LATENT POLICIES:

AN EXTENDED EXAMPLE

Richard Arnott*

Brian Sack**

and

Chong-en Bai*

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*Department of Economics
Boston College

**Department of Economics
M.I.T.

Chestnut Hill
MA 02167
USA

Cambridge
MA 02139
USA

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Abstract

Arnott and Stiglitz (1993) have argued that, in competitive insurance markets with moral hazard, equilibrium may entail firms offering latent policies **--** policies that are not bought in equilibrium but are kept in place to deter entry. This paper provides an extended example of such an equilibrium, which not only proves that latent policies can be present in equilibrium but also elucidates the mechanism which makes them potentially effective in deterring entry.

* This paper is an outgrowth of a long-term research collaboration between Arnott and Joseph Stiglitz on moral hazard in general equilibrium. Accordingly, the authors would like to acknowledge the intellectual debt they owe, in the writing of this paper, to Joseph Stiglitz.

Latent Policies: An Extended Example

There is still no consensus on how to model competitive insurance markets with moral hazard. The activities of insurance firms are shrouded in secrecy. As a result, there are few "stylized facts" to guide the model-building exercise, and the theorizing must be done largely in a vacuum. Pauly (1974) provided the first analysis of competitive equilibrium with moral hazard. He considered a stylized static economy with identical individuals (to abstract from issues relating to adverse selection) and two events, "accident" and "no accident," where an individual's probability of accident depends on her accident-prevention effort which is unobservable to the insurer (moral hazard). He considered two situations. In the first, the individual's total insurance purchases are unobservable. Pauly assumed that this would result in the market determining the price of insurance, with the individual being able to purchase as much insurance as she wants at this price. We refer to this as price insurance. Pauly argued that (with event-independent utility) the individual would purchase full insurance and expend zero effort at accident avoidance. In the second situation, the individual's total insurance purchases are observable. Pauly argued that, in this situation an insurance contract would specify both a price and a quantity of insurance, with the individual being rationed in the amount of insurance she can purchase at this price, and would prohibit her from purchasing supplementary insurance from other firms. We refer to this as quantity insurance. By providing less than full insurance, the contract would leave the individual with an incentive to expend accident-prevention effort and would in fact efficiently trade off risk-bearing and incentives.

Helpman and Laffont (1975) pointed out that, in his analysis of price insurance, Pauly neglected the nonconvexities to which moral hazard may give rise, and Arnott and Stiglitz (1994) provided an analysis of price insurance taking these nonconvexities into account.

Other researchers have investigated the middle ground where insurers may have incomplete information on their clients' purchases from other companies. Hellwig (1983) endogenized communication between insurance firms concerning their clients' purchases, and Bizer and de Marzo (1992) assumed that insurance companies can monitor clients' past purchases of insurance but not their future purchases.

Stiglitz (1983) and Arnott and Stiglitz (1993) provided a more thorough treatment of the situation where insurance firms cannot observe their clients' aggregate purchases of insurance. They argued that the form of the contracts offered by insurance companies should be derived rather than assumed. They proceeded by gradually expanding the set of admissible contracts, at each step determining the competitive equilibrium conditional on the set of admissible contracts. They employed an equilibrium concept analogous to that employed by Rothschild and Stiglitz (1976) in their analysis of competitive equilibrium with adverse selection. The equilibrium is characterized by a group of incumbent firms choosing to offer contracts that maximize expected profits conditional on deterring entry. Specifically, with the contracts offered by incumbent firms taken as fixed, any utility-improving contract offered by an entering firm must make a loss. When there is only one incumbent firm, a case Arnott and Stiglitz (1993) focus on, the incumbent firm acts as a Stackelberg leader, taking into account how potential entrants will react to the contracts it offers, but not reacting to the contracts offered by potential entrants.

Arnott and Stiglitz (1993) argued that, in these circumstances and with this definition of competitive equilibrium, incumbent firms may choose to offer latent policies -- policies that are not bought in equilibrium but protect their active policies against entry. This possibility arises due to the nonconvexities generated by moral hazard.

To elaborate, suppose that there is a single incumbent firm offering a contract which contains an active quantity policy A and a latent quantity policy L. Faced with this pair of policies, the individual purchases only policy A, which we write as $(A;\ A,L)$. And with only policy A purchased, the policy makes a profit, which we write as $\pi(A;A) \geq 0$, where the second argument refers to the total amount of insurance purchased. Now suppose that a potential entrant offers policy E, and that:

i) In the absence of the latent policy, the insured would purchase E either by itself or in combination with A, and E would make a profit; i.e.,

$$(E;A,E) \cap \left(\pi(E;E) > 0\right) \ \underline{\text{or}}\ \ (E + A;A,E) \cap \left(\pi(E;E + A) > 0\right).$$

ii) With the latent policy offered, the insured would purchase E in combination with L, and perhaps with A as well, and E would make a loss; i.e.,

$$(E + L;A,L,E) \cap \left(\pi(E;E + L) < 0\right) \ \underline{\text{or}}$$
$$(E + L + A;A,L,E) \cap \left(\pi(E;E + L + A) < 0\right).$$

Thus, in the absence of the latent policy, E would upset A as an equilibrium, but with the latent policy offered, E would not upset (A,L) as an equilibrium. The latent policy therefore protects A against E.

The latent policy exploits moral hazard -- when the insured purchases more insurance, she reduces her accident-prevention effort which increases the probability of accident. In the absence of the latent policy, the insured's aggregate insurance purchases are small enough that E makes a profit. But when L is offered, the insured's aggregate insurance purchases increase sufficiently that the probability of accident rises by enough to render E unprofitable.

