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Lateral Earth Pressure between two Parallel Rigid Retaining Walls

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Abstract - Parallel retaining walls are usually used for jetties, docks and cutoff walls. The lateral earth pressure is a significant design parameter in retaining structures and in number of foundation engineering problems. Retaining walls require quantitative estimate of the lateral pressure for either design or stability analysis. It is known that the distribution of lateral earth pressure against retaining walls is neither triangular nor linear owing to the effect of arching in the backfill. This paper presents experimental evaluation of the reaction induced by lateral earth pressure from granular soil contained between two parallel rigid retaining walls when the distance between the two walls is narrow.

A medium-scale test rig (earth tank), in which rigid retaining walls can be tested, is designed and fabricated. Two types of parallel retaining walls are tested in the earth tank: cutoff walls and isolated two parallel walls. The distance between the two parallel walls is varied and the variation in the reactive force exerted on the retaining walls from the lateral earth pressure is presented.

Suggestion for the value of optimum clear distance between two parallel retaining walls is presented depending on the reduction in the effective active earth pressure from full active wedge and on the ratio of the at-rest pressure to the active pressure.

Index Terms— Lateral earth pressure, parallel retaining walls, cutoff walls, coefficient of earth pressure.

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1 Introduction

Retaining walls are structures used to provide stability for soil where conditions disallow the mass of earth to assume its natural slope, and are commonly used to hold or support soil banks. Retaining walls are classified, based on the method of achieving stability, into the following main principal types:

- The gravity wall depends upon its weight as the name implies, for stability.
- The cantilever wall is a reinforced concrete wall that utilizes cantilever action to retain the soil mass.
- The counter fort retaining wall is similar to a cantilever retaining wall, except that it has counter forts, which tie the wall and the base together, built at intervals along the wall to reduce the bending moments and shears.

2 Earth Pressure Theories in Retaining Wall Problems

The earth pressure exerted on retaining walls by the retained soil may be greater than the fully active earth pressure, the minimum value of the active earth pressure, which occurs after sufficient movement or deflection of the retaining wall; the necessary movement is usually within the serviceability limit state of the wall, [1]. The method of plastic equilibrium as defined by Mohr rupture envelope is most generally used for estimating the lateral pressure from earth. The earth pressure theory proposed by Coulomb, about 1776, is still, however, quoted in standard references on the subject [2], [3], even though it is based on the following simplifying assumptions:

i. The soil is isotropic and homogenous and possess both internal friction and cohesion;
ii. The rupture surface is a plane surface;
iii. The friction forces are distributed uniformly along the plane of rupture;
iv. There exist wall friction; and
v. Failure is a plane-strain problem.

The principal deficiencies in Coulomb theory are in the assumption of ideal soil and that a plane defines the rupture surface.

The resultant earth pressure based on Coulomb theory for cohesionless soil is given by the following equations, [3]:

$$P_a = \frac{1}{2}\gamma H^2 K_a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$P_p = \frac{1}{2}\gamma H^2 K_p \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

in which,

γ = soil unit weight,
H = total height of retaining wall,
P_a = the resultant active earth pressure,
P_p = the resultant passive earth pressure, and
K_a and K_p = coefficients of active and passive earth pressure, given by the following equations:

$$K_a = \frac{\gamma H^2}{2}\,\frac{\sin^2(\alpha+\emptyset)}{\sin^2\alpha\,\sin(\alpha-\delta)\left[1+\sqrt{\frac{\sin(\emptyset+\delta)\sin(\emptyset-\beta)}{\sin(\alpha-\delta)\sin(\alpha+\beta)}}\right]} \quad \ldots \quad (3)$$

$$K_a = \frac{\gamma H^2}{2}\,\frac{\sin^2(\alpha+\emptyset)}{\sin^2\alpha\,\sin(\alpha+\delta)\left[1-\sqrt{\frac{\sin(\emptyset+\delta)\sin(\emptyset+\beta)}{\sin(\alpha+\delta)\sin(\alpha+\beta)}}\right]} \quad \ldots\ldots \quad (4)$$

where, the angles α and β are shown in Fig. 1, ϕ = angle of internal friction for the soil, and δ = angle of friction between soil and the retaining wall.

