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Lateral Stability of Wood Beam-and-Deck Systems

By John J. Zahn,[1] M. ASCE

Introduction

Many modern roof structures consist of a few widely spaced deep beams bridged by tongue-and-groove timber decking. Deep beams are inherently efficient in their use of material but are subject to the possibility of failure by lateral buckling. It is clear that the shear stiffness of the attached deck contributes to the stability of the beam-and-deck system, but presently available design formulas do not contain this effect. A few roof systems have failed by instability during erection before the decking was applied; this indicates that present design practice does rely upon the decking for stability, although the margin of safety in such designs cannot be estimated with precision.

The problem of assessing the influence of deck stiffness upon system stability decomes particularly acute when decks of low shear stiffness are employed. For example, 2 in. wood decking employs two nails to attach each board to the support beam; there are no interconnections between boards. Shear rigidity of 2 in. decking is derived solely from the nail couples used for attachment whereas thicker wood decks are spiked together every 30 in. What effect does such a deck have upon the stability of the deep beams? The analysis presented herein will provide an answer to such questions.

The effect of lateral restraints upon the stability of deep beams was investigated to some extent by Flint (2) who considered the effects of elastic supports and an elastic restraint at an intermediate point on the span. Schmidt (6) gave a more rigorous treatment of the same problem. The effect of continuous torsion-

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[1] Research Engr., Forest Products Lab., Forest Serv., U.S. Dept. of Agriculture, Madison, Wis.

a1 restraint was analyzed by Taylor and Ojalvo (9). Haussler modeled standing seam roofing (3) by treating the tension flange as fixed against lateral displacement and elastically restrained against rotation. Continuous lateral support provided by a roof deck has been the subject of recent experimental work at Cornell University (13) by Errera, Pincus, and Fisher. Their theoretical solutions are limited to the cases of axial load and constant moment. They include the effect of flexural stiffness of the corrugated sheet metal deck. This report neglects bending of the deck because wood systems are not so rigidly connected as to maintain a constant right angle between beam and deck. The effect of a continuous lateral support has also been previously studied by the writer (10) in a report which contained an unrealistic assumption, i.e., that the deck was attached along the centroidal axis of the beam. The present report extends those results (10) by permitting the line of deck attachment to be any distance from the centroidal axis. Jenkinson and the writer have verified experimentally (4) that the present results are realistic. In addition, this paper presents an improved variational derivation of the governing differential equations and boundary conditions. The chief virtue of this is that it automatically supplies the correct boundary conditions for each particular case of load and support, obviating the need for the special arguments that had to be adduced in Ref. 10.

Theoretical Analysis

Consider a system of deep beams whose top edges are bridged by a deck of low in-plane shear rigidity, such as a wood-frame floor or roof with a plank deck (Fig. 1). It is assumed that differential displacement of the deck planks is elastically restrained so that the deck behaves as a shear diaphragm with nonzero stiffness. Minimal attachment between beams and deck is assumed; the deck shall transmit only a lateral force to the beams. Furthermore, all beams are assumed to be equally loaded.

If the system should buckle laterally, all beams would deform congruently and the deck would displace laterally by an amount, w_D, in which w_D is a function of coordinate x measured along the length of the beams (Fig. 1). The in-plane shear strain in the deck is

$$\gamma_{\text{deck}} = \frac{dw_D}{dx} \quad \text{(1)}$$

The deck displacement can be expressed in terms of the lateral displacement, w, of the centroidal axis of the beam and the angle of twist, β, of the beam by matching displacements along the line of deck attachment:

$$w_D = w + c\beta \quad \text{(2)}$$

in which c = the distance from the centroidal axis of the beam to the line of deck attachment.

The shear stiffness of the deck tends to stabilize the system because the deck must deform in shear during buckling. Attention will be restricted to one beam of the system and a strip of deck of width S, in which S is the beam spacing (Fig. 1).

Derivation of Total Energy.—Equilibrium of the system can be expressed by minimizing the total potential energy of the system, U. Each part of the system contributes to the total energy:

$$U = U_D + U_B + U_L \quad (3)$$

in which U_D = the strain energy of the deck; U_B = the strain energy of the beam; and U_L = the potential energy of the external loads. Ordinarily, U is measured from the stress-free state as zero datum. In lateral stability problems,

