

# Leading-power contributions to $B \rightarrow \pi, \rho$ transition form factors

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We calculate the  $B \rightarrow \pi, \rho$  transition form factors in the framework of perturbative QCD to leading powers of  $1/M_B$ ,  $M_B$  being the  $B$  meson mass. We explain the basic principle by discussing the pion electromagnetic form factor. It is shown that the logarithmic and linear singularities occurring at small momentum fractions of light meson distribution amplitudes do not exist in a self-consistent perturbative analysis, which includes  $k_\perp$  and threshold resummations.

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## I. INTRODUCTION

Branching ratios of  $B$  meson two-body nonleptonic decays have been measured by the CLEOIII, BELLE, and BaBar Collaborations [1–4].  $CP$  violations in these modes may be observed in the near future. Cognizant of this point, we have presented some theoretical anticipations for the  $B \rightarrow K\pi$  [5],  $\pi\pi$ ,  $\pi\rho$  [6], and  $KK$  [7] decays in the perturbative QCD (PQCD) framework. In particular, 5–15%  $CP$  violation is expected in the  $B \rightarrow K\pi$  decays. The  $B \rightarrow \pi, \rho$  transition form factors are the integral part of two-body nonleptonic decay amplitudes. In this paper, we shall attempt to convince readers that these form factors in the large recoil region of light mesons are calculable in PQCD. This is where our approach starts to differ from other approaches to exclusive  $B$  meson decays.

According to the PQCD factorization theorem, a form factor is written as the convolution of a hard amplitude with initial-state and final-state hadron distribution amplitudes  $\phi(x)$ , where  $x$  is the momentum fraction associated with one of the partons. It has been pointed out that perturbative evaluation of the pion form factor suffers nonperturbative enhancement from the end-point region with a momentum fraction  $x \rightarrow 0$  [8]. If this is true, the hard amplitude is characterized by a low scale, such that expansion in terms of a large coupling constant  $\alpha_s$  is not reliable. More serious end-point (logarithmic) singularities have been observed in the twist-2 (leading-twist) contribution to the  $B \rightarrow \pi$  transition form factor [9,10]. The singularities even become linear at twist 3 (next-to-leading twist) [11]. Because of these singularities, it was claimed that the  $B \rightarrow \pi$  form factor is dominated by soft dynamics and not calculable in PQCD [12]. We shall argue that this conclusion is false. We shall show that at

the end points, where the above singularities occur, the double logarithms  $\alpha_s \ln^2 x$  should be resummed in order to justify perturbative expansion. The result, called threshold resummation [13,14], leads to strong Sudakov suppression at  $x \rightarrow 0$  [15]. Therefore, the end-point singularities do not exist in a self-consistent PQCD analysis.

In this work, we shall investigate contributions to the  $B \rightarrow \pi$  and  $B \rightarrow \rho$  transition form factors from twist-2 and from two-parton twist-3 distribution amplitudes, and we will argue that both contributions are of leading power in  $1/M_B$ ,  $M_B$  being the  $B$  meson mass. There exist many other higher-twist sources in  $B$  meson decays, whose contributions are indeed down by a power of  $1/M_B$ . These sources include the  $B$  meson and  $b$  quark mass difference  $\bar{\Lambda} = M_B - m_b$ , the light quark masses  $m_u$ ,  $m_d$ , and  $m_s$ , and the light pseudoscalar meson masses  $M_\pi$  and  $M_K$ . Those from three-parton distribution amplitudes are further suppressed by the coupling constant  $\alpha_s$ . Two-parton twist-4 distribution amplitudes do not contribute to the heavy-to-light form factors [16]. All these subleading contributions will be neglected in the current formalism.

In Sec. II, we illustrate the PQCD formalism by studying the pion electromagnetic form factor. We review the reasoning why one might conclude that the form factor is not calculable, and explain why these objections are not justified in QCD.

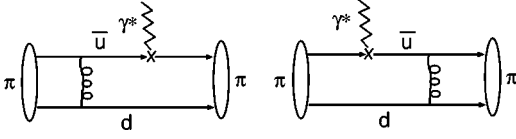
In Secs. III and IV, we derive the  $B$  meson transition form factors. It will be shown that the twist-3 contributions, which seem to be proportional to  $m_0/M_B$  or  $M_\rho/M_B$ , do not vanish in the  $M_B \rightarrow \infty$  limit. Here  $m_0$  and  $M_\rho$  are the chiral symmetry breaking scale and  $\rho$  meson mass, respectively. We record our results of the form factors at large recoil: the  $B \rightarrow \pi$  form factor  $F_+ \sim 0.3$  and the  $B \rightarrow \rho$  form factor  $A_0 \sim 0.4$ .

Meson distribution amplitudes are defined and the Sudakov factor from threshold resummation is derived in the Appendices.

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FIG. 1. Leading-order contribution to  $F_\pi(Q^2)$ .

## II. PQCD APPROACH TO FORM FACTORS

The suggestion that a hadronic form factor is calculable in PQCD was first made in Refs. [17–21]. The rough idea is summarized as follows. One expands the bound-state wave function for a pion in terms of Fock states containing on-shell partons (quarks or gluons) [17],

$$\Psi_M = \psi(q\bar{q}) + \psi(q\bar{q}g) + \psi(q\bar{q}gg) + \psi(q\bar{q}q\bar{q}) + \psi(q\bar{q}q\bar{q}g) + \dots \quad (1)$$

Define a soft function  $\Psi_M(\Lambda)$  at a typical hadronic scale  $\Lambda$  as the initial wave function,

$$\Psi_M(\Lambda) = \psi^\Lambda(q\bar{q}) + \psi^\Lambda(q\bar{q}g) + \psi^\Lambda(q\bar{q}gg) + \psi^\Lambda(q\bar{q}q\bar{q}) + \psi^\Lambda(q\bar{q}q\bar{q}g) + \dots \quad (2)$$

The wave function  $\Psi_M$  can be related to  $\Psi_M(\Lambda)$  via

$$\Psi_M = \Psi_M(\Lambda) + G^\Lambda K \Psi_M(\Lambda), \quad (3)$$

where  $K$  is an irreducible kernel and  $G^\Lambda$  is the Green function involving only hard-loop momenta.

The pion electromagnetic form factor  $F_\pi(Q^2)$  is then expressed as a convolution integral,

$$F_\pi(Q^2) = \int dx_1 dx_2 d^2k_{1\perp} d^2k_{2\perp} \psi^\Lambda(P_1, x_1, \vec{k}_{1\perp}) \times T_H(P_1, x_1, \vec{k}_{1\perp}; P_1 + q, x_2, \vec{k}_{2\perp}) \times \psi^\Lambda(P_1 + q, x_2, \vec{k}_{2\perp}) + \dots, \quad (4)$$

with  $P_1$  being the momentum of the initial-state pion,  $q$  the large momentum transfer, and  $Q^2 = -q^2$ . Here we have written the parton momenta associated with the initial state and final state as  $k_1 = (x_1 Q/2, \vec{k}_{1\perp}, x_1 Q/2)$  and  $k_2 = (x_2 Q/2, \vec{k}_{2\perp}, -x_2 Q/2)$ , respectively, in the notation  $p^\mu = (p^0, p^1, p^2, p^3)$ , and made explicit the dependence of the two-parton wave function  $\psi^\Lambda(q\bar{q}) \equiv \psi^\Lambda(P_1, x_1, \vec{k}_{1\perp})$  on  $P_1$  and  $k_1$ . The first term in Eq. (4) contains leading contributions, and ellipses represent those from higher Fock states, which are down by powers of  $1/Q^2$  in the light-cone gauge and by powers of  $\alpha_s$ . The leading diagrams are displayed in Fig. 1. It can be shown that the large momentum transfer  $Q^2$  flows through the hard amplitude  $T_H$ , and that all nonperturbative dynamics goes into wave functions. One can therefore compute  $T_H$  perturbatively.

However, it was pointed out that the above argument has a crucial problem [8]: the diagrams in Fig. 1 may be infrared-divergent, because an important contribution to the form factor comes from the region where the exchanged glu-

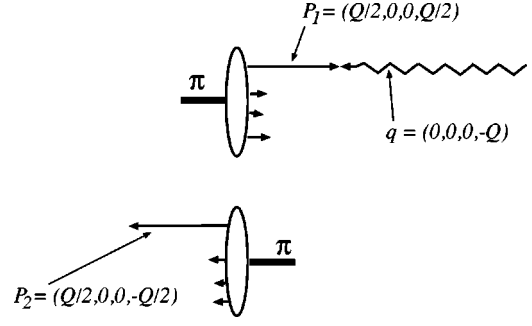


FIG. 2. Feynman's viewpoint of the dominant contribution to the pion electromagnetic form factor.

ons are soft. PQCD is therefore not applicable. Below we shall examine this difficulty in more detail. According to Eq. (4), the first diagram in Fig. 1 gives

$$\begin{aligned} & \langle \pi(P_2) | J_\mu(0) | \pi(P_1) \rangle \\ &= g^2 C_F N_c \int dx_1 dx_2 d^2k_{1\perp} d^2k_{2\perp} \\ & \times \frac{dz^- d^2z_\perp}{(2\pi)^3} \frac{dy^+ d^2y_\perp}{(2\pi)^3} \\ & \times e^{-ik_2 \cdot y} \langle \pi(P_2) | \bar{d}_\gamma(y) u_\beta(0) | 0 \rangle \\ & \times e^{ik_1 \cdot z} \langle 0 | \bar{u}_\alpha(0) d_\delta(z) | \pi(P_1) \rangle T_{H\mu}^{\gamma\beta;\alpha\delta}, \quad (5) \end{aligned}$$

with the color factor  $C_F = \frac{4}{3}$ , the number of colors  $N_c = 3$ , and the hard amplitude

$$T_{H\mu}^{\gamma\beta;\alpha\delta} = [\gamma_\sigma]^{\gamma\delta} \frac{1}{(k_2 - k_1)^2} \left[ \gamma^\sigma \frac{k_2 - P_1}{(P_1 - k_2)^2} \gamma_\mu \right]^{\alpha\beta}. \quad (6)$$

Write  $(k_2 - k_1)^2 \sim -x_1 x_2 Q^2 - |\vec{k}_{1\perp} - \vec{k}_{2\perp}|^2$ . If we ignore  $|\vec{k}_{1\perp} - \vec{k}_{2\perp}|^2$ , it has been shown that the integral in Eq. (5) is dominated by contributions from the end-point regions with  $x_1, x_2 \rightarrow 0$ . If the pion wave function does not vanish at  $x \rightarrow 0$ , the integral will be even infrared-divergent. If we somehow regulate the infrared singularity by an appropriate choice of the wave function, the running coupling constant  $\alpha_s(x_1 x_2 Q^2)$  evaluated at the hard gluon momentum  $x_1 x_2 Q^2$  is still too large to make sense out of the perturbative expansion.