When L protects A against all entering policies that would, in the absence of L, upset A as an equilibrium, we say that L protects A against entry. This is the idea behind a <u>latent policy equilibrium</u>, which is characterized by the most profitable contract that can be protected against entry by the use of latent policies.

Arnott and Stiglitz (1993) argued that latent policy equilibria are plausible. This paper provides a complete example, which proves that these equilibria can indeed arise. Furthermore, working out the example provides insight into how latent policies operate and how they exploit the nonconvexities to which moral hazard may give rise.

Section 1 presents the basic two-event moral hazard model of Arnott and Stiglitz (1988). To keep the analysis manageable, attention is restricted to the simple case with only two possible levels of effort. Section 2 investigates equilibrium with quantity insurance but without latent policies for a particular example with a single incumbent firm. Section 3 considers the same example but with latent policies admitted. Section 4 discusses the assumption that the incumbent does not react to contracts offered by potential entrants. Section 5 concludes.

1. The Basic Model

We employ the simplest possible model with moral hazard. Individuals are identical, and uncertainty is characterized by two states of nature. For each individual, either a fixed-damage accident occurs or it does not, and accident probabilities are statistically independent across individuals. Prior to insurance, if the accident does not occur the individual receives w, while if it does occur the individual receives $w-d$, where d is the accident damage. Insurance firms cannot monitor their clients' purchases of insurance from other firms. Insurance contracts are assumed to be in the form of quantity insurance. As discussed in Arnott and Stiglitz (1993), allowing only price insurance would be too restrictive since an insurance firm may attempt to influence a client's total insurance purchases by restricting the quantity of insurance it will offer her.[1] Insurance is available with a premium of β and a net payout in the event of accident of α.

[1]Most insurance policies restrict the total amount of coverage the policy offers.

The individual has the binary choice whether to exert effort to prevent the accident or not.[2] If she does exert effort, the accident probability is p^s, while if she does not, it is $p^r\left(> p^s\right)$. Her expected utility with the safe activity is

$$EU^s = \left(1 - p^s\right)u\left(w - \beta\right) + p^s u\left(w - d + \alpha\right) - e \equiv V^s\left(\alpha, \beta\right), \tag{1a}$$

where e is the disutility of effort. And her expected utility with the risky activity is

$$EU^r = \left(1 - p^r\right)u\left(w - \beta\right) + p^r u\left(w - d + \alpha\right) \equiv V^r\left(\alpha, \beta\right). \tag{1b}$$

Note, in (1a) and (1b), that expected utility is separable between effort and consumption and that the utility-from-consumption functions are event-independent. Moral hazard arises in the provision of insurance since an insurance firm is unable to observe whether or not a client exerts effort to prevent the accident.

Figure 1: The Basic Diagram

To explore this model, we employ the diagrammatic set-up developed in Arnott and Stiglitz (1988) -- see Figure 1. In $\alpha - \beta$

[2] More generally, the individual has the choice between undertaking a risky activity or a safe activity.

space, plot first the switching locus, Φ, along which $V^s(\alpha,\beta)=V^r(\alpha,\beta)$. Below the locus (in $\mathfrak{S}$) -- for low levels of insurance -- the individual chooses to take care. Above the locus (in $\mathfrak{R}$) the individual chooses to exert no accident-prevention effort. Next plot the zero-profit locus (ZPL), $\Pi(\alpha,\beta)=0$, where

$$\Pi(\alpha,\beta)=\begin{cases}\Pi^s(\alpha,\beta)=(1-p^s)\beta-p^s\alpha & \text{for } (\alpha,\beta)\in\mathfrak{S}\\ \Pi^r(\alpha,\beta)=(1-p^r)\beta-p^r\alpha & \text{for } (\alpha,\beta)\in\mathfrak{R}.\end{cases} \tag{2}$$

Denote by $\mathfrak{F}$ the feasible set -- the set of (α,β) for which $\Pi(\alpha,\beta)\geq 0$. The shaded area in Figure 1 is the complement of $\mathfrak{F}$. Next, plot a sample indifference curve, $V(\alpha,\beta)=\overline{V}$, where

$$V(\alpha,\beta)=\begin{cases}V^s(\alpha,\beta) & \text{for } (\alpha,\beta)\in\mathfrak{S}\\ V^r(\alpha,\beta) & \text{for } (\alpha,\beta)\in\mathfrak{R}.\end{cases} \tag{3}$$

Due to risk aversion, both $V^s(\alpha,\beta)=\overline{V}$ and $V^r(\alpha,\beta)=\overline{V}$ are convex. However, because the slope of $V(\alpha,\beta)=\overline{V}$ increases discontinuously across Φ, $V(\alpha,\beta)=\overline{V}$ is nonconvex. Finally, plot the full-insurance line (FIL), $\alpha+\beta=d$, along which the marginal utility of income is the same in both states. Note that the full-insurance line must lie beyond the switching locus since along the FIL, $V^r-V^s=e>0$.

2. <u>Competitive Equilibrium without Latent Policies</u>

As noted in the introduction, the nature of competitive equilibrium depends critically on whether an insurance firm can restrict its clients from purchasing supplementary insurance from other firms. If it can, then the competitive equilibrium is the exclusive contract equilibrium (described in Arnott and Stiglitz (1988)) which solves $\max_{\alpha,\beta} V(\alpha,\beta)$ s.t. $\Pi(\alpha,\beta)=0$. Here we are concerned with the alternative situation in which a firm cannot restrict its clients from purchasing supplementary insurance from other firms.

