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Layered Random Beamforming OFDMA with Fair Scheduling Algorithms

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Abstract—**This paper presents performance analysis of Layered Random Beamforming (LRB) - MIMO-OFDMA employing various resource scheduling algorithms with feedback reduction in a realistic outdoor environment. LRB enables the exploitation of spatial multi-user diversity gain, spatial multiplexing capacity gain and layer spatial multi-user diversity gain, which is achieved by enabling the multiplex of data transmitted simultaneously to different destinations. Unlike a conventional beamforming system, an LRB system only requires an effective signal to interference and noise ratio (ESINR) based numerical data rate as feedback from every spatial layer of the MIMO channel and thus has potentially lower feedback requirements than a system which requires feedback of more detailed channel information. By combining the LRB technique with OFDMA, LRB-OFDMA can achieve an additional spectral multi-user diversity gain compared to the single carrier LRB system. Various scheduling algorithms are proposed for LRB-OFDMA and they show a trade-off between maintaining fairness and minimising delay. The performance of LRB-OFDMA is evaluated using some well-established statistical channel models as well as a propagation modeling tool, which represents a realistic outdoor environment.**

***Index Terms*—Layered Random Beamforming, MIMO- OFDMA, Multi-user Diversity, Scheduling Algorithm.**

I. INTRODUCTION

RESEARCH on future generation cellular systems has focused on supporting multi-user transmission and providing higher data rates and spectral efficiency. An OFDMA system is one of the most promising PHY and multiple access candidates for future communication systems [1]. For example, the WiMAX standard (802.16) uses OFDMA as the air interface [2] and the Third Generation Partnership Project (3GPP) Long Term Evolution (LTE) has already assumed that the downlink of the air interface would be OFDMA based [3]. Performance can be further improved by employing MIMO. Eigenbeamforming [4][5] is a capacity achieving transmission scheme that utilises singular value decomposition (SVD) and requires full channel state information (CSI) at the transmitter. However, this feedback amount increases with the product of the numbers of transmit and receive antennas.

In a multi-user environment, there is likely to be at least one mobile station (MS) whose channel is near its peak at one time and/or frequency, provided different MSs experience independent fading channels. Application of a randomly generated beamforming pattern at the transmitter to achieve Opportunistic Beamforming is proposed in [6] and it can effectively exploit multi-user diversity in combination with transmit beamforming to attain the coherent beamforming capacity and only requires the feedback of signal to noise ratio (SNR) (no spatial information is required). By combining the opportunistic beamforming concept and SVD technique, [7] and [8] extend this theory to a single carrier MIMO system and develop the Random Beamforming (RB) and Layered Random Beamforming (LRB) techniques respectively. RB is capable of achieving multi-user diversity gain and spatial multiplexing gain and supports one MS transmission at any time/frequency. LRB with an MMSE linear receiver is capable of achieving further multiple access and an additional layer spatial multi-user diversity gain by allowing different spatial layers to be separated and allocated to different MSs simultaneously. The feedback of an LRB system is the numerical data calculated on the basis of an ESINR metric [8], which indicates not only the eigenvalues of the MIMO channels and the mismatch between the random precoding matrix and the unitary matrix of the actual MIMO channels, but also considers the self-interference caused by other spatial layers. Our previous work proposes a novel LRB-OFDMA system and it is shown to achieve an additional spectral multi-user diversity gain [9][10] compared to the single-carrier case. However, the sub-carrier allocation for a LRB-OFDMA system considered in the previous work is based on a greedy algorithm and the fairness of resource allocation is not considered. In addition, LRB-OFDMA systems are only verified using either numerical analysis or simulation in statistical channels in these papers.

In this paper, various resource scheduling algorithms are proposed for an LRB-OFDMA system. Simulation results are generated for performance comparison among different scheduling algorithms using statistical channel models. The LRB-OFDMA PHY simulator and outdoor propagation modelling tool are also combined in order to evaluate the coverage and throughput of a 2x2 MIMO LRB-OFDMA system in realistic multi-user environments.

This paper is organized as follows. Section II describes the PHY model of LRB-OFDMA. In Section III, the greedy algorithm, proportional fair algorithm and fair cluster algorithm are proposed for resource allocation of an LRB-OFDMA system. In Section IV, simulation performance of an LRB-OFDMA system employing various scheduling algorithms using both statistical channel models and realistic outdoor channel are presented and their performances are compared in terms of throughput and fairness. Section V concludes the paper.

