

Lattice Thermal Conductivity Within the Earth and Considerations of a Relationship Between the Pressure Dependence of the Thermal Diffusivity and the Volume Dependence of the Grüneisen Parameter

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The method proposed by Mooney and Steg (1969) for obtaining the dilatational dependence of the Grüneisen parameter from data on the pressure dependence of the thermal conductivity (or, equivalently, thermal diffusivity) is critically examined and applied to thermal diffusivity data for sodium chloride and quartz. The values obtained are $\gamma_0' (=d\gamma/d\Delta)|_{\Delta=0} = 3.0$ for sodium chloride and $\gamma_0' = 2.0$ for quartz. Corresponding values of the parameter $q (= \gamma_0'/\gamma_0)$ are 1.9 and 2.8, in reasonable agreement with values obtained by other methods. It is suggested that this method be further investigated as a means of obtaining γ_0' and q from thermal data. A model for the lattice thermal conductivity of the mantle to the core boundary is presented. The model suggests that increases in conductivity with pressure due to lattice conduction processes in the mantle are less than 2.0%/kbar or 0.7%/km. Under conditions of normal geothermal gradient in the crust and upper mantle the increase in lattice conductivity due to the pressure effect will be substantially less than the decrease due to the temperature effect. A minimum value of lattice conductivity is attained in the region of the olivine-spinel phase change, 400 km. The lattice conductivity may increase by a factor of 3 at the depth of the spinel-postspinel phase change owing to the high conductivity of the dense oxide phases. The lattice contribution to the thermal conductivity at the mantle-core boundary is ~ 0.01 cal/cm s °K.

INTRODUCTION

In the preceding paper in this volume [Kieffer *et al.*, 1976], experimental data for the pressure dependence of the thermal diffusivity of Teflon, sodium chloride, quartz, and silica were reported. The purpose of this paper is to call attention to a method by which such thermal diffusivity or conductivity data can be used to estimate the volume dependence of the Grüneisen parameter γ . The concepts developed are then used to calculate the lattice contribution to the conductivity of the earth to the core-mantle boundary.

DILATATIONAL DERIVATIVE OF GRÜNEISEN'S PARAMETER

Mooney and Steg [1969] have suggested that with certain simplifying assumptions it is possible to obtain the dilatational derivative of the Grüneisen parameter, $\gamma_0' = (d\gamma/d\Delta)|_{\Delta=0}$, where $\Delta = (v/v_0) - 1$, from thermal conductivity data. For this calculation, one must know the pressure dependence of the thermal conductivity K or, equivalently, the diffusivity κ , the Grüneisen parameter γ_0 at 1-bar pressure, and the pressure-volume equation of state of the material. For crystalline materials the relation between K , γ_0 , and γ_0' depends on the submicroscopic nature of the material—whether it is composed of perfect crystals or contains large defect concentrations. In this paper it is assumed that the perfect crystal model applies. To first order the results are similar for crystals which have strong point defect scattering [Mooney and Steg, 1969, p. 238]. At the present time there is no theory for the thermal conductivity or its pressure dependence for amorphous solids, although attempts to relate the Grüneisen parameter to the thermal conductivity and its pressure derivative for polymers and glasses have been made by Barker [1967] and Barker and Chen [1970].

There are numerous definitions of the Grüneisen parameter [e.g., reviewed by Barron, 1955]. The mode Grüneisen parameters are defined as

$$\gamma_j = -d \ln \nu_j / d \ln v \quad (1)$$

where ν_j is the frequency of the j th normal mode of vibration. On the other hand, the thermal Grüneisen parameter is the quantity obtained from thermodynamic measurements:

$$\gamma_{th} = \alpha K_s / \rho c_p \quad (2)$$

where K_s is the adiabatic bulk modulus, c_p is the specific heat, ρ is the density, and α is the thermal expansion. The thermal Grüneisen parameter is related to the mode Grüneisen parameters γ_j only through averaging procedures. The most common method of averaging is to require that the energy, pressure, and specific heat be thermodynamically consistent [Born and Huang, 1954; Barron, 1955; Anderson *et al.*, 1968], which leads to

$$\bar{\gamma} = (\sum c_j \gamma_j) / (\sum c_j) \quad (3)$$

where c_j is the specific heat of the j th mode. If it is further assumed that all of the γ_j are equal, that is, that all mode frequencies vary with volume in a similar manner, then $\bar{\gamma}$ is the thermal Grüneisen parameter of (2).

