# Lateral adaptive control for vehicle lane keeping

Mariana S. Netto, Salim Chaib
INRETS/LCPC - LIVIC
Vehicle - Infrastructure - Driver Interactions Research Unit
14, route de la Minière, 78000 Versailles, France.
mariana.netto@lcpc.fr

Said Mammar
LSC/CNRS-FRE 2494,
Université d'Évry val d'Essonne
40 rue du Pelvoux CE1455, 91025, Evry, Cedex, France.
said.mammar@iup.univ-evry.fr

*Abstract*— **In this paper we deal with the vehicle lateral control problem. More precisely, we solve this problem by properly applying the self-tuning regulator proposed in [1] to the concerned vehicle lateral model. The interest of this solution is that only the lateral displacement at a lookahead distance is used as the measure for the controller. From a practical point of view, this measure can be obtained through a vision system. Also, all the parameters are considered unknown, where it is only assumed that they belong to a known compact set. The controller is also robust to variations on curvature and lateral wind. Simulations illustrate the efficacy of the controller.**

## I. Introduction

The idea of a vehicle full automation is, among others, motivated by the necessity of increasing the capacity of the highways without reducing the security of the passengers. The traffic flow will need to be supervised in the actual infrastructure or in a new dedicated one, in the context of automated highways.

On the other hand, driver lane keeping support systems, where partial automation may take place, are a challenging and important research area since twenty years. Several passive and active systems have been developed [2], [3], [4], [5]. In the first case the driver receives a sound or a light alert when the vehicle is expected to perform lane departure [6]. Haptic interactions have also been used. In the second case, a virtual driver takes a partial or a full control of the vehicle in order help the driver to bring the vehicle to the lane.

Our work concerns vehicle lateral control that has two main goals. First, it is a step towards the full vehicle automation where lateral and longitudinal control will be coupled to form the mentioned future automated highways. Second, it is important in the context cited above of lane departure avoidance where the automat will take the full control of the car to maintain the car in the lane when a lane departure announces to appear.

Solutions for lateral control were presented in the literature from the most simple ones such as PID solutions [7], [8] to more complicated ones such as H-infinity solutions as presented for instance in [9] and [10] or sliding modes solutions also presented in [9]. Coupling effects from lateral and longitudinal modes were studied for instance in [11].

In this paper, we apply the self-tuning regulator in [1] to solve the vehicle lateral control problem. All the parameters are considered unknown (belonging to a known compact set) and only the lateral displacement at a lookahead distance is supposed to be measured. In practice, this measure can be obtained by a vision system. The solution is robust with respect to road curvature and lateral wind. In Section II we present the vehicle lateral model and state the problem. Section III is dedicated to the control design. We first review the self-tuning regulator proposed in [1]. We apply then the theory to our problem. In Section IV we present the simulations and we wrap up the paper in Section V with the conclusions.

## II. Vehicle lateral model and problem statement

In this section, we present the linearized model proposed by Ackerman [12] that is used in Section III.B for the control design. We state then the control problem.

### A. Vehicle lateral model

The model proposed by Ackerman [12] under the wind forces is as follows:

$$\dot{x} = Ax + b\delta_f + p_1\rho_{ref} + p_2 f_w \tag{1}$$

where

$$x = \begin{bmatrix} \beta \\ r \\ \Psi_L \\ y_L \end{bmatrix} \tag{2}$$

with $\beta$ representing the side slip angle, $r$ the yaw rate, $\Psi_L$ the yaw angle error and $y_L$ the lateral displacement at a lookahead distance $\ell_s$. $\rho_{ref}$ and $f_w$ represent the road curvature and the resultant of the wind forces actuating at a distance $\ell_w$ of the vehicle center of gravity, respectively. $\delta_f$, the control input to the system, is the steering angle.

The matrices $A$, $b$, $p_1$and $p_2$ are as follows:

$$A = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ v & l_s & v & 0 \end{bmatrix}; \quad b = \begin{bmatrix} \bar{b}_1 \\ \bar{b}_2 \\ 0 \\ 0 \end{bmatrix} \tag{3}$$

$$p_1 = \begin{bmatrix} 0 \\ 0 \\ -v \\ -v\ell_s \end{bmatrix}; \quad p_2 = \begin{bmatrix} h_1 \\ h_2 \\ 0 \\ 0 \end{bmatrix} \tag{4}$$

| | |
|---|---|
| $m$ | total mass of the vehicle $(1500kg)$ |
| $I_z$ | moment of inertia around the vertical axis $(2454kg.m^2)$ |
| $c_f^*,\ c_r^*$ | front and rear tire lateral stiffness - nominal values) $(57.5kN.rad^{-1})$ |
| $\mu$ | adherence coefficient $(0 < \mu \leq 1$ depending on the road conditions) |
| $c_f,\ c_r$ | $c_f = \mu c_f^*,\ \ c_r = \mu c_r^*$, with $\mu$ varying between $0$ and $1$ |
| $n_t$ | Tire contact length $(0.0113m)$ |
| $l_f,\ l_r$ | distance from center of gravity to front and rear axle $(1.0065/1.4625m)$ |
| $l'_f$ | $l'_f = l_f - n_t \approx l_f$ |
| $v$ | longitudinal speed |