The equilibrium concept we employ is similar to that used by Rothschild and Stiglitz (1976) in their analysis of competitive insurance markets with adverse selection. There is a finite number of incumbent insurance firms indexed by $i\in\mathfrak{I}$, and an arbitrarily large number of potential entering firms. Each firm offers a single contract from a set of admissible contracts, $\mathbb{C}$, which includes the null contract. There is an arbitrarily large number of identical individuals, each of whom may purchase any subset of the offered contracts, but at most one contract per firm. Equilibrium is given by a set of admissible contracts offered by incumbent firms, $\{c_i\}$, such that:

i) Each individual purchases a subset of the offered contracts that maximizes her utility.

ii) Each contract offered by incumbent firms at least breaks even.

iii) No incumbent firm can increase its profits by replacing its contract with an alternative contract, taking the contracts offered by other incumbent firms as given.

iv) With the contracts offered by incumbent firms given, no potential entrant can offer a contract that would strictly increase individuals' utility and make a profit.

We term the equilibrium competitive because there are no (conventional) barriers to entry, there is no collusion between firms, and there is a large number of (potential) buyers and sellers of insurance contracts. However, the equilibrium is not equivalent to a situation in which a large number of competitors choose their actions simultaneously. The equilibrium concept entails asymmetric treatment of incumbent firms and potential entrants.

To simplify, we shall consider only the case in which there is a single incumbent firm.[3] We assume that the incumbent firm cannot withdraw or alter its contract once it is bought by an individual, but the individual can buy additional insurance or switch to a new contract from another insurance company. Under these assumptions, the equilibrium is given by the entry-deterring contract that is profit-maximizing for the incumbent firm.

[3] Arnott and Stiglitz (1993) discuss the case of two or more incumbent firms.

In this section, we assume that the set of admissible contracts includes only simple quantity contracts of the form (α,β), which specify a premium and a net payout in the event of accident. The profit made on insurance policy (α_0,β_0) when the individual purchases aggregate insurance of (α,β) is given by

$$\pi(\alpha_0,\beta_0;\alpha,\beta)=\begin{cases}(1-p^s)\beta_0-p^s\alpha_0 & \text{for } (\alpha,\beta)\in\mathfrak{S}\\ (1-p^r)\beta_0-p^r\alpha_0 & \text{for } (\alpha,\beta)\in\mathfrak{R}.\end{cases} \tag{4}$$

The incumbent firm's optimization problem is

$$\max_{\hat{\alpha},\hat{\beta}}\ \pi\left(\hat{\alpha},\hat{\beta};\hat{\alpha},\hat{\beta}\right)\quad \text{s.t.}$$

i) $$\mathfrak{X}\left(\hat{\alpha},\hat{\beta}\right)=\left\{\left(\overline{\alpha},\overline{\beta}\right)\middle|\left(V\left(\overline{\alpha},\overline{\beta}\right)>V\left(\hat{\alpha},\hat{\beta}\right)\right)\cap\left(V\left(\overline{\alpha},\overline{\beta}\right)>V\left(\hat{\alpha}+\overline{\alpha},\hat{\beta}+\overline{\beta}\right)\right)\right.$$

$$\left.\cap\left(\pi\left(\overline{\alpha},\overline{\beta};\overline{\alpha},\overline{\beta}\right)>0\right)\right\}=\varnothing$$

(5)

ii) $$\mathfrak{Y}\left(\hat{\alpha},\hat{\beta}\right)=\left\{\left(\overline{\alpha},\overline{\beta}\right)\middle|\left(V\left(\hat{\alpha}+\overline{\alpha},\hat{\beta}+\overline{\beta}\right)>V\left(\hat{\alpha},\hat{\beta}\right)\right)\cap\left(V\left(\hat{\alpha}+\overline{\alpha},\hat{\beta}+\overline{\beta}\right)>V\left(\overline{\alpha},\overline{\beta}\right)\right)\right.$$

$$\left.\cap\left(\pi\left(\overline{\alpha},\overline{\beta};\hat{\alpha}+\overline{\alpha},\hat{\beta}+\overline{\beta}\right)>0\right)\right\}=\varnothing,$$

iii) $$\pi\left(\hat{\alpha},\hat{\beta};\hat{\alpha},\hat{\beta}\right)\geq 0$$

iv) $$V\left(\hat{\alpha},\hat{\beta}\right)\geq V(0,0)$$

where $\mathfrak{X}\left(\hat{\alpha},\hat{\beta}\right)$ is the set of profitable entering policies the individual would choose to purchase alone, conditional on $\left(\hat{\alpha},\hat{\beta}\right)$, and $\mathfrak{Y}\left(\hat{\alpha},\hat{\beta}\right)$ is the set of profitable entering policies the individual would choose to purchase in addition to $\left(\hat{\alpha},\hat{\beta}\right)$. The firm will choose the policy $\left(\hat{\alpha}^*,\hat{\beta}^*\right)$ which solves this maximization problem. For the case of a single incumbent firm, Arnott and Stiglitz (1993) term $\left(\hat{\alpha}^*,\hat{\beta}^*\right)$ the (non-exclusive) quantity equilibrium or <u>Q-equilibrium</u> contract. In the event the maximization problem has no solution since no policy satisfies the constraints, a Q -equilibrium does not exist.

