

# BOČNA NOSIVOST I POMERANJA VERTIKALNIH ŠIPOVA OPTEREĆENIH HORIZONTALNIM SILAMA

## LATERAL CAPACITY AND DEFORMATIONS OF VERTICAL PILES LOADED BY HORIZONTAL FORCES

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### 1 UVOD

Temelji graĎevinskih objekata fundiranih na šipovima uglavnom prenose vertikalno opterećenje, a šipovi su opterećeni aksijalnim silama pritiska/zatezanja [7]. Međutim, ponekad su vertikalni šipovi opterećeni i značajnim horizontalnim silama koje mogu da budu posledica stalnog opterećenja, ali i vetra i/ili zemljotresa. U takvim slučajevima, potrebno je da se odredi bočna nosivost vertikalnih šipova [13]. Ona je posledica horizontalnog pomeranja šipova i usled toga mobilisanja njihove cvrstoće i cvrstoće okolnog tla. Imajući to u vidu, bočna otpornost šipova može da bude prekoracena s obzirom na:

- nosivost okolnog tla, što je tzv. geotehnička nosivost;
- nosivost poprečnog preseka šipa, što je tzv. konstruktivna nosivost.

U ovom radu ćemo, pre svega, analizirati geotehničku nosivost šipova i - saglasno tome - obraditi sledeće metode: Rankinovu, Bromsovu i Brinch-Hansenovu. Osim toga, pokazaćemo kako se mogu odrediti horizontalne deformacije bočno opterećenih vertikalnih

### 1 INTRODUCTION

Pile foundations are mostly loaded by vertical forces, which means that they are loaded by axial compression or tension forces [7]. However, in some cases vertical piles are loaded by high horizontal forces due to the dead loads, winds or earthquakes. In such cases it is necessary to determine lateral capacity of vertical piles which is due to the horizontal displacements of piles and therefore mobilized pile strength and the strength of surrounding soil [13]. So, the ultimate resistance of piles can be reached regarding

- ultimate capacity of surrounding soil i.e. geotechnical capacity
- ultimate capacity of pile cross section i.e. structural capacity.

In this paper, the geotechnical capacity of piles will be analyzed first and then, according to the findings the following methods will be presented: Rankine's, Broms' and Brinch-Hansen's methods. Afterwards, the following methods for determining horizontal deformations of vertical piles loaded by horizontal forces will be presented: applications of elastic theory, coefficient of

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šipova i to primenom teorije elastičnosti, primenom koeficijenta horizontalne krutosti tla ili p-y krivih. Ova problematika je vrlo složena i analizirali su je brojni autori, npr. Brinch-Hansen [15], Broms [2, 3, 4], Meyerhof and Ranjan [16], Meyerhof [17], Milović i Đogo [18, 19], Poulos and Davis [22], Reese and Van Impe [26].

Kada je reč o konstruktivnoj nosivosti šipova, ona se određuje na isti način kao kod armirano-betonskih stubova opterećenih na savijanje. U vezi sa tim naglašavamo da ako je vertikalni šip opterećen istovremeno horizontalnom i vertikalnom (aksijalnom) silom onda se proračun vrši tako što se uzima u obzir interakcija momenta savijanja i aksijalne sile.

## 2 BOČNA NOSIVOST POJEDINAČNOG ŠIPA

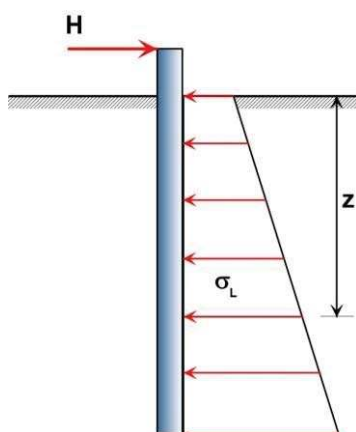
Određivanje bočne/horizontalne nosivosti vertikalnog šipa opterećenog horizontalnom silom složen je inženjerski problem koji je posledica interakcije šipa i okolnog tla [20, 21]. Ona zavisi od čvrstoće okolnog tla, krutosti i dužine šipa, kao i od načina oslanjanja njegove glave.

Stoga, za njeno određivanje, pre svega, potrebno je da se sprovedu adekvatna geotehnička istraživanja, terenska i laboratorijska, te na osnovu toga da se definiše geotehnički model terena na mestu budućeg objekta. A zatim, na tako definisanom modelu, radi se proračun bočne nosivosti šipova [12, 13, 24].

Prilikom određivanja bočne otpornosti tla oko šipa, po pravilu, čine se određena uprošćavanja, kako bi se dobilo rešenje koje je prihvatljivo za geotehničku praksu [6, 9, 16, 17]. Neka od ovih rešenja prikazaćemo u nastavku teksta.

### 2.1 Rankinova metoda

U geotehničkoj praksi (i ne samo našoj [11]), ovaj problem još uvek se tretira ravanski (ravna deformacija) i pretpostavlja se da se pomeranju šipa, od horizontalne sile  $H$ , suprotstavlja pasivni otpor tla (Slika 1), koji se može odrediti iz sledeće jednačine [25]:



Slika 1. Rankinova metoda  
Figure 1. Rankin's method

subgrade reaction or p-y curves, too. This problem is very complex and has been analysed by many authors, e.g. Brinch-Hansen [15], Broms [2, 3, 4], Meyerhof and Ranjan [16], Meyerhof [17], Milović and Đogo [18, 19], Poulos and Davis [22], Reese and Van Impe [26].

Structural capacity of the piles is determined in the same way as for reinforced concrete columns loaded by bending moments. Following that, if the vertical pile is simultaneously loaded by horizontal and vertical (axial) forces than in calculation procedure has to be included interaction between bending moment and axial force.

## 2 LATERAL CAPACITY OF A SINGLE PILE

Determining the lateral/horizontal capacity of vertical piles loaded by horizontal forces is a complex problem which is the consequence of the interaction between pile and surrounding soil [20, 21]. The interaction depends on the strength of surrounding soil, the stiffness and the length of pile and its head support conditions.

Accordingly, at first, it is necessary to make adequate geotechnical investigations, in laboratory and in situ, and on the basis of that geotechnical model of terrain under the structure has to be defined. For such defined model, lateral capacity of piles has to be calculated [12, 13, 24].

Various simplifications are necessary for providing acceptable solutions for geotechnical practice [6, 9, 16, 17]. Some of these solutions will be presented in the following text.

### 2.1 Rankine's method

In geotechnical practice, not only in Serbia [11], this is treated as a plain strain problem using passive earth pressure theory. It is assumed that horizontal movements are restricted by passive resistance of the soil (Fig. 1) which can be determined using the following equation [25]:

$$\sigma_L = \sigma_V \cdot \operatorname{tg}^2\left(45 + \frac{\varphi}{2}\right) + 2 \cdot c \cdot \operatorname{tg}\left(45 + \frac{\varphi}{2}\right) \quad (1)$$

gde je:

$\sigma_L$  – bočni otpor tla na dubini z;  
 $\sigma_V$  – vertikalni napon na dubini z;  
 c – kohezija;  
 $\varphi$  – ugao unutrašnjeg trenja.

where:

$\sigma_L$  – lateral resistance of soil at depth z  
 $\sigma_V$  – vertical stress at depth z  
 c - cohesion  
 $\varphi$  – angle of internal friction

Sumiranjem horizontalnih napona – po dubini i prečniku/širini šipa – i rešavanjem jednačina ravnoteže koje definišu ponašanje šipa, dobija se granična horizontalna sila.

Međutim, ovakav način rada predstavlja konzervativni pristup određivanju bočne nosivosti šipova, jer se prostorni problem rešava ravanski. Na taj način, zanemaruje se uticaj treće dimenzije na veličinu bočnog otpora tla. Kao posledica toga, dobijaju se znatno manje sile bočnog otpora od onih koje okolno tlo može da prihvati.

By summing the horizontal stresses over depth and diameter/width of a pile and by using equilibrium conditions which define the behaviour of a pile, the ultimate lateral force  $H_f$  should be determined.

This approach is, however, conservative because the three-dimensional problem is treated as it is two-dimensional one. In such a way the influence of the third dimension, on lateral force, is neglected. As a consequence, significantly lesser horizontal forces are obtained than the surrounding soil may withstand.