Table 1: Vehicle parameters and their nominal values.

with

$$a_{11} = -2\frac{(c_f+c_r)}{mv},\ a_{12} = -\ \ 1 + 2\frac{\left(c_r l_r - c_f l'_f\right)}{mv^2}$$

$$a_{21} = 2\frac{\left(c_r l_r - c_f l'_f\right)}{I_z},\ a_{22} = -\ \ 2\frac{\left(c_f {l'_f}^2 - c_r l_r^2\right)}{vI_z}$$

$$\bar{b}_1 = 2\frac{c_f}{mv},\ \bar{b}_2 = 2\frac{c_f l'_f}{I_z},\ h_1 = \frac{1}{mv},\ h_2 = \frac{\ell_w}{I_z}$$

where $c_f$, $c_r$, $l_r$, $l'_f$, $m$, $v$ and $I_z$ are defined in Table 1.

**Remark:** (*Observability and controlability of the model)* It can be easily shown that the model is controllable except for a longitudinal speed $v$ equal to zero. In fact, in this case, singulatities are introduced in the matrix A. For observability, only $y_L$ is indispensable.

### *B. Problem Statement*

Consider the system described by (1)-(4), under the action of wind forces. Find the steering angle $\delta_f$ to apply to the vehicle wheels, such that the state vector $x = \left[\ \beta,\ \ r,\ \ \psi_L,\ \ y_L\ \right]^T$ be bounded and the lateral displacement at a lookahead distance $\ell_s$ tends to zero, that is, $\lim_{t\to\infty}\ y_L(t) = 0$ .

## III. CONTROL DESIGN

In this section, we first present an overview of the seft-tuning regulator proposed in [1]. We apply then this theory to the lateral control problem, using the model described by (1)-(4).

### *A. Self-tuning regulator*

We consider in this section SISO uncertain systems

$$\begin{aligned} \dot{x} &= f(x\,,u) + g(x,\theta)\,u,\ x \in \Re^n,\ u \in \Re,\ \theta \in\ \Omega \subset\ \Re^p \\ y &= h(x,\theta)\ ,\quad y \in \Re \end{aligned} \qquad (5)$$

where $x$ is the state vector, $u$ is the control input, $\theta$ is a constant uncertain parameter vector belonging to a known compact set $\Omega$, $h$ is a smooth output function with $h(x_\theta, \theta) = 0$ for some vector $x_\theta, \forall\ \theta \in \Omega$, and $f$ and $g$ are smooth vector fields, with $g(x, \theta) \neq 0\ , \forall\ x \in \Re^n\ , \forall\theta \in\ \Omega$ . Only the output $y$ is assumed to be measured.

Theorem 6.1, stated in the Appendix, shows the structural geometric conditions required to the design of the self-tuning regulator[1], identifying the class of nonlinear systems to be considered.

The theorem below establishes the existence of a global self-tuning output feedback regulator for system (5) with relative degree equal or greater than 2. The control algorithm is shown in the proof of the theorem.

*Theorem 3.1:* **[1]:** Consider the system (5) of relative degree $\rho$, $2 \leq\ \rho \leq n$, $\forall\theta \in \Omega$ . If conditions (1) to (5) of Theorem 6.1 are satisfied, then there exists a global self-tuning output feedback regulator for system (5).

*Proof:* We introduce the filter:

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \vdots \\ \dot{\xi}_{\rho-1} \end{bmatrix} = \begin{bmatrix} -\lambda_1 & 1 & 0 & \cdots & 0 \\ 0 & -\lambda_2 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -\lambda_{\rho-1} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_{\rho-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \sigma(y)u \overset{\triangle}{=} A\xi + b_f\sigma(y)u\ ,\quad \xi\,(0) = \xi_0 \qquad (6)$$

Let us define recursively $\rho$ vectors denoted $d\,[\,i]\,(\theta) \in \Re^n, 1 \leq i \leq \rho$ as follows:

$$\begin{aligned} d\ [\rho]\,(\theta) &= b(\theta) \\ d\,[i-1]\,(\theta) &= A_c d\,[i]\,(\theta) + \lambda_{i-1} d\,[i]\,(\theta),\ \rho \geq i \geq 2 \end{aligned} \qquad (7)$$

where $b(\theta)$ is the vector defined in the Appendix (eq. (21)).