If $\beta = \delta = 0$ and $\alpha = 90°$ (i.e. a smooth vertical wall with horizontal backfill) equations (1) and (2) simplifies, respectively,

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to:

$$P_a = \frac{1}{2}\gamma H^2 \frac{1-\sin\emptyset}{1+\sin\emptyset} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$$P_p = \frac{1}{2}\gamma H^2 \frac{1+\sin\emptyset}{1-\sin\emptyset} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

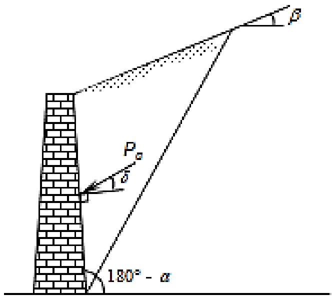

Fig. 1. General arrangement of rigid retaining wall and failure wedge

Rankine, in 1857, considered the soil in a state of plastic equilibrium and used the same assumptions as Coulomb to deduce the earth pressure coefficients K_a and K_p except that he assumed no wall cohesion or wall friction [3], [4].

The above methods are normally known as classical theories which are valid strictly for retaining walls subjected to uniform free translation. Practically all retaining walls rotate and movement of the wall could be restricted, particularly under working conditions. The lateral earth pressure on the wall often deviates from the fully active column value, i.e. there is a need for predicting the lateral earth pressure at any displacement behind a rotating wall, [5].

At-rest effective lateral earth pressures are often assumed to follow linear distribution with the effective stress σ_x' taken as a simple multiple of the vertical effective stress σ_z', [5]:

$$\sigma_x' = K_0 \, \sigma_z' \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

For normally consolidated soils K_0 is given, in terms of drained friction angle ϕ' as follows:

$$K_0 = 1 - \sin\phi' \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Practical values of K_0 are: 0.45, 0.40, and 0.35 for loose sand, medium dense sand and dense sand respectively, [5].

Fig. 2 illustrates the variation of the lateral earth pressure with respect to wall movement. Also shown on the same figure, are the usual range of earth pressure coefficients defined by the trigonometric ratios of Equations (3) and (4), [3].

3. Parallel Retaining Walls

Earth pressure on parallel retaining walls is normally affected by soil arching particularly when the clear distance between the two walls is narrow compared with the walls height, [6]. Soil arching effect and the computation of the related earth pressure had been hot topics in geotechnical engineering for many years, but there is only a little being done so far, [7], [8]. Circular and catenary always represent the shapes of minor principal stress soil arch between two parallel walls, but which shape should be used in the calculation of active earth pressure on two parallel walls remains disputed, [9], [10]. The problem arises from the fact that when the distance between the parallel walls is narrow the full active wedge will be shared by the two walls and hence the analyses will become complicated. Hereunder is description of tests conducted to evaluate such complicated lateral earth pressure.

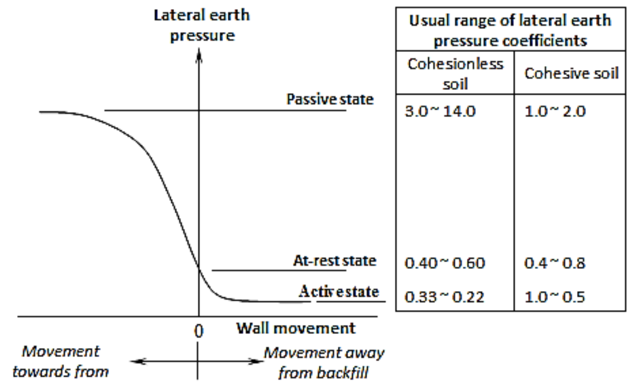

Fig. 2. Effect of retaining wall movement in the lateral earth pressure

4. Test Procedures to Identify the Lateral Earth Pressure on Parallel Retaining Walls:

4.1 The test rig:

An apparatus, in which medium scale rigid retaining walls can be tested, is designed and fabricated. The apparatus consists of: earth tank with two transparent long sides, sand feeding frame, model rigid retaining walls and deflection and reaction measuring devices. The dimensions of the earth tank are: 2.0m × 0.6m × 0.7m (L×W×H), see Plate 1.

