Laser impulse generation and interferometer detection of zero group velocity Lamb mode resonance

Dominique Clorennec, Claire Prada,[a] and Daniel Royer
Laboratoire Ondes et Acoustique, ESPCI-Université Paris 7-CNRS UMR 7587, 10 rue Vauquelin, 75231 Paris Cedex 05, France

Todd W. Murray
Department of Aerospace and Mechanical Engineering, Boston University, 110 Cummington Street, Boston, Massachusetts 02215

(Received 15 March 2006; accepted 22 May 2006; published online 10 July 2006)

In this letter, we describe experiments on the generation of the first order symmetric (S_1) Lamb mode by a pulsed yttrium aluminum garnet laser. The vibration of the plate is detected at the same point by a heterodyne interferometer. The acoustic signal is dominated by the resonance at the point of the dispersion curve where the group velocity vanishes. The time decay of the signal leads to the local attenuation coefficient of the material. The spectrum exhibits a very sharp peak, the frequency of which is sensitive to the plate thickness. For a 0.49-mm-thick Duralumin plate, thickness variations as small as 0.1 μm have been detected. Moving the detection point away from the source allows us to record the standing wave pattern resulting from the interference between the S_1 and S_{2b} Lamb waves having opposite wave vectors at the zero group velocity point. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2220010]

The propagation of Lamb waves, i.e., guided acoustic waves in a solid plate, is represented by a set of dispersion curves giving the phase velocity of each symmetric (S) and antisymmetric (A) mode versus the product of the frequency f and the plate thickness d.[1] It is now well known that some Lamb modes exhibit an anomalous behavior at frequencies where the group velocity vanishes while the phase velocity remains finite.[2] This phenomenon exists for the first order symmetric (S_1) mode in an isotropic solid having a Poisson ratio smaller than 0.45.[3] At this zero group velocity (ZGV) point and for a given energy propagation direction (from the source to the observation point), two waves can propagate spatially with opposite wave vectors, with the same phase velocity, without any energy transfer.[4] In an early paper,[5] Tolstoy and Usdin pointed out that this ZGV point "must be associated with a sharp cw resonance and ringing effects." Holland and Chimenti[6] found that this Lamb wave resonance allows efficient transmission of airborne sound waves through a plate. Using broadband, focusing, air-coupled transducers, they exploited this property for imaging discontinuities and flaws in a thick plate. Recently, Prada *et al.* reported on the generation and detection of high frequency Lamb waves in thin plates using an amplitude modulated laser diode and a Michelson interferometer.[7] With a scanning of the modulation frequency they show that the thermoelastic source is efficiently coupled to the S_1-mode ZGV resonance.

Figure 1 shows the velocity dispersion curves of the lower order symmetric and antisymmetric modes for a plate of thickness d made of a material having longitudinal and transverse bulk wave velocities equal to $V_L = 6.34$ km/s and $V_T = 3.10$ km/s, respectively. The S_1-mode ZGV resonance occurs at the frequency-thickness product $f_1 d = 2.85$ MHz mm. At this point, the S_1-mode phase velocity is equal to $V_1 = 11.20$ km/s.

In our experiments, Lamb waves were generated by a Q-switched Nd:YAG (yttrium aluminum garnet) laser providing pulses having a 20 ns duration and 4 mJ of energy. The spot diameter of the unfocused beam is equal to 1 mm. Lamb waves were detected by a heterodyne interferometer[8] equipped with a 100 mW frequency doubled Nd:YAG laser. This probe is sensitive to any phase shift along the path of the optical probe beam. The calibration factor for mechanical displacement normal to the surface (10 nm/V) was constant over the detection bandwidth (50 kHz–20 MHz). Signals detected by the optical probe were fed into a digital sampling oscilloscope and transferred to a computer.

Experiments were carried out on a commercially available Duralumin plate of average thickness $d = 0.49$ mm and lateral dimensions equal to 100 and 150 mm. The laser energy absorption heats the air at the vicinity of the surface and produces a variation of the optical index along the path of the probe beam. The resulting phase shift induces a very large low frequency voltage, which saturates the electronic detection circuit during the first 20 μs. This spurious thermal ef-

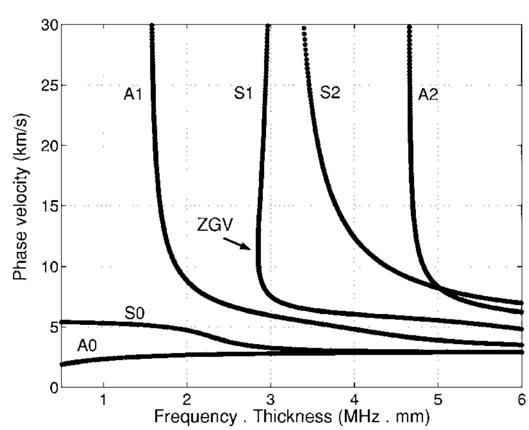

FIG. 1. Phase velocity dispersion curves for a Duralumin plate of thickness d and bulk wave velocities equal to $V_L = 6.34$ km/s and $V_T = 3.10$ km/s.

