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Latent Variables in Log-Linear Models of Repeated Observations

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Abstract

Changes in categorical characteristics may be fruitfully studied by means of log-linear models. Because unreliability of measurements is a major problem in many research settings but has notoriously disastrous consequences when studying social change, log-linear models with latent variables are particularly useful in this area.

After a very brief introduction into log-linear modeling and an explanation of the latent class model as a log-linear model with latent variables, it is shown that even a small amount of unreliability may lead to quite misleading conclusions about the underlying processes of change.

Next, measurement models for indicators measured at several points in time are discussed. The main purpose of this discussion is to show how to disentangle 'true', latent changes and observed changes caused by unreliability of measurements.

A main topic is the causal analysis of panel data. Extending Goodman's loglinear 'modified path analysis approach' to include latent variables, it is shown how to set up 'modified LISREL models' for the analyses of cross-sectional and longitudinal data.

Finally, some other possible applications of the (categorical) latent variable approach are pointed out, along with some important new developments.

1 Introduction

During the seventies and eighties, log-linear modeling has nearly conquered the world of categorical characteristics and discrete events. And even now, in the early nineties, the log-linear territory is still expanding (see, among others, Agresti, 1990 and Goodman, 1991). A potentially rich but not totally occupied area is the Latent Variable Sector. Just like its continuous counterpart, the discrete world is infected by measurement error. Therefore, corrections for all kinds of unreliability and invalidity are an essential prerequisite for drawing valid conclusions. This is especially true in longitudinal and developmental research: Measurement unreliability can make for systema-

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tically looking, but very misleading observed patterns of change that do not reflect the true state of affairs. Fortunately, log-linear models with latent variables provide the tools for effectively dealing with measurement error in categorical characteristics (see, among others, Andersen, 1990 and Hagenaars, 1990).

The misleading effects of measurement error as well as the usefulness of latent variable models will be illustrated in the next section by means of a very simple example. In Section 3, measurement models will be presented that may be employed when several related indicators have been measured at several points in time. After a brief discussion of the potentialities of log-linear models with latent variables for the investigation of particular systematic patterns of true change (Section 4), the explanation of processes of change by means of causal (log-linear) models with latent variables will be the topic of Section 5. Section 6 is a brief section on estimation and available computer programs. Concluding and evaluating remarks about present difficulties and future possibilities will be made in the last section.

2 Misleading Effects of Measurement Error: An Example

The number of children coming from families with a Noneuropean background is steadily growing in the Netherlands. Therefore, it was decided to launch a series of tv documentaries trying to enhance the level of information of white Dutch children about the cultures of relevant minority groups. The documentary was shown once a month during a whole year. An investigation was started to evaluate the effects of the series. At the beginning of the series and a couple of weeks after the last program had been shown, 1100 white Dutch children were asked, among other things, about their attitudes towards minority groups. The data are presented in Table 1.

Table 1: Attitude towards Minority Groups and Watching the Documentary Series on TV

		A. Watching the TV Documentary		
		1.Regularly	2.Not Regularly	Total
B. Attitude towards Minorities; Before	C. Attitude towards Minorities; After			
1.Favorable	1.Favorable	81	326	407
	2.Unfavorable	10	89	99
2.Unfavorable	1.Favorable	10	89	99
	2.Unfavorable	11	484	495
	Total	112	988	1100

SOURCE see text

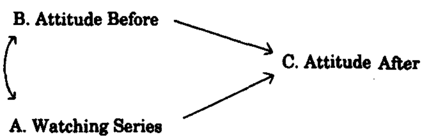
Even a rather simple table such as Table 1 can be used to answer a lot of questions about the nature and the causes of the changes in the characteristics concerned. Table 2, for example, a turnover table derived from Table 1 restricting the data to just those children that watched the series regularly shows that the vast majority (89.0%) of the children that had a favorable attitude to begin with kept that favorable attitude during the whole series, while almost half (47.6%) of those that started out with an unfavorable attitude changed that attitude towards a more favorable one.

Table 2: Turnover in Attitude towards Minorities among Those Watching Regularly.

	C. After		
	1.Favorable	2.Unfavorable	Total
B. Before			
1.Favorable	89.0%	11.0%	100%
2.Unfavorable	47.6%	52.4%	100%
Total	81.3%	18.7%	100%

NOTE horizontal percentages
SOURCE Table 1

The difference in Table 2 between the transition probabilities 11.0% and 47.6% for the two groups is indeed dramatic and highly significant: Pearson-χ^2 = 15.61, df = 1, p = .000. Such findings might be explained by the fact that among the children who watched the series regularly those with an unfavorable attitude form a minority and social pressure has been exercised towards this 'minority' to change their minds or by the fact that this 'minority' has changed under the influence of the documentaries.



1. Saturated Model {ABC}: $\hat{\lambda}^{ABC}_{111} = -.101$, s.e. = .072
2. No Effect Model {AB,C}: L^2 = 17.11, df = 2, p = .00
3. Covariance Model {AB,AC,BC}: L^2 = 1.89, df = 1, p = .17, $\hat{\lambda}^{AC}_{11} = .279$, s.e. = .075

Figure 1: Log-linear Covariance Model for Table 1

This latter possibility has to do with the very purpose of this investigation: Did the series influence children's knowledge and attitudes about Minority Groups. From the data in Table 1 it can be calculated that among those who watched the series regularly the percentage with a favorable attitude at the time of the posttest is much higher than among those who did not watch the series on a regular basis (81.3% vs. 42.0%). However, as this is not a true experiment, 'watchers' and 'nonwatchers' may have had (and actually had) different attitudes from the start. Controlling for these initial differences may be done by means of the log-linear 'covariance' model pictured in Figure 1 (Plewis, 1985, Chapter 6, Hagenaars, 1990, Section 5.3; see also Fisher 1935 (1990), Sections 56, 57).