Using the filter (6), equations (7), the changes of coordinates[2]

$$z = \zeta - \sum_{i=2}^{\rho} d[i]\xi_{i-1}$$

and

$$\begin{aligned} \eta_i &= z_{i+1} - \frac{d_{i+1}[1](\theta)}{d_1[1](\theta)}\ z_1\ , \qquad 1 \leq i \leq n-1 \\ y &= z_1 \end{aligned}$$

and defining

$$\eta_r = -\Gamma^{-1}\,(\theta)\left(\bar{\psi}(y_d\,,\theta) + y_d\beta(\theta)\right)$$

$$p(\theta) = \frac{1}{d_1[1](\theta)}\left(y_d\frac{d_2[1](\theta))}{d_1[1](\theta)} + \psi_1(y_d,\,\theta) + \eta_{r\,1}\right)$$

with $\Gamma$, $\beta$ , $\bar{\psi}$ given in the Appendix and $\eta_{r_1}$ denoting the first component of $\eta_r$, we obtain:

$$\begin{aligned} \dot{\tilde{\eta}} &= \Gamma\,(\theta)\,\tilde{\eta} + e\,\beta(\theta) + e\,\gamma(e\,y_d, \theta) \\ \dot{e} &= \tilde{\eta}_1 + \frac{d_2\,[1]\,(\theta)}{d_1\,[1]\,(\theta)}e + \ e\phi_1(e, y_d, \theta) + \\ &\quad + d_1\,[1]\,(\theta)\,(\xi_1 +\ p(\theta)) \end{aligned} \qquad (8)$$

[1]See [1] for the definition of a self-tuning regulator

[2]These changes of coordinates are done after having tranformed system (5) to the form (20).

where $e = y - y_d$ is the regulation error, $\tilde{\eta} = \eta - \eta_r$ and $\gamma$ is defined as follows[3]

$$\gamma(e, y_d, \theta) = \begin{bmatrix} \phi_2(e, y_d, \theta) - \frac{d_2[1](\theta)}{d_1[1](\theta)}\phi_1(e, y_d, \theta) \\ \vdots \\ \phi_n(e, y_d, \theta) - \frac{d_n[1](\theta)}{d_1[1](\theta)}\phi_1(e, y_d, \theta) \end{bmatrix} \quad (9)$$

Consider the first equation of (6), $\dot{\xi}_1 = -\lambda_1 \xi_1 + \xi_2$, and the change of coordinates $\tilde{\xi}_1 = \xi_1 - \xi_1^*$ with (we assume $b_\rho > 0$ without loss of generality):

$$\xi_1^* = -e\left[\hat{k}_1(t) + \hat{k}_2(t)\,\alpha_1(e, y_d) + \hat{k}_3(t)\alpha_2^2(e, y_d)\right] - \hat{p}(t) \triangleq e\phi_{\xi_1}(e,t) - \hat{p}(t) \triangleq \xi_1^*\,(e,\,t)$$

Define:

$$\begin{aligned} \xi_2^* \;=\; & \lambda_1\,\xi_1^* + \frac{\partial \xi_1^*}{\partial t} - \frac{1}{2}\left(\frac{\partial \xi_1^*}{\partial e}\right)^2 \tilde{\xi}_1(\hat{k}_2\,[1] + \\ & + \hat{k}_3\,[1]\,\alpha_1^2 + \hat{k}_4\,[1]\,\phi_{\xi_1}^2) - \hat{k}_1\,[1]\,\tilde{\xi}_1 + \\ & \frac{\partial \xi_1^*}{\partial e}\hat{p}_1\,[1] - \frac{\partial \xi_1^*}{\partial e}\,\hat{p}_2\,[1]\,\hat{p} \triangleq \xi_2^*\;\;(e\,,\xi_1\,,t) \end{aligned}$$

where $\hat{p}_1\,[1]$ is the estimate of $p_1 = d_1\,[1]\,(\theta)\,p(\theta)$, $\hat{p}_2\,[1]$ is the estimate of $p_2 = d_1\,[1]\,(\theta)$, and $\hat{k}_i\,[1]\,, 1 \leq i \leq 4$ and $\hat{k}_i, 1 \leq i \leq 3$ are estimates of the unknown constants $k_i\,[1]$ and $k_i$ that are used for the non-adaptive case [1].

Consider the adaptation laws:

$$\begin{aligned} \dot{\hat{k}}_1\,[1] &= \tilde{\xi}_1^2, & \dot{\hat{k}}_1 &= e^2 \\ \dot{\hat{k}}_2\,[1] &= \tilde{\xi}_1^2\left(\frac{\partial \xi_1^*}{\partial e}\right)^2, & \dot{\hat{k}}_2 &= e^2\alpha_1(e, y_d) \\ \dot{\hat{k}}_3\,[1] &= \tilde{\xi}_1^2\left(\frac{\partial \xi_1^*}{\partial e}\right)^2 \alpha_1^2, & \dot{\hat{k}}_3 &= e^2\alpha_2^2(e, y_d) \\ \dot{\hat{k}}_4\,[1] &= \tilde{\xi}_1^2\left(\frac{\partial \xi_1^*}{\partial e}\right)^2 \phi_{\xi_1}^2, & \dot{\hat{p}} &= e \\ \dot{\hat{p}}_1\,[1] &= -2\,\tilde{\xi}_1\frac{\partial \xi_1^*}{\partial e} \\ \dot{\hat{p}}_2\,[1] &= 2\,\tilde{\xi}_1\frac{\partial \xi_1^*}{\partial e}\,\hat{p} \end{aligned}$$

where

$$\begin{aligned} \alpha_1\,(e, y_d) &\geq |\,\phi_1(e\,, y_d, \theta)|\,, \forall \theta \in \Omega \\ \alpha_2\,(e, y_d) &\geq \|\gamma(e,\,y_d,\,\theta\|\,, \forall \theta \in \Omega \end{aligned} \quad (10)$$