Ample experimental data, however, demonstrate that the γ_j are not equal. For example, in an isotropic Debye solid there are two distinct values of γ_j which correspond to longitudinal (γ_l) and transverse (γ_t) waves [Bijl and Pullan, 1954]. These two γ 's will be equal only if Poisson's ratio does not vary with volume. However, at high temperatures, all of the longitudinal and transverse modes are excited, and the Grüneisen parameter may be taken to be an average of the two mode γ 's [Barron, 1955]:

$$\gamma^{HT} = (2\gamma_l + \gamma_t) / 3 \quad (4)$$

This γ is referred to as the high-temperature Grüneisen parameter. The definition (4) applies to monatomic isotropic substances; for anisotropic materials, appropriate spatial averages must be used.

The development of the Mooney and Steg [1969] theory depends on the assumption that for the purpose of thermal conductivity calculations the vibrational modes of a substance may be represented by a Debye spectrum. It has been demon-

strated that the vibrational spectra of real materials do not, in fact, resemble Debye spectra [Kieffer, 1971; Kieffer and Kamb, 1971], but it is possible to apply Debye theory to minerals for the purpose of thermal conductivity calculations if the appropriate Debye temperature is used. Owing to the complex crystal structure of minerals, their lattice vibrational spectra have both acoustic and optic branches. Typically the acoustic, Debyelike branches contain only a few percent of the total modes. The specific heat depends on contributions from all branches; hence a Debye model for lattice vibrational spectra is inadequate for calculation of the specific heat of minerals. However, to a good approximation only the acoustic branches contribute to the thermal conductivity, since the optic branches have a low group velocity [Roufosse and Klemens, 1974; Klemens, 1958]. To the degree that the acoustic branches may be approximated by a Debye spectrum the thermal conductivity may be described in terms of the acoustic branches alone, with an effective or acoustic Debye temperature proportionally reduced from the usual Debye temperature by the fraction of oscillators in the acoustic branches:

$$\theta_D^{ac} = (3/3n)^{1/3} \theta_D \quad (5)$$

where n is the number of atoms in a unit cell of the mineral and θ_D is the Debye temperature based on the assumption that all degrees of freedom are contained in the acoustic branches. For example, for quartz ($n = 9$, $\theta_D = 528^\circ\text{K}$) the acoustic Debye temperature appropriate for thermal conductivity calculations is 253°K , and for sodium chloride the acoustic Debye temperature, 264°K , is similarly low. For most minerals the acoustic Debye temperature is lower than room temperature, so that high temperature limits may be used for thermal conductivity calculations.

Mooney and Steg [1969] derived a simple relation between K , γ_0 , and γ_0' , applicable at temperatures greater than the acoustic Debye temperature. This theory, with a mathematical correction brought to the author's attention by T. Shankland, is briefly reviewed here.

In a crystal free from defects the thermal conductivity is [Klemens, 1958, p. 46; Mooney and Steg, 1969]

$$K \sim \text{const} (1/\gamma^2) \cdot a \cdot (\theta_D^{ac}/T) \quad (6)$$

where a is the interatomic spacing. The constant term involves Planck's and Boltzmann's constants and the mean atomic mass. This equation applies to single crystals or to nonporous polycrystalline aggregates in which the grain size exceeds the average phonon mean free path, which is typically less than $1 \mu\text{m}$. If there is no dispersion in the acoustic branches and if they have a mean sound speed, c , then the Debye temperature θ_D^{ac} is simply related to the lattice dimension:

$$\theta_D^{ac} = (3/3n)^{1/3} (k/h)(3N_0/4\pi Av)^{1/3} c \quad (7)$$

where h and k are Planck's and Boltzmann's constants, respectively, N_0 is Avogadro's number, and A is the mean atomic weight. Thus at a given temperature:

$$K = \text{const} \cdot c^3/\gamma^2 a^2 \quad (8)$$

The variables c , γ , and a are all functions of the dilatation Δ . A characteristic velocity of sound is

$$c = c_0[1 - (\gamma - \frac{1}{2}) \Delta] \quad (9)$$

(instead of $c = c_0[1 - \gamma(\Delta)]$ used by Mooney and Steg [1969, p. 237]). The average lattice spacing a is simply related to the volume v :

$$(a/a_0)^3 = v/v_0 \quad (10)$$

K may be expressed in a power series expansion:

$$K \sim K_0 + \frac{dK}{d\Delta} \Big|_{\Delta=0} \Delta + \frac{1}{2} \frac{d^2K}{d^2\Delta} \Big|_{\Delta=0} \Delta^2 \quad (11)$$

and when (8) is differentiated with respect to Δ and the derivatives of (9) and (10) are substituted, the following expression is obtained for a perfect crystal:

$$K \approx K_0 \{1 - [3\gamma_0 - \frac{1}{2} + 2(\gamma_0'/\gamma_0)] \Delta + \dots\} \quad (12)$$

where

$$\gamma_0' = \frac{d\gamma}{d\Delta} \Big|_{\Delta=0} \quad (13)$$

This expression differs from that of Mooney and Steg [1969, p. 238] by having a $(-\frac{1}{2})$ in the parentheses in place of a $(+\frac{1}{2})$. Equation (12) may be rewritten as a relation between thermal diffusivity κ and γ_0 , γ_0' as

$$\kappa \approx \kappa_0(\Delta + 1) \{1 - [3\gamma_0 - \frac{1}{2} + 2(\gamma_0'/\gamma_0)] \Delta\} \quad (14)$$

For simplicity, a first-order expression for Δ is assumed:

$$\Delta = -P/K_s \quad (15)$$

Then, to first order in pressure,

$$\kappa = \kappa_0 \{1 + [3\gamma_0 - \frac{1}{2} + 2(\gamma_0'/\gamma_0)] P/K_s\} \quad (16)$$

If the pressure dependence of κ is expressed as

$$\kappa = \kappa_0 + \beta P \quad (17)$$

a comparison of the coefficients of P in these equations gives

$$\gamma_0' = (\gamma_0/2)[(\beta K_s/\kappa_0) - 3\gamma_0 + \frac{1}{2}] \quad (18)$$

(perfect crystal).

This equation gives $\gamma_0' = 3.04$ for sodium chloride (with $\kappa_0 = 0.031 \text{ cm}^2 \text{ s}^{-1}$, $\beta = 9.5 \times 10^{-4} \text{ cm}^2 \text{ s}^{-1} \text{ kbar}^{-1}$, $K_s = 238 \text{ kbar}$, and $\gamma_0 = 1.6$ [Mooney and Steg, 1969]) and $\gamma_0' = 1.96$ for quartz (with $\kappa_0 = 0.031 \text{ cm}^2 \text{ s}^{-1}$, $\beta = 5.3 \times 10^{-4} \text{ cm}^2 \text{ s}^{-1} \text{ kbar}^{-1}$, $K_s = 377 \text{ kbar}$, and $\gamma_0 = 0.703$ [Anderson et al., 1968]). In the evaluation of (18), thermal Grüneisen parameters are used.