The covariance model {AB,AC,BC} (using the standard short-hand notation for hierarchical log-linear models) may be rendered in its multiplicative form as follows:

$$F^{ABC}_{ijk} = \eta\tau^{A}_{i}\tau^{B}_{j}\tau^{C}_{k}\tau^{AB}_{ij}\tau^{AC}_{ik}\tau^{BC}_{jk}, \qquad (1)$$

or in loglinear terms:

$$G^{ABC}_{ijk} = lnF^{ABC}_{ijk} = \theta+\lambda^{A}_{i} +\lambda^{B}_{j} +\lambda^{C}_{k} +\lambda^{AB}_{ij}+\lambda^{AC}_{ik} +\lambda^{BC}_{jk}, \qquad (2)$$

where F^{ABC}_{ijk} denotes the expected cell frequency (i,j,k) of the joint variable ABC, where superscripts refer to the variables concerned and subscripts to the categories of these variables and where τ's indicate the multiplicative and λ's the log-linear effects.

As follows from the test statistics mentioned at the bottom of Figure 1, model {AB,AC, BC} fits the data in Table 1 rather well. It is not necessary to introduce the three-variable effect λ^{ABC}_{ijk}: This effect is very small and not significant as shown in Figure 1, model {ABC}. Deleting the crucial direct effect λ^{AC}_{ik} yields a non-fitting model (model {AB,BC} in Figure 1). From the size of the effect parameter λ^{AC}_{11} in model {AB,AC,BC} (Figure 1), it may be concluded that the tv series has exercised a moderately large, statistically significant effect on the attitude towards minorities and that providing information led to a more favorable attitude.

Although in simplified forms, these are types of analyses that are rather characteristic for the ways social and behavioral scientists treat longitudinal data. However, these data (and the example) were made up according to a scheme that had nothing to do with and actually contradicts the conclusions drawn above.

In constructing these artificial data, it was assumed that 10% of the children had watched the series regularly, and 90% not or irregularly. Among those that regularly watched 90% were supposed to have a favorable attitude towards minority groups and 10% an unfavorable attitude; for the nonwatchers these percentages were 40% and 60% respectively. Furthermore, it was assumed that the individual attitudes were stable and did not change during the whole period of investigation. However, these true (and stable) attitudes were not perfectly observed. The probability of getting the correct score at the pretest was .90 for each and every individual (and, accordingly, the probability of a misclassification .10). The same probabilities were applied to the posttests regardless of the scores obtained at the pretest. Within rounding off errors, this scheme led to the data in Table 1 for N = 1100.

Figure 2 presents the 'causal' diagram of this scheme: There is a not directly

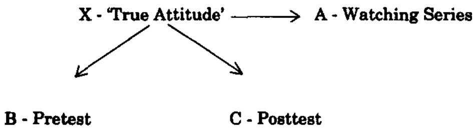


Figure 2: 'Causal' diagram

observed, latent (and stable) attitude X which 'causes' the children to watch the series and there are two not completely reliable indicators A and B of this true attitude X.

The diagram in Figure 2 represents a standard latent class model or, equivalently, a particular log-linear model with a latent variable. In terms of Lazarsfeld's original parametrization, the latent class model may be rendered as follows (Lazarsfeld and Henry, 1968, Goodman, 1974):

$$F^{ABCX}_{ijkt} = N\pi^{ABCX}_{ijkt} = N\pi^{X}_{t}\pi^{\bar{A}X}_{it}\pi^{\bar{B}X}_{jt}\pi^{\bar{C}X}_{kt}, \qquad (3)$$

where N is the sample size, π^{ABCX}_{ijkt} denotes the probability of obtaining score (i,j,k,t) on the joint variable (ABCX), $\pi^{\bar{A}X}_{it}$ indicates the conditional response probability that someone who belongs to latent class t, (X = t), obtains the score A = i, and where the other symbols have analogous meanings.

In loglinear terms, the diagram in Figure 2 corresponds with model {AX,BX,CX} (Haberman, 1979), that is, with:

$$F^{ABCX}_{ijkt} = \eta\tau^{A}_{i}\tau^{B}_{j}\tau^{C}_{k}\tau^{X}_{t}\tau^{AX}_{it}\tau^{BX}_{jt}\tau^{CX}_{kt}. \qquad (4)$$

Eq. (4) represents a special log-linear model in that the variable X is a not directly observed variable. Several algorithms and computer programs have been developed for obtaining the maximum likelihood estimates of the parameters of loglinear models with latent variables, the most important of which are mentioned in Section 6.

The results of applying model {AX,BX,CX} with a dichotomous latent variable X to the data in Table 1 are presented in Table 3.

With three dichotomous indicators, the two latent class model is exactly identified and fits the data perfectly, producing as a matter of course, the parameter estimates that were used to construct the data in Table 1, within the limits of rounding off errors.

If we had not known how these data came about, it would not have been possible to decide on empirical grounds between the latent class model in Figure 2 and the covariance model in Figure 1. However, the implications of the two competing models are very different. According to the latent class model nobody changes her or his attitude and, as there is no true change, there are also no true 'dramatic differences' in transition probabilities between those with an favorable and those with an unfavorable attitude as were observed in Table 2. And as far as there are 'true' changes in the form of random fluctuations in the characteristics concerned, that is, fluctuations that are registered as unreliability (Hagenaars, 1990, Section 4.4.1), the unreliability of the measurements as indicated by the probability of a misclassification,

Table 3: Latent Class Model Applied to Table 1

True Attitude X	$\hat{\pi}^{X}_{t}$	A.Watching Series		B.Pretest		C.Posttest	
		$\hat{\pi}^{\bar{A}X}_{it}$		$\hat{\pi}^{\bar{B}X}_{jt}$		$\hat{\pi}^{\bar{C}X}_{kt}$	
		1.Yes	2.No	1.fav.	2.unfav.	1.fav.	2.unfav.
Latent Class 1	.45	.20	.80	.90	.10	.90	.10
Latent Class 2	.55	.02	.98	.90	.10	.10	.90
		$\hat{\tau}^{AX}_{11} = 1.86$		$\hat{\tau}^{BX}_{11} = 3.00$		$\hat{\tau}^{CX}_{11} = 3.00$	
		$\hat{\lambda}^{AX}_{11} = .623$		$\hat{\lambda}^{BX}_{11} = 1.099$		$\hat{\lambda}^{CX}_{11} = 1.099$	

$L^2 = 0$ $df = 0$ $p = 0$

is the same (.10, see Table 3) for each and every child and therefore the same for those with a favorable and an unfavorable attitude.