The control law for the case $\rho = 2$ is then defined as

$$\begin{aligned} u &= \sigma^{-1}\,(y)\,\xi_2^*\,(e, \xi_1, t) \\ \dot{\xi}_1 &= -\lambda_1 \xi_1 + \sigma(y)\,u \end{aligned}$$

[3]The vector function $\phi$ is obtained in the following way. Define $\psi_D(e, y_d, \theta) = \psi(e + y_d, \theta) - \psi(y_d, \theta)$. We have that $\psi_D(0, y_d, \theta) = 0$ and since $f(x, \theta)$ is assumed to be smooth, $\psi_D(e, y_d\,, \theta)$ is also smooth. Then $\psi_D(e, y_d, \theta)$ can be written as $\psi_D(e, y_d, \theta) = e\,\phi\,(e, y_d, \theta)$ with $\phi\,(e, y_d, \theta)$ smooth (see [1] for details). Writing $\psi_D(e, y_d, \theta)$ in this way is usefull in the proof to put the system in form (8).

The boundedness of the state vector and the estimated parameters as well as the proof of the convergence of the output $y$ to the desired output $y_d$ for the case $\rho = 2$ can be carried out with the following Lyapunov function (making $\xi_2 = \xi_2^*$). See [1] for the case $\rho > 2$.

$$V_1 = \bar{V}_1 + \tilde{k}_1^2[1] + \frac{1}{2}\sum_{i=2}^{4} \tilde{k}_i^2[1] + \frac{1}{2}(\tilde{p}_1^2[1] + \tilde{p}_2^2[1])$$

with

$$\bar{V}_1 = \tilde{\eta}^T\,P(\theta)\tilde{\eta} + \frac{1}{2}e^2 + \xi_1^2 + \frac{1}{2}d_1\,[1]\,(\theta)\sum_{i=1}^{3}\tilde{k}_i^2 + \frac{1}{2}d_1\,[1]\,(\theta)\tilde{p}^2$$

where $\tilde{k}_i = k_i - \hat{k}_i$, $1 \leq i \leq 3$, $\tilde{k}_i[1] = k_i[1] - \hat{k}_i[1]$, $1 \leq i \leq 4$, $\tilde{p} = p - \hat{p}$ and $\tilde{p}_i[1] = p_i[1] - \hat{p}_i[1]$, $i = 1, 2$. *P* is the symmetric positive definite solution of $\Gamma^T(\theta)\,P + P\,\Gamma(\theta) = -2\,I$ where $\Gamma(\theta)$ is asymptotically stable $\forall\,\theta \in \Omega$. ■

**Remark 3.1:** Although the controller does not demand explicitly that the unknwon parameters belong to a known compact set, in order to compute the functions (10), we have to make this assumption.

*B. Application to the vehicle lateral control problem*

We will now apply the theory described above to the vehicle lateral control design. We begin by transforming our system (1) into the form (20) (see the Appendix). Its relative degree is equal to two and it is a minimum phase system. This can be trivially proved by anaylzing the numerator of the transfer function from the steering angle $\delta_f$ (control input) to the lateral displacement at a lookahead distance $y_L$ (output). The numerator of this transfer function is a second-order polynome which coefficients are positive for all possible values of the vehicle parameters[4]. Since the system is linear, minimum phase with known relative degree, it can be transformed into the form (20) by the change of coordinates (see remark 6.1 in the Appendix):

$$\zeta = Tx, \quad T = W * obs(A, C) \quad (11)$$

where $C = \begin{bmatrix} 0, & 0, & 0, & 1 \end{bmatrix}^T$ since the output is $y_d$, $obs(A, C)$ is the observability matrix of the $(A, C)$-pair. The matrix $W$ is given in the following:

$$W^T = \begin{bmatrix} 1 & a_1 & a_2 & a_3 \\ 0 & 1 & a_1 & a_2 \\ 0 & 0 & 1 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

with $a_1 = 2\frac{m(c_f\,l_f^2 + c_r\,l_r^2\,) + I_z(c_f + c_r)}{mvI_z}$, $a_2 = \frac{4c_f c_r l^2 + 2mv^2\,(c_r l_r - c_f l_f)}{m\,v^2\,I_z}$, $a_3 = 0$, where $l = l_r + l_f'$.

