Contents

List of Symbols xvii

Introduction 1

Part I The Electromagnetic Field 5

1 Geometrical Optics 7
1.1 General Aspects 7
1.2 Ray Transfer Matrices 9
1.2.1 One-Dimensional Optical Systems 9
1.2.2 Matrix Elements and Liouville's Theorem 19
1.2.3 Misaligned Optical Systems 29
1.2.4 Two-Dimensional Optical Systems 34
1.2.5 Rotation and Tilt 38
1.2.6 The ABCD Law for the Radius of Curvature 47
1.2.7 Eigensolutions and Eigenvalues 50
1.3 Optical Resonators and Ray Transfer Matrices 52

2 Wave Optics 57
2.1 Huygens' Principle and Kirchhoff-Integral 57
2.2 Diffraction 61
2.2.1 Rectangular Aperture 61
2.2.2 Circular Aperture 67
2.3 Collins-Integral 71
2.3.1 One-Dimensional Optical Systems 71
2.3.2 Two-Dimensional Optical Systems 73
2.4 Collins-Integral and Vanishing Ray Matrix Elements 75
2.4.1 Imaging Condition (B=0) 75
2.4.2 Fourier Transformation (A=0) 76
2.5 Gaussian Beams 80
2.5.1 Gaussian Beams in One-Dimensional Optical Systems 80
2.5.2 Elliptical Gaussian Beams 91

2.6 Intensity Moments and Beam Propagation 96
2.6.1 Stigmatic and Simple Astigmatic Beams 96
2.6.2 Generalized Astigmatic Beams 102
2.6.3 Beam Quality 107
2.7 Coherence 110
2.7.1 Temporal Coherence 110
2.7.2 Spatial Coherence 117
2.8 Diffraction Theory of Optical Resonators 134
2.8.1 Integral-Equation for the Electric Field Distribution 134
2.8.2 The Gaussian Beam as a Fundamental Resonator Mode 136
2.9 Plane Wave Presentation of Diffraction 138
2.10 Diffraction-Free Beams 143
2.11 Beam Propagation in Anisotropic Crystals 150

3 Polarization 153
3.1 General Aspects 153
3.2 Jones Matrices 156
3.2.1 Definition 156
3.2.2 Matrices for Rotated Polarizing Optics 161
3.2.3 Combination of Several Polarizing Optics 163
3.3 Eigenstates of Polarization 167
3.4 Polarization in Optical Resonators 169
3.4.1 Eigenstates of the Roundtrip Jones Matrix 169
3.4.2 Polarization and Diffraction-Integrals 170
3.5 Depolarizers 171
3.6 Momentum and Angular Momentum of a Beam 173
3.6.2 The Poynting Vector of Structured Beams 176
3.6.3 Angular Momentum 178

Part II Basic Properties of Optical Resonators 187

4 The Fabry Perot Interferometer 189
4.1 General Aspects 189
4.2 The Fabry Perot Interferometer 191
4.2.1 Passive Fabry Perot Interferometer 191
4.2.2 Applications of FPIs 197
4.2.3 Fabry Perot Interferometer with Gain - The Laser Resonator 199
4.3 Optical Coatings 204
4.3.1 Coating Design Matrix Method 204
4.3.2 Quarter Wavelength Systems 209
4.3.3 Coating Methods and Materials 214

Part III Passive Open Resonators 217

5 Stable Resonators 219
5.1 General Aspects 219
5.2 Unconfined Stable Resonators 221
5.2.1 Transverse Mode Structures 222
5.2.2 Resonance Frequencies 233
5.2.3 The TEM_{00}-Mode 235
5.2.4 Higher Order Modes 242
5.2.5 Focusability and Beam Quality 250
5.3 Aperture Limited Stable Resonators 259
5.3.1 One Aperture Limited Mirror 261
5.3.2 Two Aperture Limited Mirrors 267
5.4 Misalignment Sensitivity 271
5.4.1 One Aperture Limited Mirror 273
5.3.2 Two Aperture Limited Mirrors 277

6 Resonators on the Stability Limits 281
6.1 Resonators with $g_1g_2=1$ 281
6.2 Resonators with One Vanishing g-Parameter 285
6.3 The Confocal Resonator 287