Furthermore, according to the latent class model, there just is not any effect from watching the series (A) on the attitude at the posttest (C). In this case, the conclusions drawn from the log-linear covariance model do not reflect the (simulated) true state of affairs; they just mirror the distortions caused by even this small amount of measurement error.

Because unreliability looms everywhere (Hagenaars, 1990, p. 182), latent variable models are useful, at the very least, for drawing the researcher's attention to the possibility that the conclusions directly based on the observed data may be very wrong.

Of course, all this is not of a startling novelty. Already decades ago, a number of methodologists have warned for these kinds of misleading effects of measurement error (Thorndike, 1942, Wiggins, 1955, Maccoby, 1956, and many others since). But researchers keep on ignoring measurement errors (and methodologists fail to inform them adequately). One wonders how much of our present day 'knowledge' about the instability of small groups compared to large groups (e.g., nonvoters versus voters or voters for large versus small parties - see Barnes, 1990, Hagenaars, 1990, Chapter 5) and about the effects of particular 'nonexperimental interventions' is just based on unreliability of the measurements.

This is a regrettable situation and the more so because the models and algorithms that are now available for estimating the sizes and consequences of unreliability and invalidity are much more powerful than ever before. For continuous variables, Jöreskog's LISREL and Bentler's EQS model are excellent examples. For discrete data, log-linear models with latent variables may be put to much the same uses, as will be partly shown in the remainder of this paper (see also, Hagenaars, 1988a).

3 Measurement Models for Related Characteristics Measured Over Time

It often occurs in panel studies that related characteristics, e.g., Preference Political Party and Preference Presidential Candidate are measured in at least two waves. A common way to analyze such data is by means of Lazarsfeld's cross-lagged panel correlation technique. The main purpose of this technique is to determine which characteristic might be regarded as the cause of the other: Does Party Preference determines Candidate Preference or is it rather the other way around? References regarding origin, extensions and critiques of this technique are provided, among others by Hagenaars (1990, pp. 240-248). Along with many other assumptions, application of this technique requires perfectly reliable and valid measurements. Once the possibility of measurement error is allowed, other interesting models arise which may lead to totally different explanations of the data (Goodman, 1974b).

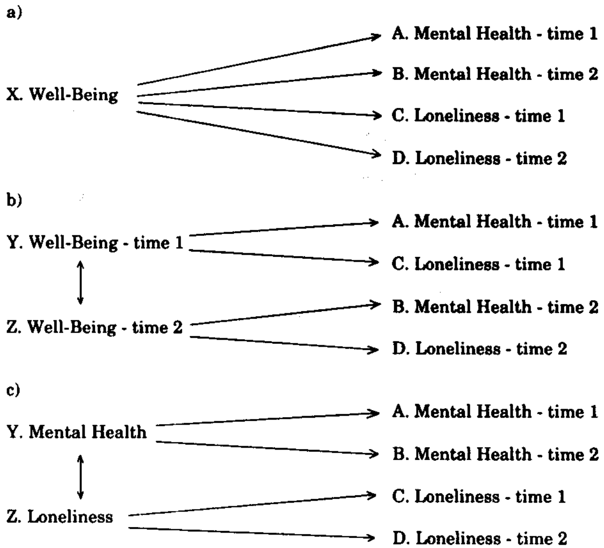
An example is presented in Table 4. The data come from a longitudinal study of pre-school children and their families and refer to the mother's mental health and feelings of loneliness; the times of measurements were 12 month apart (Moss and Plewis, 1977).

Table 4: Mothers' Mental Health and Loneliness

A.Mental Health time 1		1.Good		2.Poor		
B.Mental Health time 2		1.Good	2.Poor	1.Good	2.Poor	Total
C.Loneliness Time 1	D.Loneliness Time 2					
1.Absent	1.Absent	72	8	13	11	104
	2.Present	11	2	2	7	22
2.Present	1.Absent	15	2	4	8	29
	2.Present	7	4	11	12	34
	Total	105	16	30	38	189

SOURCE Plewis (1985, 120)

Plewis applies (the well fitting) 'cross-lagged' log-linear model {AB,AC,AD,-BC,BD,CD} to the data in Table 4. From comparing the cross-lagged effects λ^{AD}_{11} and λ^{BC}_{11} he then tentatively suggests that Mental Health causes feelings of Loneliness rather than vice versa (Plewis, 1985 pp.119-122; for a critical review of the appropriateness of this particular log-linear model for determining the causal direction, see Hagenaars, 1990, Section 5.4).



