

Lattice QCD Evidence for Exotic Tetraquark Resonance

Hideo Suganuma* and Kyosuke Tsumura

Department of Physics, Kyoto University, Kitashirakawaiwake, Sakyo, Kyoto 606-8502, Japan

E-mail: suganuma@scphys.kyoto-u.ac.jp, tmura@scphys.kyoto-u.ac.jp

Noriyoshi Ishii

Department of Physics, Tokyo Institute of Technology, Ohokayama, Tokyo 152-8551, Japan

E-mail: ishii@th.phys.titech.ac.jp

Fumiko Okiharu

Department of Physics, Nihon University, Kanda-Surugadai, Chiyoda, Tokyo 101-8308, Japan

E-mail: oharu@phys.cst.nihon-u.ac.jp

We study the manifestly exotic tetraquark $D_{s_0}^{++}(cu\bar{s}\bar{d})$ and the scalar tetraquark $f_0(ud\bar{u}\bar{d})$ in $SU(3)_c$ anisotropic quenched lattice QCD. We adopt the $O(a)$ -improved Wilson (clover) fermion at $\beta = 5.75$ on $12^3 \times 96$ with renormalized anisotropy $a_s/a_t = 4$, and investigate the correlator of the four-quark (4Q) system, $cu\bar{s}\bar{d}$, with various quark masses including the idealized $SU(4)_f$ case. For $f_0(ud\bar{u}\bar{d})$ etc., we only consider connected diagrams at the quenched level, i.e., the tetraquark $f_0(ud\bar{u}\bar{d})$ is identical with $D_{s_0}^{++}(cu\bar{s}\bar{d})$ in the idealized $SU(4)_f$ case. First, for comparison, we study the lowest $q\bar{q}$ scalar meson, and find that it has a large mass of about 1.37GeV after chiral extrapolation, which corresponds to $f_0(1370)$. Second, we investigate the lowest 4Q state in the spatial periodic boundary condition, and find that it is just a scattering state of two pseudoscalar mesons, as is expected. Third, to extract spatially-localized 4Q resonances, we use the Hybrid Boundary Condition (HBC) method, where anti-periodic and periodic boundary conditions are imposed on quarks (c, u) and antiquarks (\bar{s}, \bar{d}), respectively. By applying the HBC on a finite-volume lattice, the threshold of the two-meson scattering state is raised up, while the mass of a compact 4Q resonance is almost unchanged. In HBC, we find a nontrivial 4Q resonance state about 100 MeV below the two-meson threshold in some quark-mass region. Its chiral behavior largely differs from a two-meson scattering state. The scalar tetraquark $f_0(ud\bar{u}\bar{d})$ is found to have the mass of about 1.1GeV after chiral extrapolation, and seems to correspond to $f_0(980)$. Then, the manifestly exotic tetraquark $D_{s_0}^{++}(cu\bar{s}\bar{d})$ would exist around 1GeV in the idealized $SU(4)_f$ chiral limit. Finally, MEM analysis is applied to obtain the spectral function of the 4Q system.

XXIIIrd International Symposium on Lattice Field Theory

25-30 July 2005

Trinity College, Dublin, Ireland

*Speaker.

1. Introduction

The tetraquark [1-7] is an interesting subject in hadron physics. The tetraquark picture was first proposed for scalar mesons [3]. Recently, charmed tetraquark candidates were discovered [1, 2].

1.1 Experimental discoveries of tetraquark candidates

In recent years, various candidates of multi-quark hadrons have been experimentally observed [1, 2] and theoretically studied [4-9]. For instance, $D_{s0}^+(2317)$ [1], $D_{s1}^+(2460)$, $X(3872)$ [2] and $Y(4260)$ are expected to be tetraquark states [5, 6] from the consideration of their mass, narrow decay width and decay mode. As the unusual features of $X(3872)$, its mass is rather close to the threshold of $D^0(c\bar{u})$ and $\bar{D}^{0*}(u\bar{c})$, and its decay width is very narrow as $\Gamma < 2.3\text{MeV}$ (90 % C.L.). These facts seem to indicate that $X(3872)$ is a tetraquark [5] or a molecular state of $D^0(c\bar{u})$ and $\bar{D}^{0*}(u\bar{c})$ rather than an excited state of a $c\bar{c}$ system [6]. Also, $D_{s0}^+(2317)$ is conjectured to be not a simple meson of $c\bar{s}$ but a tetraquark from its small mass and its narrow decay width. In fact, simple quark-model assignment for $D_{s0}^+(2317)$ is $c\bar{s}({}^3P_0)$, but almost all theoretical predictions for P-wave $c\bar{s}$ states fail to reproduce its small mass [6, 10]. Furthermore, there is a puzzle in the decay mode of $D_{s0}^+(2317)$: CLEO collaboration [1] shows that the dominant decay mode is $D_s^{*+}\pi^0$ and its contribution is much larger than the radiative decay as