After applying the change of coordinates (11) , we obtain the following system:

$$\begin{aligned} \dot{\zeta} &= A_c\zeta + b(\theta)\sigma(y)\delta_f + \psi(y, \theta) \\ y &= c_c\zeta \end{aligned}$$

[4]We remind that all the parameters in Table 1 are positive.

with:

$$A_c = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$b = \begin{bmatrix} 0 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}, \quad c_c = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

where $b_2 = \frac{2c_f(I_z + l_f \ell_s m)}{mI_z}$, $b_3 = \frac{4\,c_f c_r l\,(\ell_s + l_r)}{mvI_z}$, $b_4 = \frac{4c_f c_r l}{mI_z}$ and

$$\sigma(y) = 1,$$

$$\psi(y, \theta) = -\underbrace{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}}_{a} y_L + T \underbrace{\begin{bmatrix} 0 \\ 0 \\ -v \\ -v\ell_s \end{bmatrix}}_{p_1} \rho_{ref} + $$
$$+ T \underbrace{\begin{bmatrix} h_1 \\ h_2 \\ 0 \\ 0 \end{bmatrix}}_{p_2} f_w$$

where $a_4 = 0$.

Since the goal is lane keeping, the desired output is equally to zero, that is, $y_d = 0$. Then, the regulation error is equal to the output, that is, $e = y_L - y_d = y_L$.

In order to proceed in the control design, we will now find the expressions for $\phi$ and $\gamma$. To determine $\phi(e, y_d, \theta) = \phi(y_L, \theta)$ we proceed as following.

We have that

$$\begin{aligned} \psi_D(e, y_d, \theta) &= \psi(e + y_d, \theta) - \psi(y_d, \theta) = \psi(y_L, \theta) - \psi(0, \theta) \\ &= -a y_L + T p_1 \rho_{ref} + T p_2 f_w \\ &\quad - (T p_1 \rho_{ref} + T p_2 f_w) \\ &\Rightarrow \psi_D(e, y_r, \theta) = -a y_L \end{aligned}$$

Since $\psi_D(e, y_d, \theta) = e\phi(e, y_d, \theta)$, we have

$$\phi = \begin{bmatrix} -a_1 \\ -a_2 \\ -a_3 \\ -a_4 \end{bmatrix} \quad (12)$$

From (7), we have

$$\begin{aligned} d[2] &= b \\ d[1](\theta) &= A_c d[2](\theta) + \lambda_1 d[2](\theta) = A_c b + \lambda_1 b \\ &= \begin{bmatrix} b_2 \\ b_3 + \lambda_1 b_2 \\ b_4 + \lambda_1 b_3 \\ \lambda_1 b_4 \end{bmatrix} \end{aligned}$$

According to (9), we have then:

$$\gamma(e, y_d, \theta) = \begin{bmatrix} -a_2 + \frac{b_3 + \lambda_1 b_2}{b_2} a_1 \\ -a_3 + \frac{b_4 + \lambda_1 b_3}{b_2} a_1 \\ -a_4 + \frac{\lambda_1 b_4}{b_2} a_1 \end{bmatrix} \quad (13)$$

Since the relative degree of the system is equal to two and $\sigma(y) = 1$, the filter (6) reduces to the following first-order filter:

$$\dot{\xi}_1 = -\lambda_1 \xi_1 + u \quad (14)$$

The expression of the self-tuning regulator is given by:

$$\delta_f = \xi_2^* (y_L, \xi_1, t) \quad (15)$$

where $\xi_2^* (y_L, \xi_1, t)$, after a parameter reduction (see [1]) is given by

$$\begin{aligned} \xi_2^* &= \lambda_1 \xi_1^* + \frac{d}{dt} \xi_1^* - \\ &- \hat{k}_1 \left[ \frac{\tilde{\xi}_1}{2} \left( \frac{d\xi_1^*}{de} \right)^2 \left(1 + \alpha_1^2 + \phi_{\xi_1}^2\right) + \tilde{\xi}_1 \right] + \\ &+ \frac{d\xi_1^*}{de} \left( \hat{p}_1 [1] - \hat{p}_2 [1] \, \hat{p} \right) \end{aligned} \quad (16)$$

where

$$\xi_1^* = -\hat{k} e \left(1 + \alpha_1 + \alpha_2^2\right) - \hat{p} \quad (17)$$

and

$$\phi_{\xi_1} = -\hat{k}[1 + \alpha_1 + \alpha_2^2] \quad (18)$$

with $\hat{k}$ and $\hat{k}_1$ being estimates of $k = \max_{1 \le i \le 3} k_i$ and $k_1 = \max_{1 \le j \le 4} k_j[1]$ where $k_i$ and $k_j[1]$ are constants used for the nonadaptive case (see [1]).