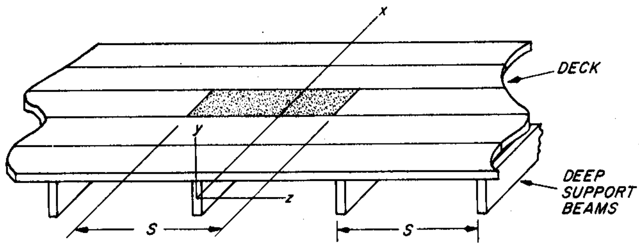

FIG. 1.—Basic Beam-and-Deck System Used for Analysis in Study

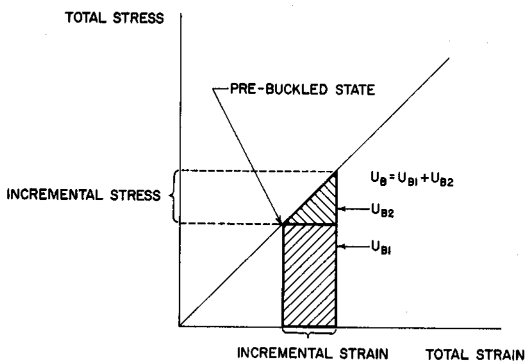

FIG. 2.—Beam Strain Energy U_B Measured from Prebuckled State as Datum

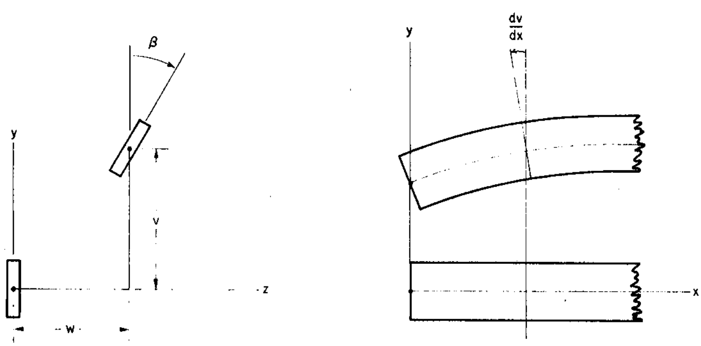

FIG. 3.—Displacement of Cross Section During Buckling

however, it is more convenient to use the deflected state at the instant of incipient buckling as the datum and to work with the incremental displacements which occur at buckling rather than With the total displacement, The relationship of the total and incremental displacements is

$$b_T = b;\ v_T = v_1 + v;\ w_T = w \quad \ldots\ldots \quad (4)$$

in which unsubscripted v and w = the incremental displacements of the centroidal axis in the y and z directions, subscript T denotes total displacement, subscript 1 denotes displacement at incipient buckling. Of course the only initial displacement is vertical.

Strain Engergy of Beam.—The strain energy of the beam, U_B, is the work done by the internal stresses during the incremental displacements (Fig. 2). This can be broken into two parts as suggested by the figure. The part labeled U_{B2} is calculated as if the incremental strains were measured from a state of zero stress. Thus it is given by the usual formula from elementary strength of materials, i.e.

$$U_{B2} = \frac{1}{2}\int_0^L \left[EI_2 \left(\frac{d^2 w}{dx^2}\right)^2 + JG\left(\frac{d\beta}{dx}\right)^2 \right] dx \quad \ldots\ldots \quad (5)$$

in which EI_2 = lateral bending stiffness; and JG = torsional stiffness of beam.

The part labeled U_{B1} is the work done by the prebuckling stresses during buckling. It is calculated as if these stresses were constant during buckling. Thus

$$U_{B1} = \int_0^L \int_0 (\sigma_x^o e_x + \tau_{xy}{}^o \gamma_{xy})\, dA\, dx \quad \ldots\ldots \quad (6)$$

in which A = the cross-sectional area; σ_x^o **and** τ_{xy}^o = prebuckling stresses; and e_x and g_{xy} = incremental strains. To obtain these strains, the displacement functions of any point on the cross section are written in terms of the centroidal displacement by the usual elementary assumption that plane sections remain plane. Then (Fig. 3):

$$\left.\begin{aligned} \bar{u}(x,y,z) &= -y\frac{dv}{dx} - z\frac{dw}{dx} \\ \bar{v}(x,y,z) &= v(x) - z\beta(x) \\ \bar{w}(x,y,z) &= w(x) + y\beta(x) \end{aligned}\right\} \quad \ldots\ldots \quad (7)$$

The incremental strains are related to these displacements by

$$\left.\begin{aligned} e_x &= \frac{\partial \bar{u}}{\partial x} + \frac{1}{2}\left(\frac{\partial \bar{v}}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial \bar{w}}{\partial x}\right)^2 \\ \gamma_{xy} &= \frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x} + \frac{\partial \bar{v}}{\partial x}\frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial x}\frac{\partial \bar{w}}{\partial y} \end{aligned}\right\} \quad \ldots\ldots \quad (8)$$

in which products of derivatives of $\bar{u}$ have been neglected.

Substituting Eq. 7 into Eq. 8 gives

$$\left.\begin{aligned} e_x &= -y\frac{d^2v}{dx^2} - z\frac{d^2w}{dx^2} + \frac{1}{2}\left(\frac{dv}{dx} - z\frac{d\beta}{dx}\right)^2 + \frac{1}{2}\left(\frac{dw}{dx} + y\frac{d\beta}{dx}\right)^2 \\ \gamma_{xy} &= -z\frac{d\beta}{dx} + \beta\frac{dw}{dx} + y\beta\frac{d\beta}{dx} \end{aligned}\right\} \quad (9)$$

The prebuckling stresses are

$$\left.\begin{aligned} \sigma_x^o &= -\frac{My}{I_1} \\ \tau_{xy}^o &= -\frac{3}{2A}\frac{dM}{dx}\left[1 - \left(\frac{y}{c}\right)^2\right] \end{aligned}\right\} \quad (10)$$

in which M = the prebuckling bending moment about the z-axis; and I_1 = the principal moment of inertia.

