

# Laws of Population Growth

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## Abstract

An important issue in the study of cities is defining a metropolitan area, as different definitions affect the statistical distribution of urban activity. A commonly employed method of defining a metropolitan area is the Metropolitan Statistical Areas (MSA), based on rules attempting to capture the notion of city as a functional economic region, and is performed using experience. The MSA is time-consuming and is typically constructed only for a subset (few hundreds) of the most highly populated cities. Here, we introduce a new method to designate metropolitan areas, denoted “City Clustering Algorithm” (CCA). The CCA is based on spatial distributions of the population at a fine geographic scale, defining a city beyond the scope of its administrative boundaries. We use the CCA to examine Gibrat’s law of proportional growth, which postulates that the mean and standard deviation of the growth rate of cities are constant, independent of city size. We find that the mean growth rate of a cluster utilizing CCA exhibits deviations from Gibrat’s law, and that the standard deviation decreases as a power-law with respect to the city size. The CCA allows for the study of the underlying process leading to these deviations, shown to arise from the existence of long-range spatial correlations in the population growth. These results have socio-political implications, such as those pertaining to the location of new economic development in cities of varied size.