Supplementary Materials

Learning-based dynamic ticket pricing for passenger railroad service providers

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1. More numerical examples

In the following two examples, the optimization model is applied in the very high season and the low season conditions to analyze its behavior facing various situations. The other parameters are similar to the previous example applied for the medium season condition. It is worth noting that the average competitors' ticket prices are different from the medium season conditions in the very high and the low seasons. The optimal global prices, sales amount, and the average competitors' prices for the two other examples are shown in Figs. S1 and S2.

It can be seen that, in a very high season condition, the optimization model prefers to set prices on the highest allowed price regardless of the competitors' pricing or the remaining days to departure (see Fig. S1). All the 300 available seats can be sold at the highest price at the end of the purchasing horizon. In low season conditions, the model starts the purchasing horizon with the low fares (See Fig. S2) to encourage the price-sensitive customers to book early. When there are about two weeks to departure, ticket prices be increased due to potential demand. The model behavior seems rational since it results in stable daily sales but higher prices. In the latest week, ticket prices are decreased to capture the higher demand volume near the departure to improve the earned revenue. It makes sense from the marketing point since a price-sensitive customer (due to the low season condition) is encouraged to book the ticket at the beginning of the purchasing horizon. However, it is not profitable for the company to stimulate early booking in the high seasons.

It is worth noting that the discussed model behavior does not follow a predefined set of actions since it depends on all the parameters, such as the demand model coefficients, competitors' prices, remaining capacity, remaining time to departure, and the allowed price range. Also, the case study is about a five-star commercial train whose main goal is maximizing the company's revenue, and the social side effects and responsibilities are not

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considered. However, some price regulator parameters (P^{lb}, P^{ub}, and MC) are embedded in the optimization model to align prices with the company's pricing policies.

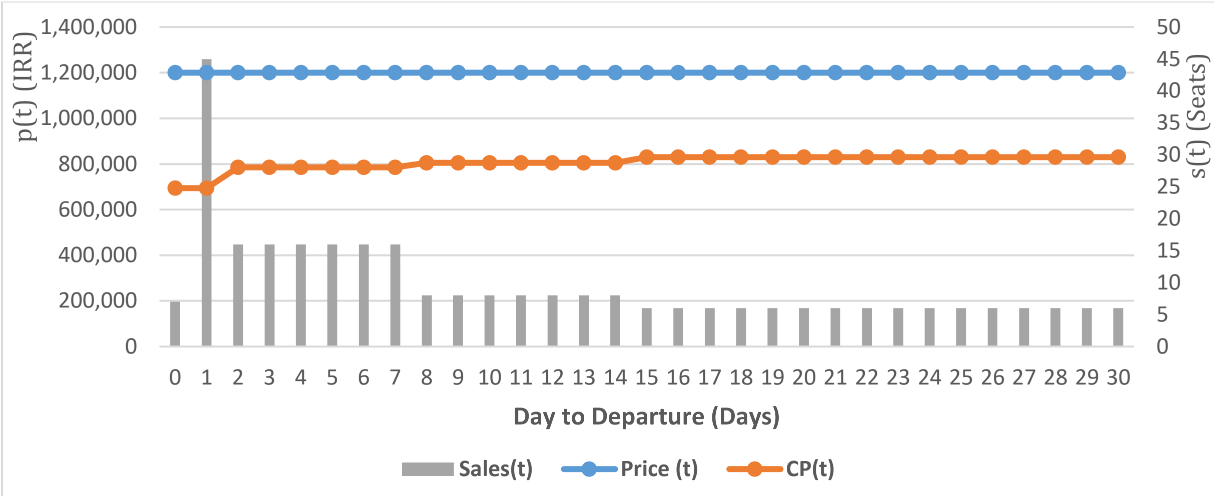

Fig. S1. Dynamic pricing for a very high season departure day in a 31-days purchasing horizon.

