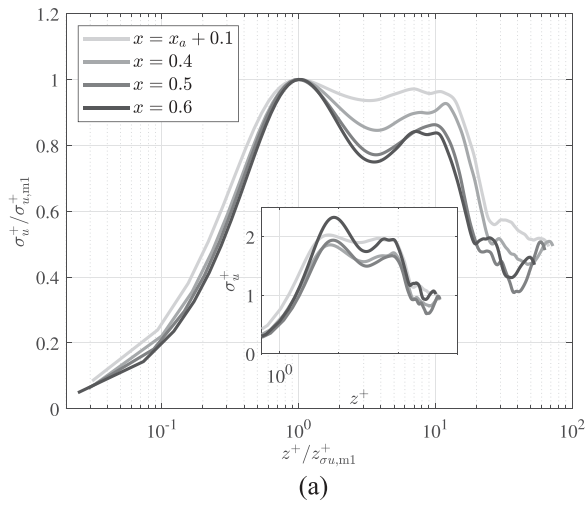
FIG. 9. The profiles of horizontal velocity fluctuation σ_u^+ normalized by the inner peak $\sigma_{u,\mathrm{m1}}^+$ and the corresponding height $z_{\sigma_{u,\mathrm{m1}}}^+$ at (a) various horizontal locations at $Ra = 5 \times 10^8$ and (b) various Ras at $x = 0.5$. The insets are the profiles free from normalization.
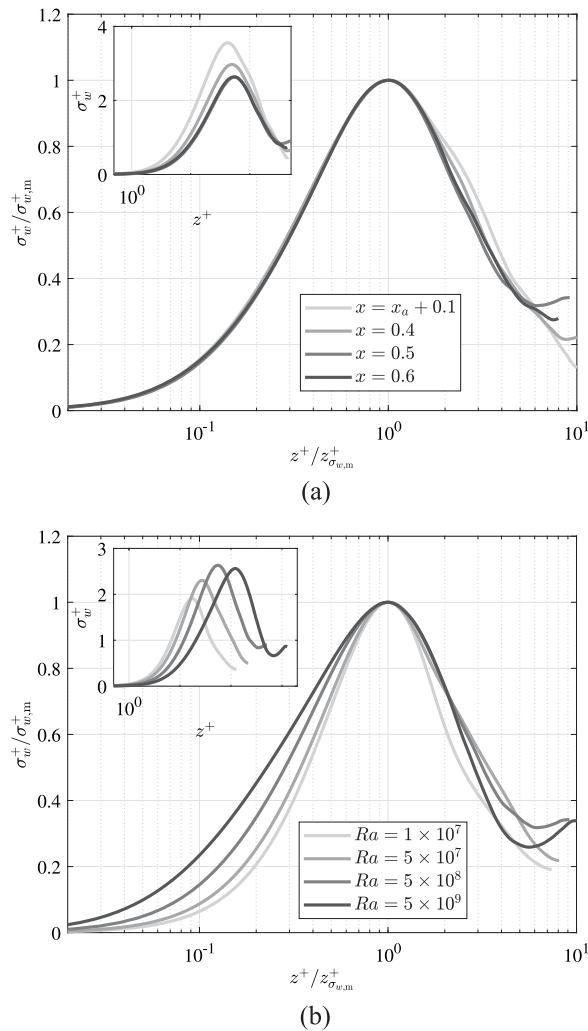
FIG. 10. (a) The height of the inner peak of horizontal velocity fluctuation $z_{\sigma_{u,\mathrm{m1}}}^+$ and (b) the magnitude of the fluctuation $\sigma_{u,\mathrm{m1}}^+$ at $x = 0.5$ as the power functions of Ra. The insets are variations at different horizontal locations for $Ra = 5 \times 10^8$ and 5×10^9.