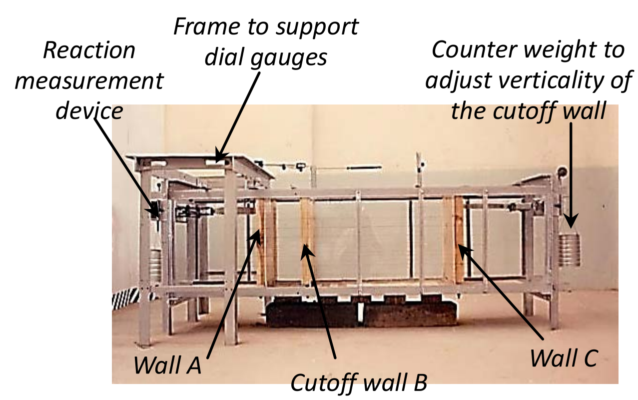

Plate 1. Apparatus for testing rigid retaining walls

The provision of transparent thick glass plate on the long sides of the earth tank made it possible to visualize and map any distortions in the retained earth mass arising from the test operations. The sand fill was placed in the earth tank in successive layers of 50mm thickness each. The sand layers were marked by depositing narrow 2mm thick darkened sand layer immediately next to the glass sides of the earth tank. The darkened sand was prepared from the same sand used in the

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tests; coloration was affected by wetting the sand in potassium permanganate solution and then let to dry. The presence of this colored sand would thus had no effect on the behavior of the retaining wall since it was of negligible amount compared to the total volume of contained sand.

Each retaining wall was made of 50mm thick timber board with dimension 0.485m × 0.450m (W×H), and free to rotate about hinges at the bottom end.

4.2 Control tests:

The rigid retaining walls were loaded by ideal sand, with practically no cohesion. This sand was prepared from a carefully selected batch of natural sand. The sand used in the tests was taken from quantity passed sieve No. 30 (size of opening = 1.18mm) and retained on sieve No. 14 (size of opening = 0.5mm), as per BS 410 test sieves.

The angle of internal friction, ϕ, of the used sand was determined from direct shear tests, the mean value found to be 33.15°. Unit weight on the same sand, γ = 14.64 kN/m^3; the mean angle of friction between the sand and the rigid retaining walls, δ = 21.7°.

4.3 Filling the earth tank:

The earth tank was filled with the prepared sand while maintaining even compaction. To achieve even compaction the sand was left to drain freely under its own weight, into the earth tank, from a constant height by the aid of funnel and rubber hose carried on an independent frame of adjustable height and capable of taking nine steps of different levels, with increment of 50mm from level to level. After deposition of every 50mm layer of sand, the thin colored sand band was poured and the funnels were raised to the next level. Note that the sand was deposited in even layers with the aid of horizontal guide-lines drawn at 50mm internals on the out surface of the glass sides of the earth tank.

4.4 Measurement of deflection and reaction:

Deflection of the retaining walls was measure at the top of the walls by mechanical dial gauges held by magnetic stands fixed on independent rigid steel frame spanning over the earth tank to insure absolute deflection readings.

The reaction force at the retaining walls was measured at height 0.4m above bottom end of the walls by two proving rings each of capacity 2.8kN. One end of the proving rings reacts on the retaining wall; the other end was fixed on a screw jack fixed to the end of the earth tank.

4.5 Identifying the full active wedge:

The rigid test walls in the earth tank, shown in Plate 1, are hinged at bottom and anchored near the top such that the anchor force is easily measured. The earth tank was filled with the prepared sand, following the above procedure; the rigid walls were continuously kept in vertical condition during sand filling operation. The so-called at rest anchor reaction is read. After a rest period Wall *C* is allowed to yield, rotate as rigid body about bottom end, by a small-calculated amount by releasing the screw jack until the readings of controlling dial gauges, located at top of the walls, reach the assigned values. The new reading of anchor force, P_C, was recorded from the proving ring after rest period of twenty-four hours. A second step of wall yielding was then allowed; the reading after 24 hours rest period was again recorded. This process is repeated until the wall pressure has dropped from the at-rest condition until well into the active state.

The presence of colored sand lines facilitated visualizing the rupture line of the full active wedge. The horizontal dimension of the active wedge at top surface, *W*, was found to be 0.274m; the result of reaction P_C is shown in Figs. 3 and 4 for *b* >> *W*.