II. PHYSICAL LAYER MODEL

For LRB-OFDMA, a unitary matrix V_r is generated from the random channel matrix H_r and it is applied to the subcarriers of the OFDMA signal on a cluster basis (a cluster is considered to consist of an integer number of sub-carriers adjacent in frequency). Different V_r is generated for different clusters of sub-carriers. The received signal after FFT and guard interval removal becomes:

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$$\begin{aligned} Y_k^s &= H_k^s V_r^s X_t^s + N_k^s \\ &= U_k^s D_k^s \left(V_k^s\right)^H V_r^s X_t^s + N_k^s \end{aligned} \tag{1}$$

where a subscript k denotes a MS index, s denotes a subcarrier index and H_k^s is a matrix containing MS k 's frequency responses of the channels between N_t transmit and N_r receive antennas at subcarrier s. D_k^s is a diagonal matrix including all the singular values of H_k^s, and U_k^s and V_k^s are the unitary matrices obtained by applying an SVD to H_k^s. X_t^s denotes an $N_t \times 1$ matrix containing the transmit signals at subcarrier s at the basestation (BS) and N_k represents the additive complex Gaussian noise with zero mean and variance σ_k^2. The MMSE filter computed by MS k for subcarrier s is:

$$G_k^s = \left(\left(H_k^s V_r^s\right)^H \left(H_k^s V_r^s\right) + SNR^{-1} I\right)^{-1} \left(H_k^s V_r^s\right)^H \tag{2}$$

The received signal is multiplied by G_k^s, and it becomes:

$$G_k^s Y_k^s = G_k^s H_k^s V_r^s X_t^s + G_k^s N_k^s \tag{3}$$

For a 2x2 MIMO system, the MIMO channels have two subspaces that can be considered as 2 data streams transmitting through 2 parallel sub-channels. For data stream q at every sub-carrier, the MS k computes the SINR (the subcarrier index is omitted):

$$ESINR_k^q = \frac{E_s}{\left|\left(B_k\right)_{qq}\right| \sigma_k^2} - 1 \tag{4}$$

where $B_k = \left(\left(H_k V_r\right)^H \left(H_k V_r\right) + SNR^{-1} I\right)^{-1}$ and E_s denotes the average symbol energy and $|\cdot|_{qj}$ indicates the element located in row q and column j. In an OFDMA system, feedback from every sub-carrier will be required by the BS. To reduce the feedback, every MS calculates the average data rate across all subcarriers in each layer-cluster (a group of adjacent sub-carriers at one spatial layer of one cluster) and sends it to the BS through the feedback channel. For layer-cluster (q,c) (the q th spatial layer of cluster c), if the index of the starting subcarrier is n and finishing subcarrier is m, the MS k calculates the average data rate on a layer-cluster basis by averaging the data rates of the sub-carriers in the frequency domain, as:

$$R_{k,c}^q = \frac{1}{m-n} \sum_{s=n}^{m} \log_2\left(1 + ESINR_{k,s}^q\right) \tag{5}$$

The BS allocates each layer-cluster to an MS according to whichever scheduling algorithm is employed.

A 2x2 MIMO architecture is considered in this paper but the analysis is readily extendible to higher MIMO orders.

III. RESOURCE SCHEDULING ALGORITHMS FOR LRB-OFDMA

By utilising the frequency response correlation of adjacent subcarriers, previous work [10] suggests the cluster size S_c can be appropriately chosen so that the feedback can be reduced without undue degradation in throughput performance. The BS then allocates each layer-cluster to one of the MSs based on one of the following scheduling algorithms.

A. Best MS Selection Criterion (Greedy Algorithm)

A greedy algorithm (GA) can be employed for an LRB-OFDMA system in order to maximise the overall system throughput: For layer-cluster (q,c), the MS k^* with the highest $R_{k^*,c}^q(t) = \max\left\{R_{1,c}^q(t), \cdots, R_{k,c}^q(t), \cdots\right\}$ is scheduled for transmission.

B. Proportional Fair Algorithm (PFA)

The drawback of GA is that the scheduler always picks the MS with the highest data rate and gives all the system resources to statistically stronger MS. This is highly undesirable especially if continuous transmission (e.g voice, video transmission) is required by any weak MS.