## 2.2 Bromsova metoda

Na osnovu rezultata terenskih opita, Broms je 1964. godine odredio bočnu nosivost vertikalnih šipova, fundiranih u homogenom koherentnom i nekoherentnom tlu [2, 3]. Pritom, kod koherentnog tla analizirao je samo slučaj nedreniranih terenskih uslova. Rezultati tih opita pokazali su da se bočni otpor tla  $\sigma_L$  može izračunati korišćenjem sledećih jednačina:

koherentno tlo:

$$\sigma_L = 9 \cdot c_u \quad (2)$$

nekoherentno tlo:

$$\sigma_L = 3 \cdot \gamma \cdot z \cdot k_p \quad (3)$$

gde je:

$c_u$  – nedrenirana kohezija;  
 $\gamma$  – zapreminska težina;  
 $k_p = \operatorname{tg}^2(45 + \varphi/2)$  - koeficijent pasivnog pritiska;  
 $\varphi$  – ugao unutrašnjeg trenja;  
 z – dubina na kojoj se traži bočni otpor.

where:

$c_u$  – undrained cohesion  
 $\gamma$  – unit weight of soil  
 $k_p = \operatorname{tg}^2(45 + \varphi/2)$  – coefficient of passive resistance  
 $\varphi$  – angle of internal friction  
 z – vertical distance from the ground surface to the location of lateral stress

Jednačine (2) i (3) uključuju trodimenzionalne uslove tla oko šipa.

Broms je u svojim radovima (1964, 1965) analizirao kratke (krute) i dugačke (fleksibilne) šipove (Slika 2) [2, 3, 4]. Pri tome:

– kod kratkih šipova maksimalno horizontalno opterećenje  $H_f$ , koje može da se nanese na šip, ograničeno je maksimalnim horizontalnim otporom koji može da mobilize tlo oko šipa;

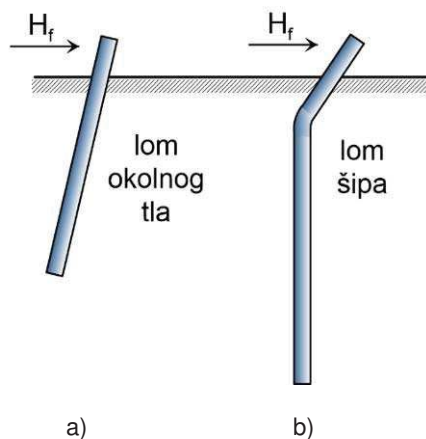
– kod dugačkih šipova maksimalno horizontalno opterećenje  $H_f$ , koje može da se nanese na šip, ograničeno je momentom savijanja koji šip može da prihvati.

The equations (2) and (3) included three-dimensional conditions of the soil surrounding the loaded pile.

In his papers (1964, 1965) Broms has analysed short (stiff) and long (flexible) piles (Fig. 2) [2, 3, 4]. So,

– for short piles, ultimate horizontal force  $H_f$  is limited by ultimate lateral resistance of the surrounding soil

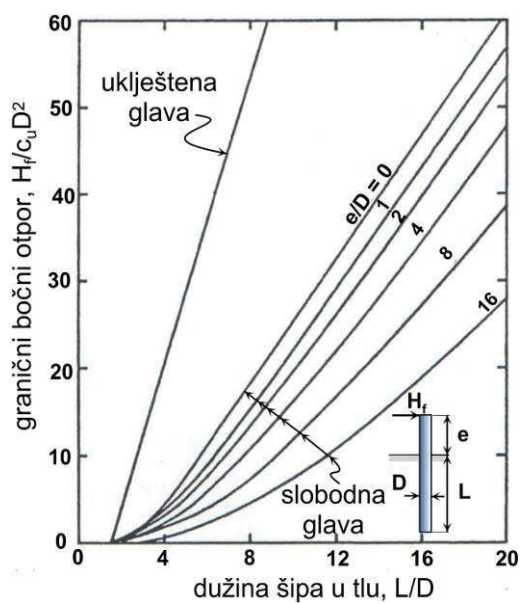
– for long piles, ultimate horizontal force  $H_f$  is limited by yield moment of pile cross-section.



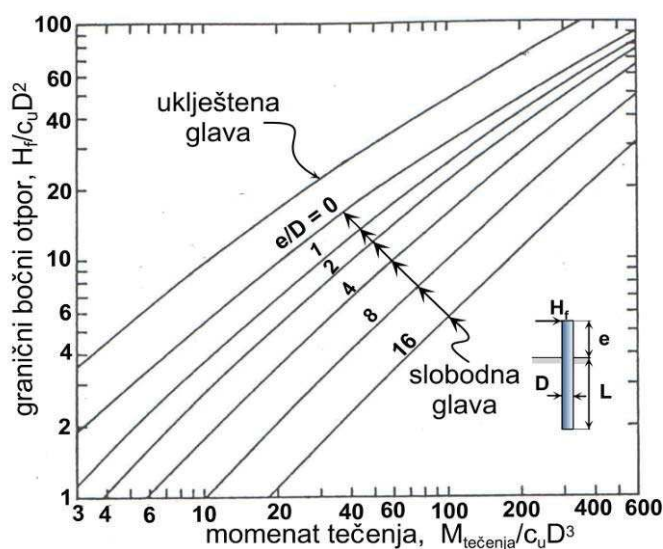
Slika 2. Lom šipa opterećenog horizontalnom silom a) kratki šip; b) dugački šip  
Figure 2. Soil/pile fails loaded by horizontal force a) short pile b) long pile

Broms je definisao načine loma i dijagrame otpornih sila koje deluju na vertikalne šipove – kako one sa slobodnom, tako i one sa uklještenom glavom. Na osnovu toga, postavljanjem odgovarajućih uslova ravnoteže, dobijaju se granične horizontalne sile. Dobijena rešenja Broms je prikazao i grafički – dijagramima na osnovu kojih se lako mogu odrediti granične horizontalne sile  $H_f$  za kratke i dugačke šipove, i u koherentnom, a i u nekoherentnom tlu (Slike 3 i 4).

For short and long vertical piles, Broms has defined the failure mechanisms and the values of lateral earth pressures. He did it for free-headed piles and for piles with restrained head as well. Therefore, ultimate lateral forces  $H_f$  were obtained from the equilibrium considerations. These values Broms presented graphically at Fig. 3 and 4.

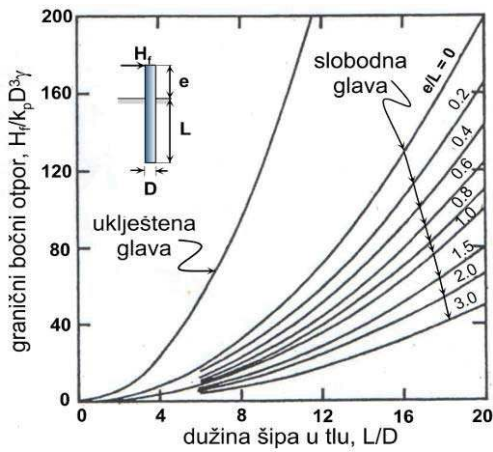


a) kratki šip (short pile)

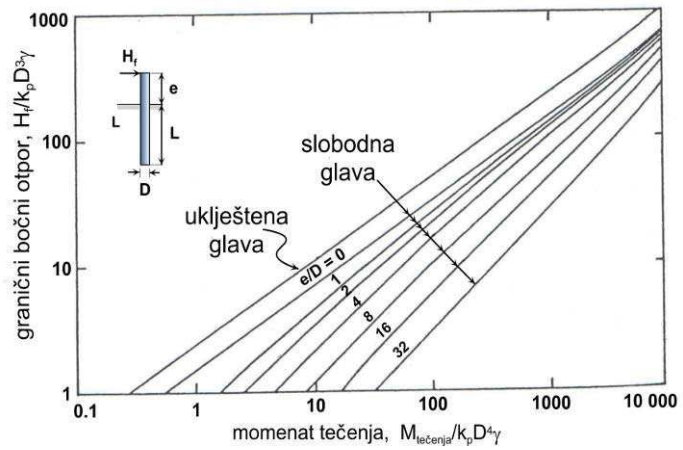


b) dugački šip (long pile)

Slika 3. Granični bočni otpor šipova u koherentnom tlu (Broms, 1964)  
Figure 3. Ultimate lateral resistance of piles in cohesion soils (Broms, 1964)



a) kratki šip (*short pile*)



b) dugački šip (*long pile*)

Slika 4. Granični bočni otpor šipova u nekoherentnom tlu (Broms, 1964)  
Figure 4. Ultimate lateral resistance of piles in cohesionless soils (Broms, 1964)

Na slikama 3 i 4: D je prečnik šipa; L – dubina ukopavanja;  $M_{tečenja}$  – moment savijanja koji izaziva tečenje/lom poprečnog preseka šipa.