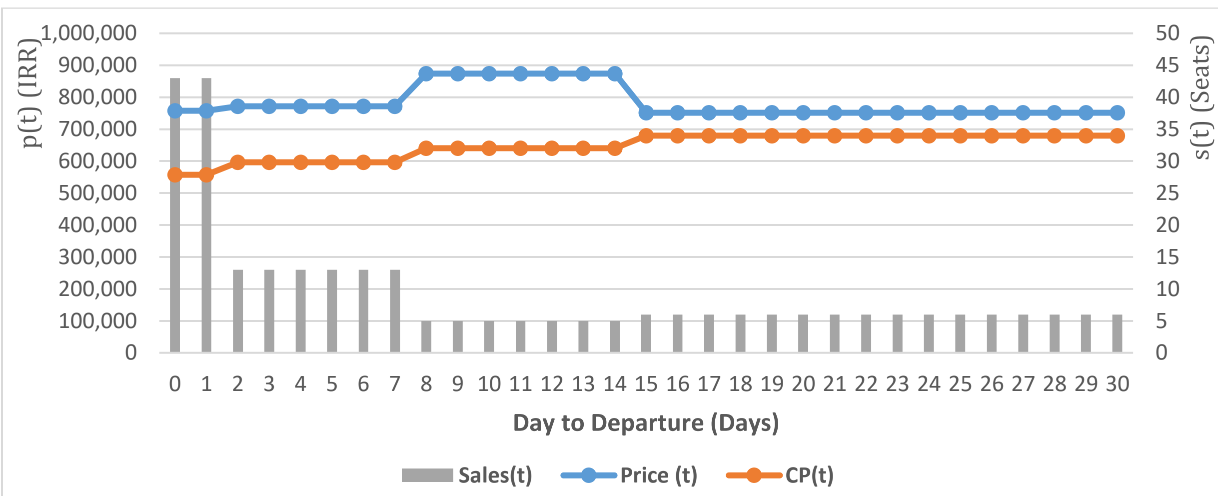

Fig. S2. Dynamic pricing for a low season departure day in a 31-days purchasing horizon.

2. Practical evaluation

The outputs of the optimization model are compared with the actual company's sales on a particular day. Therefore, a randomly selected departure day (2019-11-08), which belongs to the high season class (H), is chosen for evaluation, and the related results are presented in Table S1. This case helps to assess the practical potential of the proposed methodology in revenue enhancement.

Table S1 compares the results of the proposed methodology with the actual sales under the traditional pricing activities of the company. It shows that the revenue optimization methodology for the referred departure date could increase the revenue from 256,131,500 IRR to 434,949,987 IRR if implemented. Accordingly, the proposed RM methodology can improve the company's revenue. Also, for further evaluating the revenue potential growth by the methodology, a random sample of 30 different departure dates from 2018 and 2019 (before the Coronavirus pandemic) is chosen for statistical analysis. Table S2 shows the results.

A paired-sample *t*-test is employed to recognize whether the mean difference between two sets of observations is statistically significant. The statistical analysis is performed by Minitab 16 software, whose output is shown in Fig. S3. As the results show, at the 95% confidence level, the null hypothesis (μ_d: mean difference =0) is rejected, while the alternative hypothesis (μ_d>0) is accepted. On the other hand, the average percent of differences in Table S2 is about 23%. Hence, this implies that the proposed RM methodology has the excellent potential to improve the company's revenue.

Table S1. Practical evaluation results for a high season departure day (2019-11-08).

