Learning Impedance Control for Robotic Manipulators

Chien-Chern Cheah and Danwei Wang

***Abstract*—Learning control is a concept for controlling dynamic systems in an iterative manner. It arises from the recognition that robotic manipulators are usually used to perform repetitive tasks. Most researches on the iterative learning control of robots have been focused on the problem of free motion control and hybrid position/force control where the learning controllers are designed to track the desired motion and force trajectories. The iterative learning impedance control of robotic manipulators, however, has been studied recently. In this paper, an iterative learning impedance control problem for robotic manipulators is formulated and solved. A target impedance is specified and a learning controller is designed such that the system follows the desired response specified by the target model as the actions are repeated. A design method for analyzing the convergence of the learning impedance system is developed. A sufficient condition for guaranteeing the convergence of the system is also derived. The proposed learning impedance control scheme is implemented on an industrial selective compliance assembly robot arm (SCARA) robot, SEIKO TT3000. Experimental results verify the theory and confirm the effectiveness of the learning impedance controller.**

***Index Terms*—Convergence analysis, impedance control, iterative learning algorithm, robot force control.**

I. Introduction

MOST of today's industrial manipulators are used for tasks such as materials transfer, spray-painting, and spot welding, of which operations can be adequately handled by simple position control strategies. To expand the feasible applications of robots, it is necessary to control not only the motion but also the forces of interacting between the manipulator and the environment. Assembly, polishing, and deburring are typical examples of such tasks. Several control laws have been developed for simultaneous control of both motion and force [31], [38] of robotic manipulators. Despite the diversity of approaches, it is possible to classify most of the design procedures as based on two major approaches:

1) impedance control [19];
2) hybrid position/force control [29].

A number of researchers have proposed different implementation of hybrid position/force control and impedance control. When the structures and parameters of the robot dynamics model are known precisely, many model-based control theories and design methods, e.g., [19], [28], [36], [39] can be used to design nonlinear controllers for simultaneous motion and force control. However, due to parametric uncertainties, it is difficult to derive the full description of the dynamics. Furthermore, because of the nonlinearity of the dynamics, the identification and estimation techniques [7], [26], [30] could not be easily deployed.

Recently, there have been many studies in the topic of learning control for controlling of robotic systems in an iterative manner. In this paper, learning controllers are referred to the class of control systems that generate a control action in an iterative manner to execute a prescribed action which is defined in [4], [31]. A recent survey by Arimoto can be found in [31]. This control concept arises from the recognition that robotic manipulators are usually employed to perform repetitive tasks [4], [15]. Learning control schemes are easy to implement and do not require exact knowledge of the dynamic model. Several learning motion control laws [2], [4], [6], [15], [18], [24], [32], and learning Hybrid Position/Force control laws [1], [9], [12], [22], [37] have been developed for iterative learning control of robotic manipulators. The feedforward control inputs are learned such that the system tracks the desired motion and force trajectories as the actions are repeated. The iterative learning impedance control for robotic manipulators has been developed recently with some analytical and experimental results [10], [34], [35].

The concept of active control of a manipulator's interactive behavior is formally treated as an aspect of impedance control [19]. Hogan [19] stresses the necessity of control of the manipulator impedance based on the assertion that it is not sufficient to control position and force variables alone. Impedance control does not attempt to track motion and force trajectories but rather to regulate the mechanical impedance [19] specified by a target model. Impedance control provides a unified approach to all aspects of manipulation [19]. Both free motion and contact tasks can be controlled using a single control algorithm. It is unnecessary to switch between control modes as task conditions change. The nature of the trajectory learning formulation has prohibited the research into the impedance control problem because in impedance control, a target impedance is specified rather than the trajectory. There exists, however, another nonclassical approach of neural-network learning impedance control methods [5], [14], [17], [20], [21], [33]. However, unlike iterative learning approach [31], it is difficult to provide a theoretical framework for analyzing the learning system, guaranteeing its convergence and guiding its applications using such formulations.

In this paper, an iterative learning impedance control problem for robotic manipulators is formulated and solved. In contrast to most of the iterative learning controller designs

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in the literature, our approach allows the performance of the learning system to be specified by a reference model (or target impedance) in addition to the reference trajectory. A target impedance [19] is specified and the feedforward control input is learned such that the system follows the desired response specified by the target model as the actions are repeated. A design method for analyzing the convergence of the learning impedance control system is developed. A sufficient condition for guaranteeing the convergence of the learning impedance control system is also derived. In the experiment, an industrial selective compliance assembly robot arm (SCARA) robot, SEIKO TT3000, is used to verify the theory and to evaluate the feasibility and performance of the proposed learning impedance controller. A single learning controller was implemented without the need to switch the learning controller from non contact to and from contact tasks as needed in most of the learning controllers in the literature. Experimental results showed that the proposed learning impedance controller reduced the impedance error dramatically as the operations are repeated.

The remainder of this paper is organized as follows. Section II formulates the robot dynamic equations and control problem. Section III presents the learning impedance control for robotic manipulators, Section IV presents the application of the proposed controller to an industrial robot, and Section V concludes this paper. A preliminary version of the work in this paper was also presented in [10].

II. Robot Dynamic Equation and Problem Formulation

The equation of motion for the constrained robotic manipulator with n degrees of freedom, considering the contact force and the constraints, is given in the joint space as follows [25]:

$$\overline{M}[q_k(t)]\ddot{q}_k(t) + \overline{V}[q_k(t),\, \dot{q}_k(t)] = \tau_k(t) + f_k(t) \tag{1}$$

where $q_k(t) \in R^n$ denotes the joint angles of the manipulator at the kth operation, $\overline{M}(\cdot) \in R^{n\times n}$ is the robot inertia matrix which is symmetric and positive definite for all $q_k(t) \in R^n$, $\overline{V}(\cdot,\cdot) \in R^n$ contains the centrifugal Coriolis and gravitational forces, $f_k(t) \in R^n$ is the interaction forces/moments associated with the constraints, $\tau_k(t) \in R^n$ denotes the control inputs and $t \in [0,\, t_f]$ is the operation interval.

It is well known that when the robot's end-effector contacts the environment, a task space coordinate system defined with reference to the environment is convenient for the study of contact motion [25]. Let $X_k(t) \in R^n$ be the task space vector defined by [25]

$$X_k(t) = h[q_k(t)] \tag{2}$$

where $h(\cdot) \in R^n \to R^n$ is generally a nonlinear transformation describing the relation between the joint and task space. Then, the derivatives of $X_k(t)$ are given as

$$\begin{aligned} \dot{X}_k(t) &= J[q_k(t)]\dot{q}_k(t) \\ \ddot{X}_k(t) &= J[q_k(t)]\ddot{q}_k(t) + \dot{J}[q_k(t)]\dot{q}_k(t) \end{aligned} \tag{3}$$

where $J(\cdot) = \partial h(\cdot)/\partial q \in R^{n\times n}$ is the Jacobian matrix. It is assumed that the robotic manipulator is operating in a finite workspace such that $J(\cdot)$ is nonsingular and therefore the mapping between $q(t)$ and $X(t)$ is one-to-one by applying the implicit function theorem [27]. The equation of motion can therefore be expressed in the task space as [25]

$$M[X_k(t)]\ddot{X}_k(t) + V[X_k(t),\, \dot{X}_k(t)] = F_k(t) + T_k(t) \tag{4}$$

where

$$\begin{aligned} M[X_k(t)] &= J^{-T}[q_k(t)]\overline{M}[q_k(t)]J^{-1}[q_k(t)] \\ V[X_k(t),\, \dot{X}_k(t)] &= J^{-T}[q_k(t)]\overline{V}[q_k(t),\, \dot{q}_k(t)] - \overline{M}[q_k(t)] \\ &\quad \cdot J^{-1}[q_k(t)]\dot{J}[q_k(t)]J^{-1}[q_k(t)]\dot{X}_k(t) \\ F_k(t) &= J^{-T}[q_k(t)]f_k \\ T_k(t) &= J^{-T}[q_k(t)]\tau_k(t). \end{aligned}$$