Rather than provide a complete analysis of equilibrium under these assumptions, we explore an example. We assume that

$p^s = .25 \qquad p^r = .5 \qquad u(\cdot) = \ln(\cdot) \qquad e = .1\ln 2 \qquad w = 4.0 \qquad d = 3.0.$

Then

$$
\begin{aligned}
&V^s(\alpha,\beta) = .75\ln(4-\beta) + .25\ln(1+\alpha) - .1\ln 2 \\
&V^r(\alpha,\beta) = .5\ln(4-\beta) + .5\ln(1+\alpha) \qquad (6) \\
&FIL: \quad \alpha + \beta = 3.0 \\
&\Phi: \quad 4 - \beta - 2^{.4}(1+\alpha) = 0.
\end{aligned}
$$

Figure 2: Competitive Equilibrium without Latent Policies

It turns out that, in the example, no Q-equilibrium exists. The reasoning, which draws on Figure 2, is as follows:

i) No point outside the feasible set can be a Q-equilibrium.

ii) No point in $((\mathfrak{S} \cap \mathfrak{F}) - \Phi)$ can be a Q-equilibrium.

Suppose there is such a point, such as B in the diagram. Since the slope of an indifference curve in $\mathfrak{S}$ $\left(\frac{d\beta}{d\alpha}\Big|_{\overline{V}} = \frac{p^s u'(w-d+\alpha)}{(1-p^s)u'(w-\beta)} \right)$ exceeds $\frac{p^s}{1-p^s}$, a small, positive policy which supplements B and has price between $\frac{d\beta}{d\alpha}\Big|_{\overline{V}}$ and $\frac{p^s}{1-p^s}$ would be bought and make a profit.

iii) No point on Φ or in $\mathfrak{R} \cap \mathfrak{F}$ strictly inside the FIL can be a Q-equilibrium.

Suppose there is such a point, such as C in the diagram. Since the slope of an indifference curve in $\mathfrak{R}$ inside the FIL $\left(\frac{d\beta}{d\alpha}\Big|_{\overline{V}} = \frac{p^r u'(w-d+\alpha)}{(1-p^r)u'(w-\beta)} \right)$ exceeds $\frac{p^r}{1-p^r}$, a small, positive policy which supplements C and has price between $\frac{d\beta}{d\alpha}\Big|_{\overline{V}}$ and $\frac{p^r}{1-p^r}$ would be bought and make a profit.

iv) No point in $\mathfrak{R} \cap \mathfrak{F}$ strictly outside the FIL can be a Q-equilibrium.

If there is such a point, it must be on or above the locus $\Pi^r = 0$ (which coincides with $\pi(\hat{\alpha},\hat{\beta};\hat{\alpha},\hat{\beta}) = 0$ in $\mathfrak{R}$) such as point D in the diagram. But then a replacement policy offering a slightly smaller quantity of insurance at the same or slightly higher price would be bought by itself and make a profit.

v) Only I, the point of intersection of $\Pi^r = 0$ and the FIL, can be a Q-equilibrium.

i) - iv) rule out all points except those on the FIL. Points above I on the FIL can be ruled out since for each there is a lower-priced replacement policy on the FIL which would be bought by itself and make a profit.

vi) I is not a Q-equilibrium.

In the example, the point G and not the point I is the point of maximum utility on the price line through I. G is a profitable replacement policy. Furthermore, individuals prefer zero insurance to the proposed equilibrium I.

The general result, proved in Arnott and Stiglitz (1993), is that if a Q-equilibrium exists with a single incumbent firm, it must be at the point I, at which profits are zero.

In the next section, we shall return to the example and show that, with an appropriately-chosen latent policy, equilibrium exists and the incumbent firm's (active) policy makes a profit.

3. Competitive Equilibrium with Latent Policies

We now allow the incumbent firm to offer two policies in a contract, while maintaining the restriction that the insured may purchase only one of each type of contract. The individual has the option of choosing neither policy, either one of the two policies, or both policies. We refer to the policies as the active policy $A \equiv \left(\hat{\alpha}, \hat{\beta}\right)$ and the latent policy $L \equiv \left(\tilde{\alpha}, \tilde{\beta}\right)$. A latent policy equilibrium or L-equilibrium exists iff there is a policy pair (A, L) such that: i) individuals purchase only the active policy, ii) the active policy makes nonnegative profits, and iii) any entering policy that would be bought is not profitable. The difficulty in proving that such an equilibrium exists stems from the range of options the individual has. He may choose to purchase the entering policy on its own or in combination with only the active policy, with only the latent policy, or with both the active and the latent policy. This range of possibilities makes general analysis difficult and is the reason we have chosen to present only an example.

Formally, an L-equilibrium exists with a single incumbent firm iff that firm offers a contract containing an active policy A and a latent policy L such that:

a) Offered the policy pair, the individual's utility is maximized by choosing only the active policy, *viz.*, $V(A) \geq V(0,0)$, $V(A) \geq V(A+L)$ and $V(A) \geq V(L)$.

b) The active policy is weakly profitable, *viz*., $\pi(A;A) \geq 0$.

c) There is no entering policy which would be profitable when the individual would choose to purchase the policy by itself. That is, $\nexists$ an $E \equiv (\overline{\alpha}, \overline{\beta})$ such that:

$$\begin{aligned} &\text{i)} && V(E) > V(A) \\ &\text{ii)} && V(E) > V(A+E) \\ &\text{iii)} && V(E) > V(L+E) \\ &\text{iv)} && V(E) > V(A+L+E) \\ &\text{v)} && \pi(E;E) > 0. \end{aligned} \tag{7}$$

d) There is no entering policy which would be profitable when the individual would chose to combine the policy with the active policy. That is, $\nexists$ an E such that:

$$\begin{aligned} &\text{i)} && V(A+E) > V(A) \\ &\text{ii)} && V(A+E) > V(E) \\ &\text{iii)} && V(A+E) > V(L+E) \\ &\text{iv)} && V(A+E) > V(A+L+E) \\ &\text{v)} && \pi(E;A+E) > 0. \end{aligned} \tag{8}$$

e) There is no entering policy which would be profitable when the individual would choose to combine the policy with the latent policy. That is, $\nexists$ an E such that:

$$\begin{aligned} &\text{i)} && V(L+E) > V(A) \\ &\text{ii)} && V(L+E) > V(E) \\ &\text{iii)} && V(L+E) > V(A+E) \\ &\text{iv)} && V(L+E) > V(A+L+E) \end{aligned} \tag{9}$$

v) $\pi(E; L+E) > 0$.