(a) Model {AX,BX,CX,DX}: $L^2 = 12.56$, $df = 6$, $p = .05$ (Pearson-$\chi^2 = 12.58$);

(b) Model {YZ,AY,CY,BZ,DZ}: see text

(c) Model {YZ,AY,BY,CZ,DZ}: $L^2 = 3.17$, $df = 4$, $p = .53$ (Pearson-$\chi^2 = 2.98$);

$\hat{\lambda}^{Y\,Z}_{1\,1} = .833$ $\hat{\lambda}^{A\,Y}_{1\,1} = 1.031$ $\hat{\lambda}^{B\,Y}_{1\,1} = .783$ $\hat{\lambda}^{C\,Z}_{1\,1} = .741$ $\hat{\lambda}^{D\,Z}_{1\,1} = .775$

NOTE X,Y,Z are dichotomous latent variables; A through D observed variables

Figure 3: Measurement Models for Data in Table 4

An alternative look at the variables and data in Table 4 follows from the compound hypothesis that perhaps Loneliness and Mental Health are just imperfect indicators of one and the same dichotomous latent variable Well-Being (X) and that a person's well-being is a rather stable characteristic that does not change easily. This compound hypothesis is represented in Figure 3a and corresponds with the standard latent class model, that is, with log-linear model {AX,BX,CX,DX}. From the test statistics presented in Figure 3, model (a), it is not clear whether to accept or reject the model.

Perhaps a better result may be obtained by allowing for true, latent change to occur. If Mental Health and Loneliness are still regarded as indicators of the theoretical construct Well-Being, there will be two dichotomous latent variables Y and Z, where Y refers to Well-Being at time 1 with indicators A and C, and Z to Well-Being at time 2 with indicators B and D. The 2x2 latent turnover table YZ then shows the true changes in Well-Being between the two time points. This model is depicted in Figure 3b and corresponds with log-linear model {YZ,AY,CY,BZ,DZ}. Application of this model to Table 4 happens to yield, for these particular data, a degenerate solution, in that the two off diagonal cells YZ = 12 and YZ = 21 are empty: There is no latent change in Well-Being. There are actually just two nonempty latent classes, viz. YZ = 11 and YZ = 22, and, accordingly, for these data, the test statistics and the parameter estimates for the model in Figure 3b are identical to those obtained for the standard latent class model of Figure 3a.

Another possibility of modifying the standard latent class model in Figure 3a is to assume that there is indeed no latent change, but that Mental Health (A, B) and Loneliness (C, D) are not indicators of the same theoretical variable but refer to distinct concepts. Two dichotomous latent variables Y and Z are needed where Y now refers to the (stable) true scores of the characteristic Mental Health, with indicators A and B, and Z to the latent variable Loneliness with indicators C and D (Figure 3c). This model {YZ,AY,BY,CZ,DZ} fits the data in Table 4 excellently, and much better than the standard latent class model (a). (Note, however, that carrying out the standard conditional likelihood ratio test comparing two nested models by subtracting the L^2 (and *df*) of the less restricted model (c) from the corresponding test statistics of the more restricted model (a) is not allowed here as parameter estimates are involved that lie on the boundary of the permissible parameter space; see Bishop *et al.*, 1975, p. 510)).

In model (c), mental health and feelings of loneliness are regarded as distinct concepts rather than aspects of one underlying variable Well Being. Furthermore, there are no cross-lagged effects to compare; the observed changes solely result from measurement error. The 'reliability' of the measurements of these two distinct concepts can be estimated by means of the log-linear parameters concerning the relations between each latent variable and its indicators, that is, by means of $\lambda^{A\,Y}_{1\,1}$, $\lambda^{B\,Y}_{1\,1}$, $\lambda^{C\,Z}_{1\,1}$, and $\lambda^{D\,Z}_{1\,1}$. These coefficients are presented in Figure 3, model (c), and it follows that the scores on the observed variables A, B, C, and D are strongly, but not perfectly determined by the true scores on X; indicator A appears to be the most reliable measure.

The true log-linear association between Mental Health and Loneliness, corrected for unreliability of the measurements, is provided by λ^{YZ}_{11} and turns out to be rather strong: (.833 - Figure 3, model (c)) and in the expected direction. This association between the latent variables is much stronger than the corresponding associations between the manifest variables based on the observed two-dimensional tables (for table AC: .164, for AD: .472, for BC: .188, and for BD .195), a result that parallels the 'correction for attenuation' well-known from classical test theory (Nunnally, 1978, 237).

If the last model had failed to fit, still other models with latent variables taking measurement error into account would have been possible. For example, it seems theoretically reasonable to assume that Mental Health is a more stable phenomenon than feelings of Loneliness. So, on theoretical grounds, a model with three dichotomous latent variables might have been proposed: W - Mental health with indicators A and B; Y - Loneliness at time 1 with indicator C; Z - Loneliness at time 2 with indicator D, leading to model {WYZ,AW,BW,CY,DZ}.

This model seems to have zero degrees of freedom as it has as many parameters to be independently estimated as observed cell frequencies. However, it is not identified. (For expositions of identifiability, see Goodman, 1974a, De Leeuw *et al.*,1990, Van der Heijden *et al.*, 1992). Extra restrictions are needed to make it identifiable.

First, it is possible to impose in various ways restrictions on the 'reliabilities' (resulting in nonhierarchical log-linear models). In terms of conditional response probabilities, this usually amounts to setting the probabilities of giving the 'correct' answer in agreement with the latent score equal to each other for particular manifest variables for particular categories of the latent variables (Goodman 1974a, Mooyaart and Van der Heijden, 1992.); for example: $\pi^{AW}_{11} = \pi^{AW}_{22} = \pi^{CY}_{11} = \pi^{CY}_{22}$. Restricting the 'reliabilities' in terms of log-linear parameters means setting all or some of the two-variable parameters λ^{AW}_{iq}, λ^{BW}_{jq}, λ^{CY}_{kr}, and λ^{DZ}_{ls} equal to each other; for example: $\lambda^{AW}_{11} = \lambda^{CY}_{11}$. Computer programs are available to introduce all these kinds of restrictions routinely (Section 6). When imposing these equality constraints, the user has to be aware of the fact that, in general, restrictions on the conditional response probabilities result in different models than imposing restrictions on the loglinear two-variable parameters (although restrictions on the conditional response probabilities can be expressed in terms of (combinations of) restrictions on the log-linear one- and two-variable effects; see Hagenaars, 1990, p.111, 185 and especially, Heinen, 1993).