Clearly, in the case where the task space is the joint space, we have

$$h[q_k(t)] = q_k(t) \tag{5}$$

and hence $J[q_k(t)] = I$. It is important to note that $M(\cdot)$ is a symmetric and positive definite matrix [30]. We consider the stiffness relation between $F_k(t)$ and $X_k(t)$ at the contact point be dominated by

$$F_k(t) = K_s[X_s(t) - X_k(t)] \tag{6}$$

where $K_s \in R^{n\times n}$ is a symmetric and positive definite stiffness matrix that describes the environment stiffness. The vector $X_s(t) \in R^n$ can be seen as representing the location to which the contact point $X_k(t)$ would return in the absence of contact force. Note that in this paper, we assume that the environment stiffness K_s and the static position $X_s(t)$ are unknown. The specifications of the impedance control problem are given in terms of a reference motion trajectory and a desired dynamic relationship between the position error and the interaction force. Impedance control does not attempt to track motion and force trajectories but rather to control motion and force by developing a relationship between interaction forces and manipulator position [19], that is, the mechanical impedance. The target impedance [19] is specified as

$$\begin{aligned} M_m[\ddot{X}_d(t) - \ddot{X}(t)] &+ C_m[\dot{X}_d(t) - \dot{X}(t)] \\ &+ K_m[X_d(t) - X(t)] = -F(t) \end{aligned} \tag{7}$$

where M_m, C_m, and $K_m \in R^n$ are positive definite matrices which specify the desired dynamic relationship between the reference position error and the interaction force and $\ddot{X}_d(t) \in R^n$, $\dot{X}_d(t) \in R^n$, $X_d(t) \in R^n$ are the reference acceleration, velocity, and position, respectively. For learning impedance control design, we assume that M_m, C_m, and K_m are chosen such that $M_m^{-1}C_m$, $M_m^{-1}(K_s + K_m)$, and $M_m^{-1}(K_s + K_m)M_m^{-1}C_m$ are symmetric matrices. For instances, when all the matrices are diagonal matrices, the multiplication of the diagonal matrices will also be diagonal and symmetric.

The objective of Learning Impedance Control design is to develop an iterative learning law such that the system response satisfies the behavior of the specified target impedance (7) for all $t \in [0,\, t_f]$ as the actions are repeated [10], [11]. That is, as $k \to \infty$

$$w_k(t) \to 0 \tag{8}$$

where

$$w_k(t) = M_m[\ddot{X}_d(t) - \ddot{X}_k(t)] + C_m[\dot{X}_d(t) - \dot{X}_k(t)] + K_m[X_d(t) - X_k(t)] + F_k(t) \tag{9}$$

is defined as the impedance error.

Remark 1: In the conventional iterative learning control formulation, the controller is designed to track a desired trajectory $y_d(t)$ as the action is repeated. In general [31]

$$y_k(t) \rightarrow y_d(t), \qquad \text{as } k \rightarrow \infty \tag{10}$$

where $y_k(t)$ is the motion and/or force trajectory. In our learning approach, the control objective can be specified by a target impedance (or reference model) as seen from (8) and (9). Furthermore, since the desired motion and force trajectories cannot be derived from the reference model (7) because K_s and $X_s(t)$ are unknown, the conventional trajectory learning control cannot be applied directly for learning the desired model explicitly from the desired trajectories.

Remark 2: From (9), the reference trajectory error $X_d(t) - X_k(t)$ can be written in s-domain or Laplace domain as

$$X_d(s) - X_k(s) = [M_m s^2 + C_m s + K_m]^{-1} \cdot [-F_k(s) + w_k(s)]. \tag{11}$$

Therefore, in the special case of free motion or non contact task where the contact force is zero, the reference trajectory error also converges to zero in addition to the convergence of the reference model error $w_k(t)$ because

$$X_d(s) - X_k(s) = [M_m s^2 + C_m s + K_m]^{-1} w_k(s). \tag{12}$$

Hence, the learning impedance control scheme can be applied to both contact and noncontact tasks. Using the learning impedance approach, a unified learning controller can be developed for both contact and non contact tasks without the need to switch the learning controllers from non contact to and from contact tasks for learning control of robotic manipulator. This is important since the current iterative learning control designs provide methods to control robots during contact and free motion separately. From a practical point of view, most tasks involve a transition from free motion to contact motion and every contact task ends with a transition from contact to free motion. Therefore, when these different control schemes are applied to the robots, the learning algorithms are needed to switch from one control to another and therefore the overall control is discontinuous in nature. □

III. Learning Impedance Control

In this section, we present the learning impedance controller for robotic manipulators. We suppose that a feedback control law [25] has been designed for stability of the closed-loop system as

$$T_k(t) = K_p[X_d(t) - X_k(t)] + K_v[\dot{X}_d(t) - \dot{X}_k(t)] + K_y y_k(t) + m_k(t) \tag{13}$$

where $K_p \in R^{n\times n}$, $K_v \in R^{n\times n}$, $K_y \in R^{n\times n}$ are feedback and compensator gains to be chosen, $m_k(t) \in R^n$ is a feedforward learning control input, and $y_k(t) \in R^n$ is an intermediate state variable. In this control law, a dynamic compensator $y_k(t)$ is introduced and a learning control input $m_k(t)$ is added and updated according to an iterative rule, so that the system response is identical to the behavior of the target impedance specified by (7) as the action is repeated. This iterative learning control law is proposed as

$$m_{k+1}(t) = m_k(t) + \beta K_v z_k(t) \tag{14}$$

where

$$z_k(t) = a[X_d(t) - X_k(t)] + [\dot{X}_d(t) - \dot{X}_k(t)] + y_k(t) \tag{15}$$

is an intermediate reference model error and $\beta \in (0, 2)$ and a are positive constants. The dynamic compensator $y_k(t)$ is introduced as

$$\dot{y}_k(t) + \alpha y_k(t) = L_p[X_d(t) - X_k(t)] + L_v[\dot{X}_d(t) - \dot{X}_k(t)] + L_r F_k(t) \tag{16}$$

where $L_p \in R^{n\times n}$, $L_v \in R^{n\times n}$, $L_r \in R^{n\times n}$ are the feedback gains to be defined and α is a positive constant to be chosen. Without the introduction of $y_k(t)$ described by (16) in the control laws (13) and (14), the resulting learning system is a PI-type learning system [2]. The uniform boundedness result of the tracking errors of the PI-type learning system can be analyzed as in [2], [3] using the passivity concept. Since the feedback system described by (16) is strictly passive, the stability of the interconnected system with the feedback control laws (13) and (16) can also be studied using passivity theorem [16]. Another useful theorem for studying the stability of this interconnected system is the application of small gain theorem [16].