$\sigma_{u,\mathrm{m1}}^+$ and the wall distance z^+ is normalized with $z_{\sigma_{u,\mathrm{m1}}}^+$. The similarity of horizontal velocity fluctuation in the inner region, $z^+ \leq z_{\sigma_u,\mathrm{m1}}^+$, is presented with various x and Ra in Fig. 9(a) and 9(b), respectively. The inset of Fig. 10(a) shows that $z_{\sigma_{u,\mathrm{m1}}}^+$ seems independent of the horizontal position, and has an Ra-scaling $z_{\sigma,\mathrm{m1}}^+ = 0.42 Ra^{0.15}$. The inner peak value $\sigma_{u,\mathrm{m1}}^+$, on the other hand, presents a "U-shape" along the horizontal direction, as seen in the inset of Fig. 10(b). The strong velocity fluctuation near the stagnation point is associated with intensive plume ejecting at the corner and sweeping downward to the plate thereafter. While the fluid approaching the right sidewall, Re_{Mx} and adverse pressure gradient for $x > 0.55$ are largely increased.[10] As a result, the fluctuation increases again at the right corner of the RB cell. Moreover, the horizontal velocity fluctuation at $x = 0.5$ is approximated as a power law of Ra, $\sigma_{u,\mathrm{m1}}^+ = 0.16 Ra^{0.125}$.

Comparison of the RB and PWJs shows that $\sigma_{u,\mathrm{m1}}^+$ for the RB system is smaller than both the turbulent PWJ ($\sigma_{u,\mathrm{m1}}^+ \approx 3.0$ for $Re_{Mz} \approx 5000$)[59] and the turbulent channel (and pipe) flow ($\sqrt{3.66 + 0.642 \ln Re_\tau}$ and saturates at $\sqrt{9.2}$).[65] Due to the finite Ra and data scattering, it is hard to distinguish whether $\sigma_{u,\mathrm{m1}}^+$ satisfies square-root-log law or power law. If $\sigma_{u,\mathrm{m1}}^+$ satisfies a scaling, it can be extrapolated to $\sqrt{9.2}$, and the critical Rayleigh number would be 1.67×10^{10}.

The height of the first peak, $z_{\sigma_{u,\mathrm{m1}}}^+$, is smaller than that in the turbulent PWJ ($z_{\sigma_{u,\mathrm{m1}}}^+ \approx 20$–$40$ in Ref. 59), the turbulent channel, and pipe flow ($z_{\sigma_{u,\mathrm{m1}}}^+ \approx 14$ with the direct measurement[66,67] and theoretical value[65] $z_{\sigma_{u,\mathrm{m1}}}^+ = 9.8\sqrt[3]{2}$). By extrapolating the scaling of $z_{\sigma_{u,\mathrm{m1}}}^+$ to 14, one has the critical Rayleigh number is about 1.42×10^{10}, consistent with the extrapolation of $\sigma_{u,\mathrm{m1}}^+$. It implies that the flow is developed turbulence when Ra is larger than 1×10^{10}.

Figure 11 presents the profiles of $\sigma_w^+ = \sqrt{\overline{w'^2}^+}$ for different horizontal locations and Ras normalized with the peak value $\sigma_{w,\mathrm{m}}^+$ and the corresponding scale $z_{\sigma_{w,\mathrm{m}}}^+$. A peak appears farther away from the wall near $\sigma_{u,\mathrm{m2}}^+$.[68] Figure 11(a) shows the similarity of the vertical profiles of σ_w^+ in the range of $z^+ < z_{\sigma_{w,\mathrm{m}}}^+$. However, it seems that these two scales are not intrinsic for Ras, as seen in Fig. 11(b).