### A. Feynman's picture of a form factor

The above end-point singularity corresponds to the picture of the pion form factor Feynman had in mind. In the so-called brick-wall frame, the initial-state pion with momentum  $P_1 = (Q/2, 0, 0, Q/2)$  is struck by a spacelike current of momentum  $q = (0, 0, 0, -Q)$ , and turns around with momentum  $P_2 = (Q/2, 0, 0, -Q/2)$  as shown in Fig. 2. Feynman pointed out that the major contribution to the form factor comes from the region where one of the partons carries the full pion momentum. The remaining partons, being very small, do not know in which direction they are moving. The resulting configuration is essentially identical to the initial-

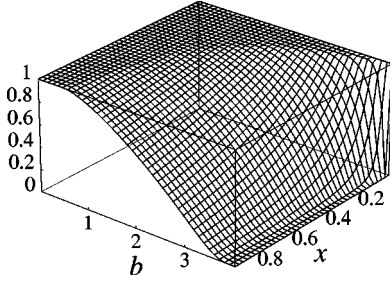


FIG. 3. The Sudakov factor  $\exp[-S(x, b, P_1)]$ . Note that its value is very small in the region  $b \sim b_{\max} = 1/\Lambda_{\text{QCD}}$ , with the QCD scale  $\Lambda_{\text{QCD}} = 250$  MeV.

state pion except that the momentum of the fast parton is reversed. Hence, Feynman claimed that the  $Q^2$  dependence of the pion form factor is related to the probability of finding a single parton carrying all the pion momentum. Feynman's picture is consistent with the statement that the form factor is dominated by the singular part of Eq. (6). Because it is singular, we cannot compute the form factor.

We argue that Feynman's picture of the pion form factor is false. Consider a QED example. When an electron undergoes hard scattering, it cannot help but emit infinitely many photons in the direction of the electron momentum. As a consequence, the elastic scattering cross section  $d/d\Omega(e^+e^- \rightarrow e^+e^-)$  at finite angle vanishes at high energy, implying that the probability for the final  $e^+e^-$  state being accompanied by no photons diminishes. In other words, the final state must be accompanied by many photons. In the QCD case of the pion form factor, when a quark inside the pion gets hit by a current, the final state will contain many gluons unless the spectator quark is nearby to shield the color charge. When one of the quarks carries all the momentum, the rest of the pion cannot shield the color charge of the fast quark, and many gluons will be emitted in arbitrary directions during the hard scattering. Thus, the final configuration ending up as a single pion is extremely unlikely. This is the so-called Sudakov suppression on exclusive processes at kinematic end points. Therefore, the contribution from Fig. 2 is negligible, and the end-point singularity does not exist.

In the above argument, we have ignored the term  $|\vec{k}_{1\perp} - \vec{k}_{2\perp}|^2$  in Eq. (5). Small  $k_{\perp}$  in momentum space corresponds to the large transverse distance  $b$  of two valence quarks. The color charge of the quark, which is struck by the current, is not shielded in this large  $b$  region, and will emit many gluons. The probability for having a single pion in the final state is then vanishingly small. That is, this configuration cannot contribute to the form factor. Hence, the momentum space with  $k_{\perp} \rightarrow 0$ , where the end-point singularity occurs, is also Sudakov suppressed [22]. The typical behavior of the Sudakov factor  $\exp[-S(x, b, P_1)]$ ,  $x = 1 - x_1$ , which is associated with the struck quark, is shown in Fig. 3. We observe that the Sudakov factor decreases fast at large  $b$  for  $x \sim 1$  ( $x_1 \sim 0$ ), which corresponds precisely to the end-point region in Eq. (6). In conclusion, the end-point singularity is absent, and the major contribution to Fig. 1 comes from the region with hard gluon exchanges.

### B. Twist-3 contributions

As derived in Appendix A, a light-cone pion distribution amplitude is written as

$$\begin{aligned} & \langle \pi^-(P) | \bar{d}_{\gamma}(y) u_{\beta}(0) | 0 \rangle \\ &= -\frac{i}{\sqrt{2N_c}} \int_0^1 dx e^{ixP \cdot y} \{ [\gamma_5 \mathbf{P}]_{\beta\gamma} \phi_{\pi}(x) + [\gamma_5]_{\beta\gamma} m_0 \phi_{\pi}^p(x) \\ & \quad + m_0 [\gamma_5 (\not{h}_+ \not{h}_- - 1)]_{\beta\gamma} \phi_{\pi}^t(x) \}, \end{aligned} \quad (7)$$

where  $P = (P^+, 0, 0_{\perp})$  is the pion momentum, the lightlike vector  $z = (0, z^-, 0_{\perp})$  is the coordinate of the  $d$  quark. The dimensionless vector  $n_+ = (1, 0, 0_{\perp})$  is parallel to  $P$ , and  $n_- = (0, 1, 0_{\perp})$  is parallel to  $z$ . Here a four-vector has been expressed in terms of light-cone coordinates,

$$p^{\mu} = \left( \frac{p^0 + p^3}{\sqrt{2}}, \frac{p^0 - p^3}{\sqrt{2}}, p_{\perp} \right). \quad (8)$$

The distribution amplitude  $\phi_{\pi}$  is twist-2, and  $\phi_{\pi}^t$  and  $\phi_{\pi}^p$ , proportional to  $m_0 = M_{\pi}^2 / (m_d + m_u) \sim 1.4$  GeV, where  $m_q$  is the current quark mass of the quark  $q$ , are twist-3. The origin of these terms can be simply understood by means of the field-current identity from chiral symmetry,

$$\bar{d} \gamma_5 u = i m_0 f_{\pi} \pi^+. \quad (9)$$

It is easy to observe that twist-3 contributions are suppressed by a power of  $m_0/Q$ . The asymptotic behaviors of  $\phi_{\pi}$ ,  $\phi_{\pi}^t$ , and  $\phi_{\pi}^p$  are known to be

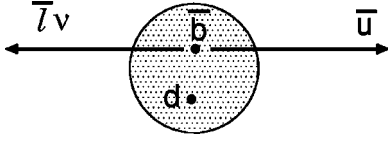
$$\phi_{\pi}(x) \propto x(1-x), \quad \phi_{\pi}^{p,t}(x) \propto 1. \quad (10)$$

As the hard amplitude in Eq. (6) is convoluted with these distribution amplitudes, we find that the twist-2 contribution is finite, while the twist-3 ones are logarithmically divergent without Sudakov suppression. The Sudakov factor then introduces an effective cutoff to the integral at  $x_c \sim \Lambda_{\text{QCD}}/Q$ , and the twist-3 contributions are proportional to  $(m_0/Q) \ln(Q/\Lambda_{\text{QCD}})$ . That is, the power counting is not altered by a logarithmic divergence in the factorization formula. As shown later, the power counting for contributions to the  $B$  meson transition form factors is modified by linear divergences in the factorization formulas. Therefore, the different end-point behavior leads to different power counting rules for the pion form factor and for the  $B$  meson transition form factors.

### III. $B \rightarrow \pi$ TRANSITION FORM FACTORS

In the  $B$  meson rest frame, we define the  $B$  meson momentum  $P_1$  and the pion momentum  $P_2$  in the light-cone coordinates:

$$P_1 = \frac{M_B}{\sqrt{2}} (1, 1, 0_{\perp}), \quad P_2 = \frac{M_B}{\sqrt{2}} (\eta, 0, 0_{\perp}), \quad (11)$$

FIG. 4. Soft contribution to  $F^{B\pi}$ .

with the energy fraction  $\eta$  carried by the pion. The spectator momenta  $k_1$  on the  $B$  meson side and  $k_2$  on the pion side are parametrized as

$$k_1 = \left( 0, x_1 \frac{M_B}{\sqrt{2}}, k_{1\perp} \right), \quad k_2 = \left( x_2 \eta \frac{M_B}{\sqrt{2}}, 0, \vec{k}_{2\perp} \right). \quad (12)$$

Note that the four components of  $k_1$  should be of the same order,  $O(\bar{\Lambda})$ , with  $\bar{\Lambda} \equiv M_B - m_b$ ,  $m_b$  being the  $b$  quark mass. However, since  $k_2$  is mainly in the plus direction with  $k_2^+ \sim O(M_B)$ , the hard amplitudes will not depend on the plus component  $k_1^+$  as explained below. This is the reason we do not show  $k_1^+$  in Eq. (12) explicitly.

Consider the configuration for the semileptonic decay  $B \rightarrow \pi \bar{l} \nu$  depicted in Fig. 4, which corresponds to soft contribution to the  $B \rightarrow \pi$  form factor  $F^{B\pi}$ . The  $\bar{u}$  quark and the lepton pair fly back to back with energy of  $M_B/2$ . The spectator quark  $d$  carries a momentum of  $O(\bar{\Lambda})$ . If this configuration is responsible for the decay, it is impossible to compute  $F^{B\pi}$  using PQCD. However, applying an argument similar to that used for the pion form factor, we know that the  $\bar{u}$  quark recoiling against the lepton pair is bound to emit infinitely many gluons. Thus, Fig. 4 in fact corresponds to the inclusive decay  $B \rightarrow X_{\bar{u}} \bar{l} \nu$ . The probability that the final state in Fig. 4 contains only a single pion is suppressed by the Sudakov form factors. A quantitative estimate of Sudakov suppression of the soft contribution to  $F^{B\pi}$  in the QCD sum rule formalism will be discussed later.

### A. Threshold and $k_{\perp}$ resummations

It has been explained that the internal  $\bar{b}$  quark involved in the hard amplitude becomes on-shell as the momentum fraction  $x$  of the  $d$  quark vanishes [15]. The contributions to the  $B \rightarrow \pi$  form factor  $F^{B\pi}$  are then logarithmically divergent at twist 2 and linearly divergent at twist 3. We argue that as the end-point region is important, the corresponding large double logarithms  $\alpha_s \ln^2 x$  need to be organized into a jet function  $S_i(x)$  as a consequence of threshold resummation [15]. This jet function vanishes as  $x \rightarrow 0, 1$ , and modifies the end-point behavior of meson distribution amplitudes effectively. This modification provides a plausible explanation for the model of the twist-3 pion distribution amplitude proportional to  $x(1-x)$ , which was adopted in [5]. Our numerical study shows that the results of the  $B \rightarrow \pi$  form factor obtained in this work are almost the same as those obtained in [5]. In the following analysis we shall employ the approximate form

$$S_i(x) = \frac{2^{1+2c} \Gamma(3/2+c)}{\sqrt{\pi} \Gamma(1+c)} [x(1-x)]^c, \quad (13)$$

where the parameter  $c \approx 0.3$  comes from the best fit to the next-to-leading-logarithm threshold resummation in moment space. Note that the jet function  $S_i$  is normalized to unity. For details of the derivation, refer to Appendix D.

Similarly, the inclusion of  $k_{\perp}$  regulates the end-point singularities, and large double logarithms  $\alpha_s \ln^2 k_{\perp}$  are produced from higher-order corrections. These double logarithms should also be organized to all orders, leading to  $k_{\perp}$  resummation [23,24]. The resultant Sudakov form factor, whose explicit expression can be found in our previous works [25,26], controls the magnitude of  $k_{\perp}^2$  to be roughly  $O(\bar{\Lambda} M_B)$  by suppressing the region with  $k_{\perp}^2 \sim O(\bar{\Lambda}^2)$ . The coupling constant  $\alpha_s(\bar{\Lambda} M_B)/\pi \sim 0.13$  is then small enough to justify the PQCD evaluation of heavy-to-light form factors [5]. We emphasize that the hard scale for heavy-to-light decays must be  $\bar{\Lambda} M_B$  in order to define a gauge-invariant  $B$  meson distribution amplitude [27]. We shall include the Sudakov factor associated with the light spectator quark of the  $B$  meson. Whether this factor is essential will be determined by the  $B$  meson distribution amplitude. Since the  $B$  meson is dominated by soft dynamics with  $x_1 \sim O(\bar{\Lambda}/M_B)$ , the associated Sudakov effect is minor compared to that from the energetic pion.