However, despite the apparent gain in degrees of freedom, restrictions on the 'reliabilities' do not necessarily make model {WYZ,AW,BW,CY,DZ} identifiable when applied to Table 4 (definitely not for the sets of restrictions that were actually tried out). Additional restrictions, resulting in an identifiable model, may be found by defining a nonsaturated loglinear model for the relations among the latent variables, for example, model {WY,YZ,AW,BW,CY,DZ}. Also these kinds of restrictions can be routinely applied making use of existing software.

Theoretical considerations are the most important when one tries to obtain well-fitting models. However, these theoretical notions may be guided by 'empirical' means, especially by the (adjusted or standardized) residual frequencies of the baseline model (Haberman, 1978, p. 78). With latent variable models, such an inspection often reveals that the strength of the observed association between particular indicators is underestimated by the proposed model. By introducing into the log-linear model direct relations between the indicators concerned (thus violating the basic latent class assumption of local independence) a tremendous improvement of fit is often obtained at the cost of just one or two degrees of freedom (Hagenaars, 1988b). Such direct effects between manifest variables point to the fact that the observed relations among the indicators concerned are not fully explained by the latent variables in the model. There are other, omitted (unmeasured, 'latent') variables that cause association among the manifest variables over and above the association caused by the included latent variables. Or, to put it in other words, there are correlated errors.

Now, it often is a plausible assumption in longitudinal and developmental research that the measurement errors of a particular characteristic are correlated over time. In such cases, improving the fit of the baseline model by introducing direct effects among the indicators may be the sensible thing to do. However, if there are no clear substantive explanations for the extra direct effects between the indicators, a purely empirical, atheoretical way of achieving a good fit should be avoided.

4 Systematic Patterns of Latent Change

The log-linear model with latent variables has been mainly treated so far as a (modified) latent class model, that is, as a measurement model for investigating the relations between the latent and the manifest variables. However, loglinear models with latent variables can also be used for finding the causes and consequences of the true, latent changes, which will be the topic of the next section, and for exploring the nature of the true, latent change, the latter to be discussed in this section. This exposition will be even more informal than the ones above, the main purpose being to give a first impression of the many possible applications of log-linear modeling with latent variables for the study of patterns of latent change and of the work that has been done in this area.

Still close to the latent class (measurement) model are a set of models that have been developed for the analysis of large turnover tables, where the variables concerned have many (five or more) categories. Clogg, Marsden and Luijkx have proposed very ingenious and useful models for such tables in the context of social mobility research; Hagenaars reviewed these procedures and applied them to turnover tables on Voting (the Dutch political system produces a lot of political parties); Van der Heijden extended some of these models into a general approach for 'latent time budget models' (Clogg, 1981b, Marsden, 1985, Luijkx, 1988, Hagenaars, 1990, Section 4.4.2, Van der Heijden *et al.*, 1992).

The basic idea behind these approaches is that the movements among the many categories in the observed turnover table (among, for example, occupations or political parties) can be explained by the fact that each respondents belongs to one of the few postulated latent classes (for example, to one of three social classes or one of the three basic political orientations) and that this latent position determines with a certain probability the belonging to particular manifest categories at each point in time. In some models, people may change their latent positions over time, in others not.

If latent change is allowed for, one might be interested in investigating whether the changes follow a particular pattern. In general, whenever a model allowing for latent change, e.g., model (b) in Figure 3, has been set up, one may want to investigate whether the latent change follows particular systematic patterns.

One might wonder, for example, whether a square latent turnover table YZ is symmetric, that is whether the entry of cell (r,s) is the same as cell frequency (s,r), for all values of s and r, $s \neq r$. Or perhaps one suspects that the square latent turnover table is quasi-symmetric, that is symmetric as far as the differences between the (latent) marginals allow it. Symmetry and quasi-symmetry models can be easily defined in terms of restrictions on the log-linear parameters (Bishop *et al.*, 1975, Chapter 8, Haberman, 1979, Chapter 8, Hagenaars, 1986).

Another interesting angle from which latent turnover tables might be viewed is (quasi)-independence (Goodman, 1968). Perhaps at the latent level, people are inclined to occupy the same latent position over time, i.e., to belong to one of the main diagonal cells of the square turnover table YZ, but once they change their latent position they may have no special preferences for particular other positions. The independence model {Y,Z} is postulated for the off-diagonal cells of the latent turnover table YZ, while no special restrictions are applied to the main diagonal cells.

Especially in developmental research, other patters of latent change may be relevant. The latent variables may involve a particular ability which once acquired will not be lost. If the latent variables Y and Z each have three categories referring to increasing levels of ability, a model such as model (b) in Figure 3 may be fitted with the additional restriction that in the latent turnover table YZ the cells (2,1), (3,1), and (3,2) are empty. If, moreover, development takes place only through successive stages, then also cell frequency (1,3) implying a jump from level 1 directly to level 3 must be restricted to zero.

Even when at the manifest level these 'impossible' cells are not empty in the observed turnover table(s), the proposed developmental model might still hold on the latent level, that is, if measurement errors are accounted for. In this way, a wide variety of latent trajectories can be defined; the parameters of such models can be estimated by means of most programs for carrying out latent class analysis.

Finally, when a particular characteristic is measured at more than two points in time, still other possibilities for modeling latent change arise. For example, when a particular characteristic has been measured in each wave of a four wave panel study, there are four manifest variables A (wave 1) through D (wave 4). Allowing for latent change to occur at each point in time yields four latent variables V (wave 1), W (wave 2), Y (wave 3), and Z (wave 4). A possible model for such data would be model {VW,WY,YZ,AV,BW,CY,DZ}, depicted in Figure 4.