A question might arise here regarding the proper value of γ to use in evaluation of (18). The initial postulates of the model that (1) the nonacoustic modes have low group velocities, so that only the Debyelike acoustic branches contribute to the thermal conductivity, and that (2) temperatures are high in relation to the acoustic Debye temperature, suggest that the appropriate γ to use might be γ^{HT} defined by (4). This γ is clearly defined, however, only for monatomic substances. Its extension to polyatomic substances relies upon the further assumption that (3) for optic modes the individual mode γ_j 's have the same values as for the corresponding acoustic modes [Schuele and Smith, 1964, p. 809]. This assumption is unrealistic for polyatomic solids [Anderson et al., 1968, p. 499], although it is commonly used. I have therefore chosen to use the empirical macroscopic thermal γ_{th} , evaluated at the temperature at which the thermal conductivity measurements were made. Schuele and Smith [1964] have demonstrated that to a very good approximation, $\gamma^{HT}(=1.51) = \gamma_{th}(=1.55)$ for NaCl, which is in many ways nearly 'monatomic' in its vibrational properties because of similarities of mass and atomic environments of the Na and Cl atoms. For quartz a larger difference between $\gamma^{HT}(=0.4)$ and $\gamma_{th}(=0.7)$ is found [Anderson et al., 1968, p. 494], presumably because of the dissimilarity of

atomic environments around the Si^{+4} and O^{-2} ions. Use of γ^{HT} instead of γ_{th} in (19) changes the calculated value of q for quartz from 2.8 to 3.2, a small change in view of the many assumptions of the model.

The dilatational derivative $\gamma'_0(\Delta)$ is closely related to the parameter $q[\equiv(\partial \ln \gamma/\partial \ln v)]_T$, which is of importance in geophysical problems. The relationship between the two parameters is

$$q = \gamma'_0(\Delta)/\gamma_0 \quad (19)$$

From this relationship and the previously calculated values of γ'_0 , $q = 1.9$ for sodium chloride and $q = 2.8$ for quartz.

The value of q obtained from the model presented in this paper may be compared with q obtained from other types of thermodynamic data, by the thermodynamic definition:

$$q = 1 + (1 + T\alpha\gamma)\delta_s - K'_s + \gamma + T(\partial \ln \gamma/\partial T)_V \quad (20)$$

[Roberts and Ruppel, 1971]. In this equation, α is the thermal expansion, δ_s is the Anderson-Grüneisen parameter, and K'_s is the temperature derivative of the bulk modulus. The value of q obtained from (20) for NaCl is 1.4–1.46 [Roberts and Ruppel, 1971], lower than the value of 1.9 obtained by the Mooney-Steg model. With the Mie-Grüneisen assumptions that $T\alpha\gamma \ll 1$ and that $(\partial \ln \gamma/\partial T)_V$ is a function of volume only, (20) gives a value of $q = 2.6$ for quartz [Anderson, 1974]. This value is in good agreement with the value $q = 2.8$ obtained here from the thermal diffusivity data.

Several approximations to the general thermodynamic equation for q are common in the geophysical literature: (1) that $q = 1$, i.e., that γ_0/v is constant [e.g., Ahrens et al., 1969]; (2) that $q = 1 + \gamma_0$, i.e., that γ and the bulk modulus are functions of the volume only [e.g., Rice, 1965]. Although the agreement between the q from thermal diffusivity measurements and the q from the general thermodynamic definition (20) for sodium chloride and quartz is not exact, it appears sufficiently good to say that the approximations $q = 1$ or $q = 1 + \gamma_0$ are inadequate for minerals and that only the full definition (or perhaps Anderson's approximation to it) should be used.

The thermal diffusivity and the Grüneisen parameter are both measures of the anharmonicity of the lattice vibrations; it is therefore not surprising that one variable is related to the other. However, in view of the complexity of theories required to predict accurately a relation between the thermal diffusivity and the Grüneisen parameter it is perhaps surprising that their pressure and volume derivatives can be related to each other through a relatively simple thermodynamic argument. Although many assumptions and simplifications are involved in this method of obtaining the volume dependence of the Grüneisen parameter from thermal diffusivity data, the apparent success of the model in predicting γ'_0 for quartz and sodium chloride suggests that it should be further investigated as a method of obtaining γ'_0 and q .