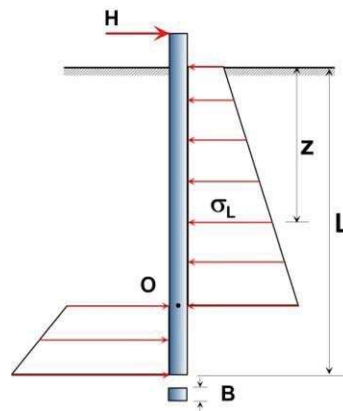
In Fig. 3 and 4 is noted: D – diameter of pile; L – length of embedment;  $M_{yield}$  – yield moment of pile cross-section.

### 2.3 Brinč-Hansenova metoda

Brinč-Hansen (1961) predložio je metodu za određivanje bočne otpornosti tla u slučaju vertikalnog šipa, širine B i dubine ukopavanja L, opterećenog horizontalnom silom H (Slika 5) [15].

### 2.3 Brinch-Hansen's method

Brinch-Hansen (1961) has presented the method for determination of ultimate lateral resistance of the soil surrounding the short vertical piles loaded by horizontal force H (Fig. 5) [15].



Slika 5. Brinč-Hansenova metoda  
Figure 5. Brinch Hansen's method

Ova metoda odnosi se na kratke - krute šipove koji se pod dejstvom sile H rotiraju oko tačke O. Bočni pritisci  $\sigma_L$  uzimaju u obzir trodimenzionalne uslove u kojima se šip nalazi i predstavljaju razliku između bočnih pritisaka ispred i iza šipa. Veličina tako definisanih bočnih pritisaka, određuje se iz sledeće jednačine:

In the state of failure pile rotates, as a rigid body, about a point O. Lateral pressures  $\sigma_L$  take into consideration three-dimensional conditions of surrounding soil and they are resultant of pressures i.e. passive minus active pressures. So defined lateral pressures  $\sigma_L$  can be determined from the following equation:

$$\sigma_L = q \cdot k_q + c \cdot k_c \quad (4)$$

gde je:

$\sigma_L$  – bočni pritisak na dubini z;  
q =  $\sigma_V$  – vertikalni napon na dubini z;

where:

$\sigma_L$  – lateral pressure at depth z  
q =  $\sigma_V$  – vertical stress at depth z



c – kohezija;  
 $k_q$  i  $k_c$  – koeficijenti bočnog pritiska tla.

c – cohesion  
 $k_q$  and  $k_c$  – coefficients of lateral pressures of soil

Veličina koeficijenata  $k_q$  i  $k_c$  određuje se iz dijagrama datih na slikama 6 i 7. Na tim dijagramima  $\varphi$  je ugao unutrašnjeg trenja.

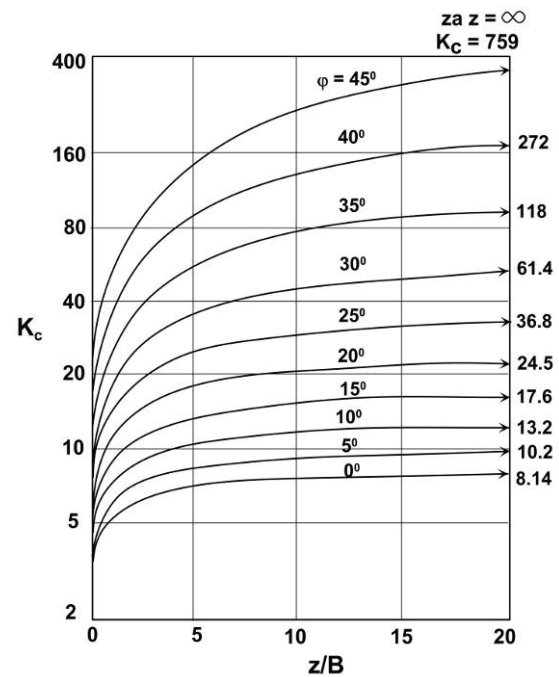
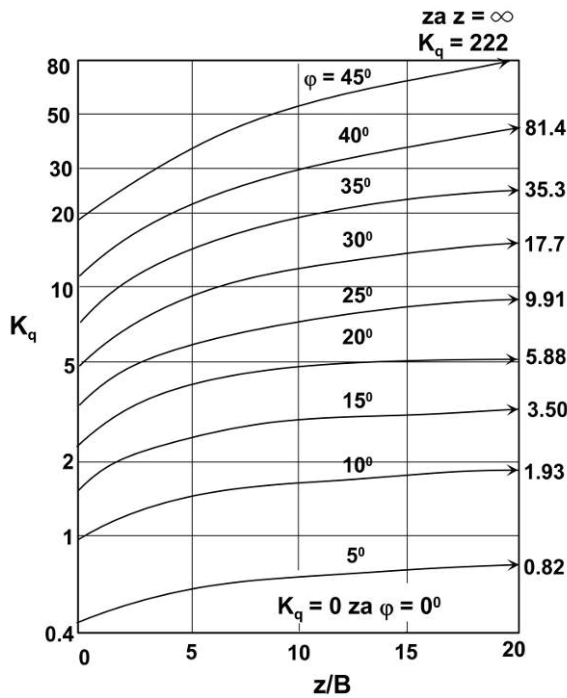
Coefficients  $k_q$  and  $k_c$  may be determined from curves given in Fig. 6 and 7. In these figures  $\varphi$  is the angle of internal friction.

Veličina granične horizontalne sile  $H_f$  – koja deluje na šip (Slika 8) – određuje se rešavanjem sledećih jednačina ravnoteže:

The ultimate horizontal force  $H_f$  (Fig. 8) is determined by means of following equilibrium conditions:

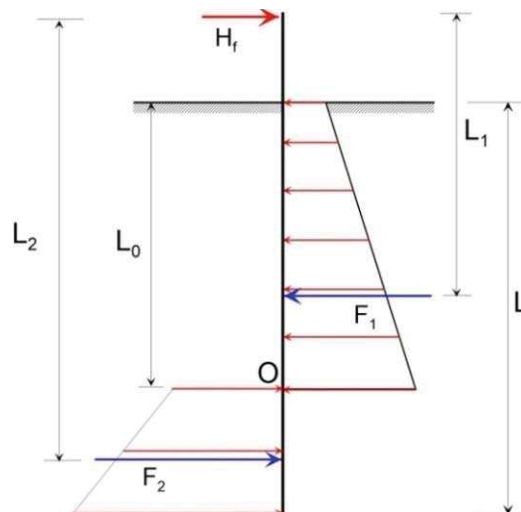
$$F_1 \cdot L_1 = F_2 \cdot L_2 \quad (5)$$

$$H_f = F_1 - F_2 \quad (6)$$



Slika 6. Koeficijent bočnog pritiska koji zavisi od vertikalnog napona (Brinch-Hansen, 1961)  
 Figure 6. Coefficient of lateral pressure which is dependent of vertical stress (Brinch-Hansen, 1961)

Slika 7. Koeficijent bočnog pritiska koji zavisi od kohezije (Brinch-Hansen, 1961)  
 Figure 7. Coefficient of lateral pressure which is dependent of cohesion (Brinch-Hansen, 1961)



Slika 8. Geotehnička nosivost šipa opterećenog horizontalnom silom  
 Figure 8. Geotechnical capacity of vertical pile loaded by horizontal force

## 2.4 Dozvoljeno bočno opterećenje

U poglavljima 2.1, 2.2 i 2.3 prikazani su postupci određivanja granične nosivosti pojedinačnog vertikalnog šipa, opterećenog horizontalnom silom. Pritom, za dobijanje dozvoljenog bočnog/horizontalnog opterećenja  $H_a$ , potrebno je da se njegova nosivost  $H_f$  redukuje faktorom sigurnosti  $F_s$ , tj.

$$H_a = \frac{H_f}{F_s} \quad (7)$$

Veličina faktora sigurnosti kreće se između  $F_s = 2$  i  $3$ .

Napominjemo i to da ukoliko je konstruktivna nosivost šipa manja od njegove geotehničke nosivosti, onda je ona merodavna za određivanje horizontalne sile koju vertikalni šip može da prihvatiti.

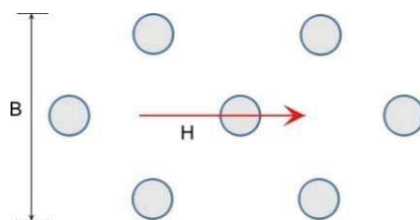
Horizontalna sila  $H$  – koja deluje na šip – mora da bude manja od dozvoljene sile  $H_a$ . Osim toga, horizontalna pomeranja šipa treba da budu u dozvoljenim granicama.