Days to Dep.		Optimization		Actual sales		Days to Dep.		Optimization		Actual sales	
(t)	$CP(t)$	$p(t)$	$s(t)$	$p(t)$	$s(t)$	(t)	CP(t)	$p(t)$	$s(t)$	$p(t)$	$s(t)$
0	655,285	1,143,792	55	828,388	40	25	719,904	1,120,065	5	850,000	0
1	674,236	1,194,466	53	848,543	47	26	669,333	1,148,213	4	850,000	2
2	663,911	1,155,087	12	828,552	29	27	692,000	1,200,000	3	850,000	2
3	657,212	1,151,448	12	938,889	9	28	734,314	1,150,039	5	850,000	4
4	681,405	1,133,726	13	783,217	23	29	777,007	1,199,999	5	850,000	0
5	679,830	1,194,599	11	807,800	30	30	819,700	1,194,309	6	656,750	4
6	650,716	1,147,917	12	773,300	10	31	764,722	1,199,998	5	850,000	4
7	635,332	1,070,463	14	873,733	15	32	849,035	1,199,999	6	850,000	21
8	625,599	1,199,999	5	887,929	7	33	813,455	1,181,318	6	850,000	3
9	667,907	1,193,840	6	842,333	9	34	780,366	1,199,999	5	850,000	0
10	642,789	1,199,999	5	850,000	4	35	780,366	1,199,999	5	850,000	0
11	658,166	1,170,815	6	850,000	2	36	780,366	1,199,999	5	850,000	0
12	750,251	1,199,999	7	850,000	6	37	780,366	1,199,999	5	850,000	0
13	716,931	1,158,200	7	850,000	0	38	780,366	1,199,999	5	850,000	0
14	683,611	1,199,999	6	805,167	6	39	780,366	1,112,494	6	850,000	0
15	675,368	1,200,000	3	850,000	1	40	747,278	1,177,004	5	850,000	2
16	686,847	1,184,642	4	850,000	0	41	850,000	1,199,999	6	754,125	4
17	698,327	1,200,000	4	850,000	2	42	850,000	1,199,999	6	850,000	6
18	691,005	1,178,059	4	850,000	0	43	656,985	1,199,999	3	828,000	4
19	683,683	1,178,059	4	754,125	4	44	774,250	1,199,999	5	850,000	2
20	754,341	1,191,696	5	754,125	0	**Total Revenue**		**434,949,987**		**256,131,500**	
21	754,341	1,058,363	6	754,125	0						
22	825,000	1,199,999	6	850,000	2						
23	797,737	1,148,625	6	850,000	0						
24	770,474	1,199,999	5	850,000	4						

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                        N       Mean      StDev   SE Mean
Optimization Revenue  30  405561717  167118069  30511445
Actual Revenue        30  333549367  136129848  24853796
Difference            30   72012350   66042548  12057664


95% lower bound for mean difference: 51524847
T-Test of mean difference = 0 (vs > 0): T-Value = 5.97  P-Value = 0.000
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Fig. S3. Paired *t*-test results to compare the optimization and the actual revenue means.

Table S2. Company's revenue for a random sample of 30 different departure dates.

Dep. Date	Actual (IRR)	Optimization	Diff. (%)	Dep. Date	Actual	Optimization	Diff. (%)
01/06/2018	655,388,000	834,762,257	27	06/11/2018	332,505,000	446,405,314	34
07/06/2018	180,723,000	265,826,908	47	17/11/2018	410,520,000	491,344,359	20
21/06/2018	405,251,500	495,544,734	22	06/12/2018	276,901,500	388,777,111	40
04/07/2018	386,750,500	465,953,377	20	15/12/2018	279,372,000	291,426,564	4
19/07/2018	369,180,500	523,015,292	42	24/12/2018	185,273,500	268,730,920	45
27/07/2018	545,177,000	501,560,534	-8	10/01/2019	255,167,500	312,971,428	23
07/08/2018	378,894,000	438,708,279	16	19/01/2019	339,233,000	419,423,253	24
24/08/2018	540,215,000	792,127,789	47	13/02/2019	312,458,500	406,240,250	30
27/08/2018	537,848,000	516,295,421	-4	18/02/2019	305,356,500	371,706,612	22
08/09/2018	388,957,500	402,023,023	3	27/02/2019	165,435,000	214,180,194	29
09/09/2018	491,484,500	673,991,976	37	10/03/2019	219,850,500	278,485,025	27
23/09/2018	225,700,000	283,042,106	25	02/04/2019	144,641,500	153,288,967	6
27/09/2018	98,845,500	117,513,523	19	05/04/2019	168,973,000	217,912,857	29
11/10/2018	410,372,500	551,026,277	34	14/04/2019	274,718,500	322,037,808	17
18/10/2018	457,973,000	405,577,811	-11	20/04/2019	263,314,500	316,951,536	20