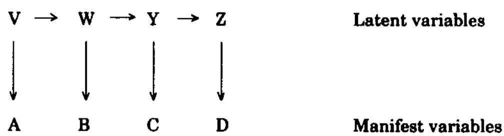


Figure 4: Four wave panel study – a possible model

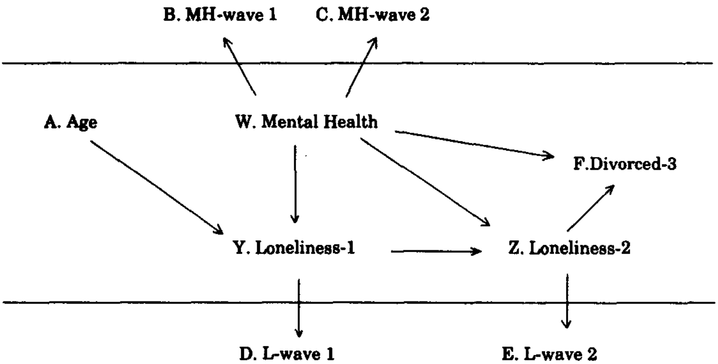
The model in Figure 4 represents a first order latent markov chain, in that the latent scores on time t are only directly influenced by the latent scores on time t-1. A standard (stationary) markov chain would require the additional restriction that the corresponding transition probabilities of all turnover tables between successive time points, i.e., of the marginal turnover tables VW, WY, and YZ, are identical. However, this particular restriction leads us outside the boundaries of log-linear modeling. Van de Pol and Langeheine (1990) discuss maximum likelihood estimation for the parameters of a very general class of markov models, including (nonstationary) loglinear models like the model in Figure 4.

So, although the log-linear model with latent variables does not encompass all possible models of discrete latent change, it is sufficiently powerful to adequately test most of a researcher's ideas about systematic patterns of true change in categorical characteristics (Hagenaars, 1990).

5 Causal Modeling with Latent Variables

In this section, loglinear models will be discussed from the viewpoint of carrying out causal analyses. Because the causal loglinear models considered in this section bear a clear resemblance to regular LISREL models, they have been termed 'modified LISREL models' (Hagenaars, 1988a). Typical for these modified LISREL models is that, because of the assumed causal order of the variables, their parameters have to be estimated, in principle, in the stepwise fashion indicated by Goodman (1973; see also Section 6).

For an illustration of the 'modified LISREL approach', the example of Table 4 on Mother's well-being has been (fictitiously) extended yielding the causal model in Figure 5. (For real world examples, see Hagenaars, 1990, 1993).



NOTE

A, B, C, D, E, and F: directly observed variables

W, Y, Z: latent variables

Structural Model: direct causal relations among A, W, Y, Z, and F

Measurement Model: direct relations between the latent variables and their indicators, i.e. between W and B, W and C, Y and D, and between Z and E.

Figure 5: Causal Models With Latent Variables

There are three latent variables in Figure 5: W - the stable Mental Health, measured at the first wave by means of B and at the second wave by means of C; Y - the true feelings of Loneliness at the first wave, with indicator D; and Z - Loneliness at the second wave, measured by means of E. Besides the indicators B, C, D, and E, there are the directly observed variables A - Age and F - Divorced or not one year after wave 2 was finished. The observed variables A and F are not indicators of some latent variable but part of the structural model, in which the assumed causal connections among A, W, Y, Z, and F are defined.

In the first instance, the model in Figure 5 may be treated as a kind of (quasi-)latent class model {XB,XC,XD,XE} with (quasi-)latent variable X and four indicators B through E. X is a joint variable having as its categories all combinations of all categories of the variables A, W, Y, Z, and F, the variables that are in the structural part of the model. The indicators B through E depend on X, i.e., on the joint variable AWYZF, in such a way that B and C only depend on W, D is only determined by Y, and E only by Z.

If all restrictions implied by the structural part of the model in Figure 5 (the part between the dashed lines) are ignored, the parameters of the resulting log-linear model {AWYZF,BW,CW,DY,EZ} can be estimated as an ordinary 'one-step' loglinear model with latent variables. However, the estimated expected frequencies of this 'one step' model obviously do not correctly reflect the hypothesized relations among A, W, Y, Z, and F: The (estimated expected) entries of marginal table AWYZF ought to mirror the structural part of Figure 5.

Although it seems to be doing the job, the intended restrictions on marginal table AWYZF are not realized by setting up loglinear model {AY,WY,YZ,WZ,WF,ZF} for this marginal table. To illustrate this point: It is assumed in Figure 5 that Age (A) and Mental Health (W) are independent of each other. Accordingly, no term {AW} occurs in model {AY,WY,YZ,WZ,WF,ZF}, i.e., λ^{AW}_{ip} is a priori set to zero. However, as, in general, loglinear parameters are partial coefficients, model {AY,WY,YZ,WZ,WF,ZF} implies that A and W are independent of each other holding the other variables (Y, Z, and F) constant. But given the causal order of the variables in Figure 5 and from the adage that what is causally posterior cannot influence what is causally prior, it follows that Age (A) and Mental Health (W) should be independent of each other without holding the other variables constant. The independence model {A,W} is assumed to hold in marginal table AW. If the independence model has to be rejected for marginal table AW, the two-variable effect λ^{AW}_{ip} in the saturated model {AW} applied to marginal table AW provides the correct estimate of the effects of Age on Mental Health and not λ^{AW}_{ip} in model {AW,AY,WY,YZ,WZ,WF,ZF} for table AWYZF.