The adaptation laws with parameter reduction (see [1]) are given by

$$\begin{aligned} \dot{\hat{k}} &= e^2 \left(1 + \alpha_1 + \alpha_2^2\right) \\ \dot{\hat{k}}_1 &= \tilde{\xi}_1^2 \left( \frac{d\xi_1^*}{de} \right)^2 \left(1 + \alpha_1 + \phi_{\xi_1}^2\right) + 2\tilde{\xi}_1^2 \\ \dot{\hat{p}}_1 [1] &= -2\tilde{\xi}_1 \frac{d\xi_1^*}{de} \\ \dot{\hat{p}}_2 [1] &= 2\tilde{\xi}_1 \frac{d\xi_1^*}{de} \hat{p} \\ \dot{\hat{p}} &= e \end{aligned} \quad (19)$$

where $\alpha_1$ and $\alpha_2$ are given by (10) with $\phi$ and $\gamma$ given by (12) and (13).

## IV. SIMULATIONS

The self-tuning regulator (14)-(19) was tested through simulations. The unknown parameter vector $\bar{\theta}$ ($\bar{\theta} \triangleq \left[m, I_z, c_f^*, c_r^*, \mu, n_t, l_f', l_r\right]^T$, where $\theta = \left[\bar{\theta}^T, v\right]^T$) was assumed to belong to the following set $\bar{\Omega}$: $1450kg \le m \le 1700kg$, $2372kg.m^2 \le I_z \le 2781kg.m^2$, $0.3 \le \mu \le 1$, $0.989m \le l_f' \le 1.189m$, $1.269m \le l_r \le 1.469m$, where $n_t$, $c_f^*$ and $c_r^*$ were fixed to $n_t = 0.0113m$, $c_f^*$

$= 57.5kN.rad^{-1}$and $c_r^* = 57.5kN.rad^{-1}$. In order to apply Theorem 3.1, we have made the standard assumption in adaptive control of constant parameters. Since the speed is available for measurement, we use it to compute the parameters $\alpha_1$ and $\alpha_2$ that, in addition, were chosen to respect inequalities (10) for all $\bar{\theta} \in \bar{\Omega}$ and $\lambda_1 = 1$. We proceeded in this way after verifying that by considering $v$ belonging to an interval $[v_{min} \leq v \leq v_{max}]$, the magnitudes of $\alpha_1$ and $\alpha_2$ can increase considerably, what may generate simulation problems. Simulation tests were also carried out to consider the case were $\alpha_1$ and $\alpha_2$ are calculated for a certain speed, with the vehicle actual speed different from this one. In order to be closer to the practice, the simulations were carried out with the nonlinear model of the vehicle (eq. (II.74) in [10])[5].

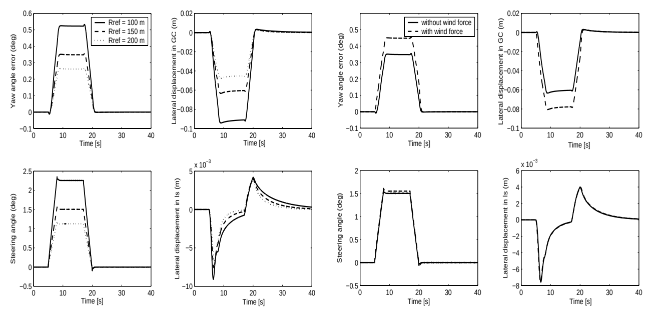
The first simulations were carried out with the nominal vehicle parameters (see Table 1). With the goal of verifying the robustness of the controller to the variations in the curvature, simulations were carried out for three different values of radius of curvature[6] ($R_{ref} = 100m$, $R_{ref} = 150m$, $R_{ref} = 200m$) for a speed of $v = 22m/s$ ($= 79,2km/h$) assuming a dry road (road adherence coefficient $\mu = 1$) (see Figure 1.a). Note that the corresponding lateral accelerations are $\sigma_{lat} = 4,84m/s^2$, $\sigma_{lat} = 3,22m/s^2$ and $\sigma_{lat} = 2,42m/s^2$. The simulations show that the controller has a good performance.We notice that for the worst case, where the lateral acceleration is $\sigma_{lat} = 4,84m/s^2$, the lateral displacement in the vehicle center of gravity ($y_{L_{CG}}$) is smaller than $10cm$, the yaw angle error ($\Psi_L$) is smaller than $0.55$ degrees and the steering angle does not depass $2.3$ degrees and then is physically implementable.

The robustness with respect to the wind was also tested. The simulation was carried out with a wind force of $f_w = 500N$, with $\ell_w = 0.5m$, and a speed of $v = 22m/s$ with $R_{ref} = 150m$ (see Figure 1.b). We can observe that the trajectory is slightly perturbed by the wind. However, we remark that errors remain small ($y_{L_{CG}} = 8cm$,$\Psi_L = 0.45\deg$).