A proportional fair algorithm (PFA) developed for a single-carrier system in [6] gives approximately the same number of time slots to all MSs in the long term and assigns the transmission to each MS when its channel condition is at its best. PFA can be applied to an LRB-OFDMA system on a per layer-cluster basis (per spatial layer per cluster basis). This approach is similar to the scheme 1 proposed in [11] for an OFDMA system. For layer-cluster (q,c), at time t, MS k sends the average data rate $R_{k,c}^q(t)$ to the BS. The PFA keeps track of the average throughput $T_{k,c}^q(t)$ in a past window of length t_c and transmits to the MS k^* with the largest $R_{k,c}^q(t) / T_{k,c}^q(t)$ at layer-cluster (q,c) at time t. The average throughput $T_{k,c}^q(t)$ is updated as follows [5]:

$$T_{k,c}^q(t+1) = \begin{cases} \left(1 - \dfrac{1}{t_c}\right) T_{k,c}^q(t) + \dfrac{1}{t_c} R_{k,c}^q(t), & k = k^* \\ \left(1 - \dfrac{1}{t_c}\right) T_{k,c}^q(t) & k \neq k^* \end{cases} \tag{6}$$

This scheme is able to take advantage of the frequency selectivity as much as possible. However, since the scheduling algorithm works on a per layer-cluster basis, for the same MS at every time instant, the feedback from different layer-clusters are not considered together, and hence resource allocation may be unfair in the short term.

C. Fair Cluster Scheduling Algorithm (FCA)

The proposed fair cluster scheduling algorithm (FCA) for LRB-OFDMA aims to allocate every MS the same number of layer-clusters N_c for transmission in both short and long term.

$$N_c = \left\lfloor \frac{N_s \times Q}{S_c \times N_{user}} \right\rfloor \tag{7}$$

where N_{user} is the number of MSs in the environment, N_s is the total number of data sub-carriers per OFDM symbol, $Q = \min(N_r, N_t)$ is the total number of spatial layers, S_c is the cluster size and $\lfloor . \rfloor$ is the floor function.

There are a few metrics defined for the FCA. $P_k(t)$ is the ratio of the scheduled data rate $a_k(t)$ based on FCA over the best data rate that MS k can possibly achieve $b_k(t)$ at time t under the constraint that every MS is allocated the same number of layer-clusters. $C_k(t)$ is the number of layer-clusters allocated to MS k.

Fig. 1 gives an example of the layer-cluster allocation process of a 2-MS system with 4 layer-clusters (2 spatial layers, 2 clusters) employing FCA.

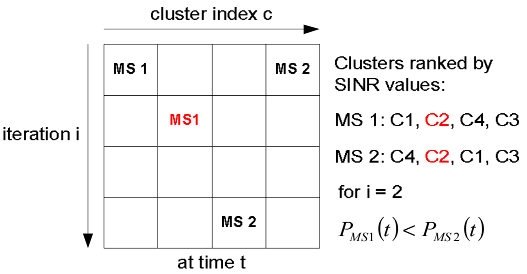

Fig. 1 Example of a 2-MS System with 4 Layer-clusters Employing FCA

Initialization: $P_k(0)=0$, $C_k(0)=0$, $a_k(0)=0$ for MSs $k=1,\ldots K$.

Step 1 (Start): For every MS, all the layer-clusters are ranked in a descending order with the computed rate $R_{k,c}^{q}$, $\{\widetilde{R}_{k,1} > \cdots \widetilde{R}_{k,i} > \cdots \widetilde{R}_{k,QC}\}$, where $\widetilde{R}_{k,i}^{q}$ is the data rate following the new index i after ranking. $b_k(t)$ is the sum of the data rate $\widetilde{R}_{k,i}$ of the best N_c clusters $b_k(t)=\sum_{i=1}^{N_c}\widetilde{R}_{k,i}$.