$$\frac{\Gamma(D_{s0}^+ \rightarrow D_s^{*+}\gamma)}{\Gamma(D_{s0}^+ \rightarrow D_s^{*+}\pi^0)} < 0.059. \quad (1.1)$$

If $D_{s0}^+(2317)$ is isoscalar ($I=0$) such as $c\bar{s}$, this decay pattern implies that the main decay mode is isospin breaking process of $O(e^4)$ in QED, which is rather anomalous. To explain this decay pattern, Terasaki proposed $I = 1$ possibility of $D_{s0}^+(2317)$ [7], which means the tetraquark picture for $D_{s0}^+(2317)$ and leads to D_{s0}^{*+} as the isospin partner.

Then, we examine manifestly exotic tetraquark $D_{s0}^{*+}(cu\bar{s}\bar{d})$ with $C = +1$, $S = +1$ and $I_3 = +1$ in anisotropic quenched lattice QCD. In this paper, we mainly report the idealized $SU(4)_f$ case.

1.2 Tetraquarks in the light-quark sector

There are five 0^{++} isoscalar mesons below 2GeV: $f_0(400-1200)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$. Among them, $f_0(1370)$ is considered as the lowest $q\bar{q}$ scalar meson in the quark model [11]. For, in the quark model, the lowest $q\bar{q}$ scalar meson is 3P_0 , and therefore it turns to be rather heavy. [$f_0(1500)$ and $f_0(1710)$ are expected to be the lowest scalar glueball or an $s\bar{s}$ scalar meson.] Then, what are the two light scalar mesons, $f_0(400-1200)$ and $f_0(980)$? This is the "scalar meson puzzle", which is unsolved even at present. As the possible answer, in 1977, Jaffe proposed tetraquark ($qq\bar{q}\bar{q}$) assignment for low-lying scalar mesons such as $f_0(980)$ and $a_0(980)$ [3].

Then, we examine the scalar tetraquark $f_0(ud\bar{u}\bar{d})$ in the light-quark sector in anisotropic quenched lattice QCD. In this paper, we only consider connected diagrams at the quenched level. Note here that, if only the connected diagram is taken account at the quenched level, the tetraquark $f_0(ud\bar{u}\bar{d})$ is identical with $D_{s0}^{*+}(cu\bar{s}\bar{d})$ in the idealized $SU(4)_f$ case.

2. Lattice QCD

We investigate the manifestly exotic tetraquark $D_{s0}^{*+}(cu\bar{s}\bar{d})$ and the scalar tetraquark $f_0(ud\bar{u}\bar{d})$ in anisotropic quenched lattice QCD mainly in the idealized $SU(4)_f$ case.

2.1 Diquark-antidiquark-type interpolating field operator

For the tetraquark $D_{s_0}^{++}(cu\bar{s}\bar{d})$ with spin $J = 0$ and isospin $I = 1$, we adopt the diquark-antidiquark-type (non-two-meson-type) interpolating field,

$$O \equiv \varepsilon_{abc}\varepsilon_{ade}(c_b^T C \gamma_5 u_c)(\bar{s}_d C \gamma_5 \bar{d}_e^T), \quad (2.1)$$

to extract the 4Q resonance rather than two-meson scattering states. Here, the roman indices denotes color indices, and $C \equiv \gamma_4 \gamma_2$ the charge conjugation matrix. In this diquark-antidiquark-type operator, the overlap with two-meson scattering states is expected to be small. The energy of the low-lying state is extracted from the temporal 4Q correlator $G(t) \equiv \frac{1}{V} \sum_{\vec{x}} \langle O(t, \vec{x}) O^\dagger(0, \vec{0}) \rangle$, where the total momentum of the 4Q system is projected to be zero. To reduce highly-excited-state components, we use a spatially-extended source of Gaussian-type with $\rho=0.4\text{fm}$ in the Coulomb gauge [8, 12]. We impose the Dirichlet boundary condition in the temporal direction [9]. We analyze the effective mass $m_{\text{eff}}(t)$ obtained from the 4Q correlator $G(t)$ as $m_{\text{eff}}(t) \equiv \ln\{G(t)/G(t+1)\}$.