Following the same kind of logic, marginal table AWY should be used to determine the effects of Age (A) and Mental Health (W) on Loneliness at wave 1 (Y). If Figure 5 is a true representation of reality, model {AW,AY,WY} should be valid for marginal table AWY. Note that model {AW,AY,WY} does contain the parameter λ^{AW}_{ip}, as it follows from the collapsibility theorem (Bishop *et al.*, 1975, 47), that given the postulated effects in Figure 5, log-linear effect λ^{AW}_{ip} will not be zero in marginal table AWY if it is zero in the collapsed table AW (see also Agresti, 1990, 152).

To determine the effects of Age (A), Mental Health (W) and Loneliness-time 1 (Y) on Loneliness-time2 (Z), the frequencies of marginal table AWYZ should be in agreement with model {AWY,WZ,YZ}. Again, all parameters referring to the independent variables of this submodel are included (by means of the term {AWY}).

Finally, to determine the effects on the ultimate dependent variable Divorce (F) of all other variables that are part of the structural model in Figure 5, model {AWYZ,WF,ZF} is postulated for marginal table AWYZF.

For models without latent variables, in which all variables are directly observed, Goodman (1973) has shown how to obtain the parameter estimates of all submodels and how to calculate the estimated expected frequencies for the whole model in such a manner that all subtables are in agreement with all postulated submodels. Actually, the simplest way to proceed in that case is to set up the appropriate observed subtables, to apply to them the submodels concerned, employing the principles outlined above, and to sum all likelihood ratio chi-square test statistics and degrees of freedom of the submodels to arrive at one overall test statistic. When there are latent variables involved, the EM-algorithm described in the next Section may be used to obtain the estimated expected frequencies for the whole model.

If the total model has to be rejected, one or more of the restrictions in one or more of the submodels may be relaxed and hierarchically nested models may be compared with each other by computing the conditional test statistics. In this way, it can be determined which restriction in what submodel is responsible for the rejection of the total causal model (although all criticisms that might be levelled against repeated and *ex post facto* testing apply here as well).

6 Estimation Procedures and Available Computer Programs

Under the (product)multinomial sampling scheme, maximum likelihood estimates for loglinear models may be obtained by means of the Newton/Raphson or the EM algorithm (Little and Rubin, 1987, Chapter 7, Haberman, 1979, Chapter 10, Goodman, 1974a,b). Programs for estimating loglinear models with latent variables are not yet part of packages such as BMDP, SAS, or SPSS^X. However, stand-alone programs most of which are rather easy to use are readily available.

Haberman's program LAT (Haberman, 1979) and its successor NEWTON (Haberman, 1988) are completely formulated in loglinear terms rather than in terms of Lazarsfeld's parametrization using conditional response probabilities. All kinds of restrictions, including equality and (curvi)linear restrictions may be imposed on the loglinear effects by means of the appropriate design matrix.

All loglinear models with latent variables discussed above can be routinely estimated by means of LAT and NEWTON, except modified LISREL models. Although using NEWTON, one can in principle apply the stepwise estimation approach needed in the modified LISREL models, a particular reparametrization has to be chosen for each particular causal model which is far from easy to handle (Winship and Mare, 1989, Appendix). On the other hand, as follows from the collapsibility theorem (Bishop *et al.*, 1975, 47), a number of causal models that seemingly have to be estimated in a stepwise fashion may be treated as ordinary 'one-step' log-linear models - the model in Figure 4 is a case in point - and no special reparametrization is required when using NEWTON.

As LAT and NEWTON make use of the scoring algorithm and a variant of the Newton/Raphson algorithm respectively, initial parameter estimates have to be rather close to the final estimates. Although much better than LAT, one still encounters difficulties, even with NEWTON, in finding the appropriate initial parameter estimates for some models and data sets.

The EM algorithm is much less sensitive to the 'quality' of the initial estimates.

(A more complete comparison of the (dis)advantages of Newton/Raphson and EM can be found in Hagenaars, 1990.) Because of this feature, a simple version of the EM-algorithm will be discussed below after mentioning a few programs that make use of this algorithm.

One of the first programs for latent class analysis in which Goodman's version of the EM-algorithm has been implemented and that uses Lazarsfeld's parametrization of the model in terms of conditional response probabilities, is Clogg's MLLSA (Goodman, 1974a,b, Clogg, 1981a). It is now part of Eliason's CDAS package (Department of Sociology, Pennsylvania State University). By means of MLLSA, the parameters of loglinear models with latent variables can be estimated, provided that the relations among all external and latent variables that make up the structural part of the model satisfy the saturated model.

Although originally written for the analysis of (latent) markov chains, Van de Pol and Langeheine's PANMARK can be used for many of the models mentioned above and work is still be done to enhance the program's applicability (Van de Pol and Langeheine, 1990).

LCAG is based on the same Goodman algorithm as MLLSA, but with the additional possibility of imposing unsaturated hierarchical models on the latent level (Hagenaars, 1990, 1993, Hagenaars and Luijkx, 1987). All models mentioned above, including the modified LISREL models can be (and have been) estimated by means of LCAG. Work is in progress to add several of the desirable features of NEWTON into LCAG.

The EM-algorithm may be seen as an extension of the Iterative Proportional Fitting (IPF) Procedure. IPF can be used to find the estimated expected frequencies F of hierarchical loglinear models without latent variables (Bishop *et al.*, 1975). In IPF, the initial estimates of F are iteratively adapted to the sufficient statistics, that is, to the observed marginal frequencies f to be reproduced by the hierarchical model. For example, in model {AB,AC,BC}, the estimated expected frequencies F^{ABC}_{ijk} are iteratively estimated in such a way that at the end the estimated expected frequencies F of the marginal tables AB, AC, BC are exactly identical to the corresponding observed frequencies f of marginal table AB, AC, and BC respectively, that is, $F^{ABC}_{ij+} = f^{ABC}_{ij+}$, $F^{ABC}_{i+k} = f^{ABC}_{i+k}$, and $F^{ABC}_{+jk} = f^{ABC}_{+jk}$.