The Ra dependence of $z_{\sigma_{w,\mathrm{m}}}^+$ and $\sigma_{w,\mathrm{m}}^+$ is shown in Fig. 12. The magnitude of $z_{\sigma_{w,\mathrm{m}}}^+$ and $\sigma_{w,\mathrm{m}}^+$ can be approximated as power laws of Ra, that is, $z_{\sigma_{w,\mathrm{m}}}^+ = 0.14 Ra^{0.30}$ and $\sigma_{w,\mathrm{m}}^+ = 0.53 Ra^{0.08}$, respectively. The $\sigma_{w,\mathrm{m}}^+$ grows with increasing Ra, and the value for the highest Ra in present cases closes to the value of high Re WJ ($\sigma_{w,\mathrm{m}}^+ \approx 3$ in Ref. 68) and much higher than those in turbulent pipe and channel flow[65,67] ($\sigma_{w,\mathrm{m}}^+ \approx \sqrt{1.29}$), which may be caused by the intensive thermal production near the plate.

(a)

(b)

FIG. 12. (a) The height of the peak of vertical velocity fluctuation $z_{\sigma_{w,\mathrm{m}}}^+$ and (b) the magnitude of the fluctuation $\sigma_{w,\mathrm{m}}^+$ at $x = 0.5$ as the power functions of Ra. The insets are variations at different horizontal locations for $Ra = 5 \times 10^8$ and 5×10^9.

(a)

(b)

FIG. 11. The profiles of vertical velocity fluctuation σ_w^+ normalized by the peak value $\sigma_{w,\mathrm{m}}^+$ and the corresponding height $z_{\sigma_{w,\mathrm{m}}}^+$ at (a) various horizontal locations at $Ra = 5 \times 10^8$ and (b) various Ras at $x = 0.5$. The insets are the profiles free from normalization.

F. TKE and dissipation rate

Turbulent transport of RB-WJ is presented by the ratio of the production to dissipation of turbulent energy, as shown in Fig. 13. The turbulence production consists of two parts: shear-induced $P_s = \overline{u_i' u_j'} \frac{\partial U_i}{\partial x_j}$ and buoyancy-induced $P_B = \overline{w' \theta'}$, that is, $P = P_s + P_B$. The dissipation is $\varepsilon = \overline{\left(\frac{\partial u_i'}{\partial x_j}\right)\left(\frac{\partial u_i'}{\partial x_j}\right)}$.

The distributions of these variables are qualitatively consistent with TBL[67] and turbulent PWJ.[68] Two peaks of the production are, respectively, located at the edge of the viscous sublayer and the bulk region. In the mixing layer, the turbulence energy is defected and transported from above and below this region.[47] Compared with TBL having a quasi-equilibrium region[67] for $z^+ > 200$, the flow in RB system is rather nonequilibrium. The TKE is relatively low in the region
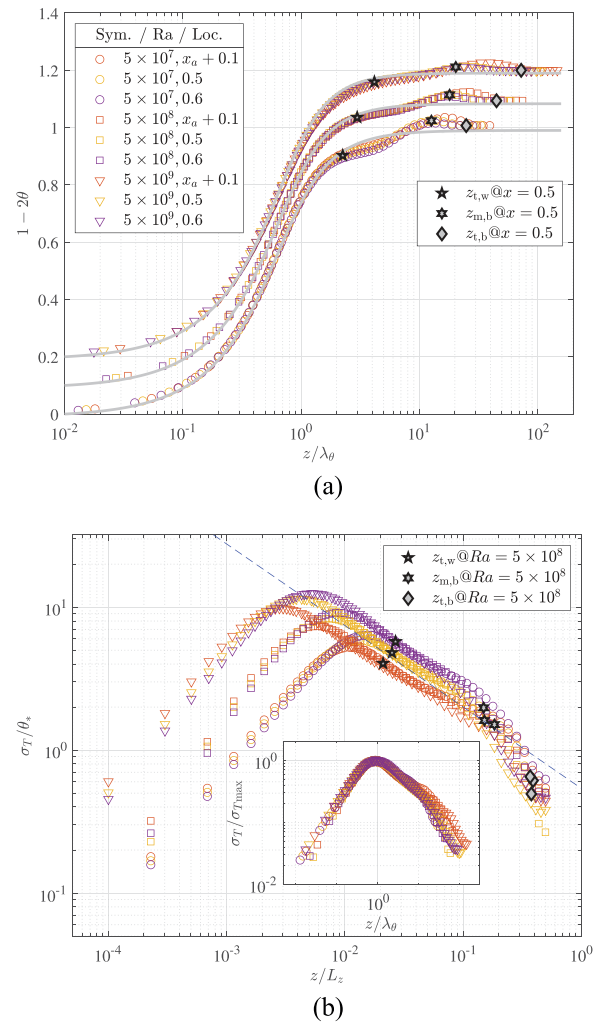
(a)