From (15), differentiate $z_k(t)$ with respect to time, we have

$$\dot{z}_k(t) = a[\dot{X}_d(t) - \dot{X}_k(t)] + [\ddot{X}_d(t) - \ddot{X}_k(t)] + \dot{y}_k(t). \tag{17}$$

Substitute (15) and (16) into the above equation to eliminate $y_k(t)$ result in

$$M_m[\ddot{X}_d(t) - \ddot{X}_k(t)] + C_m[\dot{X}_d(t) - \dot{X}_k(t)] + K_m[X_d(t) - X_k(t)] + F_k(t) = M_m[\dot{z}_k(t) + \alpha z_k(t)] \tag{18}$$

where L_p, L_v, and L_r in (16) are chosen as $L_p = M_m^{-1}K_m - a\alpha I_n$, $L_v = M_m^{-1}C_m - aI_n - \alpha I_n$, and $L_r = M_m^{-1}$. Therefore, by choosing the compensator gains L_p, L_v, and L_r appropriately, the system response converges to that specified by the target impedance (7) if $z_k(t)$ and $\dot{z}_k(t)$ converge to zero for all $t \in [0, t_f]$. Alternatively, from (11), we have

$$\overline{w}_k(s) \triangleq X_d(s) - X_k(s) + [M_m s^2 + C_m s + K_m]^{-1} F_k(s). \tag{19}$$

The learning impedance control problem can be restated as that of designing a learning controller so that

$$\overline{w}_k(t) \rightarrow 0, \qquad \text{as } k \rightarrow \infty \tag{20}$$

where $\overline{w}_k(t)$ is the inverse Laplace transformation of $\overline{w}_k(t)$ and is defined as the indirect target impedance error. From (18) and (19), we have

$$\overline{w}_k(s) = [M_m s^2 + C_m s + K_m]^{-1} M_m (s + \alpha) z_k(s). \tag{21}$$

Since (21) is stable and strictly proper linear system with the input $z_k(t)$ and output $\overline{w}_k(t)$, from the theory of linear system [16], if $z_k(t)$ converges to zero for all $t \in [0, t_f]$, the indirect target impedance error $\overline{w}_k(t)$ converges to zero for all $t \in [0, t_f]$.

To guarantee the convergence of the learning impedance control system, the controller gains K_p, K_v, K_y, L_p, L_v, and L_r have to be chosen carefully. This is made precise in the following Theorem:

Theorem: Consider the learning control systems given by (4), (6), (13), (14), and (16) with the target impedance specified by (7). Let the feedback gains and compensator gains K_p, K_v, K_y, L_p, L_v, and L_r be chosen as

$$\begin{aligned} K_p &= ak_1 I_n,\ K_y = k_1 I_n,\ L_v = M_m^{-1}C_m - aI_n - \alpha I_n \\ K_v &= k_1 I_n,\quad L_r = M_m^{-1},\ L_p = M_m^{-1}K_m - a\alpha I_n \end{aligned} \tag{22}$$

where a is a positive constant and k_1 and α are constants chosen to satisfy the following conditions:

$$\begin{aligned} l_1 &\triangleq (2-\beta)k_1 - \frac{c_2}{\gamma} \geq 0 \\ l_2 &\triangleq (2-\beta)k_1 - \frac{c_3}{c_7} - \frac{c_4^2}{\gamma l_1 c_7} \geq 0 \\ l_3 &\triangleq (2-\beta)k_1\left(1 - \frac{c_5^2}{\gamma^2\lambda_1^2 l_1 l_2}\right) \\ &\quad - \frac{c_6^2}{\gamma\lambda_1^2 c_7 l_2} - \frac{c_9}{\gamma^2\lambda_1^2 c_7 l_1 l_2} - \frac{c_8}{\gamma\lambda_1^2} \geq 0 \end{aligned} \tag{23}$$

where $\gamma < \min\{(\lambda_{12}\lambda_2)^{1/2}/b_{a1},\ \lambda_2^2/2b_{a1},\ 1\}$, $\lambda_1 = \lambda_{\min}[A_1] > 0$, $\lambda_2 = \lambda_{\min}[A_2] > 0$, $\lambda_{12} = \lambda_{\min}[A_1 A_2] > 0$, $A_1 = M_m^{-1}(K_m + K_s) - \alpha M_m^{-1}C_m + \alpha^2 I$, $A_2 = M_m^{-1}C_m - \alpha I$, b_{a1} denotes the norm bound for A_1 and $c_1 \cdots c_9$ are constants to be defined. Then, a sequence of control inputs will be generated such that the desired response specified by the target impedance (7) is reached. That is

$$\begin{aligned} w_k(t) &= M_m[\ddot{X}_d(t) - \ddot{X}_k(t)] + C_m[\dot{X}_d(t) - \dot{X}_k(t)] \\ &\quad + K_m[X_d(t) - X_k(t)] + F_k(t) \to 0 \end{aligned} \tag{24}$$

for all $t \in [0, t_f]$ as $k \to \infty$.

Proof: Refer to the Appendix. △△△

Remark 3: Equation (23) states the sufficient conditions for the convergence of the target impedance error. Note that several terms in (23) of the Theorem are inversely proportional to l_1 and l_2. Hence, increasing l_1, l_2 decreases these terms. Therefore, k_1 can be chosen such that l_1, l_2, and $l_3 \geq 0$.

Remark 4: Notice that the iterative learning impedance scheme described by (14), (15), and (16) does not require the measurement or estimation of the force derivative as in [23] or the acceleration as in [1], [9], and [13].

Remark 5: Suppose that $A_{m1} = M_m^{-1}K_m$, $A_{m2} = M_m^{-1}C_m$ are defined using the coefficient matrices of the desired model in (7) and are chosen as diagonal such that

$$A_{m2} = 2\zeta\omega_n I_n,\ A_{m1} = \omega_n^2 I_n \tag{25}$$

where ζ is the damping factor and ω_n is the undamped natural frequency. For illustration purpose, if $F(t) = 0$, the desired model is given by

$$\begin{aligned} &[\ddot{X}_d(t) - \ddot{X}_k(t)] + 2\zeta\omega[\dot{X}_d(t) - \dot{X}_k(t)] \\ &\quad + \omega^2[X_d(t) - X_k(t)] = 0. \end{aligned} \tag{26}$$

Then, from the definition of γ in the theorem, we have

$$\gamma < \min\left\{\frac{(\lambda_{12}\lambda_1)^{1/2}}{b_{a1}},\ \frac{\lambda_1^2}{2b_{a1}},\ 1\right\} = \min\{2\zeta,\ 2\zeta^2,\ 1\} \tag{27}$$

for α chosen to be zero. Hence, for a system that is sufficiently damped so that $\zeta^2 \geq \frac{1}{2}$, we have $\gamma < 1$. If system is lightly damped such that, $0 < \zeta^2 < \frac{1}{2}$, we have $\gamma < 2\zeta^2$ and hence the maximum value of γ decreases with decreasing damping factor. From (23), we can deduce that a higher controller gain is needed for a desired system response with light damping. This is because for such a system, a high overshoot arises and hence a higher controller gain is required to suppress it.

Similarly, in the presence of contact force, the desired model can be expressed by

$$\begin{aligned} &[\ddot{X}_d(t) - \ddot{X}_k(t)] + 2\zeta\omega[\dot{X}_d(t) - \dot{X}_k(t)] + (\omega^2 + K_s) \\ &\quad \cdot [X_d(t) - X_k(t)] = K_s[X_d(t) - X_s(t)]. \end{aligned} \tag{28}$$

Therefore, in the case of very stiff environment, the target impedance is a lightly damped system which required a higher controller gain to guarantee the convergence of the learning impedance system.

Remark 6: In paper [34] and [35] by Wang and Cheah, another impedance learning control scheme is developed to tackle the same problem. In comparison, the impedance learning controller in [34] and [35] uses the impedance error $w_k(t)$ directly in the iterative learning law for updating $m_k(t)$. While in the approach developed in this paper, the impedance error $w_k(t)$ is incorporated in an indirectly manner. In particular, $w_k(t)$ does not appear directly in the set of controller equations (13)–(16) and the target impedance for learning is realized in (18). Furthermore, a discrete time scheme corresponding to the approach in [34] and [35] has been developed in [8]. □

IV. Experiment

In a practical robot system, many disturbances are present. Although the robustness analysis of the learning control system to certain practical issues has been developed [2], [18], [31], implementing the proposed learning schemes in real time experiments allows the investigation of the robustness and the feasibility of the actual implementations. In this section, the proposed learning impedance controller is applied to an industrial robot and experimental results are presented.

A. Experimental Setup

The robot used in this experiment is the industrial robot SEIKO TT3000 as shown in Fig. 1. This robot is the SCARA type manipulator with three degrees of freedom as illustrated in the schematic diagram of Fig. 2. The first joint is a prismatic joint, the second and third joints are revolute joints.