## 2.5 Komentar

Uvažavajući sve što je rečeno u poglavlju 2, smatramo da Brinč-Hansenova metoda ima prednost u odnosu na druga dva prikazana postupka. Naime, ona uključuje trodimenzionalne uslove koji vladaju u tlu oko šipa, a može da se primeni u homogenom i heterogenom tlu i to u dreniranim ali i nedreniranim uslovima [29, 30]. Pri tome, ona je vrlo jednostavna za primenu, čak i u veoma složenim geotehničkim uslovima koji su često izraženi u Srbiji. To je veoma značajno prilikom fundiranja objekata, kao i prilikom sanacije klizišta [7, 8].

## 3 BOČNA NOSIVOST GRUPE ŠIPOVA

Šipovi u temeljima nikad ne dolaze pojedinačno, već kao grupa šipova koja je povezana krutom temeljnom stopom. Stoga, prilikom proračuna bočne nosivosti šipova potrebno je da se ima u vidu i njihov grupni efekat [14]. S tim u vezi, Brinč-Hansen predlaže da se, prilikom proračuna, kao ekvivalentna širina  $B$ , usvoji ukupna širina grupe šipova - upravna na pravac sile  $H$  (Slika 9) [15].



Slika 9. Ekvivalentna širina grupe vertikalnih šipova  
Figure 9. Equivalent width for group of vertical piles

Treba reći da Poulos and Davis (1980) predlažu da se bočna nosivost grupe šipova odredi kao manja od sledeće dve vrednosti [22]:

- zbir bočne nosivosti pojedinačnih šipova;

## 2.4 Allowable lateral capacity

In Chapters 2.1, 2.2 and 2.3 are presented the procedures for determining bearing capacity for single vertical pile loaded by horizontal force. Accordingly, for determining allowable lateral/horizontal force  $H_a$ , it is necessary to reduce  $H_f$  by safety factor  $F_s$  i.e.

The value of safety factor is between 2 and 3.

It is obvious that, if the structural capacity of a pile is less than geotechnical capacity of a pile, then it is proper for calculating allowable horizontal force of a pile.

Horizontal designed value  $H$  has to be less than allowable force  $H_a$ . Besides, lateral deformations of a pile have to be in allowable range.

## 2.5 Comment

In accordance with Chapter 2, Brinch-Hansen's method has a priority over the other two presented methods. Namely, it involves three-dimensional conditions of surrounding soil around the pile. Besides, it can be applied in homogenous and heterogeneous soils, in drained or undrained conditions, too [29, 30]. Moreover, it is very simple for application even in very complex geotechnical conditions which are very often in Serbia. This is highly important for foundation of structures and landslide's remedial measures, too [7, 8].

## 3 LATERAL BEARING CAPACITY OF A PILE GROUP

In the foundation structure, piles are unlikely installed as single ones, but as group of piles which are jointed by stiff foundation cap. Therefore, calculation procedure should take into account their group effects [14]. Accordingly, Brinch-Hansen suggested that an equivalent width  $B$  has to be the width of a group perpendicular to the direction of the force  $H$  (Fig. 9) [15].

In estimating the lateral bearing capacity of a pile group Poulos and Davis (1980) suggested the lesser of the following two values [22]:

- the sum of the lateral capacity of single piles

– bočne nosivosti ekvivalentnog temeljnog bloka koji obuhvata šipove i tlo između njih.

Dozvoljeno horizontalno opterećenje grupe šipova određuje se na isti način kao i u slučaju pojedinačnih šipova, odnosno redukcijom graničnog opterećenja.

#### 4 POMERANJA BOČNO OPTEREĆENIH ŠIPOVA

Prilikom projektovanja temelja na šipovima, osim bočne nosivosti šipova, treba prvenstveno da se odrede i horizontalna pomeranja glave šipova i da se proveri da li su ona, za projektovano opterećenje, u dozvoljenim granicama [18, 19]. Ta pomeranja mogu da se odrede primenom teorije elastičnosti, pomoću koeficijenta horizontalne krutosti tla ili korišćenjem p-y krivih. Ovo ćemo obraditi u nastavku teksta.

##### 4.1 Elastična analiza

Deformacije bočno opterećenog šipa u homogenom tlu, koje se može definisati kao linearno elastična sredina, mogu se odrediti primenom teorije elastičnosti [5]. Poulos and Davis (1980) horizontalno pomeranje  $\rho$  i rotaciju  $\theta$  šipa na površini terena (tačka A), usled dejstva horizontalne sile H koja deluje na visini e iznad površine terena (Slika 10), definisali su sledećim jednačinama [22]:

$$\rho = \frac{H}{E_s \cdot L} \cdot \left( I_{\rho H} + \frac{e}{L} \cdot I_{\rho M} \right) \quad (8)$$

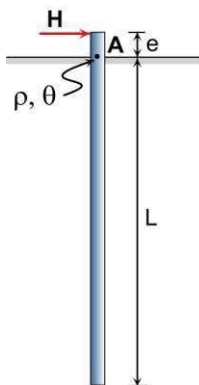
$$\theta = \frac{H}{E_s \cdot L^2} \cdot \left( I_{\theta H} + \frac{e}{L} \cdot I_{\theta M} \right) \quad (9)$$

gde je:

$E_s$  – modul elastičnosti tla;  
L – dubina ukopavanja vertikalnog šipa;  
 $I_{\rho H}$ ,  $I_{\theta H}$ ,  $I_{\rho M}$ ,  $I_{\theta M}$  – uticajni faktori.

where:

$E_s$  – modulus of elasticity of soil  
L – embedment length  
 $I_{\rho H}$ ,  $I_{\rho M}$ ,  $I_{\theta H}$ ,  $I_{\theta M}$  – influence factors



Slika 10. Rešenje Poulos-a i Davis-a (1980)  
Figure 10. Poulos and Davis solution (1980)

Vrednosti uticajnih faktora  $I_{\rho H}$ ,  $I_{\theta H}$ ,  $I_{\rho M}$ ,  $I_{\theta M}$  određuju se iz dijagrama datih na slikama 11, 12 i 13. Na tim slikama, vidi se da vrednost Poasonovog koeficijenta tla jeste  $\nu = 0.5$ , a veličine uticajnih faktora zavise od faktora savitljivosti šipa  $K_R$

– the lateral ultimate capacity of an equivalent single block containing the piles in the group and the soil between them.

The allowable lateral bearing capacity of a pile group determination is the same as for single piles i.e. by reduction of lateral ultimate capacity with safety factor.

#### 4 DEFORMATIONS OF Laterally LOADED PILES

For designing pile foundations, not only lateral bearing capacity but the horizontal displacements have to be determined, too. They have to be, for designed loads, in allowable limits [18, 19]. Lateral deformations of piles have to be estimated by elastic analysis, by application the concept of coefficient of subgrade reaction or by use p-y curves. These will be presented in the following text.

##### 4.1 Elastic analysis

On the basis of Theory of elasticity, Poulos and Davis (1980) presented solutions for lateral deflections of a single free-head pile within a linear-elastic uniform continuum [5]. The vertical pile is loaded by horizontal force H acting at a distance e above ground line (Fig. 10). Ground line displacement  $\rho$  and ground line rotation  $\theta$  (point A at Fig. 10) are expressed as [22]:

Values of influence factors  $I_{\rho H}$ ,  $I_{\theta H}$ ,  $I_{\rho M}$ ,  $I_{\theta M}$  are given in Fig. 11, 12 and 13, and the Poisson's ratio of soil is  $\nu = 0.5$ . From presented figures it is obvious that values of influence factors are functions of pile flexibility factor  $K_R$



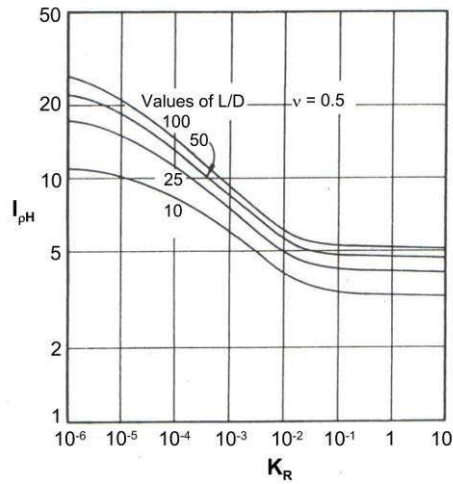
$$K_R = \frac{E_p \cdot I_p}{E_s \cdot L^4} \quad (10)$$

gde je:

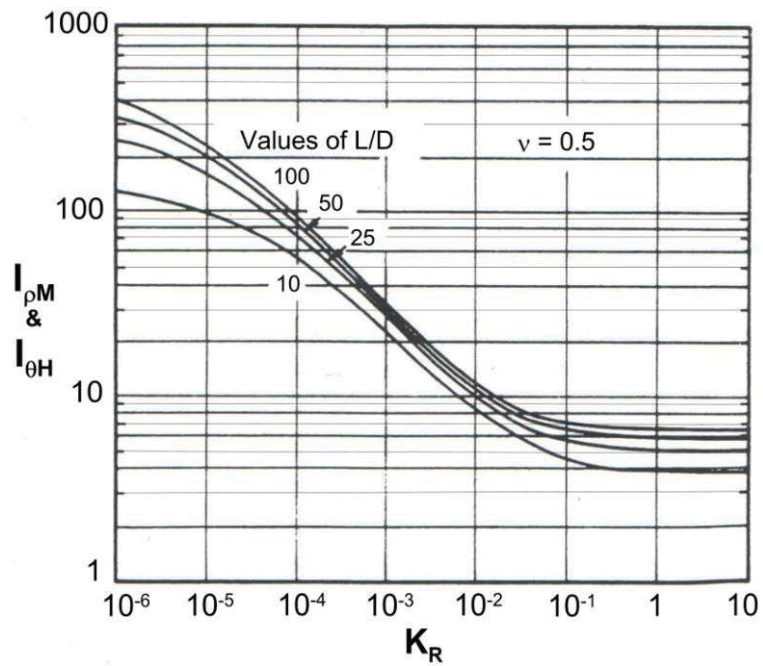
$E_p$  – modul elastičnosti šipa;  
 $I_p$  – momenat inercije šipa.

where:

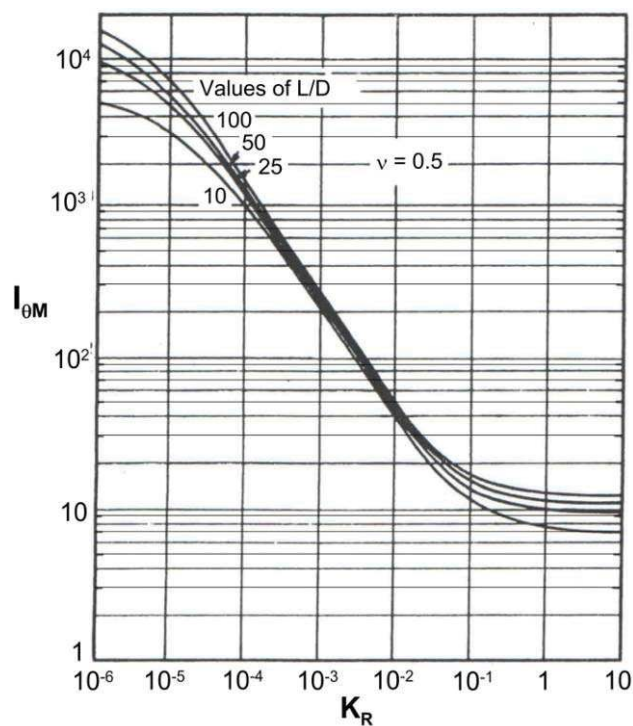
$E_p$  – modulus of elasticity of pile  
 $I_p$  – moment of inertia of pile



Slika 11. Vrednosti  $I_{pH}$  (Poulos and Davis, 1980)  
 Figure 11. Values of  $I_{pH}$  (Poulos and Davis, 1980)



Slika 12. Vrednosti  $I_{pM}$  i  $I_{\theta H}$  (Poulos and Davis, 1980)  
 Figure 12. Values of  $I_{pM}$  and  $I_{\theta H}$  (Poulos and Davis, 1980)



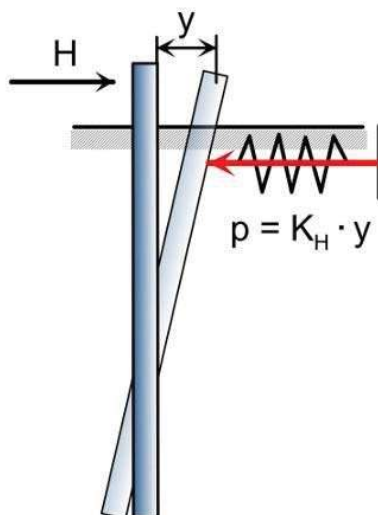
Slika 13. Vrednosti  $I_{\theta M}$  (Poulos and Davis, 1980)  
 Figure 13. Values of  $I_{\theta M}$  (Poulos and Davis, 1980)

#### 4.2 Primena koeficijenta krutosti tla

Bočna pomeranja šipa, usled dejstva horizontalne sile  $H$  (Slika 14), najčešće se sračunavaju pomoću koeficijenta horizontalne krutosti (reakcije) tla [21]

#### 4.2 Application of subgrade reaction coefficient

Lateral deformation of vertical pile, loaded by a horizontal force  $H$  (Fig. 14), may be estimated by coefficient of horizontal subgrade reaction of a soil  $K_H$  [21]



Slika 14. Horizontalno pomeranje šipa  
 Figure 14. Horizontal displacement of a pile

$$K_H = \frac{p}{y} \quad (11)$$

gde je:

$K_H$  – koeficijent horizontalne krutosti tla;

$p$  – bočni pritisak na mestu gde je pomeranje šipa jednako  $y$ ;

$y$  – horizontalno pomeranje šipa.

Koeficijent  $K_H$  ne zavisi samo od vrste tla i njegovih deformacionih karakteristika već i od prečnika/širine šipa [7].

U postupku proračuna tlo se zamenjuje serijom linearno-elastičnih opruga, s tim što se krutost svake opruge izražava koeficijentom horizontalne krutosti. Pri tome se usvaja da je njegova vrednost za koherentno tlo konstantna po dubini, a da se za nekoherentno tlo ona linearno povećava s dubinom.

Navedeni postupak proračuna deformacija u Srbiji koristi se u kompjuterskom programu TOWER.

Vrednosti koeficijenta horizontalne krutosti tla mogu da se odrede na sledeći način:

#### a) nekoherentna tla

Za nekoherentno tlo,  $K_H$  se određuje iz sledeće jednačine [28]:

$$K_H = n_h \cdot \frac{z}{D} \quad (12)$$

gde je:

$n_h$  – koeficijent koji zavisi od gustine tla (Tabela 1);

$z$  – dubina ispod površine terena;

$D$  – prečnik/širina šipa.

where:

$K_H$  – coefficient of horizontal subgrade reaction of soil

$p$  – lateral pressure at point where the displacement of a pile is  $y$

$y$  – horizontal displacement of a pile.

Coefficient  $K_H$  is dependent not only of soil type and its deformation properties but on diameter/width of a laterally loaded pile, too [7].

In calculation procedure it is assumed that the soil around a pile can be replaced by the series of horizontal linear-elastic springs and the stiffness of each spring is expresses by its coefficient of subgrade reaction. It has been assumed that its value increases linearly with depth in the case of cohesionless soils and that it is constant with depth for cohesive soils.

In Serbia this concept is incorporated in computer program TOWER.

The values of coefficient of horizontal subgrade reaction may be estimated by the following procedures:

#### a) cohesionless soil

In cohesionless soil  $K_H$  is [28]:

where:

$n_h$  – coefficient related to soil density (Table 1)

$z$  – depth below ground surface

$D$  – pile diameter/width

Tabela 1. Vrednosti koeficijenta  $n_h$  za nekoherentno tlo  
Table 1. Values of  $n_h$  for cohesionless soils (Terzaghi, 1955)

zbijenost tla <i>Soil compaction condition</i>	$n_h$ (kN/m <sup>3</sup> )	
	iznad NPV <i>above groundwater</i>	ispod NPV <i>below groundwater</i>
rastresito / <i>loose</i>	2200	1300
srednje zbijeno / <i>compact</i>	6600	4400
zbijeno / <i>dense</i>	18000	11000

NPV – nivo podzemne vode

#### b) koherentna tla

U našoj geotehničkoj praksi, za koherentno tlo, često se koeficijent horizontalne krutosti tla  $K_H$  određuje pomoću sledeće jednačine [32]

$$K_H = 0.65 \cdot \sqrt[12]{\frac{E_s \cdot D^4}{E_p \cdot I_p}} \cdot \frac{E_s}{B \cdot (1 - \nu^2)} \quad (13)$$

gde je:

$E_s$  – modul elastičnosti tla;

$E_p$  – modul elastičnosti šipa;

$\nu$  – Poasonov koeficijent tla;

$D$  – širina/prečnik šipa;

$I_p$  – momenat inercije šipa.

#### b) cohesive soil

In Serbian geotechnical practice, for cohesive soil, the value of  $K_H$  is estimate, very often, from the following equation [32]:

where:

$E_s$  – modulus of elasticity of soil

$E_p$  – modulus of elasticity of pile

$\nu$  – Poisson's ratio of soil

$B$  – pile diameter/width

$I_p$  – modulus of inertia of pile

Ova jednačina može da se koristi i za određivanje  $K_H$  za nekoherentna tla [1].