2.2 Anisotropic lattice QCD

To get detailed information on the temporal behavior of the 4Q correlator $G(t)$, we adopt anisotropic lattice QCD with the standard plaquette action and the $O(a)$ -improved Wilson (clover) fermion at $\beta \equiv 2N_c/g^2 = 5.75$ on $12^3 \times 96$ with renormalized anisotropy $a_s/a_t = 4$ [8, 12]. This is anisotropic version of the Fermilab action [13]. The scale is set by the Sommer scale $r_0^{-1} = 395\text{MeV}$, which leads to the spatial/temporal lattice spacing as $a_s^{-1} \simeq 1.10\text{GeV}$ (i.e., $a_s \simeq 0.18\text{fm}$) and $a_t^{-1} \simeq 4.40\text{GeV}$ (i.e., $a_t \simeq 0.045\text{fm}$). The spatial lattice size is $L = 12a_s \simeq 2.15\text{fm}$. We use 1827 gauge configurations, which are picked up every 500 sweeps after the thermalization of 10,000 sweeps. On the hopping parameter, we take $\kappa=0.1210\text{--}0.1240$ for light quarks and $\kappa=0.1120$ for the real charm-quark mass. We summarize the lattice parameters and related quantities in Table 1.

Table 1: The lattice parameters and related quantities. They are almost the same as in Ref.[8].

β	lattice size	a_s^{-1}	a_t^{-1}	γ_G	u_s	u_t	κ
5.75	$12^3 \times 96$	1.10GeV	4.40GeV	3.2552	0.7620	0.9871	0.1210(10)0.1240, 0.1120

2.3 Hybrid Boundary Condition (HBC) method

To extract low-lying 4Q resonances, we use the ‘‘hybrid boundary condition’’ (HBC) [8] where we impose the anti-periodic boundary condition for quarks (c, u) and the periodic boundary condition for antiquarks (\bar{s}, \bar{d}), as shown in Table 2. By applying the HBC on a finite lattice with L^3 , the two-meson threshold is raised up, while the mass of a compact 4Q resonance is almost unchanged. Then, the 4Q resonance may become visible as a low-lying state in HBC, if it exists.

Table 2: The hybrid boundary condition (HBC) to raise up the threshold of two-meson scattering states.

	c, u	\bar{s}, \bar{d}	$q\bar{q}$ -meson	2-meson threshold	tetraquark ($qq\bar{q}\bar{q}$)
PBC	periodic	periodic	periodic	$m_1 + m_2$	periodic
HBC	anti-periodic	periodic	anti-periodic	$\sum_{k=1,2} \sqrt{m_k^2 + \vec{p}_{\min}^2}$	periodic

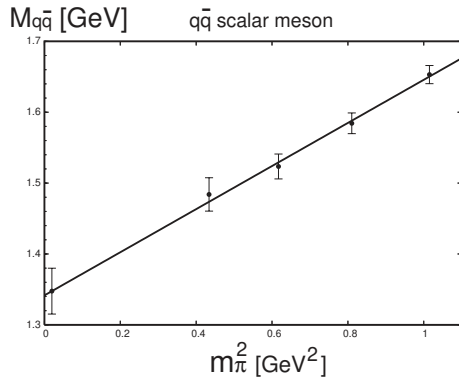


Figure 1: The lowest mass $M_{q\bar{q}}$ of the $q\bar{q}$ scalar meson plotted against m_π^2 . The $q\bar{q}$ scalar meson mass $M_{q\bar{q}}$ is extracted from the connected diagram in quenched lattice QCD.