The dynamics model of the robotic arm [25] can be described by (4) as explained in Section II. The parameters of

Fig. 1. The experimental setup.

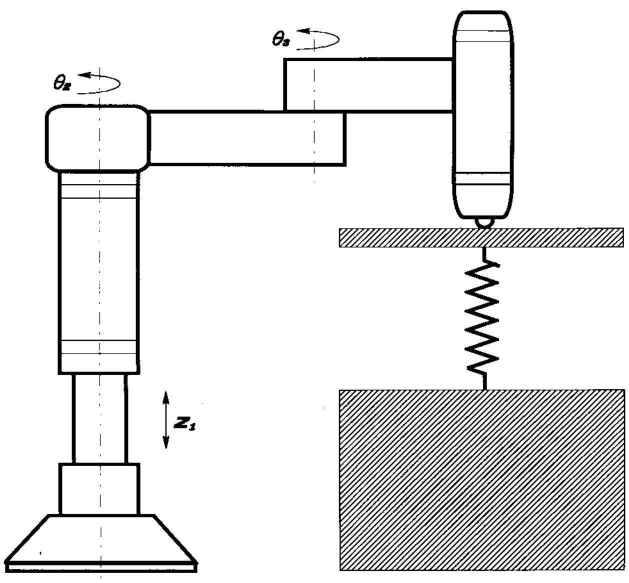

Fig. 2. A SCARA robot.

the SCARA robot can be detailed as

$$M(q) = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}, \; V(q, \dot{q}) = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$
$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}, \; f = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \tag{29}$$

where

$$\begin{aligned} m_{11} &= m_1 + m_2 + m_3 \\ m_{12} &= 0 \\ m_{13} &= 0 \end{aligned}$$

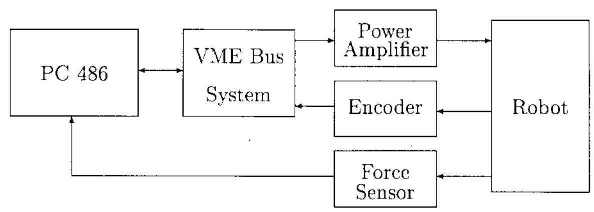

Fig. 3. Block diagram of the experimental system.

$$\begin{aligned} m_{22} &= (m_2 + m_3)a_2^2 + m_3 a_3^2 + 2m_3 a_2 a_3 \cos(\theta_3) \\ m_{23} &= m_3 a_3^2 + m_3 a_2 a_3 \cos(\theta_3) \\ m_{33} &= m_3 a_3^2 \\ v_1 &= m_{11} g \\ v_2 &= -m_3 a_2 a_3 \sin(\theta_3)(\dot{\theta}_3^2 + 2\dot{\theta}_2\dot{\theta}_3) \\ v_3 &= m_3 a_2 a_3 \sin(\theta_3)\dot{\theta}_2^2 \end{aligned} \tag{30}$$

and m_1, m_2, and m_3 are the masses of link one, two, and three, respectively, in kilograms, a_2 and a_3 are the length of link two and three, respectively, in meters, and g is the constant acceleration due to gravity in meter per second.

The hierarchical structure of the robot control system is shown in Fig. 3. At the top of the system hierarchy is the robot supervisory Computer using a PC 486 and at the lower level are the multiprocessors using a VME bus-based system. The lower level system is used for real time data collection and control. This VME bus-based system consists of the host computer MVME 147 and the target computer MVME104. The MVME 147 is a MC68030 based system with 4 MB DRAM and a 25 MHz system clock and MVME104 is a MC68010 based system with 512 kB of RAM and a 10 MHz system clock. The MVME104 is also responsible for input/output operations using four channels for the encoder inputs and four channels for the digital to analog converters.

Three encoders are employed for position measurement of each joint and a differentiator is used to estimate the velocity from the position measurements. The pulses per revolution for encoder two and three are 600 and 800, respectively. For the prismatic joint, one pulse equals 0.010 44 mm. To measure the contact force, a force sensor made by Lord is mounted on the end-effector of the robot.

B. Experimental Results

To effectively verify the proposed learning impedance control law, the end-effector was set to follow a path which involved free motion tracking, transition from free motion to contact motion, contact motion on the constraint plane with compliance, transition from contact motion to free motion, and finally free motion tracking again as illustrated in Fig. 4. Here, the joint space is chosen as the task space since the contact task in this experiment can be conveniently described by the joint axis 1 (or z axis) as shown in Figs. 2 and 4. Therefore

$$X(t) = \begin{bmatrix} z_1(t) \\ \theta_2(t) \\ \theta_3(t) \end{bmatrix}. \tag{31}$$

Mathematically, the task can be specified by the reference model (7) as

$$\begin{aligned} M_m &= diag[50, 40, 40] \\ C_m &= diag[200, 200, 200] \\ K_m &= diag[800, 1000, 1000] \end{aligned} \tag{32}$$

where the reference trajectories $X_d(t)=[z_{1d}^T(t), \theta_{2d}^T(t), \theta_{3d}^T(t)]^T$ are described by the following equations as given in (33), shown at the bottom of the page. Here, $z_{1d}(t)$ is specified in meters, $\theta_{2d}(t)$ and $\theta_{3d}(t)$ are specified in radians. The sampling frequency f_s was 244 Hz and the period t_f of the whole operation was 3600/f_s s. In this experiment, a steel ball is attached to the force sensor and hence the frictional force along the constraint plane is negligible. In another words

$$F = \begin{bmatrix} f_1(t) \\ 0 \\ 0 \end{bmatrix}. \tag{34}$$

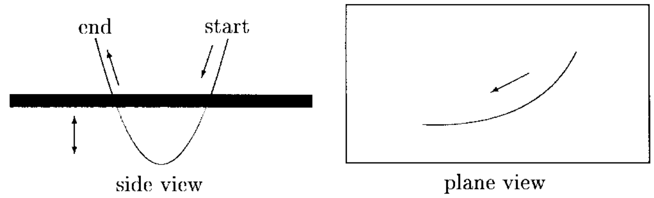

Fig. 4. End-effector path.

The impedance learning control law which described by (13), (14), and (16) were applied to the robotic system with the controller gains set as

$$\begin{aligned} K_p &= diag[10, 10, 250] \\ K_v &= K_y = diag[12, 12, 100] \\ L &= diag[12, 12, 150]. \end{aligned} \tag{35}$$

For joint two and three, $\alpha = 4$, $\beta = 1$ were chosen and a was calculated as $a = k_p/k_v = \frac{10}{12}$. For the independent joint one, $\alpha = 1$, $\beta = 1.5$ were chosen and a was calculated as $a = k_p/k_v = \frac{250}{100}$. The compensator gains L_p, L_v, and L_r were calculated based on (22). The impedance error was calculated as

$$\begin{aligned} w_k(t) = M_m[\ddot{X}_d(t) - \ddot{X}_k(t)] + C_m[\dot{X}_d(t) - \dot{X}_k(t)] \\ + K_m[X_d(t) - X_k(t)] + F_k(t) \end{aligned} \tag{36}$$

and the experimental results of the impedance errors, the trajectory errors $(X_d - X_k)$ and the contact force $f_1(t)$ are shown in Figs. 5–11. In the first trial, i.e., $k = 0$, $m_0(t)$ was also set to zero for all $t \in [0, t_f]$ and hence the controller is a feedback law with no learning control. As the operation repeated, the impedance errors decreased as shown in Figs. 5–7. From Figs. 8–10, the results also showed that the trajectory tracking errors decreased when the impedance errors decreased. It should be noted that in Fig. 8, the reference trajectory error for joint one converged to a steady state value described by (11) in the presence of contact force. Notice also that the impedance errors converged even though the contact points were changing at every iteration as shown by