[a]Electronic mail: claire.prada-julia@espci.fr

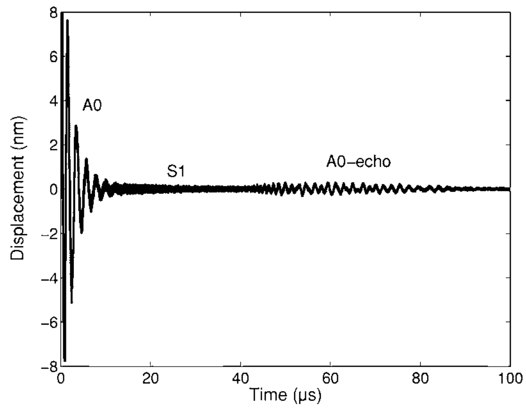

FIG. 2. Signal generated by the absorption of the YAG-laser pulse on a 0.49-mm-thick Duralumin plate and detected at the same point by an optical heterodyne interferometer.

fect was eliminated by interposing a high-pass filter having a cut-off frequency equal to 0.5 MHz. Figure 2 shows that the acoustic signal is composed of low frequency (LF) oscillations corresponding to the A_0 Lamb mode and of a high frequency (HF) part. The LF oscillations between 45 and 85 μs are ascribed to the reflections of the A_0 mode from the plate boundaries.

In Fig. 3, the fast Fourier transform computed with the first 300 μs of the signal shows the spread spectrum of the A_0 mode. However, the prominent feature is a sharp peak at 5.86 MHz. From the theoretical dispersion curve of the S_1 mode and taking into account the average plate thickness $d=0.49$ mm, the resonance is expected to occur at a frequency $f_1=5.81$ MHz, very close to the experimental value. The relative difference, smaller than 1%, lies in the uncertainty range of the material parameters and of the plate thickness. The small peak at 9.6 MHz corresponds to the thickness shear resonance at $fd=3V_T/2$ of the A_2 Lamb mode.

Owing to the high slowness of the energy propagation at frequencies f closed to the ZGV point ($V_g \leqslant 50$ m/s for $|f-f_1|<250$ Hz), it takes a long time (more than 1 ms) before the S_1-mode energy reaches the plate boundaries. In contrast to the A_0 mode signal, shown in Fig. 2, the S_1-mode resonance signal is unaffected by spurious reflections. It is thus possible to enlarge the time window up to 4 ms. As shown in Fig. 4, the signal exhibits a very slow exponential decay. Using a short-time Fourier transform, the measured decay constant was found to be $\tau=800$ μs and the Q factor was estimated to 14 700.

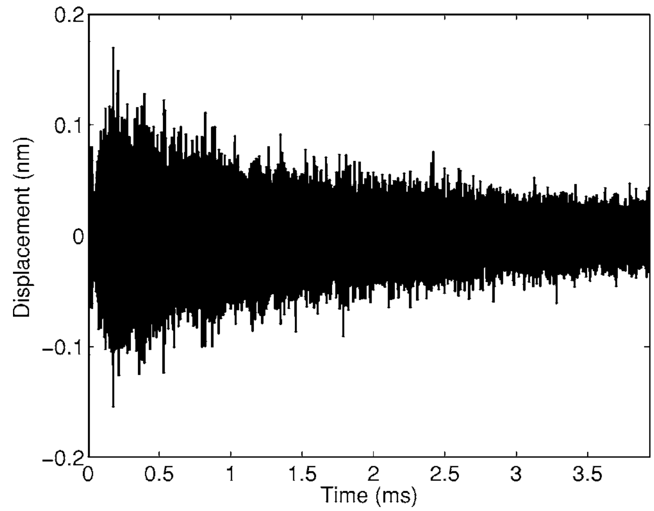

FIG. 4. 4 ms signal showing the slow decay due to the S_1-mode ZGV resonance.

Figure 5 shows, in a narrow frequency band, the spectrum of the signal in Fig. 4. The half-power bandwidth is approximately 430 Hz, giving a quality factor equal to 13 700. Since the resonance peak is slightly broadened by the signal windowing, this feature is smaller than the value previously deduced from the time domain decay. Such results are obtained if the local changes in the plate thickness d are smaller than $d/Q \cong 0.03$ μm over the laser spot diameter (1 mm). If this condition is not fulfilled, the resonance peak is either enlarged, in the case of a continuous thickness variation, or split, in the case of a step-shaped variation. Both patterns have been experimentally observed.