Now, combining Eqs. 6, 9, and 10, and noting that certain integrals vanish for a rectangular beam, U_{B1} becomes

$$U_{B1} = \int_0^L \left(M\frac{d^2v}{dx^2} - M\frac{dw}{dx}\frac{d\beta}{dx} - \frac{dM}{dx}\frac{dw}{dx}\beta\right)dx \quad (11)$$

Potential Energy of Loads.—The potential energy of the loads is the negative of the work done by external forces during buckling. In this report the external forces are a downward force P at $x = 0$ and a downward distributed load, p. Both act at the top flange of the beam. Only simply supported beams and cantilevers will be considered. The origin will be placed as shown in Fig. 4. In each case the span from $x = 0$ to $x = L$ will be considered so that for simply supported beams, the energy in that span is only half the total energy. In the simply supported case $(1/2)P$ will be used as the force at the origin so as to get only one half the potential energy of force P. Thus

$$U_L = P\left(v_o - \frac{1}{2}c\beta_o^2\right) + \int_0^L p\left(v - \frac{1}{2}c\beta^2\right)dx \quad (12)$$

in which P must be cut in half for simple beams. The subscript zero denotes evaluation at $x = 0$.

Strain Energy of Deck.—Only the shear energy will be considered. Because there are no shear strains in the deck at the datum state of incipient buckling, the strain energy of the deck is simply

$$U_D = \frac{1}{2}\int_0^S\int_0^L G_D\left(\frac{dw_D}{dx}\right)^2 dx\, dz \quad (13)$$

in which G_D = the in-plane shear stiffness (force per unit length of edge). Using Eq. 2 and integrating over z this becomes

$$U_D = \frac{SG_D}{2}\int_0^L \left[\frac{d}{dx}(w + c\beta)\right]^2 dx \quad (14)$$

Total Energy.—Now note that the displacement, v, appears only in U_{B1} and U_L. Consider these terms separately. Relating p to M and integrating by parts:

$$\int_0^L \left(M \frac{d^2v}{dx^2} + pv \right) dx + Pv_o = \int_0^L \left(M \frac{d^2v}{dx^2} - \frac{d^2M}{dx^2} v \right) dx + Pv_o$$

$$= - \left(\frac{dM}{dx} v \right)_0^L + \left(M \frac{dv}{dx} \right)_0^L + Pv_o \qquad (15)$$

For a cantilever or a simple beam, the boundary conditions associated with v are such that this expression vanishes. Therefore, all terms containing v cancel from the expression for total energy and the total energy is

$$U = \frac{1}{2} \int_0^L \left\{ EI_2 \left(\frac{d^2 w}{dx^2} \right)^2 + JG \left(\frac{d\beta}{dx} \right)^2 + SG_D \left[\frac{d}{dx} (w + c\beta) \right]^2 \right.$$

$$\left. - 2 \frac{dw}{dx} \left(M \frac{d\beta}{dx} + \frac{dM}{dx} \beta \right) - pc\beta^2 \right\} dx - \frac{1}{2} Pc\beta_o^2 \qquad (16)$$

Before proceeding further, a nondimensional notation is introduced. Let

$\theta = \dfrac{|M_{max}| L}{\sqrt{JG\, EI_2}}$ is a load parameter in which M_{max} = the maximum moment produced by the load; $\dfrac{SG_D L^2}{EI_2}$ is a deck stiffness parameter;

$\phi = \dfrac{c}{L} \sqrt{\dfrac{EI_2}{JG}}$ is a depth-to-span ratio parameter and; $\omega = \dfrac{w}{L}$;

$$\psi = \beta \sqrt{\frac{JG}{EI_2}}; \xi = \frac{x}{L} \qquad (17)$$

and let primes denote differentiation with respect to ξ. Then the total potential energy can be written

$$U = \frac{EI_2}{2L} \int_0^1 \left[(\omega'')^2 + (\psi')^2 + \tau(\omega' + \phi\psi')^2 \right.$$

$$\left. - \frac{2\theta}{|M_{max}|} \left(M\omega' \psi' + M' \omega' \psi + \frac{1}{2} pL^2 \phi\psi^2 \right) \right] d\xi - \frac{EI_2}{2L} \frac{PL}{|M_{max}|} \theta\phi\psi_o^2 \qquad (18)$$

Variation of Total Energy.—A condition of neutral equilibrium during buckling is next imposed by requiring the first variation of U to vanish:

$$\delta U = 0 \qquad (19)$$

After some integration by parts Eq. 19 can be written

$$\int_0^1 \left[\omega'''' - \tau(\omega + \phi\psi)'' + \left(\frac{M\theta}{|M_{max}|} \psi \right)'' \right] \delta\omega \, d\xi$$

$$+ \int_0^1 \left[- \psi'' - \tau\phi(\omega + \phi\psi)'' + \frac{M\theta}{|M_{max}|} \omega'' - \frac{pL^2}{|M_{max}|} \phi\theta\psi \right] \delta\psi \, d\xi$$

$$+\left\{\left[-\omega''' + \tau(\omega' + \phi\psi') - \frac{\theta}{|M_{\max}|}(M\psi)'\right]\delta\omega\right\}_{\xi=0}^{\xi=1}$$

$$+\left(\omega''\,\delta\omega'\right)_{\xi=0}^{\xi=1} + \left\{\left[\psi' + \tau\phi(\omega' + \phi\psi') - \frac{\theta}{|M_{\max}|}M\omega'\right]\delta\psi\right\}_{\xi=0}^{\xi=1}$$

$$-\left(\frac{PL}{|M_{\max}|}\theta\phi\psi\,\delta\psi\right)_{\xi=0} = 0 \quad \text{(20)}$$

Because dw, dw´, and dy are independent and arbitrary, Eq. 20 implies all of the following (Eqs. 21 through 28).