$$z_{1d}(t) = \begin{cases} -0.045\left(\dfrac{6f_s^5}{1500^5}t^5 - \dfrac{15f_s^4}{1500^4}t^4 + \dfrac{10f_s^3}{1500^3}t^3\right) & \text{for } 0 \le t < \dfrac{1500}{f_s} \\ -0.045 + 0.045\left(\dfrac{6f_s^5}{1500^5}t^5 - \dfrac{15f_s^4}{1500^4}t^4 + \dfrac{10f_s^3}{1500^3}t^3\right) & \text{for } \dfrac{1500}{f_s} \le t < \dfrac{3000}{f_s} \\ 0 & \text{for } \dfrac{3000}{f_s} \le t \le \dfrac{3600}{f_s} \end{cases}$$

$$\theta_{2d}(t) = \begin{cases} 2.017 + 0.6\left(\dfrac{6f_s^5}{3000^5}t^5 - \dfrac{15f_s^4}{3000^4}t^4 + \dfrac{10f_s^3}{3000^3}t^3\right) & \text{for } 0 \le t < \dfrac{3000}{f_s} \\ 2.617 & \text{for } \dfrac{3000}{f_s} \le t \le \dfrac{3600}{f_s} \end{cases}$$

$$\theta_{3d}(t) = \begin{cases} 1.885 - 0.5\left(\dfrac{6f_s^5}{3000^5}t^5 - \dfrac{15f_s^4}{3000^4}t^4 + \dfrac{10f_s^3}{3000^3}t^3\right) & \text{for } 0 \le t < \dfrac{3000}{f_s} \\ 1.385 & \text{for } \dfrac{3000}{f_s} \le t \le \dfrac{3600}{f_s} \end{cases} \tag{33}$$

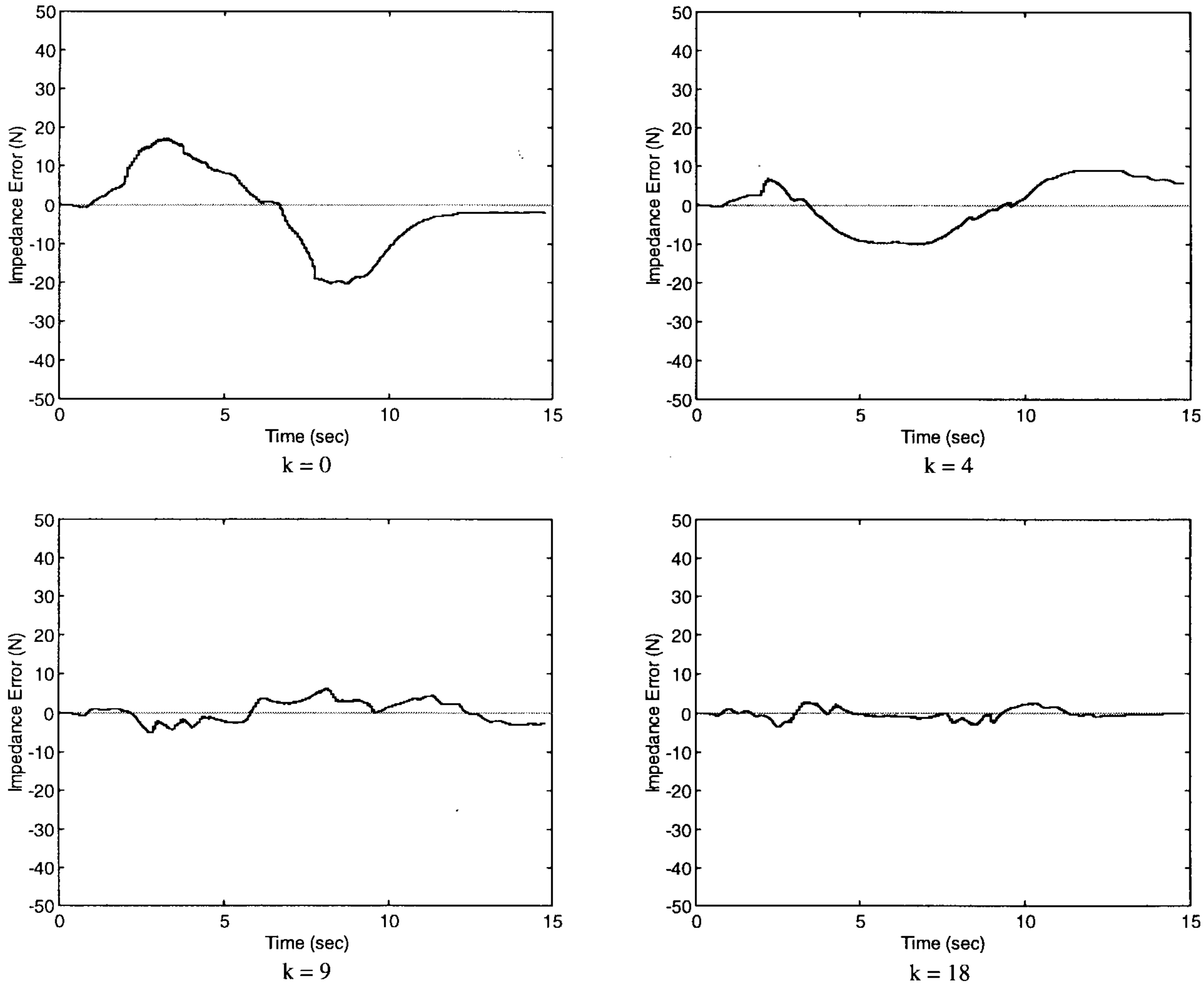

Fig. 5. The impedance error of joint one.

the contact force in Fig. 11. The experimental results illustrate the validity of the theory presented in Section III and show that the learning impedance controller reduces the impedance error tremendously. These results also illustrate the superiority of learning control as compared to no learning control on the first trial.

V. Conclusion

An iterative learning impedance control problem is formulated and solved for robotic manipulators. In contrast to most of the iterative learning controller designs in the literature, whereby a reference trajectory is given and a learning algorithm is designed to make the trajectory tracking error converges to zero as the action is repeated, our approach allows the performance of the learning system to be specified by a target impedance in addition to the reference trajectory. Given a target impedance, the learning controller is able to learn and eventually drives the closed loop dynamics to follow the response of the target impedance as the actions are repeated. A design method for analyzing the convergence of the learning impedance system is developed. A sufficient condition is also derived to guarantee the convergence of the learning controller.

The proposed learning impedance controller was applied to control of an industrial robot SEIKO TT3000 with three degrees of freedom. Experimental results verified the proposed theory and illustrated the robustness of the learning controller. A single learning controller was implemented without the need to switch the learning controller from non contact to and from contact task as needed in most of the iterative learning controllers in the literature. The development of this learning impedance control law should lead to further research and applications in learning control and force control for robot applications.

Appendix

Proof of Theorem: For clarity of the proof, the dependence of the system parameters on time is implied unless otherwise specified. Equation (7) can be rewritten as

$$\ddot{X}_d - \ddot{X}_k + A_{m2}(\dot{X}_d - \dot{X}_k) + A_{m1}(X_d - X_k) = B_m F_k \quad (37)$$

where $A_{m1} = M_m^{-1} K_m$, $A_{m2} = M_m^{-1} C_m$, and $B_m = -M_m^{-1}$. From (6) and (37), we have a desired state $[X_e^T, \dot{X}_e^T]^T$ and a desired force F_e as

$$(\ddot{X}_d - \ddot{X}_e) + A_{m2}(\dot{X}_d - \dot{X}_e) + A_{m1}(X_d - X_e) = B_m F_e \quad (38)$$