In order to estimate the variation of γ within the earth it is assumed that

$$\gamma = \gamma_0 + (\partial\gamma/\partial\Delta)d\Delta \quad (21)$$

The variation of γ according to this assumption is shown in Figure 1 (which includes the variation of γ proposed by others for olivine, periclase, and stishovite; these data are used in the discussion in the next section). The range of volumes over which γ may be represented by a linear function such as (21) is not known, but the curves almost certainly do not extend past the high-pressure polymorphic transitions: quartz to coesite,

halite NaCl structure to CsCl structure, and olivine to spinel. The curves have been truncated at the compressions at which these polymorphic changes occur. (Note that in the range of compressions appropriate for periclase and stishovite in the mantle ($-\Delta \sim 0.3$), γ of periclase decreases by only 40%, but γ of stishovite (according to the linear model) approaches zero at $-\Delta = 0.17$ – 0.33 , depending on the value chosen for $\partial \ln \gamma/\partial \ln v$. Many forms of the $\gamma - \Delta$ relation other than this simple linear relation are possible, [e.g., Knopoff and Shapiro, 1969], but measured data of Ahrens et al. [1970] also suggest that γ does tend toward a small value—less than 0.6 for $-\Delta$ greater than 0.15. Thus it is possible that the thermal γ of stishovite becomes very small or zero in the lower mantle.)

MODEL FOR LATTICE CONDUCTIVITY WITHIN THE EARTH

The results from the thermal diffusivity experiments have been extrapolated to construct a model of the lattice thermal conductivity to the mantle-core boundary. (Conductivity is specified rather than diffusivity because of its more frequent use in the literature.) In general, the lattice conductivity within the earth depends on the composition (x), pressure (P), and temperature (T), that is, $K = K(x, P, T)$. Thus, to construct a model of conductivity of the earth it is necessary to specify the composition, density, and temperature within the earth and the temperature and pressure dependences of the thermal conductivity. The parameters required in the model are shown in Tables 1 and 2. The following assumptions are made in order to specify these quantities in the model:

1. The mantle is assumed to be Mg_2SiO_4 in the olivine structure to a depth of 400 km, in the spinel structure from 400 to 750 km, and in the form of dense oxides, periclase and stishovite, below that depth. This assumption places an upper limit on the lattice conductivity of the mantle, assumed to be a forsterite-fayalite mixture, FoFa, because (1) the addition of Fe_2SiO_4 in this solid solution series lowers the lattice conductivity [Horai, 1971, p. 1299] and (2) other likely mantle components such as pyroxenes have, on the average, lower conductivities than the orthosilicates. Best estimate con-

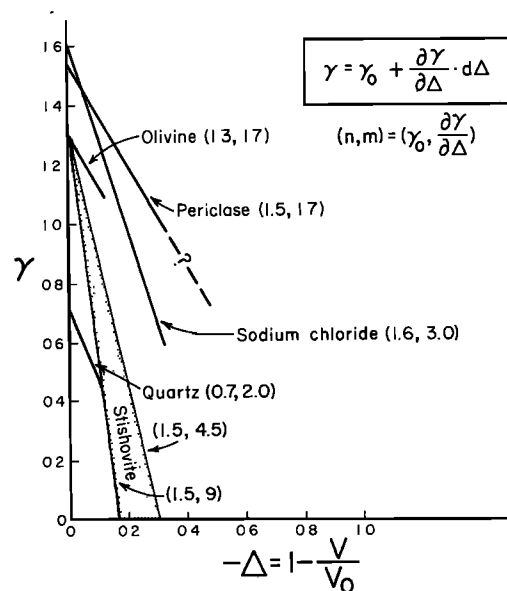


Fig. 1. Variation of the Grüneisen parameter with compression. The curves for sodium chloride and quartz are based on the theory presented in this paper and the linear extrapolation equation (21). The extrapolations for the other minerals are from data listed in Table 1 and (21).

TABLE 1. Thermal Parameters for Model Mantle

Mineral	ξ , cal ⁻¹ cm °K	η , cal ⁻¹ cm s	ρ_{00} , g/cm ³	γ_0	γ_0'
Mg ₂ SiO ₄ (olivine)	89 ^a	0.20 ^a	3.21	1.3 ^b	1.7 ^b
Mg ₂ SiO ₄ (spinel)	55 ^c	0.20 ^d	3.55	1.3 ^b	2.7 ^b
MgO	7.7 ^e	0.08 ^e	3.58 ^f	1.5 ^b	1.7 ^b
SiO ₂ (stishovite)	19 ^a	0.08 ^h	4.28 ^f	1.5 ⁱ	4.5-9 ⁱ

^a Horai and Simmons [1970, p. 980].