f) There is no entering policy which would be profitable when the individual would choose to combine the policy with both the active and the latent policy. That is, $\nexists$ an E such that:

i) $V(A+L+E) > V(A)$

ii) $V(A+L+E) > V(E)$ (10)

iii) $V(A+L+E) > V(A+E)$

iv) $V(A+L+E) > V(L+E)$

v) $\pi(E; A+L+E) > 0$.

The conditions for the existence of an L-equilibrium are rather formidable! Note that satisfaction of the above conditions establishes only that an L-equilibrium exists. The L-equilibrium contract is the <u>most profitable</u> pair (A, L) satisfying the conditions a) - f).

We perform two exercises using our example. In the first, we show that, with (A, L) = ((1.2709, .83435), (.44737, .44737)), an L-equilibrium exists. To demonstrate this, we must show that this policy pair satisfies conditions a) through f). First, note that the individual (weakly) prefers the active policy when offered the policy pair, which can be verified from the following calculations: V(1.2709, .83435) = 1, V(1.71828, 1.28172) = 1, V(.44737, .44737) = .97389, and V(0,0) = .97041. Second, the active policy is profitable: π((1.2709, .83435); (1.2709, .83435)) = .30804. Conditions a) and b) are therefore satisfied.

Conditions c) through f) are verified via a computer grid search. Let $E \equiv (\overline{\alpha}, \overline{\beta})$ denote the entering policy. Partition $\overline{\alpha} - \overline{\beta}$ space into regions in which the individual's utility would be highest if she were to purchase the entering policy alone, with the active policy, with the latent policy, and with both the active and latent policies, and if she did not purchase the entering policy. Exclude those entering policies which would not be bought. Then exclude those entering policies which would lose money. To illustrate, consider the set of contracts for which the individual's utility would be highest if she were to purchase the entering policy in addition to the latent policy. Then those E for which $(L+E) \in \mathfrak{S}$ and $(1-p^s)\overline{\beta} - p^s\overline{\alpha} < 0$ would be excluded, as would those E for which $(L+E) \in \mathfrak{R}$ and $(1-p^r)\overline{\beta} - p^r\overline{\alpha} < 0$. Repeating this exercise for all regions of $\overline{\alpha} - \overline{\beta}$ space, we find that the result for the example is the null set. Since the policy pair (A, L) makes positive profits and meets all the other conditions for the existence of an L-equilibrium, an L-equilibrium exists. The example illustrates that the presence of latent policies may indeed affect the characteristics of equilibrium. In our example, the introduction of a latent policy results in a positive profit L-equilibrium when, in the absence of a latent policy, a Q-equilibrium does not exist.

How does the introduction of a latent policy cause existence of equilibrium, when no Q-equilibrium exists in its absence? And how are positive profits sustainable in equilibrium? Recall, from the previous section, that in the absence of a latent policy many active policies could be upset as potential Q-equilibria by an entering firm offering a small, supplementary policy. In the example, this mechanism is short-circuited by the introduction of a latent policy. When such a small supplementary policy is offered, which is profitable when combined with only the active policy, the individual will choose to combine the small, supplementary policy with the latent policy and perhaps the active policy as well. As a result of purchasing the additional insurance in the latent policy, the individual switches from exerting positive effort to exerting no effort. This increases the probability of accident, which renders the small, supplementary, entering policy unprofitable. Thus, even though it is not purchased in equilibrium, the latent policy insulates the incumbent firm against entry even when it is making positive profits.

The second exercise we perform using our example is to solve for the L-equilibrium contract. This contract maximizes $\pi(A;A)$ with respect to A and L subject to constraints a) - f). Generally, this is a very complex maximization problem since it is three-tiered. Taking A, L, and E as given, the individual chooses what combination of policies to purchase. Treating A and L as given, the entering firm decides on the entering policy E to offer, taking into account how individuals will choose to combine policies. Finally, the incumbent firm chooses A and L so as to maximize profits, taking into account the responses of both the entering firm and insured individuals.

We have obtained some general results which allow us at least to winnow down the set of possibly-optimal incumbent policy pairs. Let A, A' etc. denote active policies and M, M' etc. represent the combination of the latent and the active policy, so that $L = AM = M$ - A (vector subtraction) etc. is the corresponding latent policy. For our example, the trivial case where the active policy is I and the latent policy is null can be immediately ruled out since the contract pair considered above indicates that positive profits will be sustained at the L-equilibrium. The question arises concerning the admissibility of negative policies (which pay out in the event of no accident). It can be shown that any equilibrium active policy must be positive. But we do not rule out the possibility of negative latent and entering policies.

Result 1: A and M must lie on the same indifference curve.

Since the latent policy is not bought in equilibrium, then (using obvious notation) $V(A) \geq V(M)$. Suppose the inequality is strict. Then the latent policy does not protect A against sufficiently small entering policies.

Result 2: A must lie in $\mathfrak{S}$ and M in $\mathfrak{R}$.