Assuming that the effects of nonparallel faces and surface roughness are negligible, the Q factor is only limited by the acoustic wave damping. Assuming a viscous loss mechanism, the attenuation coefficient α (m^{-1}) can be calculated from the formula

$$\alpha = \frac{\pi}{Q\lambda}, \tag{1}$$

where $\lambda = V_1/f_1$ is the acoustic wavelength at the ZGV point. With $Q=14\,700$ and since λ is of the order of 2 mm, the value found at the resonance frequency $f_1 \cong 5.86$ MHz is

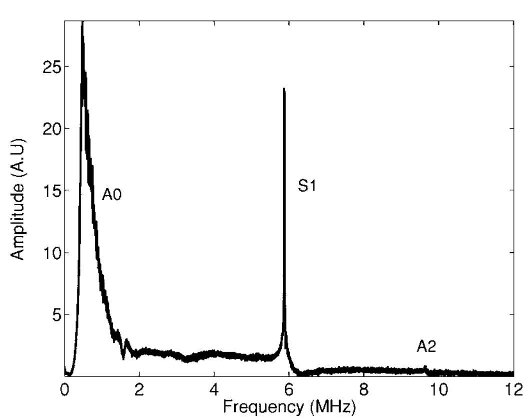

FIG. 3. Spectrum of the signal in Fig. 2. The peak at 5.86 MHz corresponds to the ZGV point of the S_1 mode.

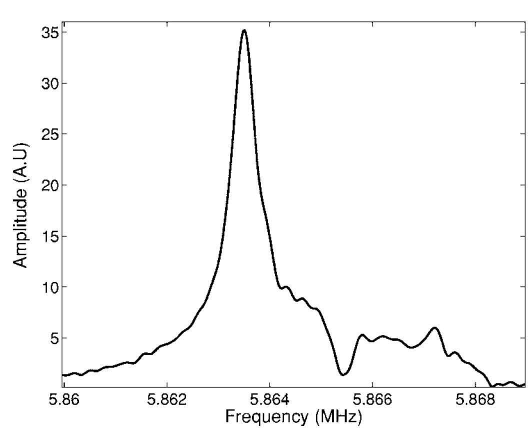

FIG. 5. Spectrum of the signal in Fig. 4.

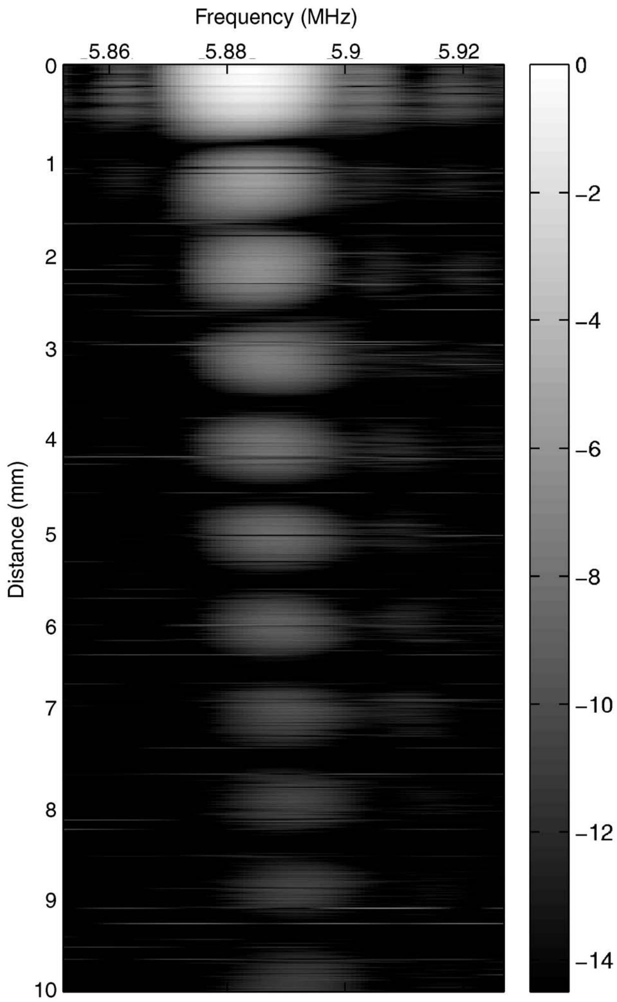

FIG. 6. Spatial distribution of the displacement amplitude (in dB) resulting from the interferences of the two counterpropagating waves S_1 and S_{2b}.

$$\alpha = 0.11 \text{ Np/m} = 0.93 \text{ dB/m}. \quad (2)$$

From the time decay of the diffuse ultrasonic field, time-damping factors as small as 2.2 dB/ms have been measured at 6 MHz in different aluminum-alloy samples by Haberer *et al.*[9] According to Weaver,[10] the diffuse field is dominated by the shear modes propagating at the velocity V_T=3.10 km/s. Then, the value found for the attenuation coefficient (0.71 dB/m) is close to our result indicating that material damping, rather than finite surface roughness effects or energy leakage to the surrounding media, is the dominant loss mechanism.