In a third simulation, the robustness with respect to the vehicle parameters was tested. We have then simulated for a set of parameters inside the defined set ($m = 1700kg$, $I_z = 2781kg.m^2$, $l_f^{'} = 1.189$, $l_r = 1.267$, $c_f^* = c_r^* = 57.5kN.rad^{-1}$, $n_t = 0.0113m$). In addition, in this same simulation, we have tested the controller for three different values of the road adherence coefficient ($\mu = 1$, $\mu = 0.7$, $\mu = 0.3$). These simulations were carried out with a speed of $v = 30m/s$ and a radius of curvature of $R_{ref} = 300m$ (see figure 2.a). We note that for a wet road ($\mu = 0.7$), the performance is still good ($y_{L_{CG}} = 28cm$,$\Psi_L = 1.2\deg$). We can also notice that oscillations appear when the road is very slippery ($\mu = 0.3$).

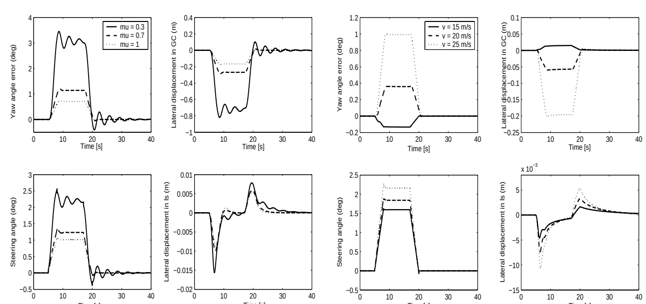
Finally, we tested the effect of changing the vehicle speed. We fixed the radius of curvature to $120m$, with $\mu = 1$, and simulated with three different speeds ($v = 15m/s$, $v = 20m/s$, $v = 25m/s$) (see Figure 2.b). This corresponds to simulating lateral accelerations of $\sigma_{lat} = 1.88m/s^2$, $\sigma_{lat} = 3,33m/s^2$ and $\sigma_{lat} = 5.21m/s^2$. Figure 2.b shows that even when the lateral acceleration is $\sigma_{lat} = 5.21m/s^2$, the errors remain small ($y_{L_{CG}} = 20cm$,$\Psi_L = 1\deg$). In the case $\sigma_{lat} = 1.88m/s^2$, we have that the lateral displacement in the vehicle center of gravity is $y_{L_{CG}} = 1.5cm$ and the yaw angle error is $\Psi_L = 0.14\deg$ and for $\sigma_{lat} = 3,33m/s^2$ we have $y_{L_{CG}} = 6cm$,$\Psi_L = 0.36\deg$, confirming the good performance of the controller.

We performed a last test concerning the robustness of the controller with respect to the choice of the parameters $\alpha_1$ and $\alpha_2$. We repeated the last simulations above with these parameters computed for a speed of $20m/s$. In other words, we carried out these simulations for the speeds of $5m/s$, $10m/s$, $15m/s$ and $25m/s$ by using $\alpha_1$ and $\alpha_2$ computed for $v = 20m/s$ (considering the vehicle nominal parameters and a dry road ($\mu = 1$)). The performance of the controller even in these conditions is quite good, where in the worst case we have that $y_{L_{CG}} = 14cm$,$\Psi_L = 0.73$.




(a) (b)

Fig. 1. Self-tuning controller: variation of road curvature (a), influence of wind forces (b).




(a) (b)

Fig. 2. Self-tuning controller: variation of road adherence coefficient (a), variation of speed (b).

## V. Conclusions

In this paper, we have applied the self-tuning regulator in [1] to solve the vehicle lateral control problem. Moreover, the considered model takes into account perturbations caused by unknown wind forces and road curvature. The simulations confirm the robustness of the controller with

[5]Note that it is still assumed linear contact forces tire-road.

[6]Note that $R_{ref} = 1/\rho_{ref}$.

respect to these parameters. In addition, all the vehicle parameters are considered unknwon, where it is only assumed that they belong to a known compact set. The simulations, carried out with a nonlinear model, confirm the efficacy of the controller, showing its robustness.

## VI. Appendix

**1. Theorem concerning structural geometric conditions for the self-tuning regulator:**

*Theorem 6.1:* **[1]:** Let system (5) be of global relative degree $\rho$, $\forall\theta \in \Omega$. Then, system (5) is transformable by a global state space diffeomorphism

$$\zeta = T(x\,,\,\theta)\ ,\qquad T(x_\theta,\,\theta) = 0\ ,\qquad \forall\theta \in \Omega$$

into

$$\begin{aligned} \dot{\zeta} &= A_c\zeta + b(\theta)\sigma(y)u + \psi(y,\theta) \\ y &= c_c\zeta \end{aligned} \tag{20}$$

with ($A_c$, $b$, $c_c$) minimum phase and in observer canonical form

$$\begin{aligned} A_c &= \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}\ ,\ b = \begin{bmatrix} 0 \\ \vdots \\ b_\rho \\ \vdots \\ b_n \end{bmatrix} \\ c_c &= \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix} \end{aligned} \tag{21}$$

with $b_\rho(\theta)$ of known and constant sign for every $\theta \in \Omega$ if, and only if, for every $\theta \in \Omega$ and for every $x \in \Re^n$:

(1) $rank\ \left\{ dh, \cdots, d\ (L_f^{n-1}h) \right\} = n$

(2) $\left[ad_f^i\ r\,, ad_f^j\,r\right] = 0,\ 0 \leq\ i\ ,\ j \leq n-1$

(3) $\left[g, ad_f^k\,r\right] = 0,\ \ 0 \leq\ k \leq n-2$

(4) There exist a smooth function $\sigma : \Re \rightarrow \Re$ and $n-\rho+1$ reals depending on $\theta$, $b_\rho\ (\theta), \ldots, b_n\ (\theta)$ such that:

$$g = (\sigma \circ h) \sum_{j=1}^{n-\rho+1} b_{n-j+1}\,(\theta) ad_{(-f)}^{j-1}\ r$$

with $b_\rho\ (\theta)\ s^{n-\rho} + \ldots + b_n\ (\theta)$ a Hurwitz polynomial with $b_\rho(\theta)$ of known and constant sign for every $\theta \in \Omega$ .

(5) The vector fields $ad_f^i\ r$ , $0 \leq\ i\ \leq n-1$, are complete, where $r$ is the vector field satisfying

$$\begin{bmatrix} \langle dh\,,\,r\rangle \\ \vdots \\ \left\langle d(L_f^{n-1}h)\,,\,r\right\rangle \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}$$

**Remark 6.1:** Any observable, minimum phase, linear time-invariant system with known relative degree $\rho$:

$$\begin{aligned} \dot{x} &= Fx + gu\ \ ,\quad x \in \Re^n \\ y &= hx\ ,\qquad y \in \Re \end{aligned}$$

with transfer function:

$$W(s) = \frac{b_\rho\ \ s^{n-\rho}\ +\ \cdots\ +\ b_n}{s^n\ +\ a_1\,s^{n-1}\ +\ \cdots\ +\ a_n}$$

is transformable, by a linear change of coordinates, into a system in observer form

$$\begin{aligned} \dot{\zeta} &= \begin{bmatrix} -a_1 & 1 & 0 & \cdots & 0 \\ -a_2 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_{n-1} & 0 & 0 & \cdots & 1 \\ -a_n & 0 & 0 & \cdots & 0 \end{bmatrix} \zeta + \begin{bmatrix} 0 \\ \vdots \\ b_\rho \\ \vdots \\ b_n \end{bmatrix} u \\ &\triangleq A_c\zeta - ay + b\,u \\ y &= \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix} \zeta = c_c\ \zeta \end{aligned}$$

In fact, for linear systems, while condition (1) in Theorem 6.1 amounts to the Kalman observability condition, conditions (2)-(5) are trivially satisfied, provided that the system is minimum phase and the sign of the high frequency gain $b_\rho$ is known.

**2. Matrices $\Gamma$, $\beta$ and $\bar{\psi}$:**

$$\Gamma = \begin{bmatrix} -d_2[1]\,(\theta)\,/d_1[1]\,(\theta) & 1 & 0 & \cdots & 0 \\ -d_3[1]\,(\theta)\,/d_1[1]\,(\theta) & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -d_{n-1}[1]\,(\theta)\,/d_1[1]\,(\theta) & 0 & 0 & \cdots & 1 \\ -d_n[1]\,(\theta)\,/d_1[1]\,(\theta) & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$\beta = \begin{bmatrix} d_3[1]\,(\theta)\,/d_1[1]\,(\theta) - (d_2[1]\,(\theta))^2\,/\,(d_1[1]\,(\theta))^2 \\ d_4[1]\,(\theta)\,/d_1[1]\,(\theta) -\ d_3[1]\,(\theta)\,d_2[1]\,(\theta)\,/\,(d_1[1]\,(\theta))^2 \\ \vdots \\ d_n[1]\,(\theta)\,/d_1[1]\,(\theta) -\ d_{n-1}[1]\,(\theta)\,d_2[1]\,(\theta)\,/\,(d_1[1]\,(\theta))^2 \\ -\,d_n[1]\,(\theta)\,d_2[1]\,(\theta)\,/\,(d_1[1]\,(\theta))^2 \end{bmatrix}$$

$$\bar{\psi} = \begin{bmatrix} \psi_2 - (d_2[1]\,(\theta)\,/d_1[1]\,(\theta))\ \ \psi_1 \\ \psi_3 - (d_3[1]\,(\theta)\,/d_1[1]\,(\theta))\ \ \psi_1 \\ \vdots \\ \psi_{n-1} - (d_{n-1}[1]\,(\theta)\,/d_1[1]\,(\theta))\ \ \psi_1 \\ \psi_n - (d_n[1]\,(\theta)\,/d_1[1]\,(\theta))\ \ \psi_1 \end{bmatrix}$$

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