LETTERS

The purpose of this Letters section is to provide rapid dissemination of important new results in the fields regularly covered by The Physics of Fluids. Results of extended research should not be presented as a series of letters in place of comprehensive articles. Letters cannot exceed three printed pages in length, including space allowed for title, figures, tables, references and an abstract limited to about 100 words.

Laser anemometer measurements of Reynolds stress in a turbulent channel flow with drag reducing polymer additives

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Turbulence measurements with a high resolution laser Doppler anemometer in a channel flow with injection of a concentrated polyethylene oxide solution are reported. With polymer injection sufficient to yield a well mixed concentration of 10 ppm the total drag is reduced by 38%, but the sum of the Reynolds stress and the conventional molecular shear stress is only $\frac{2}{3}$ of the force produced by the streamwise pressure gradient. Thus, the injected polymer solution has a significant effect on the turbulence and causes an additional retarding force equal to approximately $\frac{1}{3}$ of the total drag.

We report measurements of the effect of drag reducing polymer additives on the structure of turbulence and Reynolds stress in a fully developed channel flow using a two component, high resolution, laser Doppler anemometer (LDA).[1] The LDA optics allow the incident and scattered light to pass through spherical windows without refraction (see Fig. 1).

The two color LDA employs side scatter detection for each velocity component. Two pairs of 1.4 mm diam laser beams 30° apart are configured to measure the velocity components U_1 and U_2, inclined at $\pm 45°$ to the wall in a plane normal to the wall and parallel to the stream. Each beam passes through a 160 mm focusing lens to form a 50 μ diam waist ($1/e^2$ intensity) at the measurement volume. Two 50 μ diam images of the measurement volume are magnified (6×) and focused by 120 mm Achromat lenses on 300 μ diam pinholes in front of a blue or green laser line filter and RCA-type 4526 photomultipliers. A concentrated mixture of water and seeding particles (3 μ diam titanium dioxide in the rutile crystalline form) was injected in the settling chamber downstream of the polymer injection station at a rate to yield a single particle per 50 μ diam measurement volume. The data rate was between 1000 and 5000 samples/sec (depending on the distance from the wall). The velocity traces were reconstructed and smoothed by computer (see Wei[2] for examples). Seed particles were not injected with the polymer solution.

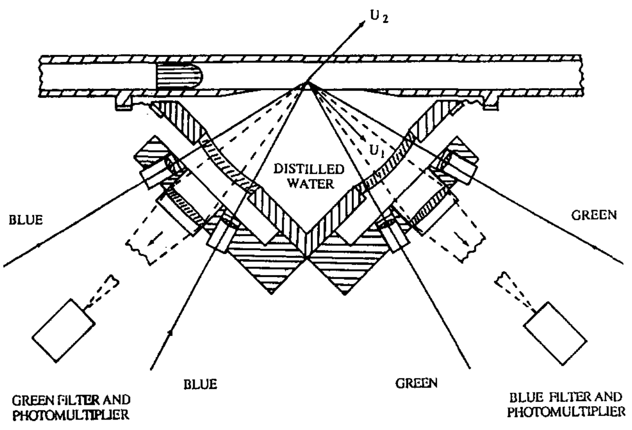

FIG. 1. Drawing of the LDA optical configuration at the measuring station.

The 1983 channel flow facility was improved to provide a more uniform fully developed flow. The 254 cm long channel is made from stainless steel panels bolted to 2.54 cm wide stainless steel bars spaced 30.48 cm apart. The channel dimensions (2.54 cm×30.48 cm) are accurate to within 0.05 mm. Manometers are installed every 15.24 cm along the length of the channel. The pressure gradient along the channel is constant. Further information and LDA alignment procedures for measurements in water at four Reynolds numbers are described by Wei.[2]

Here, LDA measurements were made of the mean and fluctuating velocity components parallel to the stream and normal to the wall for one run with water and two runs with injection of a polymer solution (1000 ppm) at four points in the settling chamber. The Reynolds number based on the average velocity and channel half-width was approximately 11 000. The polymer concentration was 10 ppm (assuming complete mixing) yielding approximately the same viscosity coefficient as water. The significant flow parameters are listed in Table I. The measurement volume diameter (waist diameter) and length (pinhole image diameter) were twice the (25 μ) viscous length scale.

The injection of polymer reduced the friction coefficient as measured from the pressure gradient by 38% (see Table I). The mean velocity profile in water agrees with the law of the wall (see Coles[3]) but was drastically changed by polymer injection (see Fig. 2). The normalized fluctuating velocity components are displayed in Fig. 3. The streamwise fluctuations are increased and the normal fluctuations reduced

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TABLE I. Channel flow parameters.