Differential Equations.—The two integrals in Eq. 20 yield two Euler-Lagrange equations, the first of which immediately integrates to

$$\omega'' = \tau\omega - \left(\frac{M\theta}{|M_{\max}|} - \tau\phi\right)\psi + C_1\xi + C_2 \quad \text{(21)}$$

in which C_1 and C_2 are integration constants, and the second of which is

$$(1 + \tau\phi^2)\,\psi'' = -2\,\phi\,\theta\psi\,\frac{\frac{1}{2}pL^2}{|M_{\max}|} + \left(\frac{M\theta}{|M_{\max}|} - \tau\phi\right)\omega'' \quad \text{(22)}$$

The integrated terms in Eq. 20 produce boundary conditions.

Boundary Conditions at x = 0

$$\left\{-\omega''' + \tau\omega' - \left[\left(\frac{M\theta}{|M_{\max}|} - \tau\phi\right)\psi\right]'\right\}\delta\omega = 0 \quad \text{(23)}$$

$$\omega''\,\delta\omega' = 0 \quad \text{(24)}$$

$$\left[(1 + \tau\phi^2)\,\psi' + \tau\phi\omega' - \frac{M\theta}{|M_{\max}|}\omega' + \frac{PL}{|M_{\max}|}\theta\phi\psi\right]\delta\psi = 0 \quad \text{(25)}$$

Boundary Conditions at x = 1

$$\left\{-\omega''' + \tau\omega' - \left[\left(\frac{M\theta}{|M_{\max}|} - \tau\phi\right)\psi\right]'\right\}\delta\omega = 0 \quad \text{(26)}$$

$$\omega''\,\delta\omega' = 0 \quad \text{(27)}$$

$$\left[(1 + \tau\phi^2)\,\psi' + \tau\phi\omega' - \frac{M\theta}{|M_{\max}|}\omega'\right]\delta\psi = 0 \quad \text{(28)}$$

Remarks.—This theory neglects certain effects, most notably a second-order torsion effect and the effect of prebuckling vertical deflections, both of which tend to increase the buckling load. Concerning torsion, an extremely deep beam would behave in torsion more like a plate than a one-dimensional member. In this case the lateral bending stiffness, EI_2, contributes a second-order term to the torsional rigidity in addition to the usual St. Venant torsional stiffness.

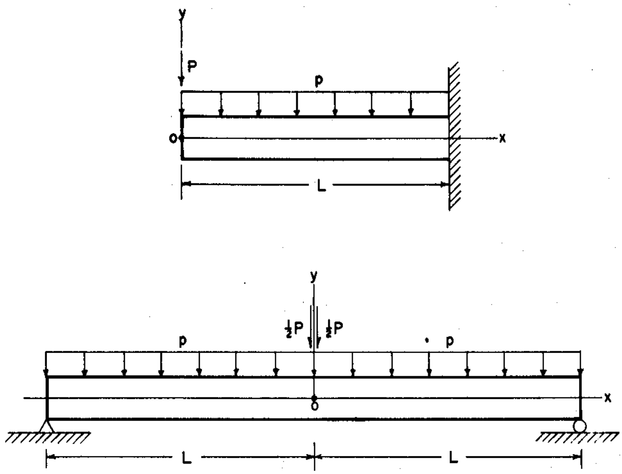

FIG. 4.—Span from Zero to *L* is: (*a*) Whole Cantilever; or (*b*) One-Half of Simple Beam, in which Case Only 1/2 *P* is Used as Force at Origin in Computing Total Energy of Half-System

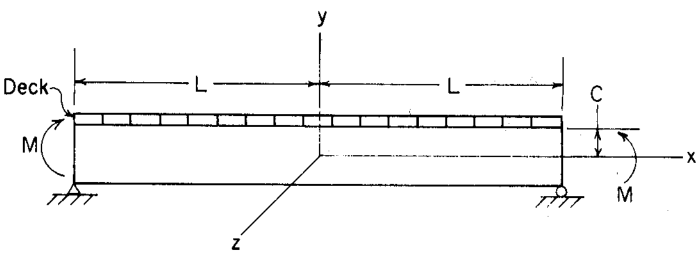

FIG. 5.—Case 1—Simply Supported Beam-and-Deck System Under Constant Moment (Ends are restrained against axial rotation)

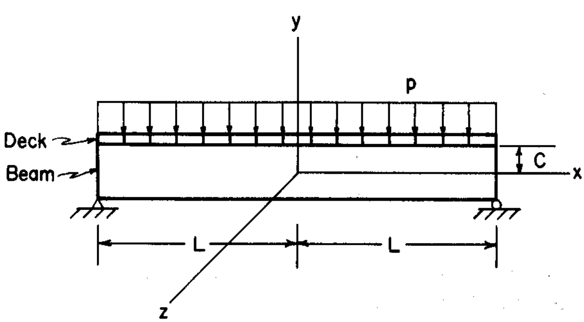

FIG. 6.—Case 2—Simply Supported Beam-and-Deck System (Uniform load is applied through deck attached at top flange. Ends are restrained against axial rotation)

For a simple beam under constant moment, e.g., this effect raises the buckling load by the factor Ö1 + (f p^2/2). The effect of prebuckling vertical deflections is to increase the buckling load by factor ÖI_1 /(I_1 - I_2) , in which I, is the larger moment of inertia of the cross section. This increase should not be relied upon if the beams have an initial camber, as is common in wood construction.