We argue that Sudakov resummation of the double logarithms is necessary for a PQCD analysis of the heavy-to-light transition at large recoil. Resummation will be demanded, if the radiative correction satisfies the condition

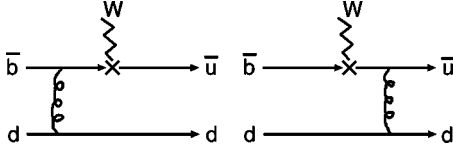
$$\frac{\alpha_s}{\pi} \ln^2 \frac{M_B}{\Lambda} \lesssim 1, \quad (14)$$

where the double logarithms appearing in  $B$  meson decays have been estimated by  $\ln(M_B/\bar{\Lambda})$ . If the above correction were larger than unity, perturbative expansion would fail. If it were much smaller than unity, resummation would not be necessary. It is trivial to confirm that for  $M_B/\bar{\Lambda} \sim 10$  and  $\alpha_s/\pi \sim 0.13$  stated before, the correction is about 0.7, and that Eq. (14) holds. That is, the resummation effects are crucial in our PQCD analysis.

With the possible order of magnitude of  $k_{\perp}^2 \sim O(\bar{\Lambda} M_B)$ , a Taylor expansion of the hard gluon propagator near the end point,

$$\begin{aligned} \frac{1}{(k_1 - k_2)^2} &\approx \frac{-1}{2k_1^- k_2^+ + |\vec{k}_{1\perp} - \vec{k}_{2\perp}|^2} \approx \frac{-1}{x_1 x_2 \eta M_B^2} \\ &+ \frac{|\vec{k}_{1\perp} - \vec{k}_{2\perp}|^2}{(x_1 x_2 \eta M_B^2)^2} + \dots, \end{aligned} \quad (15)$$

is certainly not appropriate. A more reasonable treatment is to keep  $k_{\perp}^2$  in the denominators of internal particle propagators, and to drop  $k_{\perp}^2$  in the numerators, which are power-suppressed compared to other  $O(M_B^2)$  terms. Under this prescription, the Sudakov factor from  $k_{\perp}$  resummation can be introduced into the PQCD factorization theorem without breaking gauge invariance of the hard amplitudes. For the

FIG. 5. Leading-order contribution to  $F^{B\pi}$ .

same reason, the terms proportional to  $k_1 \sim O(\bar{\Lambda})$  in the numerators should be neglected. It is then obvious from Eq. (15) that the hard amplitudes are independent of the component  $k_1^+$ . The  $k_1^+$  dependence of the  $B$  meson wave function can then be integrated out [27], leading to the parameterization in Eq. (12).

Note that the mechanism of threshold and  $k_\perp$  resummations is similar, with the former responsible for suppression in the longitudinal direction and the latter for suppression in the transverse direction. As shown below, both twist-2 and twist-3 contributions are well-behaved after including threshold and  $k_\perp$  resummations. Hence, the contributions to  $F^{B\pi}$  from Fig. 5 dominate in the large recoil region. In this configuration, the  $d$  quark gains a large momentum parallel to the  $\bar{u}$  quark momentum by exchanging a hard gluon with the  $\bar{b}$  or  $\bar{u}$  quark.

### B. Form factors

We compute the  $B \rightarrow \pi$  form factors  $F_+$  and  $F_0$  defined by the following matrix element:

$$\begin{aligned} & \langle \pi(P_2) | \bar{b}(0) \gamma_\mu u(0) | B(P_1) \rangle \\ &= F_+(q^2) \left[ (P_1 + P_2)_\mu - \frac{M_B^2 - M_\pi^2}{q^2} q_\mu \right] \\ &+ F_0(q^2) \frac{M_B^2 - M_\pi^2}{q^2} q_\mu, \end{aligned} \quad (16)$$

where  $q = P_1 - P_2$  is the lepton-pair momentum. Another equivalent definition is

$$\langle \pi(P_2) | \bar{b}(0) \gamma_\mu u(0) | B(P_1) \rangle = f_1(q^2) P_{1\mu} + f_2(q^2) P_{2\mu}, \quad (17)$$

in which the form factors  $f_1$  and  $f_2$  are related to  $F_+$  and  $F_0$  by

$$F_+ = \frac{1}{2}(f_1 + f_2), \quad (18)$$

$$F_0 = \frac{1}{2}f_1 \left( 1 + \frac{q^2}{M_B^2} \right) + \frac{1}{2}f_2 \left( 1 - \frac{q^2}{M_B^2} \right). \quad (19)$$

The factorization formula for the  $B \rightarrow \pi$  form factors is written as

$$\begin{aligned} & \langle \pi(P_2) | \bar{b}(0) \gamma_\mu u(0) | B(P_1) \rangle \\ &= g^2 C_F N_c \int dx_1 dx_2 d^2 k_{1\perp} d^2 k_{2\perp} \frac{dz^+ d^2 z_\perp}{(2\pi)^3} \frac{dy^- d^2 y_\perp}{(2\pi)^3} \\ &\times e^{-ik_2 \cdot y} \langle \pi(P_2) | \bar{d}_\gamma(y) u_\beta(0) | 0 \rangle \\ &\times e^{ik_1 \cdot z} \langle 0 | \bar{b}_\alpha(0) d_\delta(z) | B(P_1) \rangle T_{H\mu}^{\gamma\beta;\alpha\delta}. \end{aligned} \quad (20)$$

The pion distribution amplitude  $\langle \pi | \bar{d}_\gamma(y) u_\beta(0) | 0 \rangle$  has been supplied in Eq. (7), and the  $B$  meson wave function is given by (see Appendix C)

$$\begin{aligned} & \int \frac{dz^+ d^2 z_\perp}{(2\pi)^3} e^{ik_1 \cdot z} \langle 0 | \bar{b}_\alpha(0) d_\delta(z) | B(P_1) \rangle \\ &= -\frac{i}{\sqrt{2N_c}} [(\not{P}_1 + M_B) \gamma_5 \phi_B(k_1)]_{\alpha\delta}. \end{aligned} \quad (21)$$

Employing Eqs. (7) and (21), we derive, from Eq. (20),

$$\begin{aligned} f_1 &= 16\pi M_B^2 C_F r_\pi \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \\ &\times [\phi_\pi^p(x_2) - \phi_\pi^t(x_2)] E(t^{(1)}) h(x_1, x_2, b_1, b_2), \end{aligned} \quad (22)$$

$$\begin{aligned} f_2 &= 16\pi M_B^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \\ &\times \left( \left\{ \phi_\pi(x_2) (1 + x_2 \eta) + 2r_\pi \left[ \left( \frac{1}{\eta} - x_2 \right) \phi_\pi^t(x_2) \right. \right. \right. \\ &\left. \left. \left. - x_2 \phi_\pi^p(x_2) \right] \right\} E(t^{(1)}) h(x_1, x_2, b_1, b_2) \right. \\ &\left. + 2r_\pi \phi_\pi^p E(t^{(2)}) h(x_2, x_1, b_2, b_1) \right), \end{aligned} \quad (23)$$

with the ratio  $r_\pi = m_0 / M_B$  and the evolution factor

$$E(t) = \alpha_s(t) e^{-S_B(t) - S_\pi(t)}. \quad (24)$$

In the above formulas, we have dropped the terms proportional to the momentum fraction  $x_1 \sim O(\bar{\Lambda}/M_B)$  as argued before, which are power-suppressed compared to the leading terms such as  $1 + x_2/\eta$  in the form factor  $f_2$ . The explicit expressions of the Sudakov exponents  $S_B$  and  $S_\pi$  are referred to [25]. The hard function is written as

$$\begin{aligned} h(x_1, x_2, b_1, b_2) &= S_t(x_2) K_0(\sqrt{x_1 x_2 \eta} M_B b_1) \\ &\times [\theta(b_1 - b_2) K_0(\sqrt{x_2 \eta} M_B b_1) \\ &\times I_0(\sqrt{x_2 \eta} M_B b_2) \\ &+ \theta(b_2 - b_1) K_0(\sqrt{x_2 \eta} M_B b_2) \\ &\times I_0(\sqrt{x_2 \eta} M_B b_1)], \end{aligned} \quad (25)$$

where the factor  $S_t$  suppresses the end-point behaviors of the pion distribution amplitudes, especially of the twist-3 ones. The hard scales  $t$  are defined as

$$\begin{aligned} t^{(1)} &= \max(\sqrt{x_2} \eta M_B, 1/b_1, 1/b_2), \\ t^{(2)} &= \max(\sqrt{x_1} \eta M_B, 1/b_1, 1/b_2). \end{aligned} \quad (26)$$

It is obvious that by turning off threshold and  $k_\perp$  resummations with  $\alpha_s$  fixed, Eqs. (22) and (23) are infrared-divergent.

We argue that the two-parton twist-3 distribution amplitudes  $\phi_\pi^{p,t}$ , though proportional to the ratio  $m_0/M_B$ , need to be taken into account. As stated above, the corresponding convolution integrals for the  $B \rightarrow \pi$  form factor are linearly divergent without including Sudakov effects. These integrals, regulated in some way with an effective cutoff  $x_c \sim \bar{\Lambda}/M_B$ , are proportional to the ratio  $M_B/\bar{\Lambda}$ . Combining the two ratios  $m_0/M_B$  and  $M_B/\bar{\Lambda}$ , the twist-3 contributions are in fact not down by a power of  $1/M_B$ :

$$\frac{m_0}{M_B} \int_{x_c}^1 \frac{dx_2}{x_2^2} \sim O\left(\frac{m_0}{\bar{\Lambda}}\right), \quad (27)$$

and should be included in a complete leading-power analysis. We emphasize that the presence of linear divergences modifies the power-counting rules, causing the difference between the  $B$  meson transition form factors and the pion form factor.

Various computing methods have been proposed for the evaluation of the  $B \rightarrow \pi$  transition form factors  $F^{B\pi}(q^2)$  in the literature, such as the lattice technique [28], light-cone QCD sum rules [12,29], and PQCD [25,30]. Obviously, lattice calculations become more difficult in the large recoil region of the light meson. However, this region is the one where PQCD is reliable, indicating that the PQCD and lattice approaches complement each other. This complementation will be explicitly exhibited in Fig. 6 below. In light-cone sum rules, dynamics of the  $B \rightarrow \pi$  form factors have been assumed to be dominated by the large scale of  $O(m_b)$ . This is the reason twist expansion into Fock states in powers of  $1/m_b$  applies to the pion bound state. If this assumption were valid, PQCD should also be applicable to the  $B \rightarrow \pi$  form factors. Besides, large radiative correction to the  $B$  meson vertex, which reaches 35% of the full contribution, or about half of the soft (zeroth-order) contribution, has been noticed. This  $O(\alpha_s)$  correction renders the sum rule for  $f_B F^{B\pi}$ , with  $f_B$  being the  $B$  meson decay constant, quite unstable relative to the variation of input parameters [29,31]. To stabilize the sum rule, one considers another sum rule for  $f_B$  at the same time, which also receives large radiative correction to the  $B$  meson vertex. The two large vertex corrections then cancel in the ratio  $f_B F^{B\pi}/f_B$ . However, the radiative correction to  $f_B$  is then large.