Date	26 June 1986	3 July 1986	10 July 1986
Polymer (POLYOX) concentration (ppm)	0.0	9.92	10.04
Friction velocity (cm/sec)	4.5	3.8	3.8
Average velocity (cm/sec)	83.3	89.5	88.5
Maximum velocity (cm/sec)	94.73	99.31	102.7
Kinematic viscosity (cm^2/sec)	0.010	0.010	0.010
Viscous length (μm)	22.2	26.3	26.3
Re based on average velocity	10 579	11 367	11 240
Friction coefficient = τ_w/q_{avg}	0.005 84	0.003 61	0.003 89

by polymer addition. The Reynolds stress profile for water and polymer are shown in Figs. 4 and 5.

The mean, streamwise momentum equation in a two-dimensional channel can be integrated from the wall, at $y = 0$, to the center line of the channel, at $y = b$, to obtain

$$\frac{\tau}{\tau_w} = 1 - \frac{y}{b} = -\frac{\overline{uv}}{u_\tau^2} + \frac{\nu(\partial U/\partial y)}{u_\tau^2}. \qquad (1)$$

Here, τ, the sum of the Reynolds stress and molecular shear stress decreases linearly from a maximum value, $\tau_w = -b(\partial p/\partial x)$, at the wall to zero at the center of the channel. The Reynolds stress from Eq. (1) is

$$-\frac{\overline{uv}}{u_\tau^2} = 1 - \frac{y}{b} - \frac{\nu(\partial U/\partial y)}{u_\tau^2}. \qquad (2)$$

The Reynolds stress in water calculated from Eq. (2) using measurements of $\partial U/\partial y$ and pressure gradient (to give u_τ^2) is compared (in Fig. 4) to the average product of the velocity fluctuations u and v directly measured using the LDA. The directly measured Reynolds stress (open circles) in Fig. 4 lie near the curved solid line calculated from Eq. (2). This is a very good check of the accuracy of the LDA measurements.

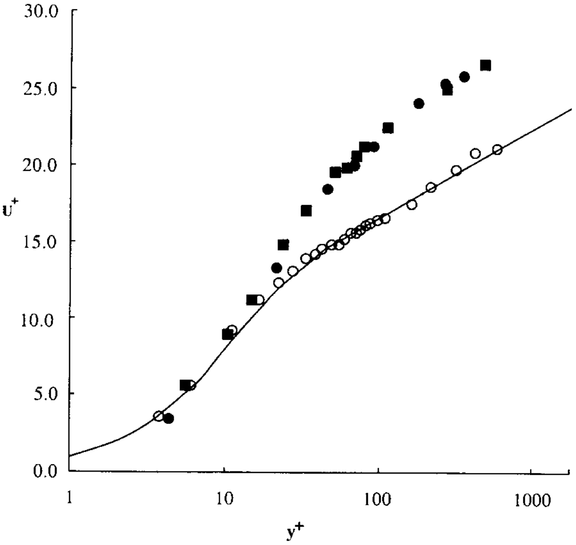

FIG. 2. Mean velocity profiles scaled on wall variables. Open circles: water, 26 June 1986; solid symbols: water with 10 ppm POLYOX; circles: 3 July 1986; squares: 10 July 1986; solid line: law of the wall (Coles[3]).

The result of the same procedure for the flow with polymer injection is that the Reynolds stress determined from the fluctuating velocity components (the solid points in Fig. 5) is *not* the same as the Reynolds stress calculated from Eq. (2) (the curved solid line). The measurements displayed in Fig. 5 indicate that at any point in the channel the total shear stress consists of a viscous stress $\mu\, \partial U/\partial y$, the Reynolds stress $-\rho\, \overline{uv}$, and an additional stress (of unknown origin) approximately equal to one-half of the Reynolds stress measured by the LDA. Further measurements are planned with different polymer concentrations.

The reviewer suggested that the results of Bewersdorff[4] who measured drag reduction in circular pipes with an injection of concentrated polymer solution on the pipe axis be cited in this Letter. Bewersdorff found significant drag reduction, but the polymer solution formed a thread which is visually observed to extend for large distances along the pipe. He also made two-component LDA measurements of the axial and radial velocity and the Reynolds stress. He shows (Fig. 22 of Bewersdorff[4]) that in a homogeneous, 20 ppm solution of polymer there is drag reduction with re-

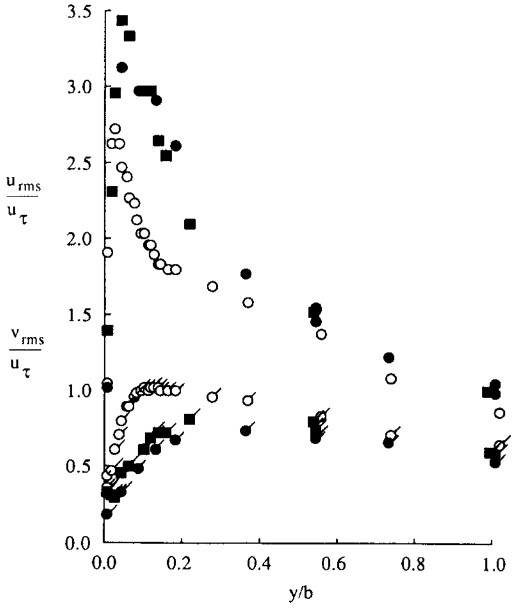

FIG. 3. Profiles of the velocity fluctuations normal to the wall (tagged points) and in the streamwise direction. Open circles: water, 26 June 1986; solid symbols: water with 10 ppm POLYOX, circles: 3 July 1986; squares: 10 July 1986.