where

$$F_e = K_s(X_s - X_e). \quad (39)$$

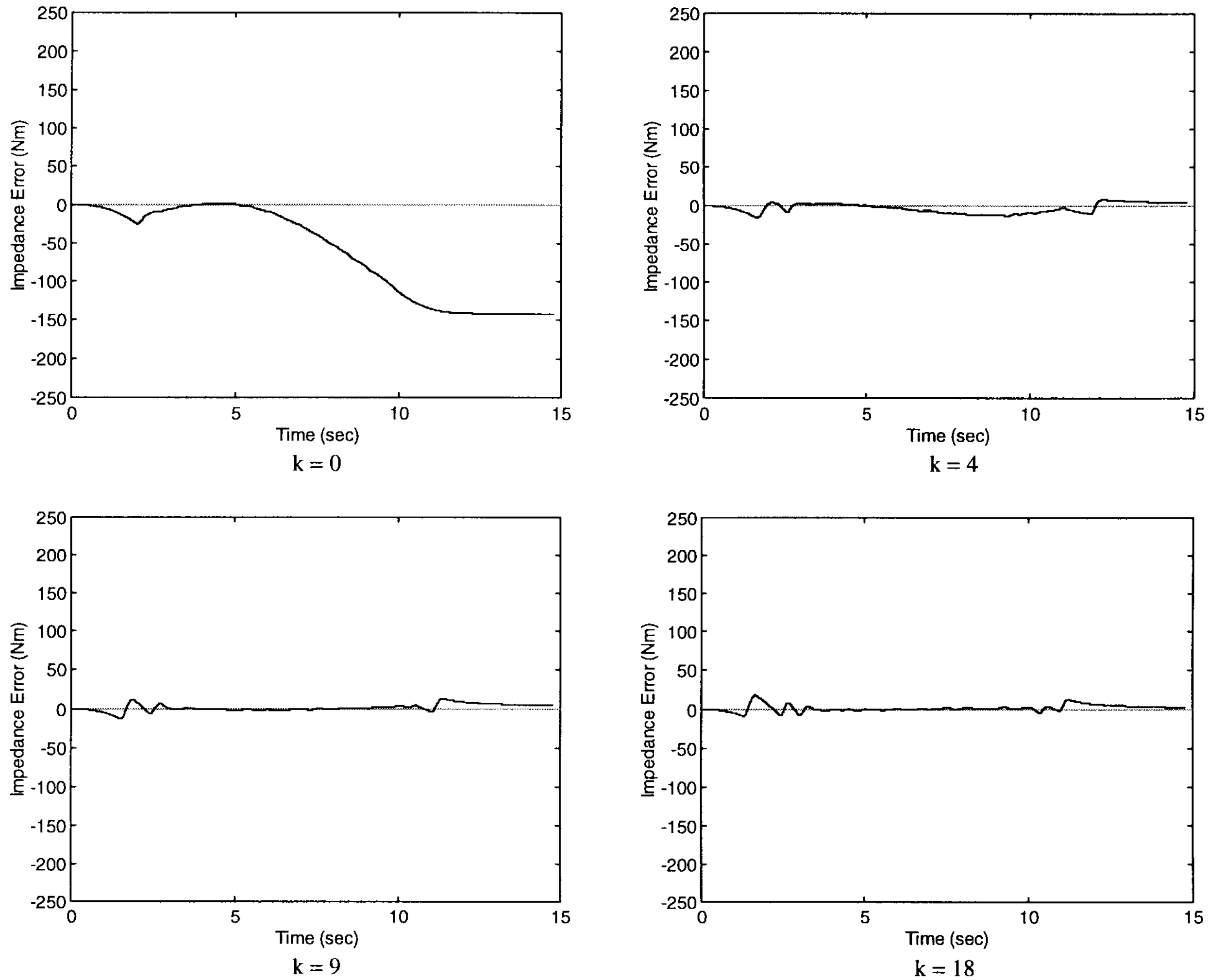

Fig. 6. The impedance error of joint two.

Similarly, from (16), we can define a desired intermediate state value y_e corresponding to the desired state X_e as

$$\dot{y}_e + \alpha y_e = (A_{m1} - a\alpha I)(X_d - X_e) + (A_{m2} - aI - \alpha I) \cdot (\dot{X}_d - \dot{X}_e) - B_m F_e \tag{40}$$

where L_p, L_v, and L_r are chosen as in (22) of the theorem. The desired state $[X_e^T, \dot{X}_e^T]^T$ exists but is unknown since F_e is an unknown desired force because X_s and K_s are unknown. Here, the definitions of the desired state and force are for analysis and are not used in the control law in actual implementation. From (18), the desired value of z_k and $\dot{z}_k$ corresponding to the desired model is given by $z_e = 0$ and $\dot{z}_e = 0$, respectively. Now, with the feedback gains K_p, K_v also chosen as in the theorem, we have the feedback control law in (13) given by

$$T_k = k_1\{a(X_d - X_k) + (\dot{X}_d - \dot{X}_k) + y_k\} + m_k. \tag{41}$$

From (4) and (6), the dynamic model can be written as

$$\ddot{X}_k = f(X_k, \dot{X}_k, t) + M^{-1}(X_k)T_k \tag{42}$$

where $f(X_k, \dot{X}_k, t) = -M^{-1}(X_k)\, V(X_k, \dot{X}_k) + M^{-1}(X_k)\, K_s\, (X_s - X)$. The interconnection of the passive robotic system [3] with a strictly passive feedback system (16) does not disturb the stability of the system as a result of the Passivity Theorem [16]. Alternatively, the stability of the system can be analyzed using Small Gain Theorem [16]. Therefore, the boundedness of the velocity variable is ensured and $M^{-1}(\cdot)$, $f(\cdot, \cdot, \cdot)$ are local Lipschitz continuous [2]. Substituting (15) and (41) into (42), we have

$$\ddot{X}_k = f(X_k, \dot{X}_k, t) + M^{-1}(X_k)k_1 z_k + M^{-1}(X_k)m_k. \tag{43}$$

Therefore, the desired control input m_e corresponding to the desired state X_e is described by

$$\ddot{X}_e = f(X_e, \dot{X}_e, t) + M^{-1}(X_e)m_e \tag{44}$$

where we note that $z_e = a(X_d - X_e) + (\dot{X}_d - \dot{X}_e) + y_e = 0$ and hence

$$y_e = a(X_e - X_d) + (\dot{X}_e - \dot{X}_d) \tag{45}$$

and

$$\begin{aligned} z_k &= a(X_d - X_k) + (\dot{X}_d - \dot{X}_k) + y_k \\ &\quad - a(X_d - X_e) - (\dot{X}_d - \dot{X}_e) - y_e \\ &= a(X_e - X_k) + (\dot{X}_e - \dot{X}_k) - (y_e - y_k). \end{aligned} \tag{46}$$