There are three other possibilities: i) A and M both lie in $\mathfrak{S}$; ii) A and M both lie in $\mathfrak{R}$; and iii) A lies in $\mathfrak{R}$ and M in $\mathfrak{S}$. Consider first possibility i). In this case L fails to protect A against small entering policies with price $\frac{\beta}{\alpha} > \frac{p^s}{1-p^s}$. It will later be demonstrated that the utility gradient becomes flatter as one moves out along an effort-fixed indifference curve. This implies that the entering policy will be bought in conjunction with A when L is positive, and in conjunction with M when L is negative. In either case, the individual continues to exert positive effort and the entering policy remains profitable.

We could run through the other cases. But it should be evident that the latent policy is valuable because it can cause the individual to (discontinuously) decrease effort in response to the introduction of a small, positive, supplementary policy.

<u>Result 3</u>: $\left(\frac{d\beta}{d\alpha}\right)_A^{\bar{V}} \leq \frac{p^r}{1-p^r}$

Suppose not. Then a small positive entering policy at a price between $\left(\frac{d\beta}{d\alpha}\right)_A^{\bar{V}}$ and $\frac{p^r}{1-p^r}$ would be bought (perhaps with the latent policy as well) and, since its price is greater than $\frac{p^r}{1-p^r}$, would be profitable.

Now take a point $A' \in \mathfrak{S} \cap \mathfrak{F}$ that is consistent with Result 3 and draw a line north-east of it with slope $\frac{p^r}{1-p^r}$. There are three possibilities. Either the line intersects the indifference curve through A' twice or not at all, or it is tangent to the indifference curve. To begin, we consider the first situation which is illustrated in Figure 3a.

Figure 3a: Reducing the set of possibly optimal incumbent policy pairs

Let M' denote the lower point of intersection, M'' the higher point of intersection, and M''' the point of intersection of the FIL with the indifference curve through A. Given the configuration, we now consider whether A' combined with some latent policy consistent with results 1 and 2 is possibly optimal. No latent policy such that M lies between Φ and M''' is consistent with equilibrium since a small positive supplementary policy at a price slightly above $\frac{p^r}{1-p^r}$ would be combined with M and make a profit. Similarly, no latent policy such that M lies beyond M''' is consistent with equilibrium since a small negative policy at a price slightly lower than $\frac{p^r}{1-p^r}$ would be combined with M and make a profit. Finally, M''' is inconsistent with equilibrium since it would be upset by a large entering policy with price above $\frac{p^r}{1-p^r}$, such as $A'J$ (which the individual would choose to combine with A' and which would be profitable).

Figure 3b: Reducing the set of possibly optimal incumbent policy pairs

Turn now to Figure 3b, which is like Figure 3a except that the line north-east from A' with slope $\frac{p^r}{1-p^r}$ does not intersect the indifference curve through A'. By the arguments in the above paragraph, only M''' cannot be ruled out as a candidate L-equilibrium. Is this configuration with $A = A'$ and $M = M'''$ therefore consistent with equilibrium? Since the portion of an indifference curve in $\mathfrak{S}$ has slope greater than $\frac{p^s}{1-p^s}$, profits increase as one moves up this portion of the indifference curve. Thus, profits are higher with a contract A further up the indifference curve as long as M''' continues to protect against entry. From the argument of the previous paragraph, we know that M''' fails to protect A when the line from A with slope $\frac{p^r}{1-p^r}$ intersects the upper portion of the indifference curve. If M''' protects A', does it continue to protect A when A is such that the line from A with slope $\frac{p^r}{1-p^r}$ is tangent to the upper portion of the indifference curve at M'''? Label the corresponding A, A''.

The answer is not necessarily. Take the following example. For the contract K, in Figure 3b, assume that $V(K) > V(K + A')$ and that the latent policy $A'M'''$ protects against K. In the absence of the latent policy, K would be bought by itself and be profitable, and hence would upset A'. That the latent policy protects against K implies either that $\left(V(K + M''') > V(K)\right) \cap \left(V(K + M''') > V(K + A'M''')\right)$ or $\left(V(K + A'M''') > V(K + M''')\right) \cap \left(V(K + A'M''') > V(K)\right) \cap \left(K + A'M''' \in \mathfrak{R}\right)$. That is, either K would be combined with both the latent and the active policies, rendering K unprofitable, or it would be combined with only the latent policy, rendering K unprofitable. But these results do not imply that A'', with $A''\,M'''$ in place, could not be upset by K. For example, the above set of inequalities does not rule out the possibility that

$$\left(V(K) > V(K + M''')\right) \cap \left(V(K) > V(K + A'')\right) \cap \left(V(K) > V(K + A''M''')\right).$$

Thus, we have established:

Proposition 1: In an L-equilibrium, the active policy must lie in $\mathfrak{S}$ and must be at a point at which: i) $\left(\frac{d\beta}{d\alpha}\right)_A^{\bar{V}} \leq \frac{p^r}{1-p^r}$, and ii) the line from A with slope $\frac{p^r}{1-p^r}$ does not intersect the portion of the corresponding indifference curve in $\mathfrak{R}$ or else is tangent to it. Also, the sum of the active and latent policies is at the point of intersection of the full insurance line and the indifference curve on which the active policy lies.

While these results do indeed winnow down the set of policy pairs that could be equilibria, the solution of the equilibrium policy pair is in general still a formidable task. For this example, however, we were able to solve for the L-equilibrium. The way we proceeded was to conjecture that the equilibrium policy pair has the characteristics of A'' and M''' in Fig. 3b, to solve for the optimum ignoring certain constraints, and then to verify that the constraints are satisfied.