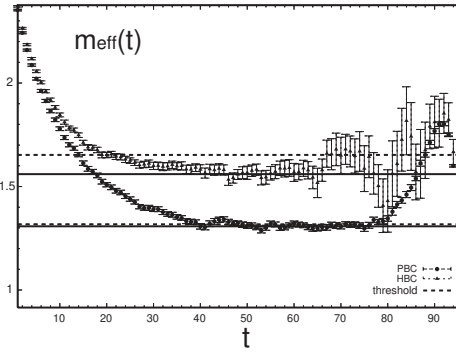


Figure 2: The effective mass plot of 4Q system in PBC and HBC. The solid lines denote the plateau of $m_{\text{eff}}(t)$, and the dashed lines the two-meson threshold. The Dirichlet boundary condition is imposed in the temporal direction.

For a compact 4Q quasi-bound state of $cu\bar{s}\bar{d}$, since it contains even number of quarks, it obeys periodic boundary condition (PBC) in HBC, and therefore its energy in HBC is almost the same as that in PBC [8]. For a two-meson scattering state, both mesons have non-zero relative momentum $\vec{p}_{\text{min}} = (\pm\frac{\pi}{L}, \pm\frac{\pi}{L}, \pm\frac{\pi}{L})$, i.e., $|\vec{p}_{\text{min}}| = \sqrt{3}\pi/L$ in HBC, while they can take zero relative momentum $\vec{p}_{\text{min}} = 0$ in PBC. In fact, the two-meson threshold $m_1 + m_2$ (m_1, m_2 : meson masses) in PBC is raised up in HBC as $E_{\text{th}} \simeq \sum_{k=1,2} \sqrt{m_k^2 + \vec{p}_{\text{min}}^2}$ with $|\vec{p}_{\text{min}}| = \sqrt{3}\pi/L \simeq 0.5\text{GeV}$ for $L \simeq 2.15\text{fm}$.

3. Lattice QCD results for tetraquarks

First, for comparison, we calculate the $q\bar{q}$ scalar meson mass in anisotropic quenched lattice QCD. Here, we only consider connected diagrams. We show in Fig.1 the lowest $q\bar{q}$ scalar meson mass plotted against m_π^2 . Our quenched lattice QCD indicates that the lightest $q\bar{q}$ scalar meson has a large mass about 1.37 GeV after the chiral extrapolation. Thus, the lightest $q\bar{q}$ scalar meson corresponds to $f_0(1370)$ or $a_0(1450)$, which is consistent with the quark-model assignment for scalar mesons. [Note that, if disconnected diagrams are dropped off, isoscalar $q\bar{q}$ mesons degenerate isovector $q\bar{q}$ mesons in quenched QCD, e.g. $m_\rho = m_\omega$ and $m(f_0) = m(a_0)$.]

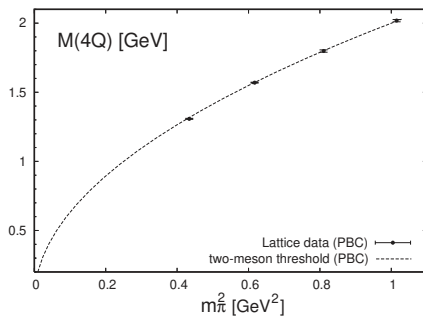


Figure 3: The lowest energy $M(4Q)$ of the 4Q system in PBC plotted against m_π^2 . The dotted curve denotes the two-meson threshold of $2m_\pi$.

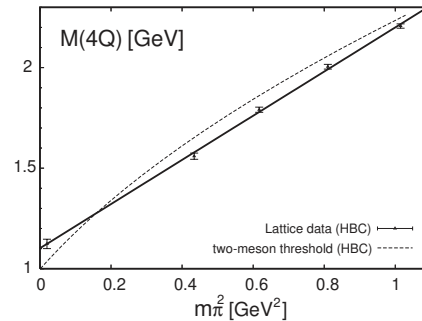


Figure 4: The lowest energy $M(4Q)$ of the 4Q system in HBC v.s. m_π^2 . The dotted curve is the two-meson threshold of $\sqrt{m_\pi^2 + 3\pi^2/L^2}$ in HBC.