We have also observed the spatial distribution of the displacement amplitude in the vicinity of the S_1-ZGV point. The pattern in Fig. 6 reveals the interference of the two waves S_1 and S_{2b} propagating in opposite directions at the velocity $V_1 \cong 11.2$ km/s ($\lambda \cong 2$ mm) and generated with comparable amplitude. Since at this point the acoustic energy does not propagate, interferences observed far from the source are caused by wave components slightly upshifted. Lamb wave components having frequencies lower than f_1 are evanescent and then do not contribute at distances from the source larger than λ.

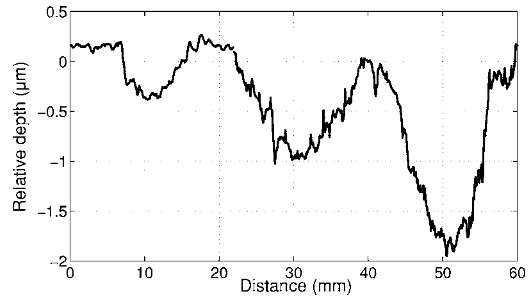

FIG. 7. Thickness variations of a Duralumin plate etched by an acid solution. The expected depths of the holes are 0.5, 1, and 1.5 μm.

From the variation Δf_1 of the S_1-mode resonance frequency, we deduced thickness variations Δd of the Duralumin plate: $\Delta d = -d\Delta f_1 / f_1$. Three holes were made on the back face of the plate by exposure to a $H_3PO_4-C_2H_5OH$ solution. The corrosion speed of this solution (0.05 μm/min on aluminium) is assumed to be the same for Duralumin. The exposure times were 10, 20, and 30 min, so that the holes are expected to be 0.5, 1, and 1.5 μm deep. A 60 mm scan of the plate with superimposed source and detection points was made before and after corrosion. Due to the surface roughness of the plate, the acquisition time was limited to 40 μs. The thickness variations Δd, proportional to the variations of the resonance frequency between the two scans, are plotted in Fig. 7, revealing holes of maximum depths of 0.5, 1, and 1.8 μm. These measurements are close to the expected values and confirm the potential applications of this method.

It was shown that the ZGV resonance that occurs at the minimum frequency of the S_1 Lamb mode is efficiently generated in the thermoelastic regime by a laser pulse. Quality factors of about 15 000 were observed on a commercially available Duralumin plate. We have shown that measuring the time decay of the resonance allows for a local measurement of the intrinsic attenuation coefficient of the material. From the shift of the resonance frequency, thickness variations as small as 0.1 μm have been detected. Similar zero group velocity resonance also occurs for higher modes depending on the Poisson ratio ν. For example, the A_2 mode exhibits a similar behavior for material of $\nu < 0.31$.[3] Application of this phenomenon to local attenuation and thickness measurements without any mechanical contact appears very promising.

[1]D. Royer and E. Dieulesaint, *Elastic Waves in Solids* (Springer, Berlin, 1999), Vol. 1.
[2]A. H. Meitzler, J. Acoust. Soc. Am. **38**, 835 (1965).
[3]K. Negishi, Jpn. J. Appl. Phys., Suppl. **26**, 171 (1987).
[4]M. Germano, A. Alippi, M. Angelici, and A. Bettuci, Phys. Rev. E **65**, 046608 (2002).
[5]I. Tolstoy and E. Usdin, J. Acoust. Soc. Am. **29**, 37 (1957).
[6]S. D. Holland and D. E. Chimenti, Appl. Phys. Lett. **83**, 2704 (2003).
[7]C. Prada, O. Balogun, and T. W. Murray, Appl. Phys. Lett. **87**, 194109 (2005).
[8]D. Royer and E. Dieulesaint, *Proceedings of the 1986 IEEE Ultrasonics Symposium* (IEEE, New York, 1986), p. 527.
[9]B. Haberer, M. Paul, H. Willems, and W. Arnold, in *Nondestructive Characterization of Materials VI*, edited by R. E. Green, K. J. Kozaczek, and C. Ruud (Plenum, New York, 1994), pp. 317–324.
[10]R. L. Weaver, J. Acoust. Soc. Am. **71**, 1608 (1982).