(b)
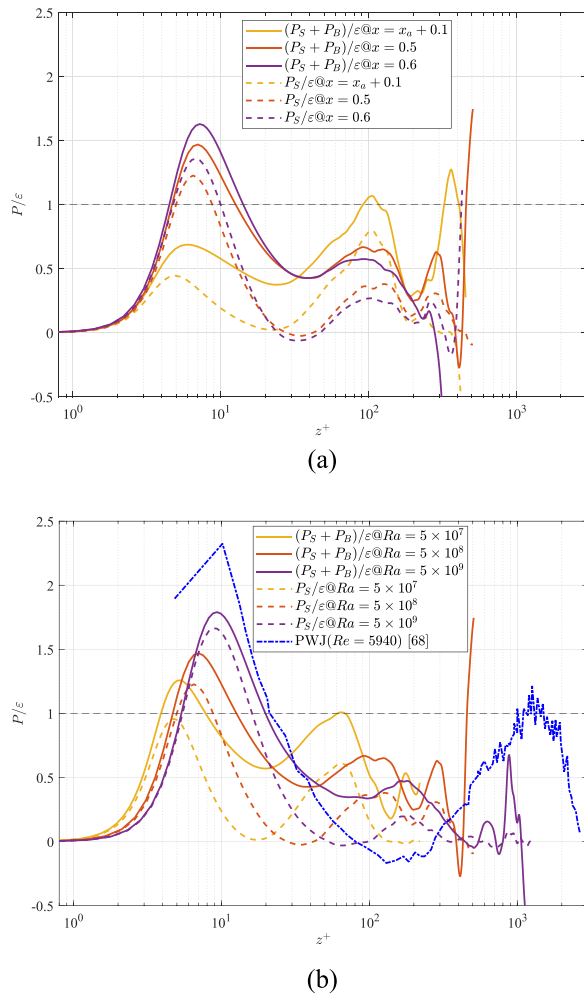
FIG. 13. The ratio of turbulent energy production ($P_s + P_B$ and P_s) to dissipation ε at (a) various horizontal locations with $Ra = 5 \times 10^8$ and (b) various Ra at $x = 0.5$. The dashed line of $P/\varepsilon = 1$ represents the balance between production and dissipation.

(a)

(b)

FIG. 14. Vertical profiles of mean temperature and temperature fluctuation: (a) mean temperature profiles at various horizontal locations and Ras. The gray curves are the thermal BL model proposed by Shishkina et al.[71] The curves for each Rayleigh number have been shifted upward with increment of 0.1. (b) Temperature fluctuation (rms) profiles normalized by θ_* at various locations and Ras. The dashed line is the power law from Refs. 74 and 75, $\sigma_T/\theta_* = 0.54(z/L_z)^{-0.57}$. The inset is the temperature fluctuations normalized by their maxima and the height normalized by λ_θ.

near the stagnation point. Due to the well-developed shear, the energy in the stagnation region is supplied by the overflow of upstream plume emission. When the flow develops and Ra increases, P/ε correspondingly increases in the near-wall region but decreases in the bulk. It becomes a distribution more like turbulent PWJ. Moreover, the buoyancy-induced production, P_B, has a remarkable effect on TKE at moderate Ra[69,70] in contrast to the shear-induced production, P_S, dominating the TKE at higher Ra.