A careful look at the light-cone-sum-rule analyses indicates that the soft contribution is more sensitive to the end-point ( $x \rightarrow 0$ ) behavior of the pion distribution amplitude than the  $O(\alpha_s)$  correction [31]. Hence, if the end-point behavior of the pion distribution amplitude is modified by the

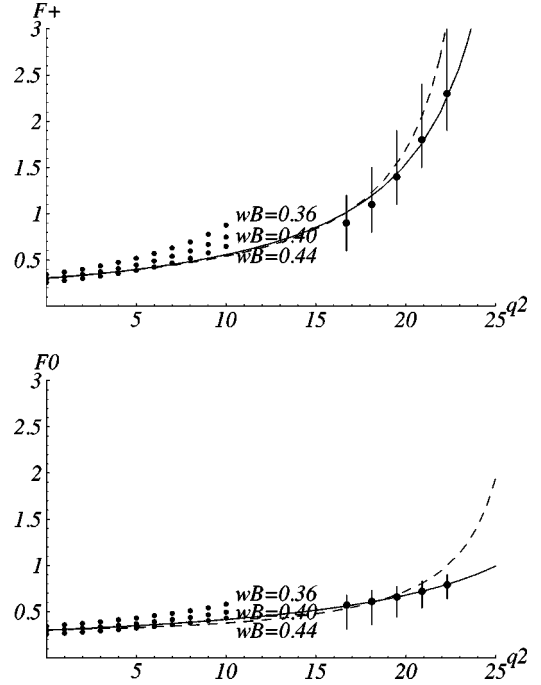


FIG. 6. The  $B \rightarrow \pi$  form factors  $F_+$  and  $F_0$  as functions of  $q^2(\text{GeV}^2)$ . PQCD results for  $\omega_B = 0.36, 0.40$ , and  $0.44$  GeV are shown as dots. The solid lines correspond to fits to the lattice QCD results with errors. The dashed lines come from light-cone sum rules.

Sudakov factor in this work, such that the end-point contribution is not important, perturbative contribution can become dominant. The Sudakov effect on the soft contribution to  $F^{B\pi}(0)$  has been investigated in the QCD sum-rule formalism [32] (without twist expansion for the pion bound state). In this analysis, the soft contribution without Sudakov suppression was estimated to be between 0.15 (corresponding to  $f_B \sim 190$  MeV) and 0.22 (corresponding to  $f_B \sim 130$  MeV). The soft contribution to  $f_B F^{B\pi}$  obtained in [31] is consistent with the above range. It was then shown that the Sudakov effect decreases the soft contribution by a factor 0.4–0.7, depending on infrared cutoffs for loop corrections to the weak decay vertex. Therefore, the soft contribution turns out to be about 0.06–0.15. Compared with the lattice results  $F^{B\pi}(0) \sim 0.3$ , it is reasonable to conclude that the soft contribution amounts to about 30%, which is consistent with the observation made in [25]. It is a fair opinion that the estimate of the soft contribution is more model-dependent than the perturbative one. For example, the perturbative contribution is less sensitive to the pion distribution amplitude or to other input parameters such as the Borel mass in light-cone sum rules [31]. In the PQCD approach, we calculate the perturbative contribution to  $F^{B\pi}$ , which is more model-independent, and show that the result can more or less saturate the value predicted by the lattice technique.

For the  $B$  meson distribution amplitude, we adopt the model

$$\phi_B(x, b) = N_B x^2 (1-x)^2 \exp\left[-\frac{1}{2} \left(\frac{xM_B}{\omega_B}\right)^2 - \frac{\omega_B^2 b^2}{2}\right], \quad (28)$$

TABLE I. Contributions to  $F_+(q^2)$  from the twist-2 and two-parton twist-3 pion distribution amplitudes.

$q^2$ (GeV <sup>2</sup> )	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
twist 2	0.120	0.128	0.138	0.148	0.159	0.172	0.188	0.204	0.223	0.243	0.270
twist 3	0.177	0.193	0.210	0.230	0.253	0.279	0.308	0.344	0.385	0.432	0.487
total	0.297	0.321	0.348	0.378	0.412	0.451	0.496	0.548	0.608	0.675	0.757

with the shape parameter  $\omega_B = 0.4$  GeV [5]. The normalization constant  $N_B$  is related to the decay constant  $f_B = 190$  MeV through the relation

$$\int dx_1 \phi_B(x_1, 0) = \frac{f_B}{2\sqrt{2N_c}}. \quad (29)$$

It is easy to find that Eq. (28) has a maximum at  $x \sim \bar{\Lambda}/M_B$ . We employ the models for the pion [33],

$$\begin{aligned} \phi_\pi(x) = & \frac{3f_\pi}{\sqrt{2N_c}} x(1-x) [1 + 0.44C_2^{3/2}(2x-1) \\ & + 0.25C_4^{3/2}(2x-1)], \end{aligned} \quad (30)$$

$$\begin{aligned} \phi_\pi^p(x) = & \frac{f_\pi}{2\sqrt{2N_c}} [1 + 0.43C_2^{1/2}(2x-1) + 0.09 \\ & \times C_4^{1/2}(2x-1)], \end{aligned} \quad (31)$$

$$\phi_\pi^t(x) = \frac{f_\pi}{2\sqrt{2N_c}} (1-2x) [1 + 0.55(10x^2 - 10x + 1)], \quad (32)$$

with the pion decay constant  $f_\pi = 130$  MeV. The Gegenbauer polynomials are defined by

$$\begin{aligned} C_2^{1/2}(t) = \frac{1}{2}(3t^2 - 1), \quad C_4^{1/2}(t) = \frac{1}{8}(35t^4 - 30t^2 + 3), \\ C_2^{3/2}(t) = \frac{3}{2}(5t^2 - 1), \quad C_4^{3/2}(t) = \frac{15}{8}(21t^4 - 14t^2 + 1), \end{aligned} \quad (33)$$

whose coefficients correspond to  $m_0 = 1.4$  GeV.

We first investigate the relative importance of the twist-2 and twist-3 contributions to  $F_+(q^2)$ , and the results are listed in Table I. It is observed that the latter are in fact larger than the former, consistent with the argument that the twist-3 contributions are not power-suppressed. The light-cone sum rules also give approximately equal weights to the twist-2 and higher-twist contributions to  $F_+$  [31]. We then compare our results of  $F_+(q^2)$  and  $F_0(q^2)$  for  $q^2 = 0 - 10$  GeV<sup>2</sup> with those derived from lattice QCD [34] and from light-cone sum rules [29] in Fig. 6, where lattice results have been extrapolated to the small  $q^2$  region. Different extrapolation methods cause uncertainty only of about 5% [35]. The good agreement among these different approaches at large recoil is explicit. The fast rise of the PQCD results at slow recoil indicates that the perturbative calculation gradually becomes unreliable. The values of  $F_+(0) = F_0(0) \equiv F(0)$  from PQCD for the parameter  $\omega_B = 0.40 \pm 0.04$  GeV are listed in Table II. The resultant range  $F_+(0) = 0.30 \pm 0.04$  is in agreement with

$F(0) \sim 0.3$  obtained in [29,34]. We shall adopt the same range of  $\omega_B$  in the evaluation of the  $B \rightarrow \rho$  transition form factors below. We also examine the uncertainty of our predictions from the parametrization of the jet function in Eq. (13). The values of  $F_+(q^2)$  vary about 15% for the choices of  $c = 0.2$  and  $0.4$  as shown in Table III. The variation for  $F_0(q^2)$  is similar. In a future work we shall incorporate the exact jet function into a convolution integrand in moment space.

#### IV. $B \rightarrow \rho$ TRANSITION FORM FACTORS

Consider the semileptonic decay  $B \rightarrow \rho \bar{l} \nu$  in the fast recoil region of the  $\rho$  meson [36]. We define the  $B$  meson momentum  $P_1$  as in Eq. (11), the momentum  $P_2$  and the polarization vectors  $\epsilon$  of the  $\rho$  meson in light-cone coordinates as

$$P_2 = \frac{M_B}{\sqrt{2}\eta} (\eta^2, r_\rho^2, 0_\perp), \quad (34)$$

$$\epsilon_L = \frac{1}{\sqrt{2}r_\rho\eta} (\eta^2, -r_\rho^2, 0_\perp), \quad \epsilon_T = (0, 0, 1, 0) \text{ or } (0, 0, 0, 1),$$

with the ratio  $r_\rho = M_\rho/M_B$ , and the energy fraction  $\eta$  carried by the  $\rho$  meson. We first keep the  $r_\rho^2$  dependence of the kinematic variables in Eq. (34), and extract the twist-3 terms proportional to  $r_\rho$ . The parametrization of  $P_2$  and  $\epsilon$  is chosen to make this extraction straightforward.

The  $B \rightarrow \rho$  form factors are defined through the following decompositions of hadronic matrix elements:

$$\begin{aligned} \langle \rho(P_2, \epsilon^*) | \bar{b}(0) \gamma^\mu u(0) | B(P_1) \rangle \\ = \frac{2iV(q^2)}{M_B + M_\rho} \epsilon^{\mu\nu\rho\sigma} \epsilon_\nu^* P_{2\rho} P_{1\sigma}, \end{aligned} \quad (35)$$

$$\begin{aligned} \langle \rho(P_2, \epsilon^*) | \bar{b}(0) \gamma^\mu \gamma_5 \mu(0) | B(P_1) \rangle \\ = 2M_\rho A_0(q^2) \frac{\epsilon^* \cdot q}{q^2} q^\mu + (M_B + M_\rho) A_1(q^2) \\ \times \left[ \epsilon^{*\mu} - \frac{\epsilon^* \cdot q}{q^2} q^\mu \right] - A_2(q^2) \frac{\epsilon^* \cdot q}{M_B + M_\rho} \\ \times \left[ P_1^\mu + P_2^\mu - \frac{M_B^2 - M_\rho^2}{q^2} q^\mu \right]. \end{aligned} \quad (36)$$

To calculate the form factors  $V$ ,  $A_0$ ,  $A_1$ , and  $A_2$ , we adopt the following procedures. First, only the transverse polarization vectors  $\epsilon_T$  are involved in Eq. (35) and associated with

TABLE II. Values of  $F_+(0)=F_0(0)\equiv F(0)$  for given  $\omega_B$ .

$\omega_B$ (GeV)	0.36	0.37	0.38	0.39	0.40	0.41	0.42	0.43	0.44
$F(0)$	0.345	0.334	0.321	0.309	0.297	0.287	0.277	0.268	0.259

the definition of  $A_1$  in Eq. (36), through which we evaluate the form factors  $V$  and  $A_1$ , respectively. Both the structures associated with  $A_1$  and  $A_2$  are orthogonal to the lepton pair momentum  $q$ . Contracting Eq. (36) with  $q_\mu$ , we have

$$\langle \rho(P_2, \epsilon^*) | \bar{b}(0) \not{q} \gamma_5 u(0) | B(P_1) \rangle = 2M_\rho A_0(q^2) \epsilon^* \cdot q, \quad (37)$$

which implies that only the form factor  $A_0$  is relevant in two-body nonleptonic decays such as  $B \rightarrow \rho \pi(K)$ . We calculate  $A_0$  from Eq. (37) using the distribution amplitudes associated with a longitudinally polarized  $\rho$  meson.