4.6 Effect of presence of cutoff walls:

Sometimes, cutoff walls might become necessarily employed when the height of the retained soil is too large. Therefore, the use cutoff walls, in this case will be limited to proportionally reduce the magnitude of the lateral earth reassure on the original retaining wall. Hence, a second experiment was performed simultaneously at the far end on the same earth tank, to determine the horizontal reaction on Wall *A* when a rigid cutoff Wall *B* was placed parallel to retaining wall *A*. In this test, the distance, *b*, between wall *A* and wall *B* was varied such that *b* = 0.25*W*, *b* = 0.5*W* and *b* = 0.75*W*, where *W* = the maximum horizontal length of the full active wedge of the soil behind a typical rigid retaining wall, wall *C* described in the above paragraph. Wall *A* yielding is then allowed, the readings after 24 hours rest period were recorded for each step of wall *A* release. Fig. 3 illustrates the results of this test.

Note that Wall *B* is kept in vertical position during the test by continuously zeroing the readings at dial gauges reacting on the top of the wall by the aid of screw balance and counterweight hanging through a pulley from the rear end of the earth tank; see Plate 1.

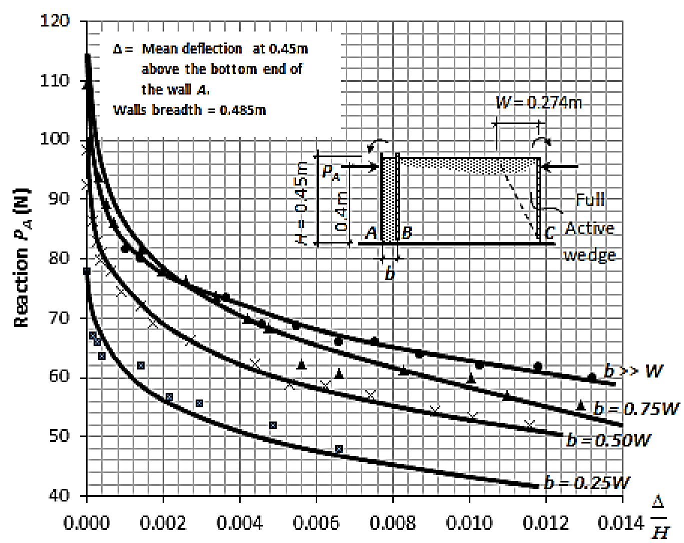

Fig. 3. Test results of soil pressure behind model rigid wall *A* in presence of cutoff wall *B*, for the cases when *b* = 0.25*W*, *b*= 0.50*W*, *b* =0.75*W*, and *b* >>*W*

4.7 Earth pressure between free standing two parallel retaining walls

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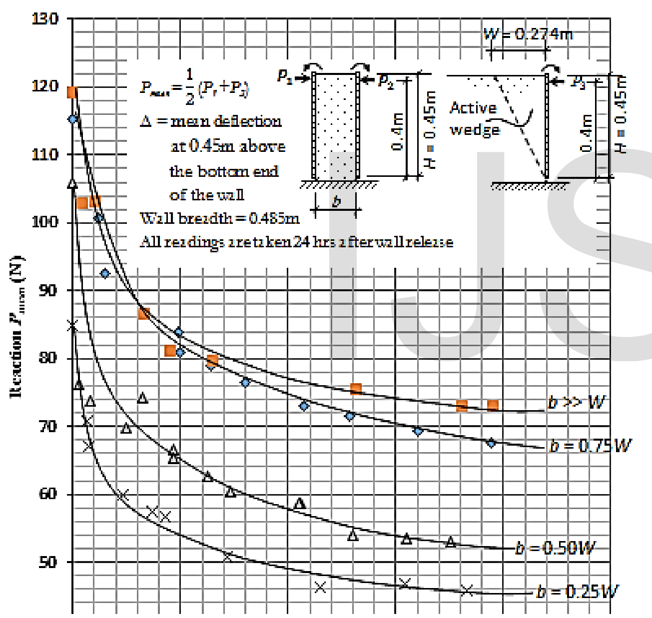
The use of two parallel retaining walls is sometimes required for jetties, elevated carriageways … etc. The first experiment is repeated filling the space between walls *A* and *B* with sand while leaving the rest of the earth tank empty and employing support for wall *B* similar to wall *A* support.

In these tests, the distance, *b*, between the two parallel walls was varied such that $b = 0.25W$, $b = 0.5W$ and $b = 0.75W$.