Figure 4: The L-equilibrium in the example

The L-equilibrium is shown in Figure 4. An important point to note is that the lowest possible level of utility for the L-equilibrium is that at which the indifference curve is tangent to the $\Pi^r = 0$ locus. At a lower level of utility, there are utility-improving entering policies with price greater than $\frac{p^r}{1-p^r}$, which would therefore be profitable whatever policy combination the individual chooses. In this example, the contract pair corresponding to the lowest level of utility possibly consistent with an L-equilibrium is upset by an entering policy that would be bought alone. The L-equilibrium is instead at a slightly higher level of utility. It is found by increasing the level of utility from its lower bound while checking the most profitable contract on each indifference curve consistent with Proposition 1 -- having the configuration shown in the Figure -- and then stopping at the first contract to satisfy conditions a) - f) (point A in Figure 4). Then we check all contracts on lower-utility indifference curves with profits greater than profits at A, finding that none satisfied conditions a) - f). The solution has $A=\left(\hat{\alpha},\hat{\beta}\right)$ = (1.23415, .88502), $L = AM=\left(\tilde{\alpha},\tilde{\beta}\right)$ = (.44042, .44042), Π= .35522, and V = .98379.

A few words concerning the welfare properties of the L-equilibrium are in order. If, as in the example, an L-equilibrium exists while a Q-equilibrium does not, the two situations cannot be utility-ranked since utility with non-existence of equilibrium is not well-defined. If, however, both a Q-equilibrium and an L-equilibrium exist, then utility is unambiguously higher in the L-equilibrium than in the Q-equilibrium,[4] even though profits are positive in the L-equilibrium but zero in the Q-equilibrium. In these circumstances, therefore, the incumbent's being able to offer a latent policy results in an unambiguous welfare improvement.

Earlier we asserted that latent policies exploit the nonconvexities to which moral hazard may give rise. Return to Figure 4. A necessary condition for the latent policy L to be effective in deterring entry is that the individual prefer to combine a small supplementary policy with M rather than with A. This in turn requires that the utility gradient be steeper at M than at A. Suppose

[4]Suppose the contrary. Then an entrant could upset the L-equilibrium by offering a positive profit contract arbitrarily close to I.

that the small, supplementary policy is infinitesimal and has $\frac{\bar{\beta}}{\bar{\alpha}} = \tau$; *viz.* $(\bar{\alpha},\bar{\beta}) = (d\alpha, d\beta) = (d\alpha, \tau d\alpha)$. For the supplementary policy to be bought when combined with A, τ has to be less than the slope of the indifference curve at A, and for the supplementary policy to be profitable when combined with A, τ must be greater than $\left(\frac{p}{1-p}\right)_A$. Thus, $\tau \in \left(\left(\frac{p}{1-p}\right)_A, -\left(\frac{V_\alpha}{V_\beta}\right)_A\right)$. The slope of the utility gradient in the direction τ is

$$\left.\frac{dV}{d\alpha}\right|_\tau = V_\alpha + V_\beta \left.\frac{d\beta}{d\alpha}\right|_\tau = V_\alpha + \tau V_\beta. \tag{11}$$

Now consider how the slope of the utility gradient changes with movement up an indifference curve:

$$\begin{aligned}\left.\frac{d\left(\left.\frac{dV}{d\alpha}\right|_\tau\right)}{d\alpha}\right|_{\bar{V}} &= \frac{\partial\left(\left.\frac{dV}{d\alpha}\right|_\tau\right)}{\partial\alpha} + \frac{\partial\left(\left.\frac{dV}{d\alpha}\right|_\tau\right)}{\partial\beta}\left.\frac{d\beta}{d\alpha}\right|_{\bar{V}} \\ &= V_{\alpha\alpha} + \tau V_{\alpha\beta} + \left(V_{\alpha\beta} + \tau V_{\beta\beta}\right)\left(-\frac{V_\alpha}{V_\beta}\right) \\ &= \left\{V_{\alpha\alpha} - 2\frac{V_\alpha}{V_\beta}V_{\alpha\beta} + \left(\frac{V_\alpha}{V_\beta}\right)^2 V_{\beta\beta}\right\} \\ &\qquad + \left[\left(\tau + \frac{V_\alpha}{V_\beta}\right)\left(V_{\alpha\beta} - \frac{V_\alpha}{V_\beta}V_{\beta\beta}\right)\right].\end{aligned} \tag{12}$$

In the situation where effort is continuously variable, (12) can be decomposed into a negative risk aversion effect and a positive moral hazard effect (in the two-activity case, the moral hazard effect manifests itself as a discontinuous increase in the slope of the utility gradient across Φ). Thus, we may say without ambiguity that the utility gradient is steeper at M than at A because of moral hazard. The term in curly brackets in (12) is proportional to the curvature of an indifference curve, and it too can be decomposed into a negative risk aversion effect and a positive moral hazard effect. Thus, nonconvexity of the indifference curve is closely related to an increase in the slope of the utility gradient with movement up the indifference curve, but neither strictly implies the other. It is in this loose sense that we say that latent policies exploit the nonconvexities to which moral hazard may give rise.

4. Commitment

In the model, the incumbent offers insurance contracts to individuals before any contracts are offered by entering firms and does not react to contracts offered by potential entrants. These assumptions give rise to the advantage of incumbency. Rothschild and Stiglitz (1976) argue that these assumptions are appropriate in a competitive market such as the one we are considering. However, other equilibrium concepts that impose more foresight on the part of firms could be considered. Should entry occur, the incumbent firm may have an incentive to renege on its threat to maintain the entry-deterring contract and change to a new contract that improves its profit. Alternative equilibrium concepts such as a Wilson equilibrium[5] may require that an entering firm consider this when choosing the contract it will offer. The issue then becomes the credibility of the incumbent firm's commitment to maintain its original contract offer.