IV. HEAT TRANSFER OF RB-WJ

We focus on the heat transfer of the RB-WJ in this section. The mean and rms of temperature profiles are analyzed at various horizontal locations and Ras. An OIJ-based heat transfer model is developed to describe the spatial distribution of the Nusselt number on the plate and the scaling law of the maximum Nusselt number (Nu_m).

A. Self-similar thermal BL solution

The WJ forms a thermal boundary layer on the plate. Figure 14(a) shows the temperature profile at various locations x with the length scale normalized by the thermal BL thickness $\lambda_\theta = L_z/(2Nu_x)$. The temperature within $z/\lambda_\theta < 4$ is described by the self-similar thermal BL equation with sublayer fluctuation proposed by Shishkina et al.,[71]

$$\theta = \int_0^\xi (1 + a^3\xi^3)^{-c} d\eta, \qquad (19)$$

where $a = \sqrt[3]{\frac{\overline{w'\theta'}}{\kappa\partial\theta/\partial z}}/\xi = \Gamma(1/3)\Gamma(c - 1/3)/(3\Gamma(c))$ is the coefficient of eddy thermal diffusivity in the thermal sublayer. The

coefficient a weakly decreases with Ra, from 1.34 at $Ra = 1 \times 10^7$ to 1.24 at $Ra = 5 \times 10^9$. We note that the model from Shishkina et al.[71] was derived under the assumption of thermal BL nested within the viscous BL, that is, $Pr \geq 1$, and empirically obtained $a \sim 1.2$ for $Pr \approx 1$. As Pr in the present study is slightly smaller than unity, the extrapolation of the original model in the manner of Ref. 72 is performed here, without introducing other parameters. Note that the error of the model becomes remarkable at the edge of the thermal BL.

Wellness of the thermal BL model for $z/\lambda_\theta < 4$ indicates that the thermal BL formed by the WJ is well developed in the region below $z_{t,w}$.[43,73] The inset of Fig. 14(a) shows that the temperature residue exists outside $z_{t,w}$, though it is used to be considered uniform in RBC. The uneven temperature profile in the bulk region has been observed by Zhou et al.[29] We marked the critical positions of $z_{t,w}$, $z_{m,b}$ and $z_{t,b}$ in Fig. 14. One can see that they have a close correlation with the temperature profile, indicating that the residue outside the thermal BL is mainly induced by the WJ flow.

The temperature fluctuation (rms) profiles, σ_T, at various Ras are shown in Fig. 14(b). The fluctuation of temperature is normalized by convective temperature $\theta_* = Nu_x^{2/3} Ra^{-1/3} Pr^{-1/3} \Delta T$ as suggested by Deardorff,[76] He and Xia,[74] and Xie et al.[75] The length is scaled by the height of the cell L_z. The location, $z_{t,w}$, is marked in the figure representing the thickness of the mixing layer. The profile at the central line [$x = 0.5$, red symbol in Fig. 14(b)] is consistent with the result from He et al., $\sigma_T/\theta_* = 0.54(z/L_z)^{-0.57}$. The power law distribution of σ_T consistent with Adrian's theory[77] is derived from the assumption of matching fluctuating velocities of the bulk region and mixing layer. For the inner scale, the σ_T is normalized by maximal temperature fluctuation, σ_{Tmax}, and the length scale normalized by λ_θ suggested by Wang et al.[78,79] The inset of Fig. 14(b) shows that the similarity of thermal fluctuation exists within λ_θ over the WJ region for all simulated Ras.