For the longitudinal polarization vector  $\epsilon_L$ , the structures of  $A_1$  and  $A_2$  are in fact proportional to each other:

$$\begin{aligned} & \frac{\epsilon^* \cdot q}{M_B + M_\rho} \left[ P_1^\mu + P_2^\mu - \frac{M_B^2 - M_\rho^2}{q^2} q^\mu \right] \\ &= \frac{(\epsilon^* \cdot q)^2 (M_B^2 - M_\rho^2 - q^2)}{(M_B + M_\rho)[(\epsilon^* \cdot q)^2 + q^2]} \left[ \epsilon^{*\mu} - \frac{\epsilon^* \cdot q}{q^2} q^\mu \right], \end{aligned} \quad (38)$$

which can be easily derived via the relation

$$P_1 + P_2 = \frac{M_B^2 - M_\rho^2 - q^2}{(\epsilon^* \cdot q)^2 + q^2} \epsilon^* \cdot q \epsilon_L^* + \frac{M_B^2 - M_\rho^2 + (\epsilon^* \cdot q)^2}{(\epsilon^* \cdot q)^2 + q^2} q. \quad (39)$$

Contracting Eq. (36) with  $\epsilon_\mu^* - \epsilon^* \cdot q q_\mu / q^2$ , we obtain

$$\begin{aligned} & \langle \rho(P_2, \epsilon^*) | \bar{b}(0) \left[ \not{\epsilon}^* - \frac{\epsilon^* \cdot q}{q^2} \not{q} \right] \gamma_5 u(0) | B(P_1) \rangle \\ &= \frac{2P_2 \cdot q}{M_B + M_\rho} \frac{(\epsilon^* \cdot q)^2}{q^2} \left[ A_2 - \frac{(M_B + M_\rho)^2}{2P_2 \cdot q} \right. \\ & \quad \left. \times \left( 1 + \frac{q^2}{(\epsilon^* \cdot q)^2} \right) A_1 \right], \end{aligned} \quad (40)$$

from which the form factor  $A_2$  can be computed. It turns out that  $A_1$  and  $A_2$  have a simple relation, since the left-hand side of Eq. (40) is power-suppressed.

We derive the leading-power factorization formulas,

$$\begin{aligned} V &= 8\pi M_B^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \\ & \quad \times \left( \left\{ \phi_\rho^T(x_2) + r_\rho \left[ \left( \frac{2}{\eta} + x_2 \right) \phi_\rho^a(x_2) \right. \right. \right. \\ & \quad \left. \left. \left. - x_2 \phi_\rho^v(x_2) \right] \right\} E(t^{(1)}) h(x_1, x_2, b_1, b_2) \right. \\ & \quad \left. + r_\rho [\phi_\rho^v(x_2) + \phi_\rho^a(x_2)] E(t^{(2)}) h(x_2, x_1, b_1, b_2) \right), \end{aligned} \quad (41)$$

$$\begin{aligned} A_0 &= 8\pi M_B^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \\ & \quad \times \left( \left\{ (1 + \eta x_2) \phi_\rho(x_2) + r_\rho \left[ (1 - 2x_2) \phi_\rho^t(x_2) \right. \right. \right. \\ & \quad \left. \left. \left. + \left( \frac{2}{\eta} - 1 - 2x_2 \right) \phi_\rho^s(x_2) \right] \right\} E(t^{(1)}) h(x_1, x_2, b_1, b_2) \right. \\ & \quad \left. + 2r_\rho \phi_\rho^s(x_2) E(t^{(2)}) h(x_2, x_1, b_2, b_1) \right), \end{aligned} \quad (42)$$

$$\begin{aligned} A_1 &= 8\pi M_B^2 C_F \eta \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \\ & \quad \times \left( \left\{ \phi_\rho^T(x_2) + r_\rho \left[ \left( \frac{2}{\eta} + x_2 \right) \phi_\rho^v(x_2) \right. \right. \right. \\ & \quad \left. \left. \left. - x_2 \phi_\rho^a(x_2) \right] \right\} E(t^{(1)}) h(x_1, x_2, b_1, b_2) \right. \\ & \quad \left. + r_\rho [\phi_\rho^v(x_2) + \phi_\rho^a(x_2)] E(t^{(2)}) h(x_2, x_1, b_2, b_1) \right), \end{aligned} \quad (43)$$

$$A_2 = \frac{A_1}{\eta}, \quad (44)$$

with the evolution factor  $E(t)$  the same as in Eq. (24). Taking the fast recoil limit with  $\eta \rightarrow 1$  and assuming the

TABLE III. Values of  $F_+(q^2)$  for  $c=0.2, 0.3$ , and  $0.4$ .

$q^2$ (GeV <sup>2</sup> )	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
$c=0.2$	0.347	0.376	0.406	0.442	0.482	0.527	0.580	0.639	0.709	0.790	0.886
$c=0.3$	0.297	0.321	0.348	0.378	0.412	0.451	0.496	0.548	0.608	0.675	0.757
$c=0.4$	0.260	0.280	0.303	0.330	0.359	0.392	0.432	0.475	0.527	0.588	0.659



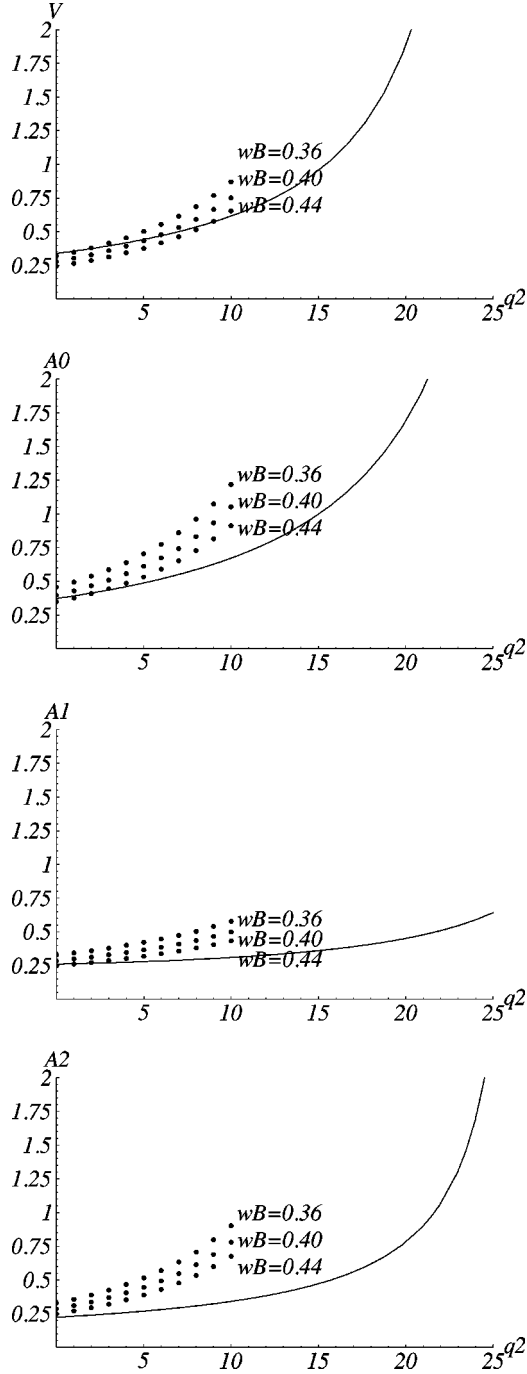


FIG. 7. The  $B \rightarrow \rho$  form factors  $V$ ,  $A_0$ ,  $A_1$ , and  $A_2$  as functions of  $q^2$ . PQCD results are given as dots. The solid lines come from light-cone sum rules.

asymptotic behavior  $\phi_\rho^v = \phi_\rho^a$ , the above form factors are found to obey the symmetry relations [11,37]

$$V = A_1, \quad A_2 = A_1 - 2r_\rho A_0, \quad (45)$$

where the term  $-2r_\rho A_0$ , being higher power, does not appear in Eq. (44). Note that the form factors, treated as non-perturbative objects, are not calculated in [11]. Instead, the diagrams we have calculated above are regarded as perturbative corrections to the relations in Eq. (45).

We adopt the  $\rho$  meson distribution amplitudes given in Appendix B [38],

$$\phi_\rho(x) = \frac{3f_\rho}{\sqrt{2N_c}} x(1-x)[1 + 0.18C_2^{3/2}(2x-1)], \quad (46)$$

$$\begin{aligned} \phi_\rho^t(x) = \frac{f_\rho^T}{2\sqrt{2N_c}} \{ & 3(2x-1)^2 \\ & + 0.3(2x-1)^2[5(2x-1)^2 - 3] \\ & + 0.21[3 - 30(2x-1)^2 + 35(2x-1)^4] \}, \end{aligned} \quad (47)$$

$$\phi_\rho^s(x) = \frac{3f_\rho^T}{2\sqrt{2N_c}} (1-2x)[1 + 0.76(10x^2 - 10x + 1)], \quad (48)$$

$$\phi_\rho^T(x) = \frac{3f_\rho^T}{\sqrt{2N_c}} x(1-x)[1 + 0.2C_2^{3/2}(2x-1)], \quad (49)$$

$$\begin{aligned} \phi_\rho^v(x) = \frac{f_\rho}{2\sqrt{2N_c}} \{ & \frac{3}{4}[1 + (2x-1)^2] + 0.24[3(2x-1)^2 - 1] \\ & + 0.12[3 - 30(2x-1)^2 + 35(2x-1)^4] \}, \end{aligned} \quad (50)$$

$$\phi_\rho^a(x) = \frac{3f_\rho}{4\sqrt{2N_c}} (1-2x)[1 + 0.93(10x^2 - 10x + 1)], \quad (51)$$

with the decay constants  $f_\rho = 200$  MeV and  $f_\rho^T = 160$  MeV. The  $q^2$  dependence of the form factors  $V$  and  $A_{0,1,2}$ , with the same  $B$  meson distribution amplitude in Eq. (28) and  $M_\rho = 0.77$  GeV employed, is displayed in Fig. 7. Our results are consistent with those from light-cone QCD sum rules [39] at small  $q^2$ .

It is found that the symmetry relation  $V = A_1$  in Eq. (45) holds very well:  $A_1$  is larger than  $V$  only by 2% in the large recoil region, even after considering the preasymptotic forms of  $\phi_\rho^v$  and  $\phi_\rho^a$  in Eqs. (50) and (51), respectively. To compare our results with the second symmetry relation, we include next-to-leading power terms in Eq. (40), obtaining

$$\begin{aligned} A_2 = & \frac{1 + 2r_\rho}{\eta} A_1 - 8\pi M_B^2 C_F \frac{2r_\rho}{\eta} \int dx_1 dx_2 \\ & \times \int b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \\ & \times \left( \left\{ (1 + \eta x) \phi_\rho(x_2) + r_\rho \left[ \left( \frac{3}{2\eta} - 1 \right) \phi_\rho^t(x_2) \right. \right. \right. \\ & \left. \left. \left. + (1 - 2x_2) \phi_\rho^s(x_2) \right] \right\} E(t^{(1)}) h(x_1, x_2, b_1, b_2) \right. \\ & \left. + 2r_\rho \phi_\rho^s(x_2) E(t^{(2)}) h(x_2, x_1, b_2, b_1) \right). \end{aligned} \quad (52)$$

Because of the cancellation of the term  $2r_\rho A_1/\eta$  and the second term in the above expression, the values of  $A_2$  only slightly deviate from those in Eq. (44). The numerical study shows that  $A_2$  is larger than  $A_1 - 2r_\rho A_0$  by about 40%, which can be regarded as the estimate of the symmetry-breaking effect.