If A and B are regarded as causes of C, stepwise estimation may be necessary. For example, if it is assumed that A and B are statistically independent, submodel {A,B} should be applied to marginal table AB. If it is further postulated that A and B both influence C, but without affecting each other's influence on C, submodel {AB,AC,BC} should be valid for table ABC. Goodman (1973) has shown that the estimated expected frequencies F^* for the whole model, assuming that both submodels are true can be obtained as follows:

$$F^{*ABC}_{ijk} = F^{AB}_{ij} \, (F^{ABC}_{ijk}/F^{ABC}_{ij+}) = F^{AB}_{ij} \, \pi^{ABC}_{ijk}, \tag{5}$$

where F^{AB}_{ij} refers to the estimated expected frequencies for model {A,B} applied to marginal table AB and F^{ABC}_{ijk} and F^{ABC}_{ij+} to the estimated expected frequencies for model {AB,AC,BC} for table ABC.

For hierarchical loglinear models with latent variables, the complete table including the latent variables is not an observed table and some or all of the sufficient statistics are not observed, not known. The basic idea behind the EM-algorithm is very simple: try to get good estimates of the unobserved sufficient statistics and then proceed as if the resulting estimated sufficient statistics are just ordinary sufficient statistics.

The EM-algorithm essentially consists of a repetition of two steps, the E- and the M-step.

These steps will be exemplified using an observed table SEABC, in which S(ex) and E(ducation) are external variables, and A, B, and C indicators of latent variable Y. We want to fit model {SE,SY,EY,YA,YB,YC} to table SEABC. Because the complete table {SEYABC} is not observed, the sufficient statistics, the 'observed' frequencies of marginal tables SY, EY, YA, YB, and YC are not known.

In order to estimate the sufficient statistics, first, initial estimates $F^{SEYABC}_{ijrklm}(0)$ have to be found that satisfy the restrictions implied by the postulated model. These estimates can be obtained by roughly estimating the parameters of the model. Then, in the E-step, the estimated observed frequencies f^{SEYABC}_{ijrklm} are calculated by using F(0) as estimates of F:

$$\begin{aligned} f^{SEYABC}_{ijrklm} &= f^{SEABC}_{ijklm} \left(F^{SEYABC}_{ijrklm} / F^{SEYABC}_{ij+klm}\right) \\ &= f^{SEABC}_{ijklm} \; {}^{SEYABC}_{ijrklm}. \end{aligned} \tag{6}$$

In the M-step, the estimated expected frequencies F obtained so far -F(0) at the first M-step- are improved by means of IPF, treating the estimated observed frequencies f as if they were regular observed frequencies f. In this example, for model {SE,SY,EY,YA,YB,YC}, F^{SEYABC}_{ijrklm} is successively adjusted to reproduce the estimated observed marginal frequencies f^{SEYABC}_{ij++++}, f^{SEYABC}_{i+r+++}, f^{SEYABC}_{+jr+++}, f^{SEYABC}_{++rk++}, f^{SEYABC}_{++r+l+}, and f^{SEYABC}_{++r++m}.

If a modified LISREL model is defined (which is not true in this example) and a stepwise estimation procedure is needed during the M-step, the appropriate marginal tables f and F are set up, and the postulated submodels are fitted to the estimated observed marginal tables. The estimated expected frequencies F obtained for each submodel are combined to obtain the estimated expected frequencies F^* for the whole modified LISREL model in the manner of Eq (5).

The estimated expected frequencies that come out of the M-step are used in the E-step to get new and better estimates of the estimated observed frequencies f, which in turn are used in the M-step to improve the estimates F, etc., until the outcomes converge.

7 Conclusions

As exemplified above, log-linear models with latent variables are extremely useful for answering a large number of questions that arise in the context of longitudinal research. However, log-linear modeling shares with all forms of categorical data analysis the problems caused by sparse tables. Even relatively 'small' models like the one in Figure 5 require very large samples to obtain reliable and robust parameter estimates and to carry out significance tests. Many proposals have been made to solve the sparse table problem, but no really satisfying solution yet exists (Read and Cressie, 1988, Agresti, 1990). An interesting and promising approach involving a renewed interests in (Fisher's) exact tests has come up within the context of graphical models (Agresti, 1992, Whittaker, 1990).

Because of this problem of small sample size, it is important to include as many cases as possible and not to waste a respondent because he or she failed to answer just one particular item. Excellent theoretical work in this area has been done by Little and Rubin (1987). Log-linear models with latent variables taking missing data into account can be handled using LCAG or NEWTON (Winship and Mare, 1989, Hagenaars, 1990, Section 5.5.1).

Although researchers often do it to avoid sparse tables and it was done here also, although mainly for reasons of simplicity of exposition, dichotomizing all variables is not really necessary. Within the context of loglinear models, polytomous manifest and/or latent variables can be handled, in principle, as easily as dichotomous variables.

An interesting class of polytomous variables form the polytomous variables whose categories are ordered. Croon (1990, 1993) has developed some interesting models taking the ordered character of the categories into account by imposing inequality restrictions on particular loglinear parameters, thus obtaining monotonically increasing or 'umbrella'-like relationships between the latent and the manifest variables.

Others have assigned interval level scores to the polytomous manifest or latent variables and restrict the loglinear parameters in such a way that linear relationships among latent and manifest variables result (Haberman, 1979, McCutcheon, *this volume*, Formann, 1992). This latter development has made it clear that interesting relationships exist between latent trait and latent class models (Langeheine and Rost, 1988, Heinen *et al*, 1988, Heinen, 1993). The fusing of these two models will extend even more the range of applicability, and therewith the usefulness of log-linear models with latent variables.

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