7 Unstable Resonators 295
7.1 General Aspects 295
7.1 Geometric-Optical Description of Unstable Resonators 296
7.2.1 Beam Propagation 296
7.2.2 Focusability 303
7.3 Diffraction Theory 311
7.3.1 Mode Structure, Beam Quality, and Losses 311
7.3.2 Applications of Unstable Resonators 317
7.4 Misalignment Sensitivity 317
7.5 Unstable Resonators in Off-Axis Geometry 323
7.6 Unstable Resonators with Homogenous Output Coupling 328
7.7 Unstable Resonators with Graded Reflectivity Mirrors 330
7.7.1 Resonator Properties 330
7.7.2 Production of VRMs 334
7.7.3 Laser Performance of VRM Unstable Resonators 337

8 Resonators with Internal Lenses 341
8.1 Resonators with Internal Lenses 341
8.2 Resonators with Polarizing Optics 344
8.2.1 The Twisted Mode Resonator 346
8.2.2 Resonators with Variable Output Coupling 347
8.2.3 The Pockels Cell Resonator 349

8.2.4 Resonators with Radially Birefringent Elements 351
8.2.5 Resonators with Azimuthally Birefringent Elements 353
8.2.6 Resonators with Radial-Azimuthally Birefringent Elements 355
8.2.7 Compensation of Radial-Azimuthal Birefringence 358

Part IV Open Resonators with Gain 365

9 The Active Medium 367
9.1 General Aspects 367
9.2 Effective Length of a Resonator 368
9.3 Amplification and Efficiencies 370
9.4 The Laser Equations 374
9.5 Line Broadening and Hole Burning 382
9.5.1 Homogeneous and Inhomogeneous Line Broadening 382
9.5.2 Spatial Hole Burning 386
9.6 Spectral Gain Distribution and Frequency Pulling 388
9.7 The Spectral Linewidth of Laser Modes 391

10 Output Power of Laser Resonators 393
10.1 Output Power of Stable Resonators 393
10.1.1 Linear Resonators 393
10.1.2 Folded Resonators without Beam Overlap 402
10.1.3 Folded Resonators with Beam Overlap 403
10.1.4 Ring Resonators 408
10.2 Output Power of Unstable Resonators 410

11 Influence of Gain on Mode Structure and Loss 413
11.1 General Aspects 413
11.2 Stable Resonators 414
11.2.1 Fundamental Mode Operation 414
11.2.2 Transverse Multimode Operation 422
11.3 Unstable Resonators 425
11.3.1 Mode Structure and Loss 425
11.3.2 Optimum Extraction Efficiency 427
11.4 Mode Structure and Steady State Condition 431

12 Q-switched Laser Resonators 433
12.1 General Aspects 433
12.2 Rate Equations for Q-switching 436
12.2.1 Inversion Densities 436
12.2.2 Energy, Pulse Duration and Peak Power 438
12.3 Optimum Output Coupling 441
12.4 Repetitive Q-switching 443

13 Resonators with Variable Internal Lenses 451
13.1 General Aspects 451
13.1.1 Thermal Lensing in Solid State Lasers 451
13.1.2 Ray Transfer Matrices 454
13.2 Stable Resonators 457
13.2.1 Fundamental Mode Operation 457
13.2.2 Transverse Multimode Operation 460
13.2.3 Beam Radii, Divergence, and Beam Quality 465
13.2.4 Output Power and Beam Quality 467
13.2.5 Output Power in Fundamental Mode Operation 473
13.2.6 Spherical Aberration 476
13.3 Unstable Resonators 488
13.3.1 Beam Propagation 488
13.3.2 Positive Branch Confocal Unstable Resonators 490
13.3.3 Rod-Imaging Unstable Resonator 494
13.3.4 Near Concentric Unstable Resonator 497
13.3.5 Beam Quality and Focusing 500

14 Resonators with Several Active Media 505
14.1 General Aspects 505
14.2 Output Power and Efficiency 507
14.2.1 Oscillator Arrangement 507
14.2.2 Oscillator-Amplifier Arrangement 508
14.3 Multirod Solid State Lasers 509
14.3.1 The Equivalent g-Diagram 509
14.3.2 Beam Quality and Output Power 511
14.3.3 Multirod Resonators with Variable Reflectivity Mirrors 515