Stability Criteria for Particular Cases

Five cases are considered: (1) Pure bending and simple support; (2) uniform load and simple support; (3) end-loaded cantilever; (4) uniformly loaded cantilever; and (5) concentrated center load and simple support. In Case 1 the differential equations have constant coefficients and a closed form solution is obtained. In the other four cases the differential equations have variable coefficients and a power series solution is employed. The essential details are outlined for Case 2. Solutions for Cases 3, 4, and 5 are similar to Case 2 and only the results are presented.

In every case, the results presented herein reduce to previously known solutions (9), when the deck stiffness is taken to be zero.

Case 1—Pure Bending and Simple Support.—In Fig. 5 the length of the beam is 2*L* and the ends x = ± *L* are restrained from rotating about the x-axis. For the case of pure bending, the loads, *p* and *p*, are zero. The internal moment, *M*, is constant and equal to the applied external moment. Therefore in this case

$$\theta = \frac{ML}{\sqrt{JG\,EI_2}} \qquad (29)$$

Boundary Conditions.—Let the subscript zero denote evaluation at x = 0 and subscript one denote evaluation at x = 1. There are three geometric boundary conditions

$$\omega'_o = 0 \qquad (30)$$

$$\omega_1 = 0 \qquad (31)$$

$$\psi_1 = 0 \qquad (32)$$

and three natural boundary conditions. Because dw$_o$ ¹ 0, Eq. 23 requires that

$$-\omega'''_o + \tau\omega'_o - (\theta - \tau\phi)\,\psi'_o = 0 \qquad (33)$$

Because dy$_o$ ¹ 0, Eq. 25 requires that

$$(1 + \tau\phi^2)\,\psi'_o + (\tau\phi - \theta)\,\omega'_o = 0 \qquad (34)$$

and because dw$'_1$ ¹ 0, Eq. 27 requires that

$$\omega''_1 = 0 \qquad (35)$$

Differential Equations.—Differentiate Equation 21, evaluate it at x = 0, and apply Eq. 33 to find that C_1 = 0. Then evaluate Eq. 21 at x = 1 and apply Eqs. 31, 32, and 35 to find that C, = 0. Thus Eqs. 21 and 22 reduce to

$$\omega'' = \tau\omega - (\theta - \tau\phi)\,\psi \qquad (36)$$

$$(1 + \tau\phi^2)\,\psi'' = (\theta - \tau\phi)\,\omega'' \qquad (37)$$

Eliminating y between Eqs. 36 and 37 yields

$$\omega'''' + \left[\frac{(\theta - \tau\phi)^2}{1 + \tau\phi^2} - \tau\right]\omega'' = 0 \qquad (38)$$

whose solution is

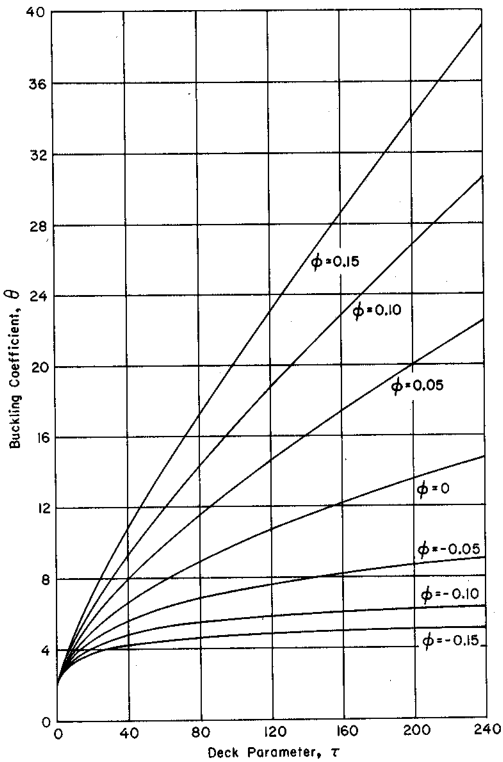

FIG. 7.—Buckling Coefficients for Case 2

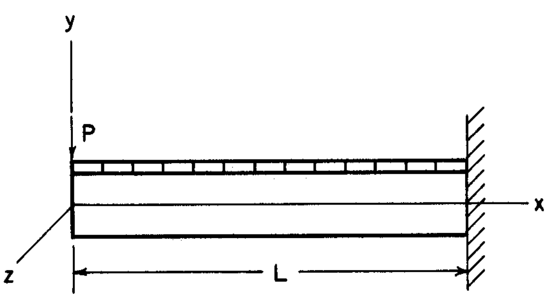

FIG. 8.—Case 3—Cantilever Beam-and-Deck System (End load *P* is applied through deck attached to top flange)

$$\omega'' = C_3 \sin \lambda\xi + C_4 \cos \lambda\xi \quad (39)$$

in which $$\lambda^2 = \frac{(\theta - \tau\phi)^2}{1 + \tau\phi^2} - \tau \quad (40)$$