Inače, u slučaju nedreniranih uslova u tlu, koristi se i sledeća jednačina [10]

This equation may be used for estimation of  $K_H$  in cohesionless soils, too [1].

In the case of undrained conditions in soil,  $K_H$  may be estimated as [10]

$$K_H = \frac{67 \cdot S_u}{D} \quad (14)$$

gde je:

$S_u$  – nedrenirana čvrstoća smicanja tla;  
 $D$  – prečnik šipa.

where:

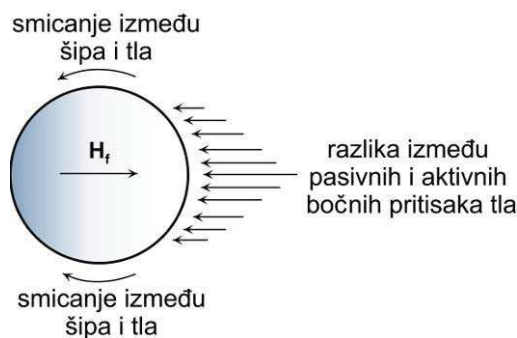
$S_u$  – undrained shear strength of the soil  
 $D$  – pile diameter/width

c) komentar

Na kraju, posebno naglašavamo da vrednosti  $K_H$  izračunate u ovom poglavlju treba duplirati prilikom projektovanja šipova [1]. To je posledica znatnog otpora smicanja između šipa i okolnog tla (Slika 15) [27].

c) comment

Finally, it has to emphasize that the values of  $K_H$ , estimated in this Chapter, should be doubled for pile design [1]. This is a consequence of considerable side shear resistance between pile and surrounding soil (Fig. 15) [27].



Slika 15. Otpor tla kod bočno opterećenog šipa (Smith, 1989)  
 Figure 15. Soil resistance to a lateral pile load (Smith, 1989)

### 4.3 Koncept p-y krive

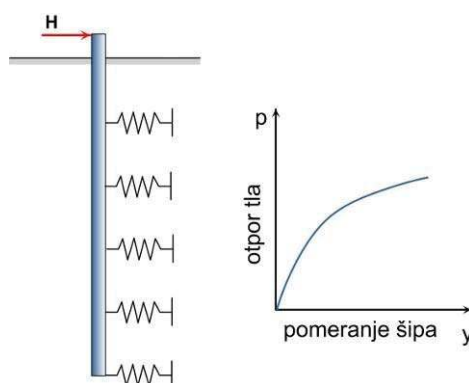
Ovom metodom se tlo oko šipa prikazuje serijom nelinearnih opruga, s tim što svaka opruga definiše zavisnost između bočnog otpora tla  $p$  i njegovog bočnog pomeranja  $y$  – na određenoj dubini ispod površine terena. Ta zavisnost određena je p-y krivama (Slika 16) [26, 31].

Ukoliko je tlo oko šipa višeslojno, onda se p-y krive određuju posebno za svaki sloj. One se mogu odrediti na osnovu rezultata laboratorijskih ili terenskih opita. Za brojna tla p-y krive već su određene i uključene u odgovarajuće kompjuterske programe (npr. LPILE) [26]. Na osnovu toga mogu da se dobiju horizontalna pomeranja šipova.

### 4.3 Concept of p-y curve

In this method the surrounding soil is simulated by using series of nonlinear horizontal springs. The each spring represents the relationship between horizontal soil resistance  $p$  and horizontal displacement  $y$  – at the particular depth under the ground line. This relationship is defined by p-y curve (Fig. 16) [26, 31].

If the surrounding soil is heterogeneous, than p-y curve has to estimate for each layer of soil. These curves may be determined by the results of laboratory or in situ tests. For different soils they had been already determined and were incorporated into adequate computer programs (e.g. LPILE) [26]. Based on that, horizontal displacements of piles can be obtained.



Slika 16. Koncept p-y krive  
 Figure 16. Concept p-y curve

## 5 BOČNA NOSIVOST I POMERANJA ŠIPOVA ZA SILOS KLINKERA U BEOČINU

Objekti za skladištenje klinkera, u okviru fabrike cementa „Lafarge” B.F.C. BEOČIN, sadrže tri vertikalna silosa. Dva silosa klinkera izgrađena su ranije i imaju kapacitet od 35.000 t, dok treći silos ima kapacitet od 50.000 t. Za potrebe izgradnje ovog trećeg silosa, „Hidrozaovod DTD” iz Novog Sada izveo je geotehnička istraživanja terena (četiri istražne bušotine SB-1 do SB-4 dubine od po 15 m, iz kojih je uzeto i laboratorijski ispitano 30 uzoraka tla).

Na osnovu obavljenih istraživanja, teren na lokaciji silosa raščlanjen je na tri sredine: dobro do loše granulisane srednje zbijene peskove i dobro granulisane šljunkovite srednje zbijene peskove debljine od 6.0 do 7.0 m (SW/SP i GW/SW), loše granulisane peskove s proslojcima šljunka, debljine 7.0-8.0 m (SP, SP/GW), dok su na dubini od oko 14 m utvrđeni lapori. Kako su fizičko-mehaničke karakteristike prva dva sloja vrlo slične, formiran je pojednostavljeni geotehnički model terena, koji je poslužio da se uradi i numerička analiza geotehničke nosivosti i pomeranja bočno opterećenih šipova (Slika 17) [23, 24].

Intenzitet horizontalne sile, koju može da prihvati betonski šip prečnika  $D = 0.90$  m i dužine  $L = 10$  m, odredićemo primenom Brinč-Hansenove metode. Vrednosti bočnih pritisaka  $\sigma_L$  po 1 m prečnika šipa, prikazane su u Tabeli 2 i na Slici 17.

## 5 LATERAL BEARING CAPACITY AND HORIZONTAL DISPLACEMENTS OF PILES FOR CLINKER BIN IN BEOCIN

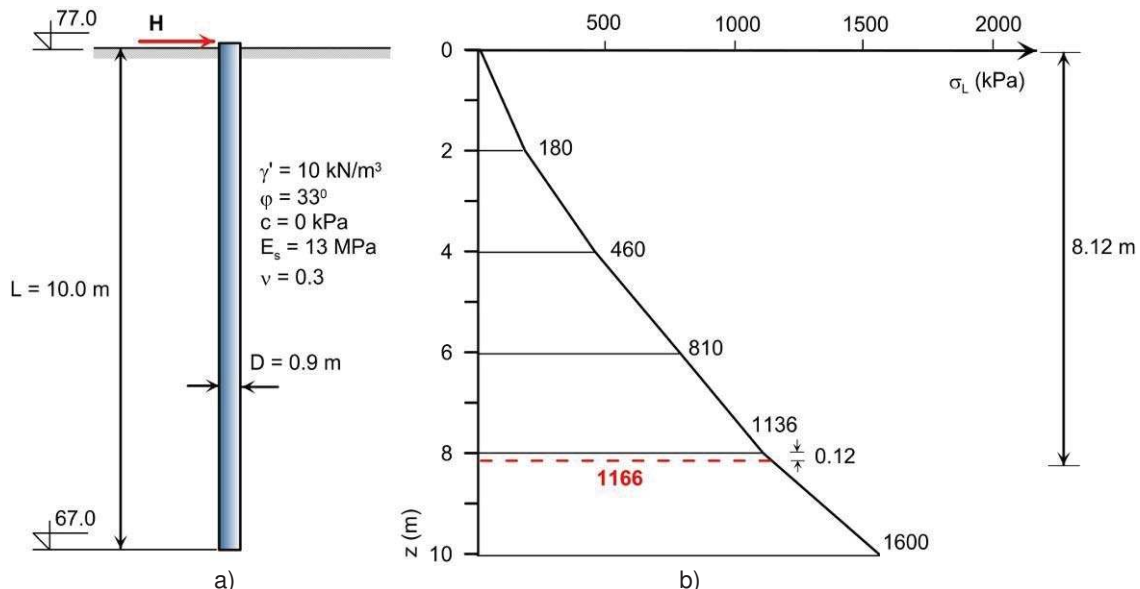
There are three vertical cement bins for binning clinkers in the area of cement factory “Lafarge” BFC in Beocin. Two cement bins, already, have been constructed and have the capacity of 35000 t each. A third one has the capacity of 50000 t. For constructing the third one, “Hidrozaovod DTD” from Novi Sad made geotechnical investigations of terrain (4 boreholes SB-1 to SB-4 with depths of 15 m; from these boreholes 30 samples of soil were tested in laboratory).