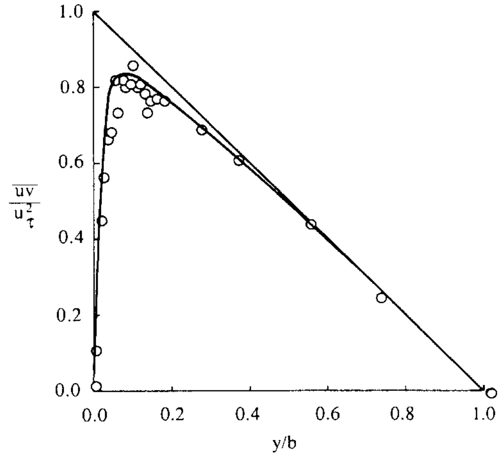

FIG. 4. Profiles of the Reynolds stress in flow with water. Open circles: 26 June 1986 direct measurements with LDA; straight line: total shear stress [Eq. (2)]; curved line: calculated from Eq. (3).

duced Reynolds stress, but there is significantly greater drag and Reynolds stress reduction when concentrated polymer solution is injected. Bewersdorff[4] did not compare the directly measured Reynolds stress to the value calculated from Eq. (2). The reviewer (who has analyzed data of Bewersdorff[4] unavailable to us) says, "As an example for an injection of 0.3% polyacrylamide solution to give 20 ppm of well mixed polymer, there is a difference in Reynolds stress calculated from Eq. (2) and the measured uv average in the range $20 < y^+ < 200$. At $y^+ = 50$ which corresponds to $y/R = 0.13$, the measured Reynolds stress is 50% of that calculated from Eq. (2). Beyond $y^+ = 200$ or $y/R = 0.5$ both methods yield the same result."

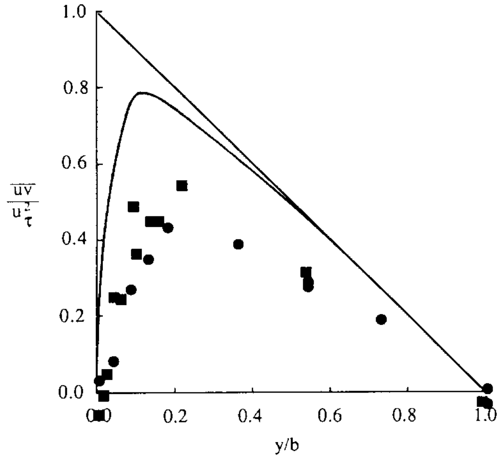

FIG. 5. Profiles of the Reynolds stress in flow with ≈ 10 ppm POLYOX. Solid circles: 3 July 1986; solid squares: 10 July 1986 direct measurements with LDA; straight line: total shear stress [Eq. (2)]; curved line: calculated from Eq. (3).

Our measurements show that the Reynolds stress is less than the calculated value from Eq. (2) all across the channel, but the measurements of Bewersdorff[4] show that the Reynolds stress is unaffected for distances from the wall greater than 0.4 times the pipe radius. This suggests to us that in our experiments with concentrated polymer injection at four points in the settling chamber a matrix of unmixed polymer threads may be present in the fluid, somewhat analogous to a composite material, which has unusual stress producing properties. We emphasize that in our measurements the seed particles were not placed in the injected polymer. If the polymer solution has formed threads which are not well mixed with the water the velocity of injected polymer solution is not included in our LDA measurements. We would welcome comments or suggestions for further measurements and interpretation of the results described above.

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[2] T. Wei, Ph.D. thesis, University of Michigan, 1986. See, also, T. Wei and W. W. Willmarth, Bull. Am. Phys. Soc. **31**, 10, BA4 (1986).

[3] D. E. Coles, in *50 Jahre Grenzschichtforschung*, edited by H. Goertler and W. Tollmien (Vieweg, Braunschweig, 1955), p. 153.

[4] H. W. Bewersdorff, Reologica Acta **23**, 522 (1984).

The velocity skewness measured in grid turbulence

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The slow decay of turbulent kinetic energy downstream of the grid is shown to contribute to the skewness of the velocity fluctuations. Estimates are given that show that energy decay has much stronger influence on the odd-order moments than the even-order moments.

In a number of experimental studies of homogeneous turbulence downstream of a grid the turbulent velocity fluctuations have been observed to have significant skewness. For isotropic turbulence the velocity skewness should be zero. Even when care is taken to ensure that the grid generated turbulence is approximately isotropic significant skewness is still observed. Various explanations for this apparent departure from isotropy have been advanced. Frenkiel and