Step 2: For $i=1$ to QC

```
    For MS $k=1,\cdots,K$
    {
    If $C_{k^*}(t) < N_c$ (MS $k^*$ has not been allocated to $N_c$ layer-clusters)
    Find the corresponding layer-cluster $(q,c)$ for $k^*$ at this iteration $i$ ($R_{k^*,c}^{q}=\widetilde{R}_{k^*,i}$)
      - If $(q,c)$ is unoccupied, allocate it to $k^*$
      - If $(q,c)$ has been allocated to another MS $k$ ($R_{k,c}^{q}=\widetilde{R}_{k,i}$, i.e. more than one MS select $(q,c)$ as its $i$th best layer-cluster including MS $k^*$), priority is given to the MS
          - With the lowest $P_k(t)$
          - For MSs with the same $P_k(t)$, allocate $(q,c)$ to the MS with the highest $R_{k,c}^{q}$
    Update $C_{k^*}$ and $a_{k^*}(t)$ ($a_{k^*}(t)=a_{k^*}(t)+R_{k^*,c}^{q}$)
    }
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Step 3: If there are any unoccupied layer-cluster

Allocate the layer-cluster to MS k^* having the highest data rate ($R_{k^*,c}^{q} \geq R_{k,c}^{q}$)

Step 4 (Allocation): The BS transmits signals to MSs based on the scheduling result. At BS, metric $P_k(t^*+1)$ of MS k at time t^*+1 is computed as:

$$P_k(t^*+1)=\sum_{t=1}^{t^*}a_k(t)\Big/\sum_{t=1}^{t^*}b_k(t) \qquad (8)$$

The metric shows the ratio of the average estimated rate that actually transmitted over the average ideal rate for the best N_c layer-clusters during the time interval t^* for MS k.

IV. PERFORMANCE ANALYSIS

The key parameters used in the simulation of the system [12] are shown in table I. 6 PHY operating modes with different combinations of modulation scheme and coding rate are considered here [12], as shown in table II, which also shows the maximum achievable data rate for the overall system.

TABLE I Parameters for the Proposed OFDMA System

Operating Frequency	5 GHz
Bandwidth	100 MHz
FFT Size	1024
Useful Sub-carriers	768
Guard Interval Length	176
Useful Symbol Duration	10.24 µs
Total Symbol Duration	12.00 µs
Channel Coding	Punctured 1/2 rate convolutional code, constraint length 7, $\{133,171\}_{octal}$

TABLE II Transmission Modes and Data Rates

Mode	Modulation	Coding Rate	Coded Bits (subcarrier)	Max. Data Rate (R) Overall
1	BPSK	½	1	64 Mbps
2	QPSK	½	2	128 Mbps
3	QPSK	¾	2	192 Mbps
4	16 QAM	½	4	256 Mbps
5	16QAM	¾	4	384 Mbps
6	64 QAM	¾	6	576 Mbps

An uncorrelated MIMO implementation of the statistical channel model E of the ETSI BRAN channel models [13] is adopted for system simulation in this paper. Channel model E have a sampling period of $T_s=10ns$ and the rms delay spread τ_{rms} of 250ns.

The proposed system is also evaluated using 2-transmit, 2-receive antenna MIMO channel data from deterministic, ray traced simulated channel models. The model uses a site-specific multi-element ray tracing model that is capable of supporting a wide range of propagation mechanisms [14]. The ray tracing scenario considers an outdoor, urban environment in central Bristol, UK, (illustrated in Figure 3) and consists of a BS located on a building top (23m above ground level) and a grid of possible MS locations (1.7m above ground). The BS uses 2 patch elements (20 λ spacing). All the MSs use 2 monopole elements (0.6 λ spacing). The transmit power is 25dBm.

A. Performance of LRB-OFDMA Employing GA, PFA and FCA using Statistical Channel Models

Three scheduling algorithms - GA, PFA and FCA - are considered for resource allocation of an LRB-OFDMA system and are evaluated using statistical MIMO channel E with 12 MSs in the environment. Since statistical channels for different MSs have the same fading characteristics, all the proposed scheduling algorithms lead to a fair resource allocation in the long term. Based on the feedback clustering feedback reduction scheme [10], the cluster size S_c is chosen to be 8. The total numerical data rate of MS k at time t is the sum of the numerical data rate of all the sub-carriers at all spatial layers allocated to MS k.