Second, we investigate the 4Q system of $cu\bar{s}\bar{d}$ with various quark masses in PBC and in HBC. Here, we show the idealized $SU(4)_f$ case, where $D_s^{++}(cu\bar{s}\bar{d})$ is identical with $f_0(ud\bar{u}\bar{d})$ as far as only connected diagrams is considered. Figure 2 shows the effective mass $m_{\text{eff}}(t)$ of $cu\bar{s}\bar{d}$ (or $ud\bar{u}\bar{d}$) in PBC and in HBC for $\kappa=0.1240$. In PBC, the plateau of $m_{\text{eff}}(t)$ agrees with the two-pseudoscalar-meson threshold, i.e., $2m_\pi$. As shown in Fig.3, the lowest energy of the 4Q system in PBC plotted is just a π - π scattering state, as is expected. [Note that this lowest 4Q level is raised up in HBC, while a compact 4Q resonance does is almost unchanged in HBC.]

Third, we analyze the lowest 4Q state in HBC. In HBC, there appears a plateau below the π - π threshold by about 100MeV as shown in Fig.3. Figure 4 shows the lowest energy of the 4Q system in HBC plotted against m_π^2 . Note that its chiral behavior is largely different from the π - π scattering state. Thus, this 4Q state is considered to be a nontrivial 4Q resonance instead of a two-meson scattering state. After the simple linear chiral extrapolation, the mass of the 4Q state is estimated to be about 1.1GeV, which seems to correspond to $f_0(980)$ or $a_0(980)$ in the light-quark scalar sector. This also indicates that the manifestly exotic tetraquark $D_{s0}^{++}(cu\bar{s}\bar{d})$ would exist around 1GeV in the idealized $SU(4)_f$ chiral limit.

4. MEM analysis for point-point 4Q correlation

Finally, using the maximum entropy method (MEM) [14], we extract the spectral function $A(\omega)$ from the temporal correlator $G(t)$, which satisfies

$$G(\tau) = \int_0^\infty d\omega K(\tau, \omega) A(\omega), \quad K(\tau, \omega) \equiv \frac{e^{-\tau\omega} + e^{-(\beta-\tau)\omega}}{1 - e^{-\beta\omega}}, \quad \beta \equiv N_t a_t. \quad (4.1)$$

From the lattice data of the 4Q correlator $G(t)$, we obtain the spectral function $A(\omega)$ of the 4Q system with MEM. At present, we perform the MEM analysis for point-source point-sink 4Q correlator $G(t)$ both in PBC and in HBC, although this correlator includes large excited-state contamination. For the point-point correlator $G(t)$, the lowest-order default function $m(\omega)$ is calculated as

$$m(\omega) = \frac{4N_c}{2^8 \Gamma(5) \Gamma(6) \pi^6} \omega^8. \quad (4.2)$$

Unlike the previous section, we here use $\kappa = 0.1240$ for u, d, s -quarks, and $\kappa = 0.1120$ for c -quark, and impose the periodic boundary condition in temporal direction. Figures 5 and 6 show the MEM analysis for the 4Q correlator $G(t)$ and the obtained spectral function $A(\omega)$ for the 4Q system.

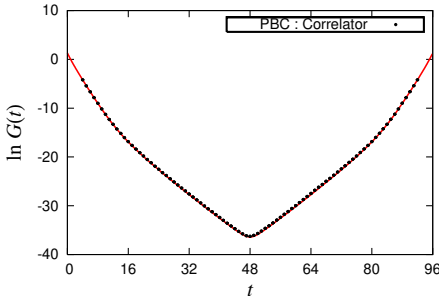


Figure 5: The 4Q correlator of point-source point-sink correlator $G(t)$ in PBC. The symbols denote the lattice data. The solid curve is the temporal correlator reconstructed by $A(\omega)$ in Fig.6.

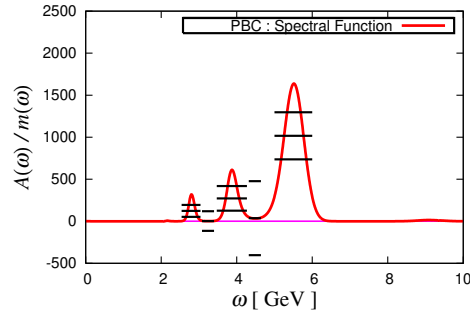


Figure 6: The spectral function $A(\omega)$ obtained with MEM from the point-source point-sink 4Q correlator in PBC.

The lattice data of 4Q correlator $G(t)$ can be well reconstructed with the spectral function $A(\omega)$ obtained by MEM through Eq.(4.1). [In this point-point correlator $G(t)$, highly-excited contamination is so large that low-lying structure of $A(\omega)$ cannot be seen clearly. To clarify the low-lying structure of the 4Q spectrum, we are performing the MEM analysis for extended-source extended-sink 4Q correlators, where excited-state components are reduced.]