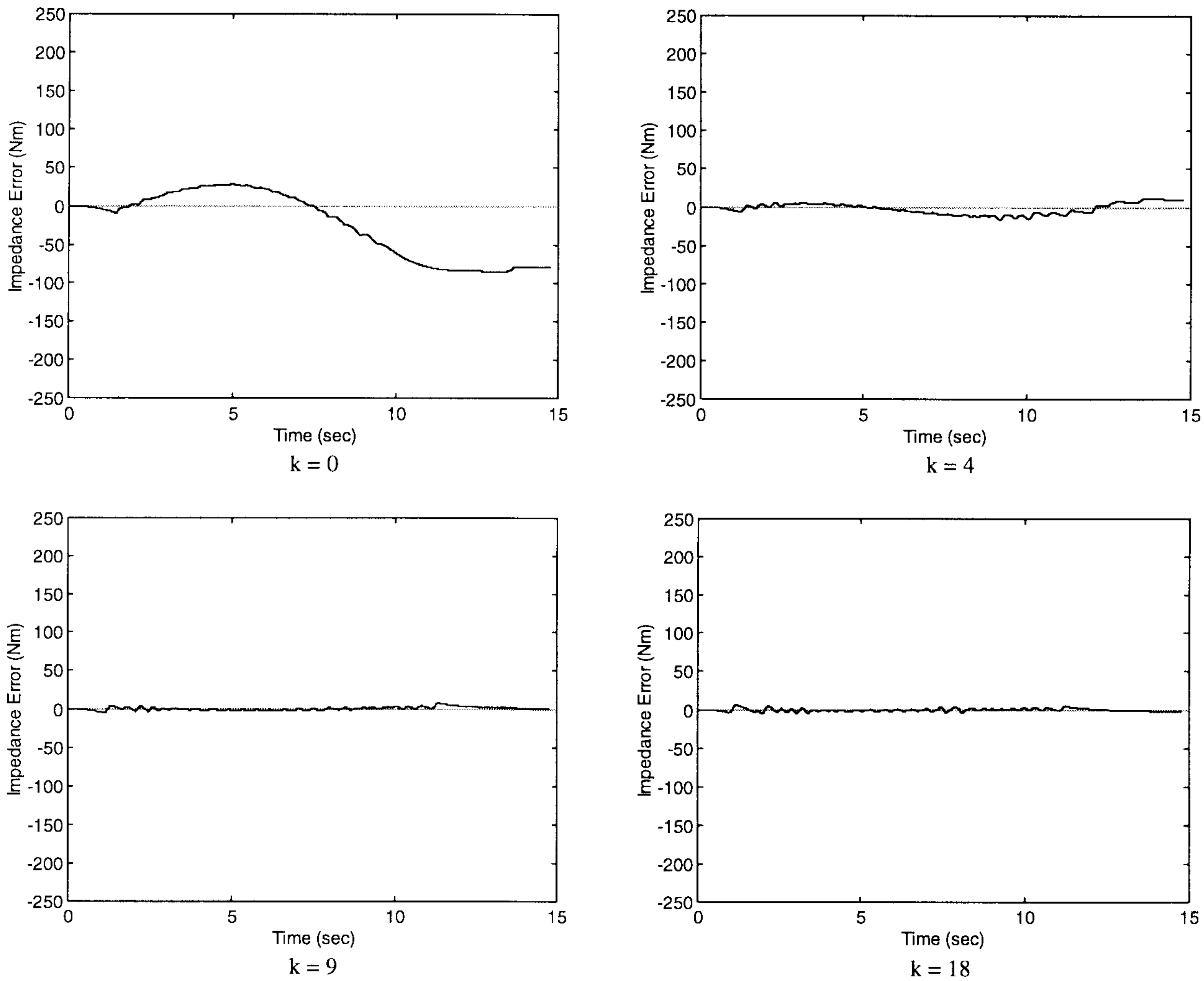

Fig. 7. The impedance error of joint three.

Subtracting (43) from (44), we have the error dynamic equation given by

$$\delta\ddot{X}_k = -k_1 M^{-1}(X_k) z_k + \delta h(X_k, t) + M^{-1}(X_k)\delta m_k \quad (47)$$

where $\delta m_k = m_e - m_k$, $\delta h(X_k, t) = [f(X_e, \dot{X}_e, t) - f(X_k, \dot{X}_k, t)] + [M^{-1}(X_e) - M_{-1}(X_k)]m_e$. Similarly, from (16) and (40), we have

$$\begin{aligned}\delta\dot{y}_k + \alpha\delta y_k = &-(A_{m1} - a\alpha I)\delta X_k - (A_{m2} - aI - \alpha I)\\ &\cdot \delta\dot{X}_k - B_m\delta F_k\end{aligned} \quad (48)$$

where $\delta X_k = X_e - X_k$, $\delta\dot{X}_k = \dot{X}_e - \dot{X}_k$, $\delta\dot{y}_k = \dot{y}_e - \dot{y}_k$, and $\delta F_k = F_e - F_k$. Furthermore, from (6) and (39), we have

$$\delta F_k = -K_s\delta X_k. \quad (49)$$

Therefore, substitute (49) into (48) results in

$$\delta\dot{y}_k + \alpha\delta y_k = -(\hat{A}_{m1} - a\alpha I)\delta X_k - (\hat{A}_{m2} - aI - \alpha I)\delta\dot{X}_k \quad (50)$$

where $\hat{A}_{m1} = A_{m1} - B_m K_s = M_m^{-1}(K_m + K_s)$, $\hat{A}_{m2} = A_{m2} = M_m^{-1}C_m$. From (46), (47), and (50), we have

$$\begin{aligned}\dot{z}_k = &(\hat{A}_{m1} - a\alpha I)\delta X_k + (\hat{A}_{m2} - \alpha I)\delta\dot{X}_k + \alpha\delta y_k\\ &- k_1 M^{-1}(X_k) z_k + \delta h(X_k) + M^{-1}(X_k)\delta m_k\end{aligned} \quad (51)$$

which implies that

$$\begin{aligned}\delta m_k = &M(X_k)\{\dot{z}_k + k_1 M^{-1}(X_k) z_k - [\hat{A}_{m1}\delta X_k + \hat{A}_{m2}\delta\dot{X}_k\\ &+ \delta f(X_k, t) + \delta M^{-1}(X_k) m_e]\}\end{aligned} \quad (52)$$

where $\delta f(X_k, t) = f(X_e, t) - f(X_k, t)$, $\delta M^{-1}(X_k) = M^{-1}(X_e) - M^{-1}(X_k)$, and $\delta h(X_k, t) = \delta f(x_k, t) + \delta M^{-1}(X_k) m_e$. Let us define an index function $V_k(t)$ as

$$V_k(t) = \int_0^t \frac{1}{k_1\beta}\,\delta m_k^T(\tau)\delta m_k(\tau)\,d\tau \quad (53)$$

for all $t \in [0, t_f]$. We assume that a m_0 exists at $k = 0$ such that

$$V_0(t) = \int_0^t \frac{1}{k_1\beta}\,\delta m_0^T(\tau)\delta m_0(\tau)\,d\tau < \infty \quad (54)$$

where $\delta m_0 = m_e - m_0$, for all $t \in [0, t_f]$. For example, if $m_0 = 0$, we have

$$V_0(t) = \int_0^t \frac{1}{k_1\beta}\,m_e^T(\tau) m_e(\tau)\,d\tau < \infty. \quad (55)$$