^b Anderson [1974].

^c Schatz [1971], estimated from Fujisawa *et al.* [1968].

^d Assumed to be same as for olivine.

^e From data in work by Clark [1966, p. 474].

^f I assume that the volume fraction of stishovite is 0.4 and that the weighted average zero-pressure density of the lower mantle oxides is 3.84 g/cm³.

^g Soga *et al.* [1972].

^h Assumed to be same as for MgO.

ⁱ Davies [1972].

ductivities are obtained by reducing the values of lattice conductivity calculated for pure forsterite by the reduction observed in room temperature values when the composition is varied from pure Fo₁₀₀ to the likely mantle composition of Fo₈₅Fa₁₅. This reduction factor is 0.85.

2. A model of density and temperature within the earth must be assumed; it is shown in Table 2. For the upper mantle, temperatures are from Schatz and Simmons [1972, p. 6979]; for the lower mantle, temperatures are extrapolated linearly to a core-mantle boundary temperature of 4400°K. Densities are from Verhoogen *et al.* [1970, p. 620]. Temperature variations of several hundred degrees would not appreciably influence the results, and choice of a lower thermal gradient would increase the magnitude of the pressure effect in relation to the temperature effect.

3. It is assumed that the temperature and pressure corrections to K can be made separately; that is, that $\partial K/\partial T$ is independent of pressure and $\partial K/\partial P$ is independent of temperature. (The Mg₂SiO₄ thermal diffusivity data of Fujisawa *et al.* [1968, p. 4731] suggest that $\partial\kappa/\partial P$ in fact decreases slightly with increasing temperature, so it is possible that $\partial K/\partial P$ is also somewhat temperature dependent.)

4. The temperature dependence of the lattice thermal conductivity at 1-bar pressure (P_0) is assumed to be given by [Schatz and Simmons, 1972]:

$$1/K(T, P_0) = \xi + \eta(T - 300) \quad (22)$$

where T is in degrees Kelvin and the parameters ξ and η are given in Table 1.

5. The pressure dependence of K is assumed to be given by (12). In the use of this equation for the earth model it is assumed that $\Delta = (\rho_{00}/\rho) - 1$ appropriate to mantle conditions as given in Table 2; ρ_{00} is the zero-pressure density of the polymorph under consideration, given in Table 1. It should be noted that (12) was obtained for perfect crystals, and although it may be applicable to pure Mg₂SiO₄, it would probably not be applicable to a Fo₈₅Fa₁₅ solid solution. For such a solution the Mooney and Steg calculations for a solid with strong point defect scattering may be more appropriate. However, in this paper the equation for a perfect crystal is used, with the empirical correction described in assumption 1 to obtain an estimate of the value of conductivity for the solid solution. Assumed values of γ_0 and γ_0' are given in Table 1; the behavior of γ is shown in Figure 1.

6. In the postspinel region the thermal conductivity is assumed to be that of a mixture of periclase and stishovite. The conductivity of the mixture can be estimated from the conductivity of the components by the use of Hashin and Shtrikman's [1962] formulas which give the upper and lower bounds, K_U^* and K_L^* of the conductivity of two-phase mixtures in terms of the phase conductivities and the volume fractions present:

$$K_U^* = K_{st} + (1 - v) \left(\frac{1}{K_{MgO} - K_{st}} + \frac{v}{3K_{st}} \right)^{-1} \quad (23)$$

$$K_L^* = K_{MgO} + v \left(\frac{1}{K_{st} - K_{MgO}} + \frac{1 - v}{3K_{MgO}} \right)^{-1}$$