5. Summary and conclusions

We have studied the manifestly exotic tetraquark $D_{s_0}^{++}(cu\bar{s}\bar{d})$ and the scalar tetraquark $f_0(ud\bar{u}\bar{d})$ in $SU(3)_c$ anisotropic quenched lattice QCD mainly in the idealized $SU(4)_f$ case. First, we have found that the lowest $q\bar{q}$ scalar meson has a large mass of about 1.37GeV after chiral extrapolation, which corresponds to $f_0(1370)$. Second, we have found that the lowest 4Q state is just a scattering state of two pseudoscalar mesons. Third, using the Hybrid Boundary Condition (HBC) method, we have found a nontrivial 4Q resonance state whose chiral behavior is different from a two-meson scattering state. The scalar tetraquark $f_0(ud\bar{u}\bar{d})$ is found to have the mass of about 1.1GeV after chiral extrapolation, and seems to correspond to $f_0(980)$. Then, the manifestly exotic tetraquark $D_{s_0}^{++}(cu\bar{s}\bar{d})$ is expected to exist around 1GeV in the idealized $SU(4)_f$ chiral limit.

References

- [1] BABAR Collaboration (B. Aubert *et al.*), Phys. Rev. Lett. **90**, 242001 (2003).
Belle Collaboration (P. Krokovny *et al.*), Phys. Rev. Lett. **91**, 262002 (2003).
CLEO Collaboration (D. Besson *et al.*), Phys. Rev. **D68**, 032002 (2003).
- [2] Belle Collaboration (S. K. Choi *et al.*), Phys. Rev. Lett. **91**, 262001 (2003).
BABAR Collaboration (B. Aubert *et al.*), Phys. Rev. Lett. **93**, 041801 (2004).
CDF II Collaboration (D. Acosta *et al.*), Phys. Rev. Lett. **93**, 072001 (2004).
D0 Collaboration (V. M. Abazov *et al.*), Phys. Rev. Lett. **93**, 162002 (2004).
- [3] R.L. Jaffe, Phys. Rev **D15**, 267 (1977).
- [4] F. Okiharu, H. Suganuma and T.T. Takahashi, Phys. Rev. **D72**, 014505 (2005); Phys. Rev. Lett. **94**, 192001 (2005); hep-lat/0410021, in "Pentaquark04", July 2004, p.339 (World Scientific, 2005).
- [5] F.E. Close and P.R. Page, Phys. Lett. **B578**, 119 (2004).
- [6] T. Barnes and S. Godfrey, Phys. Rev. **D69**, 054008 (2004); S. Godfrey, hep-ph/0508078.
- [7] K. Terasaki, Phys. Rev. **D68**, 011501 (2003); AIP Conf. Proc. **CP717**, 556 (2004);
K. Terasaki and B.H.J. McKellar, Prog. Theor. Phys. **114**, 205 (2005).
- [8] N. Ishii, T. Doi, H. Iida, M. Oka, F. Okiharu and H. Suganuma, Phys. Rev. **D71**, 034001 (2005);
N. Ishii, T. Doi, Y. Nemoto, M. Oka and H. Suganuma, hep-lat/0506022, Phys. Rev. **D72** (2005).
- [9] T.T. Takahashi, T. Umeda, T. Onogi and T. Kuhlhiro, Phys. Rev. **D71**, 114509 (2005).
- [10] R. Lewis and R.M. Woloshyn, Phys. Rev. **D62**, 114507 (2000).
- [11] Particle Data Group (K. Hagiwara *et al.*), Phys. Rev. **D66**, 010001 (2002).
- [12] H. Matsufuru, T. Onogi and T. Umeda, Phys. Rev. **D64**, 114503 (2001);
Y. Nemoto, N. Nakajima, H. Matsufuru and H. Suganuma, Phys. Rev. **D68**, 094505 (2003).
- [13] A.X. El-Khadra, A.S. Kronfeld and P.B. Mackenzie, Phys. Rev. **D55**, 3933 (1997).
- [14] M. Asakawa, T. Hatsuda and Y. Nakahara, Prog. Part. Nucl. Phys. **46**, 459 (2001).