## V. CONCLUSION

In this paper, we have presented a complete leading-power and leading-order PQCD evaluation of the  $B \rightarrow \pi, \rho$  transition form factors in the large recoil region. It has been shown that under Sudakov suppression arising from  $k_\perp$  and threshold resummations, the end-point singularities (logarithmic at twist 2 and linear at twist 3) do not exist. The soft contribution to the form factors, being Sudakov-suppressed, becomes smaller than the perturbative contribution. The physical picture for the mechanism of Sudakov suppression has been discussed. We have emphasized that the twist-3 contributions are in fact not power-suppressed in the  $M_B \rightarrow \infty$  limit. The treatment of the parton transverse momenta  $k_\perp$  and the light spectator momentum  $k_1$  in the  $B$  meson in the computation of the hard amplitudes has been clearly explained: the hard amplitudes should not be expanded in powers of  $k_\perp^2$  as the end-point region is important. Using the light meson distribution amplitudes derived from QCD sum rules, and choosing an appropriate  $B$  meson distribution amplitude, we have derived reasonable results for the  $B \rightarrow \pi, \rho$  form factors, which are in agreement with those from light-cone QCD sum rules and from lattice calculations. Our study indicates that in a self-consistent perturbative analysis, the heavy-to-light form factors are calculable.

The jet function from threshold resummation needs more thorough exploration. We shall investigate the relevant subjects, such as factorization theorem in moment space, threshold resummation up to next-to-leading logarithms, application to nonleptonic  $B$  meson decays [16], and numerical effects elsewhere. Note that if considering only  $k_T$  resummation [40], twist-3 contributions, though infrared-finite, are still too large to give reasonable heavy-to-light transition form factors, because the large double logarithms  $\alpha_s \ln^2 x$  have not yet been organized.

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## APPENDIX A: PION DISTRIBUTION AMPLITUDES

It has been shown [27] that the factorization in fermion flow between the pion distribution amplitude and the hard amplitude is achieved by inserting the Fierz identity,

$$I_{ij}I_{lk} = \frac{1}{4}I_{ik}I_{lj} + \frac{1}{4}(\gamma_5)_{ik}(\gamma_5)_{lj} + \frac{1}{4}(\gamma_\mu)_{ik}(\gamma^\mu)_{lj} + \frac{1}{4}(\gamma_5\gamma_\mu)_{ik}(\gamma^\mu\gamma_5)_{lj} + \frac{1}{8}(\sigma_{\mu\nu})_{ik}(\sigma^{\mu\nu})_{lj}, \quad (\text{A1})$$

into the quark and antiquark lines of the pion, where  $I$  represents the identity matrix, and  $\sigma^{\mu\nu}$  is defined by  $\sigma^{\mu\nu} = i[\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu]/2$ . The insertion of Eq. (A1) then leads to various nonlocal matrix elements,

$$\langle 0|\bar{u}(0)\gamma_5\gamma_\mu d(z)|\pi^-(P)\rangle, \quad \langle 0|\bar{u}(0)\gamma_5 d(z)|\pi^-(P)\rangle, \\ \langle 0|\bar{u}(0)\gamma_5\sigma_{\mu\nu}d(z)|\pi^-(P)\rangle, \dots \quad (\text{A2})$$

each of which is characterized by different twists. The light-like vector  $z = (0, z^-, 0_\perp)$  is the coordinate of the  $d$  quark, and  $P = (P^+, 0, 0_\perp)$  is the pion momentum.

The general expressions of the relevant matrix elements are, quoted from [33],

$$\langle 0|\bar{u}(0)\gamma_\mu\gamma_5 d(z)|\pi^-(P)\rangle = i\frac{f_\pi}{N_c}P_\mu \int_0^1 dx e^{-ixP \cdot z} \phi_v(x) \\ + \frac{i}{2}\frac{f_\pi}{N_c}M_\pi^2 \frac{z_\mu}{P \cdot z} \\ \times \int_0^1 dx e^{-ixP \cdot z} g_\pi(x), \quad (\text{A3})$$

$$\langle 0|\bar{u}(0)\gamma_5 d(z)|\pi^-(P)\rangle = -i\frac{f_\pi}{N_c}m_0 \\ \times \int_0^1 dx e^{-ixP \cdot z} \phi_p(x), \quad (\text{A4})$$

$$\langle 0|\bar{u}(0)\gamma_5\sigma_{\mu\nu}d(z)|\pi^-(P)\rangle = \frac{i}{6}\frac{f_\pi}{N_c}m_0 \left(1 - \frac{M_\pi^2}{m_0^2}\right) \\ \times (P_\mu z_\nu - P_\nu z_\mu) \\ \times \int_0^1 dx e^{-ixP \cdot z} \phi_\sigma(x), \quad (\text{A5})$$

where  $\phi$  and  $g_\pi$  are the distribution amplitudes of unit normalization,  $M_\pi$  is the pion mass, and  $x$  is the momentum fraction associated with the  $d$  quark. It should be understood that a path-ordered exponential exists between the two quark fields separated by a distance  $z$ , and the above matrix elements are gauge-invariant. For the derivation of the path-order exponential, refer to [27]. It is easy to observe that the contribution from  $\phi_v$ , independent of the pion mass, is twist 2, and the contribution from  $g_\pi$  is twist 4 because of the factor  $M_\pi^2$ . The contributions from  $\phi_p$  and  $\phi_v$ , proportional

to  $r_\pi = m_0/M_B$ , are twist 3. We shall neglect the twist-4 terms and the term  $(M_\pi/m_0)^2$  in Eq. (A5).

It is straightforward to read off the pseudovector and pseudoscalar structures of the pion distribution amplitudes from Eqs. (A3) and (A4). To derive the pseudotensor structure from Eq. (A5), we need more effort. Using integration by parts, Eq. (A5) is rewritten as

$$\begin{aligned} & \langle 0 | \bar{u}(0) \gamma_5 \sigma_{\mu\nu} d(z) | \pi^-(P) \rangle \\ &= \frac{1}{6} \frac{f_\pi}{N_c} m_0 \left( 1 - \frac{M_\pi^2}{m_0^2} \right) \epsilon_{\mu\nu} \int_0^1 dx e^{-ixP \cdot z} \frac{d}{dx} \phi_\sigma(x), \end{aligned} \quad (\text{A6})$$

with the antisymmetric tensor  $\epsilon_{\mu\nu}$ ,  $\epsilon^{+-} = 1$ . The tensor  $\epsilon_{\mu\nu}$  in Eq. (A6) contracts to the spin structure  $\sigma^{\mu\nu} \gamma_5 / 2$  in the evaluation of the corresponding hard amplitude. The factor  $\frac{1}{2}$  comes from the extra factor  $\frac{1}{2}$  associated with the pseudotensor structure compared to other structures in Eq. (A1). We have

$$\frac{1}{2} \epsilon_{\mu\nu} \sigma^{\mu\nu} \gamma_5 = -\frac{i}{2} (\gamma^+ \gamma^- - \gamma^- \gamma^+) \gamma_5 = -i(\not{h}_- \not{h}_+ - 1) \gamma_5. \quad (\text{A7})$$

Therefore, up to twist-3, the initial-state  $\pi^-$  meson distribution amplitudes are written as

$$\begin{aligned} & \langle 0 | \bar{u}(0)_j d(z)_l | \pi^-(P) \rangle \\ &= -\frac{i}{\sqrt{2N_c}} \int_0^1 dx e^{-ixP \cdot z} \{ [\not{P} \gamma_5]_{lj} \phi_\pi(x) \\ &+ [\gamma_5]_{lj} m_0 \phi_\pi^p(x) + m_0 [\gamma_5(\not{h}_- \not{h}_+ - 1)]_{lj} \phi_\pi^t(x) \}, \end{aligned} \quad (\text{A8})$$

with

$$\begin{aligned} \phi_\pi(x) &= \frac{f_\pi}{2\sqrt{2N_c}} \phi_v(x), \quad \phi_\pi^p(x) = \frac{f_\pi}{2\sqrt{2N_c}} \phi_p(x), \\ \phi_\pi^t(x) &= \frac{f_\pi}{12\sqrt{2N_c}} \frac{d}{dx} \phi_\sigma(x). \end{aligned} \quad (\text{A9})$$

For the final-state  $\pi^-$  meson, we consider the adjoints of Eqs. (A3), (A4), and (A5):

$$\begin{aligned} & \langle \pi^-(P) | \bar{d}(z) \gamma_\mu \gamma_5 u(0) | 0 \rangle \\ &= -i \frac{f_\pi}{N_c} P_\mu \int_0^1 dx e^{ixP \cdot z} \phi_v(x), \end{aligned} \quad (\text{A10})$$

$$\begin{aligned} & \langle \pi^-(P) | \bar{d}(z) \gamma_5 u(0) | 0 \rangle \\ &= -i \frac{f_\pi}{N_c} m_0 \int_0^1 dx e^{ixP \cdot z} \phi_p(x), \end{aligned} \quad (\text{A11})$$

$$\begin{aligned} & \langle \pi^-(P^-) | \bar{d}(z) \sigma_{\mu\nu} \gamma_5 u(0) | 0 \rangle \\ &= -\frac{f_\pi}{6N_c} m_0 \left( 1 - \frac{M_\pi^2}{m_0^2} \right) \epsilon_{\mu\nu} \int_0^1 dx e^{ixP \cdot z} \frac{d}{dx} \phi_\sigma(x). \end{aligned} \quad (\text{A12})$$

It is observed that the pseudotensor structure in Eq. (A12) acquires an extra minus sign, compared to the other two structures. The pseudotensor structure is then given by  $-\gamma_5(\not{h}_- \not{h}_+ - 1) = \gamma_5(\not{h}_+ \not{h}_- - 1)$ . Therefore, up to twist 3, we have Eq. (7) for the final-state  $\pi^-$  meson. Note that there is an extra term in the definition of  $\phi_\pi^t$ , which contains a differential operator applying to hard amplitudes [11]. This term, being power-suppressed, is negligible here. The distribution amplitudes  $\phi_\pi$  and  $\phi_\pi^p$  are normalized according to

$$\int_0^1 dx \phi_\pi(x) = \frac{f_\pi}{2\sqrt{2N_c}}, \quad \int_0^1 dx \phi_\pi^p(x) = \frac{f_\pi}{2\sqrt{2N_c}}. \quad (\text{A13})$$

The tensor distribution amplitude is normalized to zero, because of

$$\int_0^1 dx \frac{d}{dx} \phi_\sigma(x) = \phi_\sigma(1) - \phi_\sigma(0) = 0, \quad (\text{A14})$$

if  $\phi_\sigma$  vanishes at the end points of the momentum fraction.