Because w is even in x by symmetry, $C_3 = 0$. From Eqs. 35 and 39

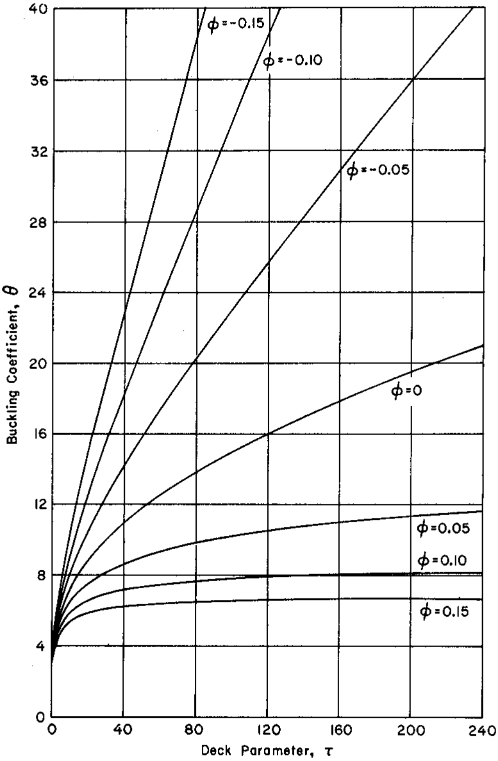

FIG. 9.—Buckling Coefficients for Case 3

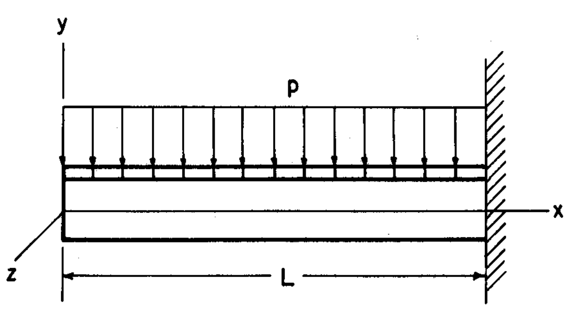

FIG. 10.—Case 4—Cantilever Beam-and-Deck System (Uniformly distributed load *p* is applied through deck attached to top flange)

$$\left.\begin{array}{l}\cos\lambda = 0\\ \text{or}\quad \lambda = \dfrac{\pi}{2}\end{array}\right\} \qquad (41)$$

Solving for q from Eqs. 40 and 41 yields the buckling criterion for this case:

$$\theta = \tau\phi \pm \sqrt{\left(\tau + \frac{\pi^2}{4}\right)(1 + \tau\phi^2)} \qquad (42)$$

in which the positive sign is taken when the deck is attached to the compression flange of the beam and vice versa. This agrees with the result obtained by Errera, Pincus, and Fisher (1).

Case 2—Uniform Load and Simple Support (see Fig. 6).—Again L is the half length as in Case 1.

$$\left.\begin{array}{l}\text{Because}\quad M_{max} = \dfrac{1}{2}\,pL^2\\ \text{the result is}\quad \dfrac{M}{|M_{max}|} = 1 - \xi^2\end{array}\right\} \qquad (43)$$

$$\text{and}\quad \theta = \frac{\dfrac{1}{2}\,pL^3}{\sqrt{JG\,EI_2}} \qquad (44)$$

Boundary Conditions.—The boundary conditions are the same as in Case 1.

Differential Equations.—The differential equations (Eqs. 21 and 22) reduce to

$$\omega'' = \tau\omega - [\theta(1 - \xi^2) - \tau\phi]\,\psi \qquad (45)$$

$$(1 + \tau\phi^2)\,\psi'' = -2\phi\theta\psi + [\theta(1 - \xi^2) - \tau\phi]\,\omega'' \qquad (46)$$

Power Series Solution.—Assume w and y have series expansions even in x:

$$\left.\begin{array}{l}\omega = a_o \displaystyle\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} A_{mn}\,\theta^m\,\xi^{2n} + b_o \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} B_{mn}\,\theta^m\,\xi^{2n}\\ \psi = a_o \displaystyle\sum_{n=1}^{\infty}\sum_{m=0}^{\infty} C_{mn}\,\theta^m\,\xi^{2n} + b_o \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} D_{mn}\,\theta^m\,\xi^{2n}\end{array}\right\} \qquad (47)$$

in which A_{mn}, B_{mn}, C_{mn}, and D_{mn} are functions of t and f. Here a_o and b_o are integration constants and the boundary conditions given by Eqs. 31 and 32 remain to be satisfied.

Substituting Eq. 47 into Eqs. 45 and 46 and equating coefficients of a_o q^m x^{2n} and of b_o q^m x^{2n} yields recursion relations and initial values by which one can generate the four arrays A_{mn}, B_{mn}, C_{mn}, and D_{mn} from prescribed values of t and f. Finally, the two remaining boundary conditions, Eqs. 31 and 32, are applied and the integration constants, a_o and b_o are required to be indeterminate. This yields

$$(\Sigma\Sigma A_{mn}\,\theta^m)\,(\Sigma\Sigma D_{mn}\,\theta^m) - (\Sigma\Sigma B_{mn}\,\theta^m)\,(\Sigma\Sigma C_{mn}\,\theta^m) = 0 \quad . . . . . (48)$$