The rigid test walls in the earth tank, shown at top right corner of Fig. 4 are hinged at bottom and anchored near the top such that the anchor force is easily measured. The two rigid walls were kept in vertical condition during sand filling operation. The so-called at rest anchor reaction is recorded. After a rest period the wall is allowed to yield simultaneously, rotate as rigid body about bottom end, by a small calculated amount. The new reading of anchor force was recorded after rest period of twenty-four hours. A second step of wall yielding is then allowed; the reading after 24 hours rest period was again recorded. The process is repeated until the walls pressure has dropped from the at-rest condition until well into the active state. Fig. 4 illustrates the results of this test.



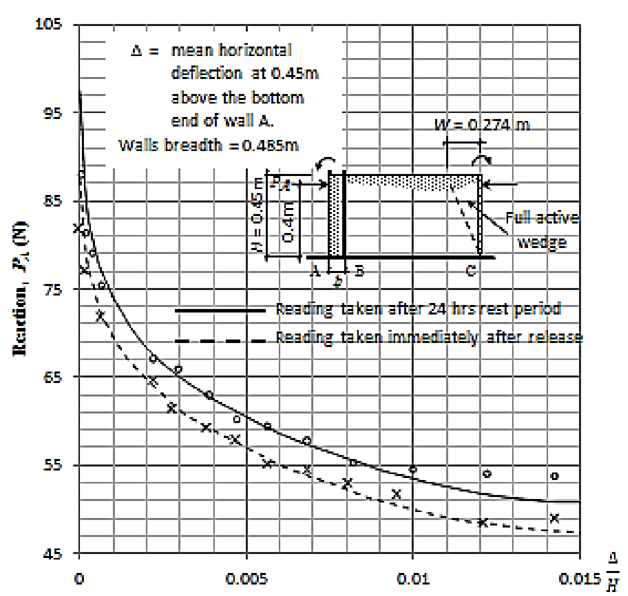


Fig. 5. Test results of soil pressure behind wall *A*, effect of time dependent increase in pressure for the case when $b = 0.5W$

4.8 Time dependent increase in lateral earth pressure:

It is worthwhile mentioning that from the observations on the measurements of the reaction forces, on all the tested retaining walls, it was noticed that there was always an increase in the readings between those immediately taken after each step of wall rotation and the readings taken after a rest period of 24 hours, e. g. see Fig. 5. The readings show no significant change after the 24 hours rest period. For this reason, all conclusions drawn in this paper depended on readings taken 24 hours rest period after each anchor release.

The above phenomenon, of time-dependent increase in earth pressure, is encountered but briefly in the literature, [3], [11], with scanty notions on its causes and working mechanisms. It is not intended in this paper to ascertain the extent of these increases in wall pressure loading; in as far as they affect design of bridge abutments, retaining walls and sheet-piles. No doubt such research would seem very much welcome in the future. However, our purpose here is to demonstrate how such phenomenon could influence experimental results if the tests were prolonged over a long period of time.

Pending the results of such further research, it would seem prudent to allow for the above increases in the wall pressure by making a suitable estimate of the lateral earth pressure coefficient, higher than, the active state coefficient.

4.9 At-rest and active pressures:

The following Tables 1 and 2 show the values of reactions R_0 and R_A on the two types of parallel retaining walls for the at-rest and active conditions, respectively. The active conditions are assumed to be mobilized when $\Delta = 0.001H$ for the cutoff walls and $\Delta = 0.002H$ for the free stand parallel walls, where Δ = mean deflection the top of the walls and *H* = walls height. The obtained values of $\frac{R_0}{R_A}$ for the above releases seem to match values obtained by other researchers, [8], [12].

In Figs. 6 and 7 the variation of lateral earth pressure coefficients for the at-rest state and active state on the two types of parallel walls with respect to the distance between the walls is presented. The use of equivalent earth pressure coefficients is suggested by assuming triangular earth pressure distribution on the parallel walls and taking moments about the hinges at the bottom ends of the walls.