## APPENDIX B: $\rho$ MESON DISTRIBUTION AMPLITUDES

We choose the  $\rho$  meson momentum  $P$  with  $P^2 = M_\rho^2$ , which is mainly in the plus direction. The polarization vectors  $\epsilon$ , satisfying  $P \cdot \epsilon = 0$ , represent one longitudinal polarization vector  $\epsilon_L$  and two transverse polarization vectors  $\epsilon_T$ . Their explicit expressions in light-cone coordinates have been given in Eq. (34). To arrive at the factorization in fermion flow, we insert the Fierz identity into the quark and antiquark lines of the  $\rho$  meson. The spin structures in Eq. (A1) lead to the following nonlocal matrix elements:

$$\begin{aligned} & \langle \rho^-(P, \epsilon) | \bar{d}(z) \gamma_\mu u(0) | 0 \rangle, \quad \langle \rho^-(P, \epsilon) | \bar{d}(z) \sigma_{\mu\nu} u(0) | 0 \rangle, \\ & \langle \rho^-(P, \epsilon) | \bar{d}(z) I u(0) | 0 \rangle, \quad \langle \rho^-(P, \epsilon) | \bar{d}(z) \gamma_\mu \gamma_5 u(0) | 0 \rangle, \end{aligned} \quad (\text{B1})$$

characterized by different twists. The definition of  $z$  is the same as that for the pion distribution amplitudes in the preceding Appendix.

The general expressions of the above matrix elements are, quoted from [38],

$$\begin{aligned} & \langle \rho^-(P, \epsilon) | \bar{d}(z) \gamma_\mu u(0) | 0 \rangle \\ &= \frac{f_\rho}{N_c} M_\rho \left\{ P_\mu \frac{\epsilon \cdot z}{P \cdot z} \int_0^1 dx e^{ixP \cdot z} \phi_\parallel(x) \right. \\ &+ \epsilon_{T\mu} \int_0^1 dx e^{ixP \cdot z} g_T^{(v)}(x) - \frac{1}{2} z_\mu \frac{\epsilon \cdot z}{(P \cdot z)^2} M_\rho^2 \\ &\times \left. \int_0^1 dx e^{ixP \cdot z} [g_3(x) - \phi_\parallel(x)] \right\}, \end{aligned} \quad (\text{B2})$$

$$\begin{aligned}
& \langle \rho^-(P, \epsilon) | \bar{d}(z) \sigma_{\mu\nu} u(0) | 0 \rangle \\
&= -i \frac{f_\rho^T}{N_c} \left\{ (\epsilon_{T\mu} P_\nu - \epsilon_{T\nu} P_\mu) \right. \\
&\quad \times \int_0^1 dx e^{ixP \cdot z} \phi_T(x) + (P_\mu z_\nu - P_\nu z_\mu) \frac{\epsilon \cdot z}{(P \cdot z)^2} M_\rho^2 \\
&\quad \times \int_0^1 dx e^{ixP \cdot z} h_{\parallel}^{(t)}(x) + \frac{1}{2} (\epsilon_{T\mu} z_\nu - \epsilon_{T\nu} z_\mu) \frac{M_\rho^2}{P \cdot z} \\
&\quad \left. \times \int_0^1 dx e^{ixP \cdot z} [h_3(x) - \phi_T(x)] \right\}, \quad (\text{B3})
\end{aligned}$$

$$\begin{aligned}
& \langle \rho^-(P, \epsilon) | \bar{d}(z) I u(0) | 0 \rangle \\
&= -\frac{i}{2N_c} \left( f_\rho^T - f_\rho \frac{m_u + m_d}{M_\rho} \right) \epsilon \cdot z M_\rho^2 \\
&\quad \times \int_0^1 dx e^{ixP \cdot z} h_{\parallel}^{(s)}(x), \\
&= \frac{1}{2N_c} \left( f_\rho^T - f_\rho \frac{m_u + m_d}{M_\rho} \right) \frac{\epsilon \cdot z}{P \cdot z} M_\rho^2 \int_0^1 dx e^{ixP \cdot z} \frac{d}{dx} h_{\parallel}^{(s)}(x), \quad (\text{B4})
\end{aligned}$$

$$\begin{aligned}
& \langle \rho^-(P, \epsilon) | \bar{d}(z) \gamma_5 \gamma_\mu u(0) | 0 \rangle \\
&= -\frac{1}{4N_c} \left( f_\rho - f_\rho^T \frac{m_u + m_d}{M_\rho} \right) \\
&\quad \times M_\rho \epsilon_\mu^{\nu\alpha\beta} \epsilon_{T\nu} P_{2\alpha} z_\beta \int_0^1 dx e^{ixP \cdot z} g_T^{(a)}(x), \\
&= -\frac{i}{4N_c} \left( f_\rho - f_\rho^T \frac{m_u + m_d}{M_\rho} \right) \frac{M_\rho}{P \cdot n_-} \epsilon_{\mu\nu\rho\sigma} \epsilon_T^\nu P^\rho n_-^\sigma \\
&\quad \times \int_0^1 dx e^{ixP \cdot z} \frac{d}{dx} g_T^{(a)}(x), \quad (\text{B5})
\end{aligned}$$

where  $f_\rho$  and  $f_\rho^T$  are the decay constants of the  $\rho$  meson with longitudinal and transverse polarizations, respectively, and  $x$  is the momentum fraction associated with the  $d$  quark. We adopt the convention  $\epsilon^{0123} = 1$  for the Levi-Civita tensor  $\epsilon^{\mu\nu\alpha\beta}$ . The distribution amplitudes  $\phi$ ,  $g$ , and  $h$  are normalized to unity.

Following similar procedures, we derive the  $\rho$  meson distribution amplitudes up to twist 3,

$$\begin{aligned}
& \langle \rho^-(P, \epsilon_L) | \bar{d}(z)_j u(0)_l | 0 \rangle \\
&= \frac{1}{\sqrt{2N_c}} \int_0^1 dx e^{ixP \cdot z} \{ M_\rho [\epsilon_L]_{lj} \phi_\rho(x) \\
&\quad + [\epsilon_L P]_{lj} \phi_\rho^t(x) + M_\rho [I]_{lj} \phi_\rho^s(x) \}, \quad (\text{B6})
\end{aligned}$$

$$\begin{aligned}
& \langle \rho^-(P, \epsilon_T) | \bar{d}(z)_j u(0)_l | 0 \rangle \\
&= \frac{1}{\sqrt{2N_c}} \int_0^1 dx e^{ixP \cdot z} \left\{ M_\rho [\epsilon_T]_{lj} \phi_\rho^u(x) \right. \\
&\quad + [\epsilon_T P]_{lj} \phi_\rho^T(x) + \frac{M_\rho}{P n_-} i \epsilon_{\mu\nu\rho\sigma} \\
&\quad \left. \times [\gamma_5 \gamma^\mu]_{lj} \epsilon_T^\nu P^\rho n_-^\sigma \phi_\rho^a(x) \right\}, \quad (\text{B7})
\end{aligned}$$

for longitudinal polarization and transverse polarization, respectively. We have dropped the terms proportional to  $r_\rho^2$  (twist-4) and the terms  $(m_u + m_d)/M_\rho$  in Eqs. (B4) and (B5). The definitions of the above distribution amplitudes are

$$\begin{aligned}
\phi_\rho &= \frac{f_\rho}{2\sqrt{2N_c}} \phi_{\parallel}, & \phi_\rho^t &= \frac{f_\rho^T}{2\sqrt{2N_c}} h_{\parallel}^{(t)}, & \phi_\rho^s &= \frac{f_\rho^T}{4\sqrt{2N_c}} \frac{d}{dx} h_{\parallel}^{(s)}, \\
\phi_\rho^T &= \frac{f_\rho^T}{2\sqrt{2N_c}} \phi_T, & \phi_\rho^v &= \frac{f_\rho}{2\sqrt{2N_c}} g_T^{(v)}, \\
\phi_\rho^a &= \frac{f_\rho}{8\sqrt{2N_c}} \frac{d}{dx} g_T^{(a)}. \quad (\text{B9})
\end{aligned}$$

### APPENDIX C: B MESON DISTRIBUTION AMPLITUDES

According to [41], the nonlocal matrix element associated with the  $B$  meson is written as

$$\begin{aligned}
& \int \frac{d^4 z}{(2\pi)^4} e^{ik_1 \cdot z} \langle 0 | \bar{b}_\alpha(0) d_\delta(z) | B(P_1) \rangle \\
&= \frac{i}{\sqrt{2N_c}} \left\{ (\mathbf{P}_1 + M_B) \gamma_5 \left[ \frac{\not{h}_+}{\varphi} \phi_B^+(k_1) + \frac{\not{h}_-}{\sqrt{2}} \phi_B^-(k_1) \right] \right\}_{\delta\alpha}, \\
&= -\frac{i}{\sqrt{2N_c}} \left\{ (\mathbf{P}_1 + M_B) \gamma_5 \left[ \phi_B(k_1) \right. \right. \\
&\quad \left. \left. - \frac{\not{h}_+ - \not{h}_-}{\sqrt{2}} \bar{\phi}_B(k_1) \right] \right\}_{\delta\alpha}, \quad (\text{C1})
\end{aligned}$$

with the wave functions

$$\phi_B = \frac{1}{2} (\phi_B^+ + \phi_B^-), \quad \bar{\phi}_B = \frac{1}{2} (\phi_B^+ - \phi_B^-). \quad (\text{C2})$$

Because the light meson momenta have been chosen in the plus direction, the hard amplitudes for the heavy-to-light transition form factors are independent of the component  $k_1^+$  as explained in Sec. III. We construct the  $B$  meson distribution amplitude,

$$\phi(x_1, b) = \int dk_1^+ d^2 k_{1\perp} e^{i\vec{k}_{1\perp} \cdot \vec{b}} \phi(k_1), \quad (\text{C3})$$

with  $x_1 = k_1^- / P_1^-$ .

The two  $B$  meson distribution amplitudes  $\phi_B^+(x) = \phi_B^+(x,0)$  and  $\phi_B^-(x) = \phi_B^-(x,0)$  are related by the equation of motion [41]

$$\phi_B^+(x) = -x \frac{d}{dx} \phi_B^-(x). \quad (\text{C4})$$

Assuming that  $\phi_B^-$  vanishes at both ends of the momentum fraction,  $x \rightarrow 0$  and  $x \rightarrow 1$ , we derive

$$\begin{aligned} \int_0^1 dx \phi_B^+(x) &= \int_0^1 dx \phi_B^-(x) = \frac{f_B}{2\sqrt{2N_c}}, \\ \int_0^1 dx x \phi_B^+(x) &= 2 \int_0^1 dx x \phi_B^-(x) \sim \frac{2\bar{\Lambda}}{M_B} \frac{f_B}{2\sqrt{2N_c}}. \end{aligned} \quad (\text{C5})$$

Therefore,  $\bar{\phi}_B$  is normalized to zero.