The fairness performance can be evaluated by calculating the Coefficient of Variation (CoV) of the numerical data rate across MSs. CoV is the ratio of the standard deviation to mean and is commonly used as metric to show fairness with small variation

[15] when MSs have the similar received SNRs. Table III shows the CoV of the data rate across different MSs when the LRB-OFDMA system employs different scheduling algorithms. A low CoV indicates a better fairness performance. GA has the worst fairness performance and PFA improves the fairness in resource allocation as the window length decreases. FCA offers the best fairness performance.

TABLE III: Overall System Throughput and CoV of Data Rate of LRB-OFDMA Employing Different Scheduling Algorithms Averaged Over 1000 Time Slots (Eb/No=12dB)

Algorithms	CoV Across Different MSs	System Throughput
GA	0.4553	561.02 Mbps
PFA (wl=100)	0.4340	537.70 Mbps
PFA (wl=10)	0.3164	380.74 Mbps
FCA	0.0478	536.54 Mbps

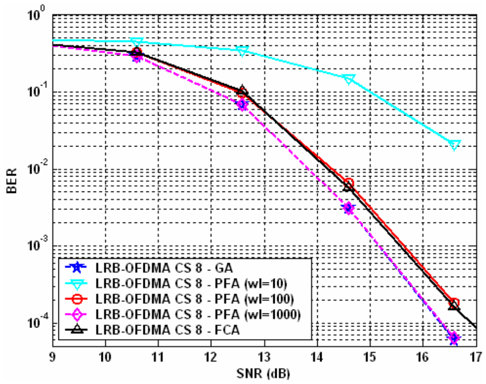

Fig. 2 BER Performance of LRB-OFDMA Employing Different Scheduling Algorithms in a 12-MS Environment (Cluster Size=8)

Fig. 2 shows the BER simulation performance of a mode 6 LRB-OFDMA system employing GA, FCA and PFA with different window lengths (10, 100 and 1000). A simple approximation of the link throughput when retransmission is employed is given by: Data Throughput = R(1-PER), where R and PER are the maximum data rate of all the assigned sub-carriers and packet error rate for a specific mode of a MIMO scheme respectively. Optimal link adaptation selects the transmission mode that achieves the highest data throughput. Tables III also shows the average system link throughput at $Eb/No = 12dB$. Although the GA has the worst fairness performance, it achieves the best overall system BER performance and throughput since it always allocates every layer-cluster to the best MS. PFA improves the system fairness as the window length decreases, but the overall system throughput and BER performance degrades at the same time. While other schemes operate in mode 6, PFA (window length=10) fails to operate in mode 6 and therefore switches to mode 5. The PFA system shows a trade-off between improving fairness and increasing overall throughput. Although both PFA and FCA consider the MS scheduling history, PFA is applied to different layer-cluster independently. Hence, even with a short window length, PFA only allows fair allocation in the long term but not in the short term or across the frequency domain. This can be improved by adopting FCA, which forces MSs to be allocated the same number of layer-clusters at every time instant. FCA is developed to maintain a good throughput performance as well as both short term and long term fairness. The overall BER performance of FCA is very close to PFA with a high window length at 100 and FCA distributes the resources more fairly than PFA with a window length of 10.

B. Performance of LRB-OFDMA Employing GA, PFA and FCA in a Realistic Channel Environment

Using the ray tracing model, complex MIMO channel impulse response data is derived on a point-to-multipoint basis for the entire area under consideration. From this data, the received SNR is determined wherein areas with brighter colour experience higher received SNRs) shown in Fig. 3. A 12-MSs outdoor environment is considered for simulation and each MS moves along a pre-defined and independent route as shown in Fig. 3. Since the average rms delay spread of the selected area is around 250ns, which is similar to the statistical channel model E, the cluster size is chosen to be 8 based on the frequency clustering feedback reduction analysis in [10].

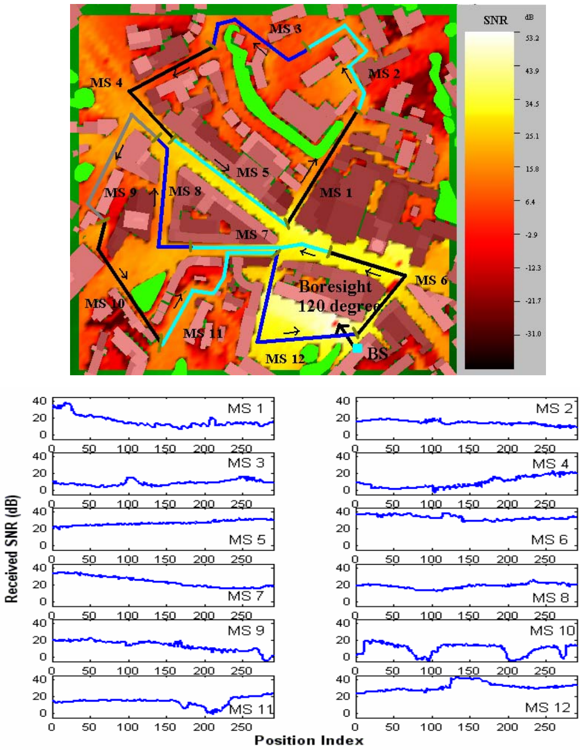

Fig. 3 Received SNR for 12 MSs along Their Routes (Transmit Power=25dBm)

Error performance is obtained by simulating LRB-OFDMA using the channels generated by the ray tracing software. The PER for every MS at every position is calculated based on the average of 1000 PER simulations for the channel generated by the ray tracing model combined with AWGN. The link throughput results of the overall system employing GA, PFA (window length=10) and FCA for the complete route are listed in TABLE IV.