TABLE 2. Physical Parameters for Model Mantle

Depth, km	P , kbar	Phase	T , °K	ρ , g/cm ³	$\Delta((\rho_{00}/\rho) - 1)$
0	0	crust	300	2.6	0
100	30	olivine	1400	3.3	-0.03
200	70		2000	3.5	-0.09
300	100		2400	3.7	-0.13
400	170		2600	3.8	-0.15
500	180	spinel	2800	4.0	-0.11
600	240		3000	4.2	-0.15
700	280		3250	4.4	-0.19
800	320	2MgO-SiO ₂	3300	4.5	-0.15*
1000	400		3500	4.6	-0.17
1500	580		3800	4.8	-0.20
2000	930		4000	5.1	-0.25
2500	1160		4200	5.4	-0.29
2865	1350		4400	5.6	-0.31

*Refers to a weighted average density for the oxide mixture, $\rho_{00} = 3.84$ g/cm³.

where v is the volume fraction of stishovite. It is assumed that the arithmetic mean of K_U^* and K_L^* gives the mean value of conductivity for the mixture:

$$\bar{K} = \frac{1}{2}(K_U^* + K_L^*) \quad (24)$$

Horai [1971] has discussed the validity of this approach for mineral-water mixtures. It is assumed that the volume fraction of stishovite is 0.4 throughout the lower mantle.

The results of the calculations are shown in Figure 2. The general form of the mantle lattice conductivity curve is in agreement with the curve predicted by Lubimova [1967] and agrees generally quite well with the lattice conductivity curve calculated by Schatz [1971, p. 32], shown in Figure 2 for comparison. The differences which occur arise mainly from different assumptions about the pressure and temperature dependences. The model leads to the following conclusions:

1. Increases in conductivity with pressure due to lattice conduction processes in the mantle are less than 2%/kbar or 0.7%/km. Under conditions of normal geothermal gradient in the crust and upper mantle (above 700 km) the increase in lattice conductivity due to the pressure effect is substantially less than the decrease due to the temperature effect.

2. In the earth's crust the pressure effect is important only under optimal circumstances, e.g., in a downgoing lithospheric plate, in which the temperature at 100-km depth (35-kbar pressure) might be as low as 500°C [Griggs, 1972]. Under such conditions the increase in conductivity due to pressure may be of the order of 50–70%. The conductivity may decrease by a factor of 2 or 3 owing to the temperature dependence [Clark, 1966, p. 459]. Therefore in geological situations where material is at high pressures and is relatively cold (so that radiative heat transfer is negligible), the increase in thermal conductivity due to pressure may be significant but will not dominate the decrease due to the temperature effect.

3. A minimum value of lattice conductivity is attained in the region of the olivine-spinel phase change.

4. The lattice conductivity may increase by a factor of 3 at

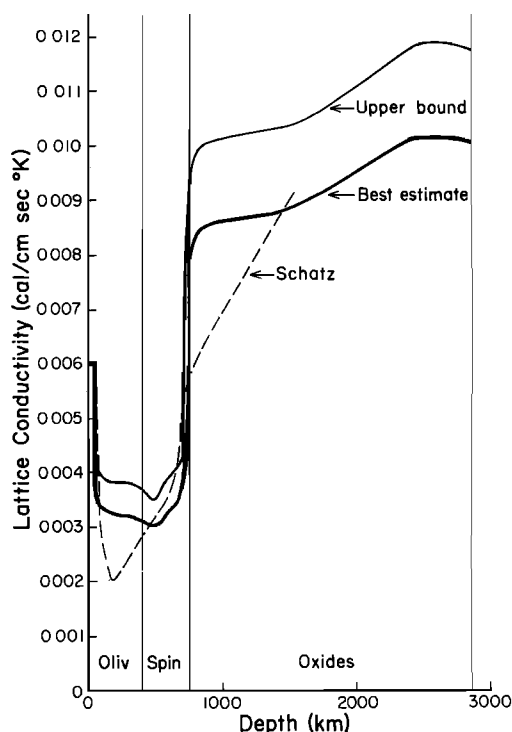


Fig. 2. Thermal conductivity for model mantle.

the depth of the spinel-postspinel phase change owing to the high conductivity of the dense oxide phases.

5. The lattice contribution to the thermal conductivity at the mantle-core boundary is ~ 0.01 cal/cm s °K.

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