On the basis of geotechnical investigations, the terrain under the third bin is divided in three layers: well to weak grained sands and compact well grained sandy-gravels with depths of 6,0-7,0 m (SW/SP and GW/SW), weak ground sands with interbeds of gravels, depths of 7,0-8,0 m(SP, SP/GW). At the depth of about 14,0 m there are marls. As the physical-mechanical properties of two upper layers are very similar, simplified geotechnical model of terrain was created. Numerical analysis was made in it for the calculation of geotechnical capacity and horizontal displacement of laterally loaded piles (Fig. 17) [23, 24].

Horizontal force intensity, which can be sustained by a pile with a diameter  $D = 0,90$  m and length  $L = 10$  m, will be determined by using Brinch-Hansen's method. Values of laterally pressures  $\sigma_L$  for diameter of pile 1,0 m are presented in Table 2 and in Fig. 17.

Tabela 2. Bočni pritisci na šip  
Table 2. Lateral pressures for pile

z (m)	0	2	4	6	8	10
z/D	0	2.2	4.4	6.7	8.9	11.1
$k_q$	0	9	11.5	13.5	14.2	16
q (kPa)	0	20	40	60	80	100
$\sigma_L = q \cdot k_q$ (kPa)	0	180	460	810	1136	1600



Slika 17. a) geotehnički model terena; b) dijagram bočnih pritisaka na šip  
Figure 17. a) geotechnical model of terrain; b) lateral pressure diagram for pile

Rešavanjem jednačine (5) određuje se položaj centra rotacije šipa, odnosno dužina  $L_0$ . U našem slučaju je  $L_0 = 8,12$  m. Tako da je  $F_1 = 3762$  kN, a  $F_2 = 2332$  kN. Iz uslova ravnoteže horizontalnih sila (jednačina 6) određuje se granična horizontalna sila  $H_f = 1430$  kN. Ako usvojimo da je  $F_s = 2,5$  onda je dozvoljena horizontalna sila  $H_a = 572$  kN. Ona je višestruko veća od stvarne horizontalne sile koja deluje na šip i iznosi 166 kN. Napominjemo da se primenom Bromsove metode dobija  $H_f = 1525$  kN i  $H_a = 610$  kN [8].

Horizontalno pomeranje glave šipa  $\rho$ , određeno je primenom teorije elastičnosti (jednačina 8). Usvojeno je da je modul elastičnosti šipa  $E_p = 30\ 000$  MPa, tako da je krutost šipa  $K_R = 0,00743$  a  $I_{pH} = 4,8$ . Delovanje horizontalne sile  $H = 166$  kN izaziva horizontalno pomeranje glave šipa  $\rho = 6,13$  mm.

Horizontalna pomeranja, bočno opterećenog šipa, određujemo i pomoću koeficijenta horizontalne krutosti okolnog tla. Na slikama 18 i 19 prikazan je  $K_H$  koncept za numeričku analizu bočno opterećenog šipa. U postupku proračuna uzeta je u obzir smičuća otpornost između šipa i tla i stoga su duplirane vrednosti  $n_h$  iz Tabele 1 tj.  $n_h = 2 \times 4400 = 8800$  kN/m<sup>3</sup>, kao i vrednosti  $K_H$  iz jednačine 13 tj.  $K_H = 2 \times 12520 = 25040$  kN/m<sup>3</sup>. Numerička analiza urađena je primenom kompjuterskog programa TOWER.

Na ovim slikama vidi se da horizontalna pomeranja glave šipa, usled dejstva sile  $H = 166$  kN, iznose  $\rho = 6,41$  mm (Slika 18) i  $\rho = 6,73$  mm (Slika 19). Te vrednosti dobro se slažu s pomeranjem koje je prethodno dobijeno elastičnom analizom.

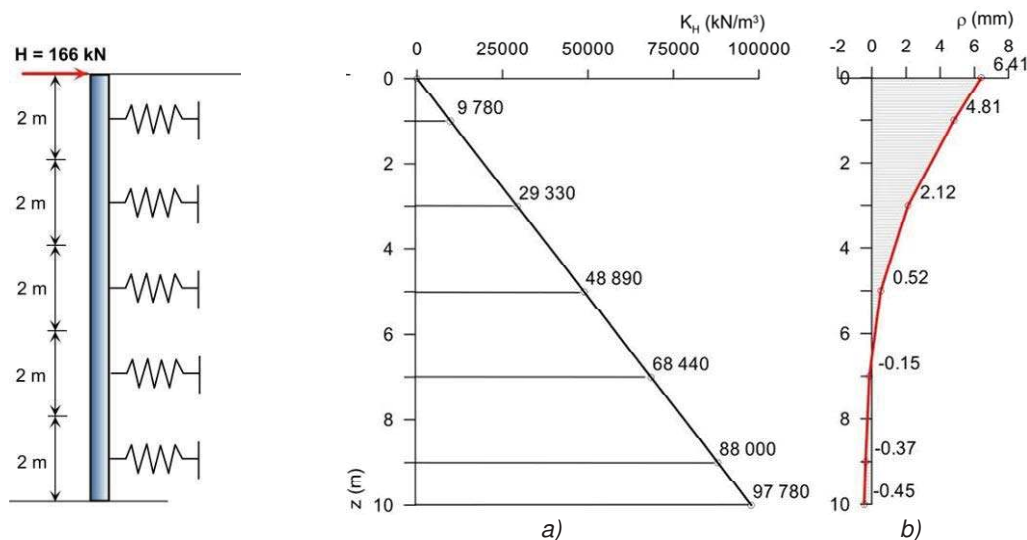
From the equation (5) the position of the rotation centre of the pile can be calculated i.e. the length  $L_0$ . In this case  $L_0 = 8,12$  m. So,  $F_1 = 3762$  kN and  $F_2 = 2332$  kN.

From the equilibrium conditions of horizontal forces (eq. 6) ultimate horizontal force  $H_f = 1430$  kN can be calculated. Allowable horizontal force, for safety factor  $F_s = 2,5$ , is  $H_a = 572$  kN. It is much higher than the designed horizontal force  $H = 166$  kN. It should be said that, by using Broms' method, it was estimated that  $H_f = 1525$  kN and  $H_a = 610$  kN [8].

Horizontal displacement of pile head  $\rho$  will be determined by elastic analysis (eq. 8). It was assumed that modulus of elasticity of a pile is  $E_p = 30000$  MPa. In such a way, the pile flexibility factor is  $K_R = 0,00743$  and influence factor is  $I_{pH} = 4,8$ . So, designed horizontal force  $H = 166$  kN causes horizontal displacement of the pile head  $\rho = 6,13$  mm.

The horizontal displacements of the laterally loaded pile will be estimated by coefficient of horizontal subgrade reaction of a surrounding soil, too. In Fig. 18 and 19 is presented  $K_H$  concept for numerical analysis of a laterally loaded pile. Calculation procedure assumes that, because of considerable side shear resistance between pile and soil, the values  $n_h$  from Table 1 and  $K_H$  from equation 13 should be doubled i.e.  $n_h = 2 \times 4400 = 8800$  kN/m<sup>3</sup>, and  $K_H = 2 \times 12520 = 25040$  kN/m<sup>3</sup>. The numerical analysis has been performed by computer program TOWER.

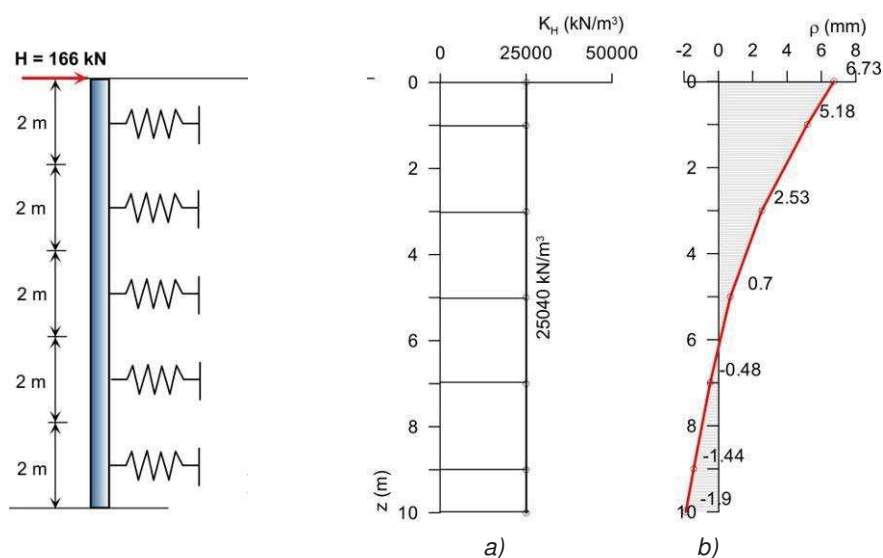
From these figures may be observed that horizontal displacements of the pile head, caused by horizontal force  $H = 166$  kN are  $\rho = 6,41$  mm (Fig. 18) and  $\rho = 6,73$  mm (Fig. 19). These values are in good agreement with previously obtained displacement by an elastic analysis.