B. Approximation of heat transfer distribution

The local Nusselt number on the plate, Nu_x, is calculated to examine the effect of the OIJ on heat transfer near the plate. Figure 15(a) demonstrates the local heat flux on the bottom conducting plate. It shows that the jet impinging at an angle on the left side of the conducting plate forms an asymmetrical heat flux distribution. The heat flux in the region of the WJ exhibits a similar trend as that in the corner roll. On the WJ side, the heat flux decreases while moving away from the stagnation point. Heat transfer in this region is attributed to the LSC sweeping over the plate where the fluid follows the path on the flat plate. Outside this region near the vertical wall, the heat flux exhibits a similar trend due to the existence of CR, as seen in Fig. 1(a). It is noteworthy that the vortices at $x < 0.3$ also play a role in heat transfer at corners. The influence of CR on heat transfer in RBC has been investigated by Zhou et al. in Ref. 29.

Figure 15(b) shows the time-averaged heat flux in dimensionless form along the horizontal midline at different Rayleigh numbers. The convective motions are strong near the sidewalls due to uprising plumes. Thermal plumes are hardly seen at the cell center, and hence, the convective heat transfer is weak there. Due to the sweeping effect of the large-scale circulation, hot plumes fall downward and cold plumes rise near the sidewalls; therefore, negative heat transfer might occur herein.

(a)

(b)
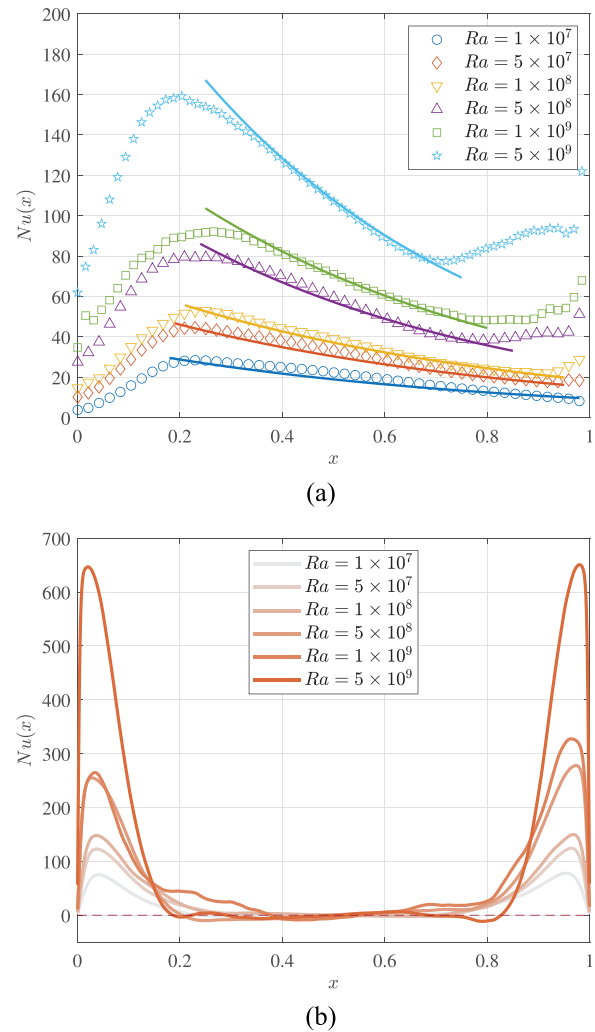
FIG. 15. The local Nusselt number Nu_x (a) at the bottom plate $z = 0$ and (b) at the middle height $z = 0.5$ with various Ras. The solid curves in (a) are the results of Eq. (20).

The horizontal distribution of the heat transfer on the plate can be described by an exponential function

$$Nu_x = Nu_m e^{-\tilde{x}}, \quad (20)$$

where $\tilde{x} = \Delta x/L_w$, Nu_m is the maximum Nusselt number on the plate, $\Delta x = x - x_{a0}$, and $L_w = L_x - x_{a0} - \Delta x_{ej}$, where Δx_{ej} is the width of the plume ejecting region at the right corner of the RB cell. The parameter x_{a0} is found to be a constant $x_{a0} = 0.282$ for the simulated Rayleigh numbers. The Nusselt number as a function of the streamwise location is then approximated as the exponential function of Eq. (20). The error of the function is not more than 15.3%, as seen in Fig. 15(a).