TABLE 1
At-rest and Active Reactions for Free Standing Parallel Two Walls, when $\Delta = 0.001H$

$\frac{b}{W}$	R_0 (kN)	R_A (kN)	$\frac{R_0}{R_A}$
0.25	85	54	1.57
0.50	105	65	1.62
0.75	116	82	1.41
>>1.0	120	84	1.43

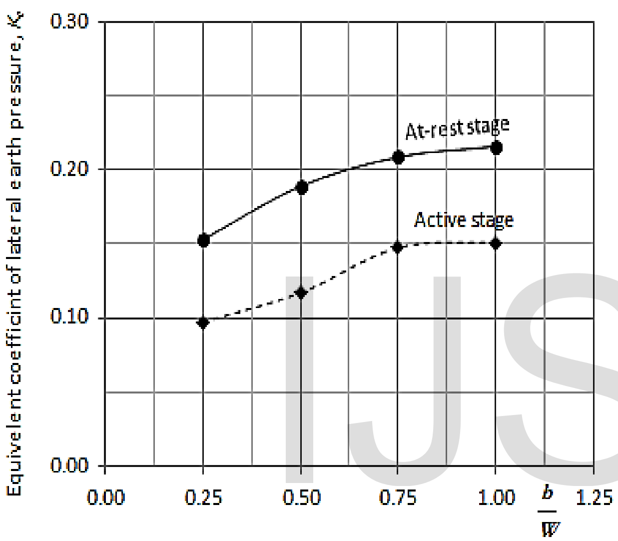


Fig. 6. Effect of distance between wall and cutoff parallel wall on the coefficients of lateral earth pressue

TABLE 2
At-rest and Active Reactions for Cutoff Wall, when $\Delta = 0.002H$

$\frac{b}{W}$	R_0 (kN)	R_A (kN)	$\frac{R_0}{R_A}$
0.25	78	56	1.39
0.50	100	69	1.45
0.75	110	78	1.41
>>1.0	115	77	1.49

5.CONCLUSIONS

The following conclusions may be drawn from this paper:

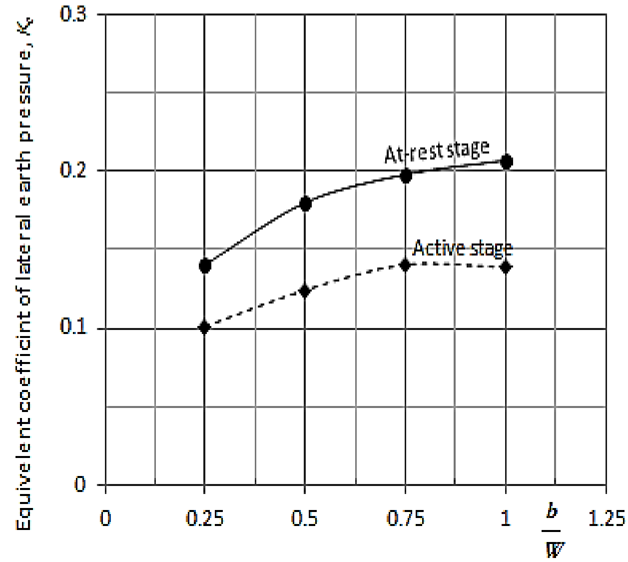


Fig. 7. Effect of distance between free standing parallel walls on the coefficient of lateral earth pressue

1. The deflection sufficient to mobilize the active earth pressure state is related to the type of parallel walls. According to the findings of this paper the active conditions are suggested to be estimated to be when $\Delta = 0.001H$ for the cutoff walls and $\Delta = 0.002H$ for the free stand parallel walls.
2. Clear distance between cutoff wall adjacent to retaining wall by greater than 0.75W does not show differences in the overall lateral earth pressure. Hence cutoff wall are optimally better be placed at distance from the main retaining equals 0.5 ~ 0.6 time W, the maximum horizontal distance of the fully active wedge, with reduction in active earth pressure reaches 50% of the at–rest pressure exerted at retaining when no cutoff wall is used.
3. Measurements of reaction force taken twenty four hours after each increment of anchor release indicate an average increase of 5% in the anchor force. This increase in pressure prompted the development of further tests to evaluate the phenomenon of time dependent rise in the earth loading. Such tests, conducted on rigid walls, indicated substantial growth in the lateral earth pressure loading with time.
4. Due to possible time dependent increase in the earth pressure, behind earth retaining structures, it is recommended to use a higher coefficient of lateral earth pressure than that given by active pressure state. From the limited test conducted in this paper it would seem that, for design, the pressure coefficient could approach the at rest pressure coefficient, especially in structures where tapping behind the retaining wall is expected, from traffic and other loading conditions, such as would occur in quay walls, wharves, bridge abutment and earth retaining structures. It may thus seem justified in design to use a coefficient of lateral earth pressure, higher than the active, perhaps, more near the at rest state coefficient K_0, (where $K_0 = 1.0 - \sin\varphi'$) for cohesionless soils).

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