We shall argue that the contribution from the distribution amplitude  $\bar{\phi}_B$  is negligible compared to that from  $\phi_B$ . Consider the reasonable parametrizations,

$$\begin{aligned} \phi_B(x) &= \frac{f_B}{2\sqrt{2N_c}} \left[ \delta\left(x - \frac{\bar{\Lambda}}{M_B}\right) - \frac{\bar{\Lambda}}{2M_B} \delta'\left(x - \frac{\bar{\Lambda}}{M_B}\right) \right. \\ &\quad \left. + O\left(\frac{\bar{\Lambda}^2}{M_B^2}\right) \right], \\ \bar{\phi}_B(x) &= \frac{f_B}{2\sqrt{2N_c}} \left[ -\frac{\bar{\Lambda}}{2M_B} \delta'\left(x - \frac{\bar{\Lambda}}{M_B}\right) + O\left(\frac{\bar{\Lambda}^2}{M_B^2}\right) \right], \end{aligned} \quad (\text{C6})$$

whose moments satisfy Eq. (C5). As shown in Sec. III, the hard amplitudes are approximated by  $\ln(1/x_1)$  at small  $x_1$ . A simple estimation indicates that the contribution from  $\phi_B$ , proportional to  $\ln(M_B/\bar{\Lambda})$ , is numerically larger than that from  $\bar{\phi}_B$ , proportional to a constant. Hence, after taking into account Eq. (C4), we consider only a single  $B$  meson distribution amplitude in this work.

#### APPENDIX D: THRESHOLD RESUMMATION

In this appendix, we supply details of the derivation of the Sudakov factor in Eq. (13). Threshold resummation introduces a jet function  $S_t(x)$  into the PQCD factorization of the  $B \rightarrow \pi$  form factors near the end points [15],

$$S_t(x) = \int_{a-i\infty}^{a+i\infty} \frac{dN}{2\pi i} \frac{S_t(N)}{N} (1-x)^{-N}, \quad (\text{D1})$$

where  $a$  is an arbitrary real constant larger than all the real parts of poles involved in the integrand. The factor  $1/N$  comes from Mellin transformation of the initial condition  $S_t^{(0)}(x) = 1$ ,

$$\int_0^1 dx (1-x)^{N-1} S_t^{(0)}(x) = \frac{1}{N}, \quad (\text{D2})$$

The Sudakov factor  $S_t(N)$  in the moment ( $N$ ) space has been derived explicitly to the accuracy of leading logarithms (LL) [15],

$$S_t^{(\text{LL})}(N) = \exp\left[-\frac{1}{4} \gamma_K^{(\text{LL})} \ln^2 N\right], \quad (\text{D3})$$

with the anomalous dimension  $\gamma_K^{(\text{LL})} = \alpha_s C_F / \pi$ . The contour integral in Eq. (D1) leads to

$$\begin{aligned} S_t^{(\text{LL})}(x) &= -\exp\left(\frac{\pi}{4} \alpha_s C_F\right) \int_{-\infty}^{\infty} \frac{dt}{\pi} (1-x)^{\exp(t)} \\ &\quad \times \sin\left(\frac{1}{2} \alpha_s C_F t\right) \exp\left(-\frac{\alpha_s}{4\pi} C_F t^2\right), \end{aligned} \quad (\text{D4})$$

which vanishes at  $x \rightarrow 0$  since the integrand is an odd function in  $t$ , and at  $x \rightarrow 1$  due to the factor  $(1-x)^{\exp(t)}$ .

In this paper we consider threshold resummation up to next-to-leading logarithms. At this level of accuracy, the anomalous dimension  $\gamma_K$  contains two-loop contributions, and the coupling constant  $\alpha_s$  is running. The Sudakov factor  $S_t(N)$  is then given by

$$S_t(N) = \exp\left[\frac{1}{2} \int_0^{1-1/N} \frac{dz}{1-z} \int_{(1-z)}^{(1-z)^2} \frac{d\lambda}{\lambda} \gamma_K[\alpha_s(\lambda M_B^2/2)]\right], \quad (\text{D5})$$

with

$$\gamma_K = \frac{\alpha_s}{\pi} C_F + \left(\frac{\alpha_s}{\pi}\right)^2 C_F \left[ C_A \left(\frac{67}{36} - \frac{\pi^2}{12}\right) - \frac{5}{18} n_f \right], \quad (\text{D6})$$

$n_f$  being the number of quark flavors and  $C_A = 3$  is a color factor. The anomalous dimension  $\gamma_K$  is the same as that for  $k_T$  resummation [42].

It can be shown that  $S_t(x)$  still vanishes at the end points  $x \rightarrow 0$  and  $x \rightarrow 1$ . To simplify the analysis, we propose the parametrization

$$S_t(x) = \frac{2^{1+2c} \Gamma(3/2+c)}{\sqrt{\pi}(1+c)} [x(1-x)]^c, \quad (\text{D7})$$

whose end-point behavior satisfies the above requirement. In the  $\alpha_s \rightarrow 0$  ( $c \rightarrow 0$ ) limit, i.e., without QCD effects, Eq. (D7) approaches unity. Mellin transformation of  $S_t(x)$  gives

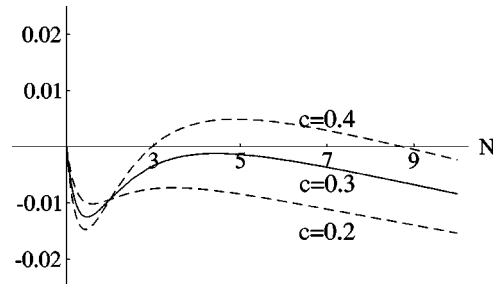


FIG. 8. Difference between the jet function and its parametrization in the moment space.

$$\frac{S_i^{\text{fit}}(N)}{N} = \frac{2^{1+2c}\Gamma(3/2+c)}{\sqrt{\pi}\Gamma(1+c)} B(c+1, c+N). \quad (\text{D8})$$

The variable  $N$  should be large enough to justify threshold resummation up to the next-to-leading logarithms  $\alpha_s \ln N$ , and small enough to avoid the divergent running coupling

constant  $\alpha_s[M_B^2/(2N^2)]$  in Eq. (D5). Performing the best fit of Eq. (D8) to  $S_i(N)/N$  for  $3 < N < 7$ , we determine the parameter  $c=0.3$ . The difference  $S_i(N)/N - S_i^{\text{fit}}(N)/N$  is shown in Fig. 8 for  $c=0.2, 0.3$ , and  $0.4$ . Equation (D7) implies that threshold resummation modifies the end-point behavior of the meson distribution amplitudes, causing them to vanish faster at  $x \rightarrow 0$ .

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- [1] CLEO Collaboration, Y. Kwon *et al.*, hep-ex/9908039.
- [2] CLEO Collaboration, D. Cronin-Hennessey *et al.*, hep-ex/0001010.
- [3] Belle Collaboration, talk presented at the 4th International Workshop on  $B$  Physics and  $CP$  Violation, Ise, Japan, 2001.
- [4] Babar Collaboration, talk presented at the 4th International Workshop on  $B$  Physics and  $CP$  Violation, Ise, Japan, 2001.
- [5] Y. Y. Keum, H.-n. Li, and A. I. Sanda, Phys. Lett. B **504**, 6 (2001); Phys. Rev. D **63**, 054008 (2001).
- [6] C. D. Lü, K. Ukai, and M. Z. Yang, Phys. Rev. D **63**, 074009 (2001); C. D. Lü and M. Z. Yang, hep-ph/0011238.
- [7] C. H. Chen and H.-n. Li, Phys. Rev. D **63**, 014003 (2001).
- [8] N. Isgur and C. H. Llewellyn-Smith, Nucl. Phys. **B317**, 526 (1989).
- [9] A. P. Szczepaniak, E. M. Henley, and S. J. Brodsky, Phys. Lett. B **243**, 287 (1990).
- [10] R. Akhouri, G. Sterman, and Y. P. Yao, Phys. Rev. D **50**, 358 (1994).
- [11] M. Beneke and T. Feldmann, Nucl. Phys. **B592**, 3 (2000).
- [12] A. Khodjamirian and R. Ruckl, Phys. Rev. D **58**, 054013 (1998).
- [13] G. Sterman, Phys. Lett. B **179**, 281 (1986); Nucl. Phys. **B281**, 310 (1987).
- [14] S. Catani and L. Trentadue, Nucl. Phys. **B327**, 323 (1989); **B353**, 183 (1991).
- [15] H.-n. Li, hep-ph/0102013.
- [16] C. H. Chen, Y. Y. Keum, and H.-n. Li, hep-ph/0107165.
- [17] G. P. Lepage and S. J. Brodsky, Phys. Lett. **87B**, 359 (1979).
- [18] G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980).
- [19] A. V. Efremov and A. V. Radyushkin, Phys. Lett. **94B**, 245 (1980).
- [20] V. L. Chernyak, A. R. Zhitnitsky, and V. G. Serbo, JETP Lett. **26**, 594 (1977).
- [21] V. L. Chernyak and A. R. Zhitnitsky, Sov. J. Nucl. Phys. **31**, 544 (1980); Phys. Rep. **112**, 173 (1984).
- [22] H.-n. Li and G. Sterman, Nucl. Phys. **B381**, 129 (1992).
- [23] J. C. Collins and D. E. Soper, Nucl. Phys. **B193**, 381 (1981).
- [24] J. Botts and G. Sterman, Nucl. Phys. **B225**, 62 (1989).
- [25] H.-n. Li and H. L. Yu, Phys. Rev. Lett. **74**, 4388 (1995); Phys. Rev. D **53**, 2480 (1996).
- [26] T. W. Yeh and H.-n. Li, Phys. Rev. D **56**, 1615 (1997).
- [27] H.-n. Li, Phys. Rev. D **64**, 014019 (2001).
- [28] S. Aoki, Int. J. Mod. Phys. A **15** (S1), 657 (2000).
- [29] P. Ball, J. High Energy Phys. **09**, 005 (1998); Z. Phys. C **29**, 637 (1985).
- [30] M. Dahm, R. Jakob, and P. Kroll, Z. Phys. C **68**, 595 (1995).
- [31] A. Khodjamirian *et al.*, Phys. Lett. B **410**, 275 (1997).
- [32] A. P. Szczepaniak and A. V. Radyushkin, Phys. Rev. D **59**, 034017 (1999).
- [33] V. M. Braun and I. E. Filyanov, Z. Phys. C **48**, 239 (1990); P. Ball, J. High Energy Phys. **01**, 010 (1999).
- [34] K. C. Bowler *et al.*, Phys. Lett. B **486**, 111 (2000).
- [35] D. Becirevic, Nucl. Phys. B (Proc. Suppl.) **94**, 337 (2001).
- [36] H.-n. Li and H. L. Yu, Phys. Lett. B **353**, 301 (1995).
- [37] J. Charles *et al.*, Phys. Rev. D **60**, 014001 (1999).
- [38] P. Ball, V. M. Braun, Y. Koike, and K. Tanaka, Nucl. Phys. **B529**, 323 (1998).
- [39] P. Ball and V. M. Braun, Phys. Rev. D **58**, 094016 (1998).
- [40] D. Du, D. Yang, and G. Zhu, Phys. Lett. B **509**, 263 (2001); Phys. Rev. D **64**, 014036 (2001).
- [41] A. G. Grozin and M. Neubert, Phys. Rev. D **55**, 272 (1997).
- [42] H.-n. Li, Phys. Lett. B **454**, 328 (1999); Chin. J. Phys. (Taipei) **37**, 569 (1999).