The exponential decay in x-direction near the stagnation points is equivalent to the exponential growth of thermal BL. The development of the thermal BL is consistent with the similarity analysis of BL

by Cantwell[80] and Shishkina *et al.*[71] On the other hand, Goldstein *et al.* have proposed an empirical formula in form of an exponential function in studying OIJ,[37]

$$Nu = A \mathrm{Re}^{0.7} e^{-(B+C\cos\theta_i)(r/D)^n},\qquad(21)$$

where D is the width of the jet, θ_i is the impinging angle, and A, B, C, n are parameters. For the turbulent OIJ, the empirical values are $A = 0.107$, $B = 0.47$, $C = 0.23$, and $n = 0.75$ for the distance from the jet nozzle to the surface $L_0/D = 10$. Here, only two parameters for RBC need to be determined, that is, the stagnation point heat transfer Nu_m and the length scale L_w to characterize the OIJ in RBC. Those two parameters should be related to Ra or Re, and need more data to examine the influence.

The scenario of OIJ makes it possible to understand the Nu_x distribution. Figure 15(a) shows that Nu decreases in the wind direction, which seems to conflict with the view of forced convective BL. For TBL, the heat flux is closely associated with the shearing on the plate. As the flow develops, the shearing is strengthened, and thus, Nu increases. However, due to the WJ being induced by OIJ, the jet impingement at the stagnation point dominates heat transfer rather than local shear stress, and the heat flux decays as the fluid convecting downstream.

C. Derivation of maximum Nusselt number

Equation (20) shows that the heat transfer is characterized by the stagnation heat transfer Nu_m. The relation between Nu_m and Ra number is derived by scaling analysis of the temperature equation for OIJ in this section. The temperature equation with respect to the WJ-formed thermal BL is as follows:

$$u\frac{\partial\theta}{\partial x} \sim \kappa\frac{\partial^2\theta}{\partial z^2}.\qquad(22)$$

The left side of the above equation is convection term, in which the velocity is the characteristic velocity of OIJ, $U_{m,b}$, as discussed in Sec. III. The length scale in horizontal direction is similar to the scale of slip line (or the scale of CR), r_{cr}. The right side of the equation is the diffusion term with the scale of thermal BL, λ_θ. It is interpreted that the convection term $\frac{U_{m,b}\Delta T/2-0}{r_{cr}}$ represents the heat flux of OIJ passing through the slip line, impinging onto the plate and dissipating in the thermal BL. Based on this fact, we have

$$\frac{U_{m,b}}{r_{cr}} \sim \kappa\frac{1}{\lambda_\theta^2}\qquad(23)$$

and

$$Nu = \frac{L_z}{2\lambda_\theta} \sim \frac{1}{2}\sqrt{\frac{U_{m,b}L_z^2}{\kappa r_{cr}}}$$
$$= \frac{1}{\sqrt{2}}Re_{lsc}^{1/2}\mathrm{Pr}^{1/2}(r_{cr}/L_z)^{-1/2}.\qquad(24)$$

The Reynolds number of LSC, $Re_{lsc} = U_{m,b}L_z/(2\nu)$, is estimated by Re_m in Eq. (7) and $Re_{lsc} \sim Ra^{0.50}$. One the other hand, the scaling of the CR size, $r_{cr}/L_z \sim Ra^{-0.085}$, was suggested in Ref. 29. Finally, we have
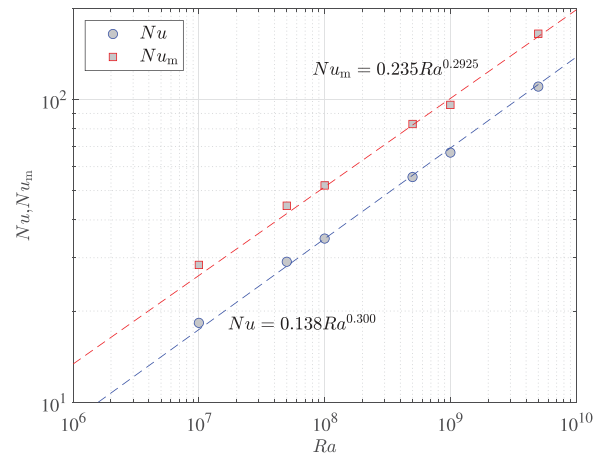