For MSs with different received SNR and fading statistics, the level of fairness of different scheduling algorithms can be demonstrated statistically and the best fairness scheduling algorithm maximises the following metric proposed in [14]:

$$\log\left(\prod_{k=1}^{K} M_k\right) \qquad (9)$$

Where M_k is the throughput of MS k. The average values of this fairness metric for GA, PFA (window length=10) and FCA for the entire route are also listed in table IV.

TABLE IV: Overall System Throughput and Level of Fairness of LRB-OFDMA Employing Different Scheduling Algorithms

Algorithms	Fairness Metric Value	Average Throughput of the Route
GA	9.27	405.17 Mbps
PFA (wl=10)	14.77	263.10 Mbps
FCA	15.32	299.31 Mbps

As expected, the GA achieves the highest average throughput of the route by unfairly allocating most of the sub-carriers to the MSs which are close to the BS and have good channel conditions. PFA (window length=10) improves the fairness significantly compared to GA. However, transmission delay for some MSs may still happen occasionally as a result of the great fluctuation of the throughput. The average throughput of the route degrades significantly compared to the GA and it can be improved by increasing the window length but the fairness of resource allocation among different MSs decreases. By adopting FCA as the scheduling algorithm, LRB-OFDMA enables every MS to have a reasonably good level of throughput instantaneously. Consistent with the statistical channel simulation results, the average throughput of FCA outperforms the system employing PFA with short window length. The difference in average throughput performance of the route between GA and FCA shows that the trade-off between maximising the system throughput and fairness in resource allocation becomes more significant as the difference in channel fading statistics (e.g. received SNR) among different MSs increases. MS 3 is selected from 12 MSs (Fig. 4) to show an example of the individual MS's throughput performance when different scheduling algorithms are employed. Since MS3 is far away from the BS and experiences a medium to low received SNR, with GA, more than 90% of the time MS 3 has no signal transmission. PFA (window length=10) significantly improves the fairness and at more than 90% of the locations, MS 3 achieves 5Mbps, which is doubled when FCA is employed.

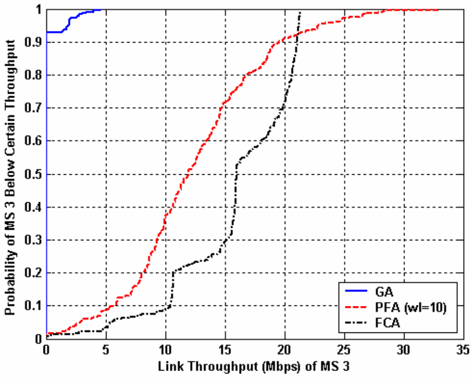

Fig. 4 Probability of MS 3 to be Below a Certain Throughput (Mbps) for Different Scheduling Algorithms

V. CONCLUSIONS

By allocating different spatial layers of MIMO channels to different MSs for transmission, a next generation layered random beamforming OFDMA system, which only requires ESINR based feedback from every spatial layer, would be capable of effectively exploiting spatial multiplexing capacity gain as well as spatial, layer and spectral multi-user diversity gain. A greedy algorithm, a proportional fair algorithm and a fair cluster algorithm considered for LRB-OFDMA are shown to have increasing fairness. The GA is capable of achieving the best possible overall throughput, but may cause severe transmission delays to individual MSs experiencing weak channel conditions. The PFA enables a fair resource allocation in the long term but MSs may still experience severe drops in throughput in the short term. As the window length increases, the overall throughput performance of the PFA approaches that of the GA but becomes less fair. The FCA can achieve a good balance between the overall throughput and both short and long term fairness, but overall throughput may be degraded while maintaining a fair resource allocation as the difference in fading statistics of MSs becomes more significant.

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