15 Misalignment Sensitivity of Output Power 517
15.1 General Properties 517
15.2 Stable Resonators in Multimode Operation 519
15.2.1 Without Thermal Lensing 519
15.2.2 With Thermal Lensing 523
15.3 Stable Resonators in Fundamental Mode Operation 529
15.4 Unstable Resonators 531
15.4.1 Without Thermal Lensing 531
15.4.2 With Thermal Lensing 535

16 Resonators with Nonlinear Elements 539
16.1 General Aspects 539
16.2 Intracavity Second Harmonic Generation 540
16.2.1 Basic Properties of SHG 540
16.2.2 Efficiency of Intracavity Second Harmonic Generation 548
16.2.3 Phase Mismatch, Axial Modes, and Conversion Efficiency 552

16.2.4 Resonator Configurations 554
16.3 Intracavity Third Harmonic Generation 558
16.3.1 General Properties of Third Harmonic Generation 558
16.3.2 Properties of Intracavity Third Harmonic Generation 563
16.4 Resonators with Phase-Conjugate Mirrors 566
16.4.1 General Properties of a Phase-Conjugate Mirror 566
16.4.2 Optical Resonators with a Phase-Conjugate Mirror 567
16.4.3 Phase-Conjugate Resonators using SBS 574

Part V Special Resonator Concepts 583

17 Prism Resonators 585
17.1 Porro Prism Resonator 585
17.2 Corner Cube Prism Resonator 591

18 Fourier Transform Resonators 595
18.1 Self-Filtering Unstable Resonators 595
18.2 Stable Fourier Transform Resonators 600

19 Hybrid Resonators 605
19.1 General Aspects 605
19.2 Unstable-Stable Resonators 606
19.3 Waveguide Resonators 607
19.3.1 Motivation 607
19.3.2 Eigenmodes of Hollow Waveguides 609
19.3.3 Properties of Waveguide Resonators 622
19.3.4 Properties of Slab Waveguide Lasers 638
19.3.5 Solid State Waveguide Lasers 643

20 Resonators for Annular Gain Media 647
20.1 Characteristics of Annular Gain Lasers 647
20.2 Stable Resonators with Toric Mirrors 649
20.2.1 Transverse Mode Structure 649
20.2.2 Beam Quality 651
20.3 Herriot Cell Resonators 654
20.4 Unstable Resonators 658
20.4.1 Toric Unstable Resonators 658
20.4.2 Azimuthally Unstable Resonators 660
20.4.3 Spherical Unstable Resonators 663

21 Ring Resonators 665
21.1 General Properties of Ring Resonators 665

21.2 Unstable Ring Resonators 670
21.3 Nonplanar Ring Resonators 673

22 Single Mode Laser Resonators 675
22.1 Axial Mode Spectrum of Lasers 675
22.2 Axial Mode Selection with Intracavity Elements 677
22.3 Axial Mode Selection in Coupled Resonators 679
22.4 Resonators for Homogeneously Broadened Lasers 682
22.5 Micro-optical Resonators 684

Part VI Measurement Techniques 687

23 Measurement of Laser Parameters 689
23.1 Measurement of Losses, Gain, and Efficiency 689
23.1.1 Findlay-Clay Analysis 689
23.1.2 Delay-Time Analysis 697
23.1.3 Measurement of Diffraction Losses 701
23.1.4 Measurement of the Saturation Intensity 703
23.2 Measurement of Thermal Lensing 704
23.2.1 Focusing of an Expanded Probe Beam 705
23.2.2 Deviation of a Collimated Probe Beam 707
23.2.3 Change in Laser Properties 708

24 Measurement of Laser Beam Parameters 713
24.1 Measurement of Beam Quality 713
24.1.1 The Beam Propagation Factor 713
24.1.2 ISO Standardized Methods 715
24.1.3 Beam Attenuation 722
24.1.4 Beam Quality Analyzers 722
24.2 Measurement of Polarization 724
24.3 Measurement of the Phase 727

References 731

Index 787