FIG. 16. The maximum Nusselt number Nu_m and the global Nusselt number Nu as the power functions of Ra. The red dashed line is the scaling law for Nu_m as described by Eq. (25), and the blue dashed line is the scaling law for Nu.

$$Nu_m \sim Re_m^{1/2}(r_{cr}/L_z)^{-1/2} \sim Ra^{0.2925}.\qquad(25)$$

Figure 16 shows that Nu_m of DNS is in agreement with the scaling model. Note that the relation between Nu and Re with regard to the scale of CR gives an effective exponent 0.585 larger than 0.5 in Ref. 81, but smaller than both the impinging jet (0.7) in Ref. 37 and TBL (0.8). This indicates that within the range of the simulated Rayleigh numbers, the impinging jet is still under development throughout the domain. Furthermore, Nu_m and the global Nu have the same scaling[14,82] as seen in Fig. 16. This indicates that the WJ characterizes the flow near the plate, and the global Nu is determined by the heat transfer of the WJ.

V. CONCLUSIONS

In this paper, an oblique impinging jet is found to dominate momentum and heat transfer on the plate in the Rayleigh–Bénard convection. The OIJ originates from the plume-emission region and contains abundant cold/warm fluid, which drives the motion of LSC. When the jet impingement occurs, the flow develops into a wall jet. The knowledge of turbulent plane wall jet is conducive to understanding both the oblique impinging jet and the wall jet of RBC.

We found that the wall jet in the RB system conforms to the nature of the turbulent plane wall jet along with finite Reynolds number effects. The similarity of the velocity along the plate indicates the coherent motion of the wall jet, which consists of three layers, respectively, called the viscous sublayer, the mixing layer, and the bulk. The layers are, respectively, scaled by the wall friction velocity u_τ and the wall unit for the viscous sublayer, the first peak velocity $U_{m,b}$ and the half-height $z_{t,b}$ for the bulk, the second peak velocity $U_{m,w}$, and its height $z_{t,w}$ for the mixing layer. The characteristic parameters, $U_{m,w}$, $z_{t,w}$, and u_τ, being normalized by the local momentum rate M_w, preserve power laws with Ra as they are in turbulent plane wall jet.

The streamwise similarity of the thermal boundary layer in the wall jet region is analyzed, and the temperature profiles are well described by the thermal boundary layer model from Shishkina for $z < 4\lambda_\theta$. Further analysis reveals that the heat transfer exponentially decays in x-direction, $Nu_x = Nu_m\exp(-\tilde{x})$. We derived the power

law of the maximum Nu with Ra by scaling analysis, $Nu_m \sim Ra^{0.2925}$, consistent with DNS.

The oblique impinging jet and the wall jet provide insights into the nature of inhomogeneous heat transfer of a turbulent RB system. The second peak of horizontal velocity fluctuation σ_u^+ is the property of turbulent plane wall jet, and the temperature residue in the wind-shearing region is attributed to the bulk flow. The occurrence of the maximum heat transfer is interpreted as the result of the energy-containing oblique jet impinging on the plate. The exponential decay of heat transfer along the plate is the nature of the wall jet in the RB system. All results indicate that the near-wall flow and heat transfer of turbulent RBC at moderate Rayleigh numbers depend on the behaviors of the oblique impinging jet and the wall jet.

ACKNOWLEDGMENTS

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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