Slika 18. Primena  $K_H$  u analizi bočno opterećenog šipa (Terzaghi, 1955)  
a) vrednosti  $K_H$  za  $n_h = 8800$  kN/m<sup>3</sup>, b) horizontalna pomeranja šipa

Figure 18. Application of  $K_H$  for numerical analysis of laterally loaded pile (Terzaghi, 1955)  
a)  $K_H$  values for  $n_h = 8800$  kN/m<sup>3</sup>, b) horizontal displacements of a pile





Slika 19. Primena  $K_H$  u analizi bočno opterećenog šipa (Vesić, 1961)  
a) vrednosti  $K_H = 25040 \text{ kN/m}^3$ , b) horizontalna pomeranja šipa

Figure 19. Application of  $K_H$  for numerical analysis of laterally loaded pile (Vesić, 1961)  
a)  $K_H = 25040 \text{ kN/m}^3$ , b) horizontal displacements of a pile

## 6 ZAKLJUČAK

Objekti koji su fundirani na šipovima često su izloženi značajnim horizontalnim silama. U tom slučaju, treba odrediti bočnu nosivost šipova i njihova horizontalna pomeranja. U ovom radu, pre svega, analizirali smo geotehničku nosivost šipova tj. bočnu nosivost koja je posledica loma okolnog tla, kao i deformacije bočno opterećenih šipova. Polazeći od toga, prvenstveno treba da se sprovedu adekvatna geotehnička istraživanja terena i da se formira – na osnovu dobijenih rezultata – geotehnički model terena na mestu budućeg objekta.

Na ovako definisanom modelu terena radi se proračun geotehničke bočne nosivosti šipova. S tim u vezi, treba voditi računa o tome da je reč o trodimenzionalnom problemu kao i da su u našoj zemlji često izraženi i složeni geotehnički uslovi. Uzimajući sve to u obzir, smatramo da je Brinč-Hansenova metoda vrlo pogodna za određivanje geotehničke bočne nosivosti šipova. Naravno, ukoliko je konstruktivna nosivost šipova manja od geotehničke nosivosti, onda je ona merodavna za određivanje maksimalne horizontalne sile koju šip može da prihvati.

Kod proračuna temelja oslonjenih na grupu šipova potrebno je da se uzme u obzir i grupni efekat šipova.

Prilikom određivanja deformacija bočno opterećenih šipova, u slučaju homogenog tla, mogu da se primene rešenja teorije elastičnosti. U složenim terenskim uslovima, međutim, pogodno je da se okolno tlo definiše odgovarajućim koeficijentima horizontalne krutosti ili p-y krivama i da se na osnovu toga odrede horizontalna pomeranja bočno opterećenih šipova.

Horizontalna pomeranja šipova treba da budu u dozvoljenim granicama. Ona su, pre svega, uslovljena karakteristikama objekta koji se fundira na šipovima.

U radu je prikazana i numerička analiza određivanja geotehničke nosivosti i horizontalnog pomeranja bočno opterećenih šipova. Budući da je tlo u kome se fundiraju šipovi homogeno i nekoherentno, urađen je proračun

## 6 CONCLUSION

Building structures which are founded with vertical piles are frequently loaded by high horizontal forces. In such cases, lateral bearing capacity and horizontal displacements of vertical piles have to be calculated. In this paper geotechnical capacity of piles i.e. lateral capacity which is governed by the strength of surrounding soil is analysed first. Accordingly, at first, it was necessary to make adequate geotechnical investigations, in laboratory and in situ and on the basis of the obtained results geotechnical model of terrain under the building structure had to be defined.

On such defined model geotechnical lateral capacity of piles is determined. In regard to that, it has to be considered that it is three-dimensional problem. Besides, in Serbia, there are very often complex geotechnical conditions. Accordingly, Brinč-Hansen's method is quite appropriate for determining geotechnical lateral bearing capacity. Surely, if the structural capacity of the piles is lesser than geotechnical capacity, maximum horizontal force that a pile can withstand should be estimated.

In calculation of lateral bearing capacity for a group of piles, their group effect has to be taken into account.

In the homogeneous soil, deformations of laterally loaded piles can be determined by elastic analysis. In complex geotechnical conditions, however, it is appropriate to define surrounding soil by coefficient of horizontal subgrade reaction or by p-y curves, too.

Such obtained deformations have to be in allowable limits which are restricted, at first, by structural characteristics of a building that is founded by the pile.

A numerical analysis for calculation geotechnical capacity and horizontal displacement of laterally loaded piles is presented in the paper. Taking into consideration that foundation soil is homogeneous and cohesionless, lateral bearing capacity is calculated not only by Brinč-Hansen's but Broms' method, too. The obtained results are in good agreement.

bočne nosivosti i po metodi Brinč-Hansena i po metodi Bromsa i dobijena su dobra slaganja. Osim toga, izračunata su i horizontalna pomeranja glave šipa elastičnom analizom, kao i pomoću koeficijenta horizontalne krutosti tla. Razlike u rezultatima su u uskim granicama, sasvim prihvatljivim za inženjersku praksu.

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In addition, the pile head displacements are determined by elastic analysis and application theory of subgrade reaction. The calculated values are in narrow limits, quite acceptable for engineering practice.

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## REZIME

### BOČNA NOSIVOST I POMERANJA VERTIKALNIH ŠIPOVA OPTEREĆENIH HORIZONTALNIM SILAMA

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Temelji na šipovima često su izloženi značajnim horizontalnim silama. U takvim slučajevima, važno je da se odredi bočna nosivost vertikalnih šipova. Ona je uslovljena čvrstoćom okolnog tla (geotehnička nosivost) odnosno čvrstoćom poprečnog preseka šipa (konstruktivna nosivost). U radu je prvenstveno analizirana geotehnička nosivost šipova i primenjene su sledeće metode za određivanje bočne nosivosti pojedinačnih šipova: Rankinova, Bromsova i Brinč-Hansenova metoda. S tim u vezi, polazeći od složenih geoloških uslova koji su česti u Srbiji, smatramo da Brinč-Hansenova metoda ima prednost u odnosu na druge dve metode. Naime, ona može da se primeni i u homogenom i u heterogenom tlu i to za drenirane, kao i za nedrenirane uslove. To je veoma važno prilikom fundiranja objekata i prilikom sanacije klizišta. Zato je u radu prikazano i kako se u proračun uvodi grupno dejstvo šipova. Horizontalna pomeranja bočno opterećenih šipova mogu da se, u slučaju homogenog tla, odrede primenom teorije elastičnosti. U slučaju složenih geoloških uslova, međutim, ta pomeranja se određuju primenom koeficijenta horizontalne krutosti okolnog tla ili korišćenjem p-y krivih. Na kraju rada data je numerička analiza određivanja geotehničke nosivosti i horizontalnog pomeranja glave bočno opterećenih šipova koji se koriste za fundiranje silosa klinkera u Beočinu.

**Ključne reči:** pojedinačni šipovi, grupa šipova, bočna nosivost, dozvoljeno bočno opterećenje, bočne deformacije.

## SUMMARY

### LATERAL CAPACITY AND DEFORMATIONS OF VERTICAL PILES LOADED BY HORIZONTAL FORCES

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Pile foundations are frequently loaded by horizontal forces. In such cases, it is important to calculate lateral capacity of vertical piles. It is governed by the strength of the surrounding soil i.e. geotechnical capacity or pile strength parameters i.e. structural capacity of a pile. In this paper, geotechnical capacity is analysed first, and then the Rankine's, Broms' and Brinch-Hansen's methods for calculating ultimate bearing capacity of a single pile under lateral loads are presented. In accordance with complex geological conditions, which are very often in Serbia, Brinch-Hansen's method has an advantage over the other two methods. It can be applied both to uniform and layered soils under drained or undrained conditions. This is highly important for foundation of structures and landslide's remedial measures, too. Accordingly, load capacity calculation of a pile group is presented as well. In the case of homogenous surrounding soil, deformations of laterally loaded piles may be determined by elastic analysis. However, in the case of complex geological conditions, these deformations may be calculated by the concept of coefficient of subgrade reaction or by p-y curves, too. Finally, numerical analysis for calculation of geotechnical capacity and pile head displacement of laterally loaded piles for foundation of Clinker Bin in Beocin is presented.

**Key words:** single piles, pile groups, ultimate lateral capacity, allowable lateral load, lateral deformations.