It appears that, in the absence of commitment, the incumbent would always change its original contract offer. For example, in the L-equilibrium, an entering policy that results in individuals undertaking the risky activity renders the incumbent's policy pair unprofitable. The incumbent can avoid loss by withdrawing both the active and the latent contracts. However, the insured individuals are made worse off by the withdrawal of these contracts.

The fact that the effectiveness of latent policies depends critically on the incumbent firm's ability to commit should not be discouraging. If we make the plausible assumption that an insurance

[5]In a Wilson equilibrium an entering firm would assume the withdrawal of all policies that become unprofitable as a result of the entering policy. Note that this equilibrium concept has been considered unnatural since it allows incumbent firms to respond by withdrawing policies but not by offering new ones.

contract requires the consent of both the firm and the policy holder to be changed, the firm may not be able to change the contract to improve its profit: contract changes that improve the profit of the firm may not be acceptable to the insured individuals and therefore will not be made. In other words, clients help an incumbent firm commit to its entry-deterring contract.

One possible equilibrium concept with latent policies that incorporates such considerations of commitment is as follows:

An <u>L-C</u> (latent policy with commitment) <u>equilibrium</u> is the most profitable policy pair (A, L) satisfying the following conditions:[6]

a) Offered the policy pair, the insured individual chooses A alone

b) The active policy is weakly profitable

c) For every entering contract E, the entrant makes negative profit given the incumbent's optimal feasible response. An incumbent's response, R (possibly the combination of A and L that the insured chooses given E), is feasible if

i) Given E, the incumbent (weakly) prefers R to (A,L), and

ii) Given E, the insured (weakly) prefers R to (A,L).

If condition c) is satisfied, entry is deterred. Unfortunately, analysis of the game implicit in this equilibrium definition is considerably more complicated than that for the game considered in the paper, and is left for future research.

5. <u>Conclusion</u>

Arnott and Stiglitz (1993) investigated competitive equilibrium in insurance markets with moral hazard. They considered the

[6]The extensive form of the game underlying this equilibrium concept is as follows. The incumbent leads by offering (A, L). The entrant then offers an entering policy, E, to which the incumbent responds by offering R. The insured individual chooses among A, L, E, and R. The $L-C$ equilibrium is the incumbent's optimal choice of (A, L) that deters entry.

situation where insurance firms cannot monitor their clients' purchases of insurance from other firms and employed a particular definition of equilibrium similar to that employed by Rothschild and Stiglitz (1976) in their analysis of insurance markets with adverse selection. They argued that in these circumstances an insurance firm might choose to offer a pair of policies -- a conventional active policy and a latent policy that would not be bought in equilibrium but would protect the active policy against entry. But they did not demonstrate conclusively that such an equilibrium can occur.

This paper presents an extended example in which there is a competitive equilibrium, as defined by Arnott and Stiglitz, with a single incumbent insurance firm offering a pair of policies, an active policy and a latent policy which deters entry. The example demonstrates that an effective latent policy protects an active policy by exploiting the nonconvexities to which moral hazard may give rise. It also illustrates the importance of considering latent policies when characterizing competitive equilibrium with moral hazard and unobservable insurance purchases. In the example, when latent policies are excluded, an equilibrium does not exist since all profitable policies offered by the incumbent firm would be upset by policies offered by entering firms. But allowing the incumbent firm to protect its active policy by using a latent policy results in an equilibrium in which individuals purchase positive amounts of insurance and the firm makes positive profits.

Two questions come to mind. First, do insurance firms in fact offer latent policies? Second, if they do not, why not?

With regards to the first question, we have no evidence that insurance firms offer latent policies. This may be due to a lack of data on insurance firms. But we are inclined to the view that insurance firms do not in fact offer latent policies.

If not, why not? There would seem to be three classes of possibilities: the model omits some essential feature of real-world insurance markets; the model makes unrealistic behavioral or informational assumptions about firms and individuals; and the equilibrium concept employed is inappropriate. Our intuition suggests that the following considerations are the most important. First, the calculation of the latent policy is informationally and computationally demanding. An exact knowledge of tastes and of accident probabilities is needed, as well as exact computation of the latent policy. A latent policy that is not just right will either not protect against entry or will be bought and cause the firm to lose money. It is doubtful that firms can grope toward the latent policy since groping in the presence of nonconvexities may lead to large losses. Second, it is certainly not obvious that latent policies would remain effective in the presence of unobservable differences between individuals in their risk behavior, which would give rise to adverse selection phenomena. And third, the equilibrium concept we have employed may be inappropriate. We attempted to capture in a static model what is in fact a dynamic game between firms and insured individuals. Furthermore, the issue of whether incumbent firms can commit themselves to their policies may become richer in the dynamic context. In particular, it may become important that the entering firm can impose discrete losses on the incumbent while sustaining only infinitesimal losses itself.[7]

Therefore, although the model is based on standard assumptions, it may mislead us in understanding how competitive insurance markets with moral hazard operate. Are latent policies simply a theoretical curiosum? We may not know until there is more empirical evidence concerning the operation of insurance markets.

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[7]To accomplish this, the entering firm should offer an infinitesimal policy with price $\frac{\bar{\beta}}{\bar{\alpha}} = -\left(\frac{V_\alpha}{V_\beta}\right)_A$.

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