as the stability criterion for this case. The critical value of q is obtained by

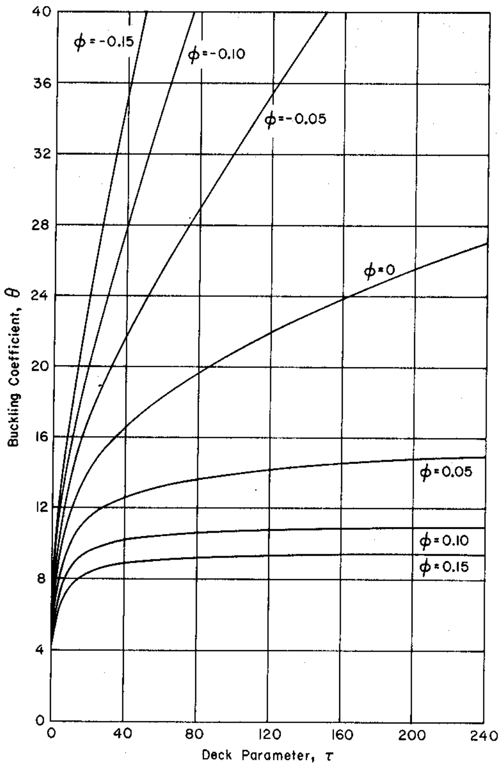

FIG. 11.—Buckling Coefficients for Case 4

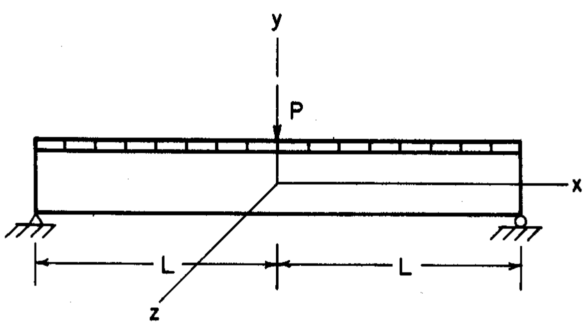

FIG. 12.—Case 5—Simply Supported Beam-and-Deck System with Concentrated Load *P* applied at Midspan Through Deck Attached at Top Flange (Ends are restrained against axial rotation)

trial from Eq. 48 for prescribed t and f, with the aid of a digital computer. The results of such a calculation are shown in Fig. 7.

TABLE 1.—Typical Effect of Deck Shear Stiffness on Buckling Load[a]

Deck material (1)	In-plane shear stiffness, G_D, in pounds per inch[b] (millinewtons per meter) (2)	Buckling parameter, q (3)	Increase in elastic buckling load, as a percentage (4)
None	0	1.8	—
2 in. wood	400 (70)	2.9	60
1/2 in. plywood	3,000 (525)	7.9	340
3 in. wood	20,000 (3,500)	28.0	1,450

[a]The beam used for this example is 30 ft long, 18 in. deep and 5-1/8 in. wide.
[b]Assumed values. Exact values would depend on details of deck construction. See Ref. 11 for example.

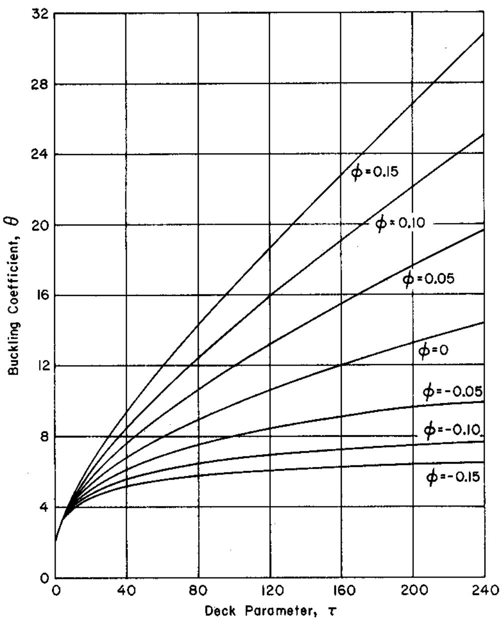

FIG. 13.—Buckling Coefficients for Case 5

Case 3—End-Loaded Cantilever (see Fig. 8).—Here *L* is the total length of the beam and *P* is the end load. The maximum moment is *PL* and thus

$$\theta = \frac{PL^2}{\sqrt{JG\,EI_2}} \quad \text{(49)}$$

Critical values of *P* can be obtained from the eigenvalues of q:

$$P_{cr} = \theta \frac{\sqrt{JG\,EI_2}}{L^2} \quad \text{(50)}$$

Values of the buckling coefficient, 8, are presented in Fig. 9.

Case 4—Uniformly Loaded Cantilever (see Fig. 10).—Again L is the length of the cantilever. Critical values of the distributed load, *p*, can be obtained from

$$p_{cr} = 2\,\theta \frac{\sqrt{JG\,EI_2}}{L^3} \quad \text{(51)}$$

and values of the buckling coefficient, 8, are presented in Fig. 11.

Case 5—concentrated Center Load and Simple Support (see Fig. 12).—Again L is the half length as in Case 1. Critical values of the center load P can be obtained from

$$P_{cr} = 2\,\theta \frac{\sqrt{JG\,EI_2}}{L^2} \quad \text{(52)}$$

and values of the buckling coefficient, 8, are presented in Fig. 13.

RESULTS

Example.—The main results of this report are in Eq. 42 and Figs. 7, 9, 11, and 13, which present the buckling coefficient, 8, in terms of the deck stiffness parameter, t, and depth-span parameter f. Their use is illustrated by an example.

Suppose a large building is to have timber beams spanning 30 ft (approx 9.1 m) and spaced every 10 ft (approx 2.0 m). The required beam cross section has been calculated to be 18 in. (460 mm) deep by 5-1/8 in. (130 mm) wide for a condition of uniform load. The design is to be checked for stability. The accepted procedure (8) is to check the slenderness factor which is defined as

$$\text{"slenderness factor"} \quad C_s = \sqrt{\frac{l_e d}{b^2}} \quad \text{(53)}$$

in which I, = the effective length (1.92 times span length for uniform load, simple support, and no deck). If C_s exceeds 10, the allowable stress must be reduced for slenderness.

In this example, the slenderness factor is

$$C_s = \sqrt{\frac{(1.92)\,(360)\,(18)}{(5.125)\,(5.125)}} = 21.8 \quad \text{(54)}$$

greater than 10, indicating that the system is unsafe if it cannot rely upon deck stiffness for stability (8). Assume, for rectangular wood beams, that

$$E = 1.8 \times 10^6 \text{ psi (approx 12 GN/m}^2) \qquad (55)$$

and $$JG \approx \frac{1}{4} EI_2 \qquad (56)$$

Then Fig. 7 may be used with L = 180 in., (approx 4.6 m) f = 0.1, and various deck stiffnesses to yield the results shown in Table 1. It is seen that even the relatively soft 2 in. deck is capable of providing a significant increase in the buckling load, although it is far less stiff than plywood or internailed decks, whose great stiffness effectively prevents elastic buckling as a mode of failure.

Remarks

Meaning of f *= 0 Curve.*—Parameter f has been referred to as a depth-to-span ratio parameter because its numerator contains *c*, the distance from the centroidal axis to the line of deck attachment, and the deck is ordinarily attached to the top flange of the beam. Thus in practical terms, *c* is the half depth. Note that negative values of f denote a deck attached to the bottom flange. The curve for f = 0 can only be interpreted as a deck attached at the centroidal axis because a zero depth would be meaningless; the curve is useful for interpolation purposes.

Cantilevers.—Note that cantilevers with a deck on top are laterally supported along the tension flange, and that such restraint does far less to prevent lateral buckling than when the compression flange is supported. Thus the curves for positive f lie below those for negative f in Cases 3 and 4. Whenever the tension flange is supported, the curves in every case approach a horizontal asymptote as the deck stiffness approaches infinity. This limit is the buckling load of a beam whose tension flange is hinged to a rigid deck.

Summary and Conclusions

The effect of attached decking upon the lateral stability of beams has been analyzed by a variational approach which includes the deck as a shear-resisting element in the beam-and-deck system. Design curves or formula are presented for five cases of loading and support. Because stability does not often govern design except when overly restrictive rule-of-thumb limits are imposed, it was not deemed necessary to analyze a large variety of cases. It is hoped that this work will provide enough information to facilitate the safe design of light-weight economical structures.

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APPENDIX I.—REFERENCES

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APPENDIX II.—NOTATION

The following symbols are used in this paper:

$A_{mn}, B_{mn}, C_{mn}, D_{mn}$ = series coefficients;

$a_o, b_o; C_1, C_2$ = integration constants;

c = distance from centroidal axis to line of deck attachment, in inches (meters);

EI_2 = lateral bending stiffness, in pound-inches squared (newtons-meters squared);

e_x = incremental strain in beam due to buckling;

G_D = in-plane shear stiffness of deck, in pounds per inch (newtons per meter);

JG = torsional stiffness, in pound-inches squared (newtons-meters squared);

L = length of cantilever or half-length of simple beam, in inches (meters);

M = internal bending moment about z-axis, in inch-pounds (newtons-meters);

P = concentrated load at origin, pounds (newtons);

p = uniform load, in pounds per inch (newtons per meter);

S = beam spacing, in inches (meters);

U, U_D, U_B, U_L = energy;
$\bar{u}, \bar{v}, \bar{w}$ = displacements of point at (x, y, z);
v = vertical displacement of centroidal axis, in inches (meters);
w = lateral displacement of centroidal axis, in inches (meters);
w_D = lateral displacement of line of deck attachment, in inches (meters);
x, y, z = coordinate axes (see Fig. 1);
β = angle of twist;
γ_{deck} = deck shear strain;
γ_{xy} = incremental shear strain in beam due to buckling;
$\theta, \xi, \tau, \phi, \psi, \omega$ = see Eq. (17); and
$\sigma_x{}^o, \tau_{xy}{}^o$ = prebuckling stresses.

9829 LATERAL STABILITY OF WOOD BEAM-AND-DECK

KEY WORDS: **Beams (supports); Floors; Lateral stability; Roofs; Stability; Structural engineering; Timber construction;** Timbers; Wood

ABSTRACT: Modern timber structures frequently employ large wood beams bridged by 2 in. wood decking. Economy of material dictates the use of deep narrow beams whose design is limited by considerations of lateral instability. Such constructions must rely upon the in-plane shear rigidity of the attached deck to prevent collapse, yet when decks of low rigidity such as 2 in. decks are employed, the stability of the resulting structure is seriously in question. This report quantitatively answers the question of how much shear rigidity a deck must possess in order to provide adequate lateral support when attached to the top of a set of deep beams. A variational method of derivation is employed, and buckling curves are presented for five cases of load and support. The use of these results by structural engineers will facilitate the safe design of lightweight economical wood structures.

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