From (14) and (15), we have

$$\delta m_{k+1} = \delta m_k - \beta k_1 z_k. \quad (56)$$

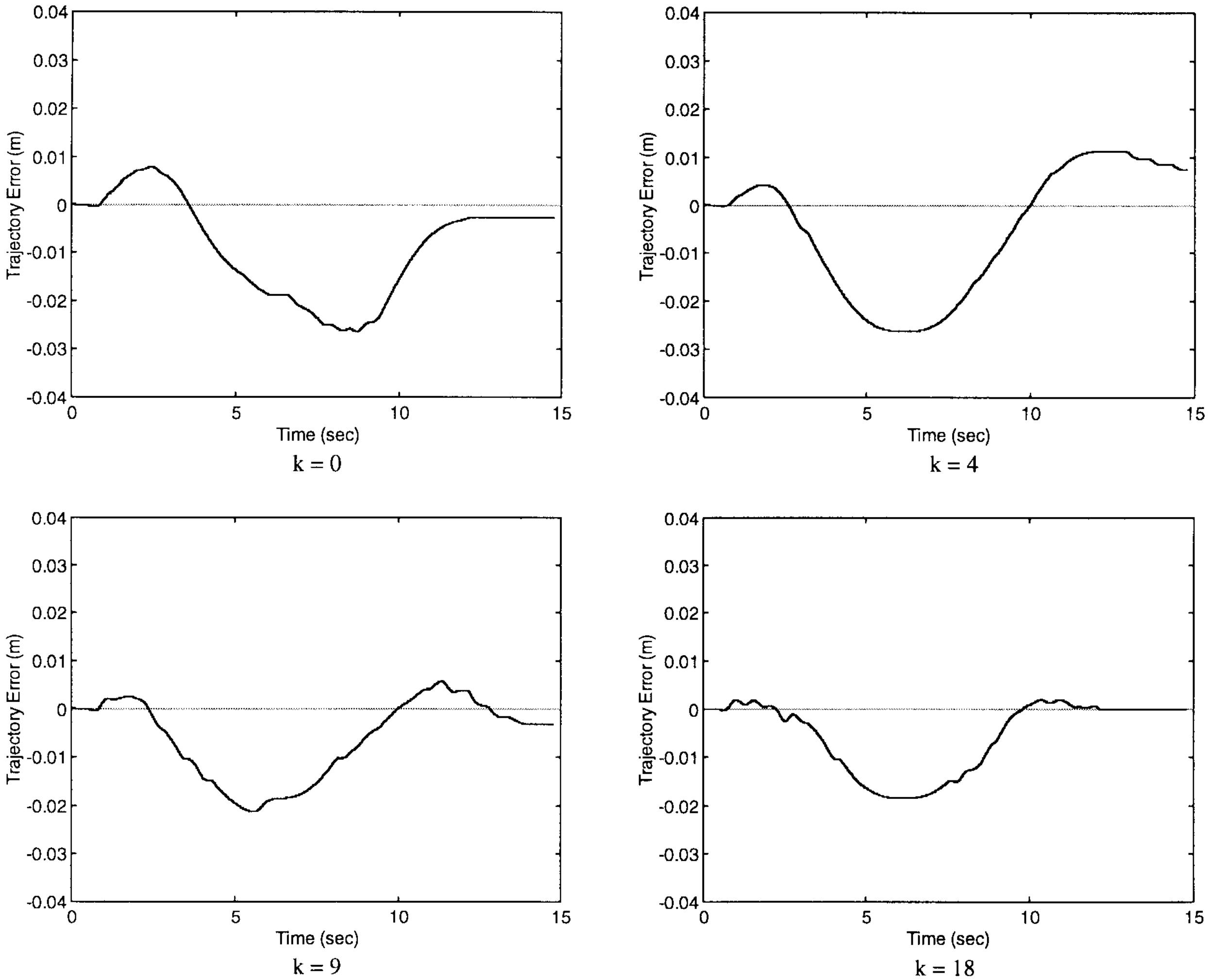

Fig. 8. The reference trajectory error of joint one.

Define $\Delta V_k = V_{k+1} - V_k$, we have, from (53) and (56), that

$$\Delta V_k = \beta k_1 \int_0^t z_k^T(\tau) z_k(\tau)\, d\tau - 2 \int_0^t z_k^T(\tau)\delta m_k(\tau)\, d\tau. \tag{57}$$

For simplicity of the following presentation, the dependence of the functions on their arguments is implied. Substitute (52) into the above equation, and integrating by parts, we have

$$\begin{aligned}\Delta V_k =& -z_k^T M z_k - \int_0^t z_k^T(\tau)[(2-\beta)k_1 I - \dot{M}] z_k(\tau)\, d\tau \\ &+ 2\int_0^t z_k^T(\tau) M[(\hat{A}_{m1} - a\alpha I)\delta X_k(\tau) + \alpha \delta y_k \\ &+ \delta f + (\hat{A}_{m2} - \alpha I)\delta \dot{X}_k(\tau) + \delta M^{-1} m_e]\, d\tau \end{aligned} \tag{58}$$

where $\dot{M}(\cdot) = dM(\cdot)/dt$. Integrating (50) gives

$$\begin{aligned}\delta y_k =& -(\hat{A}_{m1} - \alpha \hat{A}_{m2} + \alpha^2 I)\xi_k(t) \\ &- (\hat{A}_{m2} - aI - \alpha I)\delta X_k \end{aligned} \tag{59}$$

where $\xi_k(t) \triangleq \int_0^t e^{-\alpha(t-\tau_1)} \delta X_k(\tau_1)\, d\tau_1$. Therefore, from (46) and (59), we have

$$z_k = \delta \dot{X}_k + A_2 \delta X_k + A_1 \xi_k \tag{60}$$

where $A_1 = \hat{A}_{m1} - \alpha \hat{A}_{m2} + \alpha^2 I$, $A_2 = \hat{A}_{m2} - \alpha I$. Note that M_m, C_m, K_m, and α can be chosen such that $\lambda_{\min}[A_1]$, $\lambda_{\min}[A_2]$, and $\lambda_{\min}[A_1 A_2]$ are nonzero. Let $L = (2-\beta)k_1 I$ and substituting (60) into the second term of (58), we have

$$\begin{aligned} &-\int_0^t z_k^T(\tau)(L - \dot{M}) z_k(\tau)\, d\tau \\ &= -\int_0^t \{\delta \dot{X}_k^T(\tau)(L-\dot{M})\delta \dot{X}_k(\tau) \\ &\quad + \delta X_k^T(\tau) A_2 (L-\dot{M}) A_2 \delta X_k(\tau) \\ &\quad + \xi_k^T(\tau) A_1 (L-\dot{M}) A_1 \xi_k(\tau) \\ &\quad + \delta \dot{X}_k^T(\tau)(L-\dot{M}) A_2 \delta X_k(\tau) \\ &\quad + \delta \dot{X}_k^T(\tau) A_2 (L-\dot{M}) \delta \dot{X}_k(\tau) \\ &\quad + \delta \dot{X}_k^T(\tau)(L-\dot{M}) A_1 \xi_k(\tau) \\ &\quad + \xi_k^T(\tau) A_1 (L-\dot{M}) \delta \dot{X}_k(\tau) \\ &\quad + \delta X_k^T(\tau) A_2 (L-\dot{M}) A_1 \xi_k(\tau) \\ &\quad + \xi_k^T(\tau) A_1 (L-\dot{M}) A_2 \delta X_k(\tau)\}\, d\tau. \end{aligned} \tag{61}$$

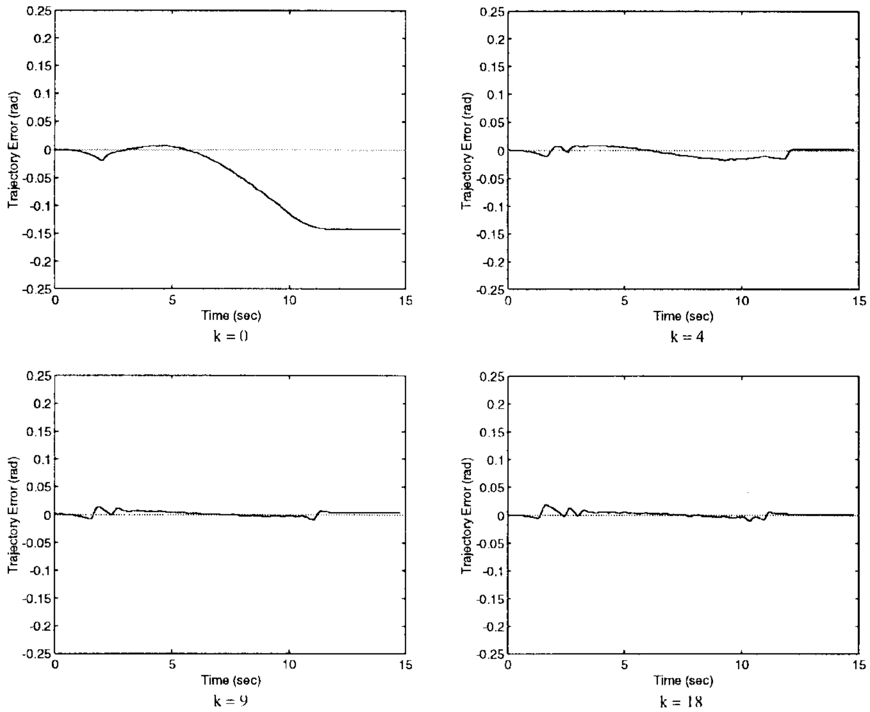

Fig. 9. The reference trajectory error of joint two.

Similarly, substituting (59) and (60) into the last term of (58), we have

$$\begin{aligned}2 \int_0^t & z_k^T(\tau) M[(\hat{A}_{m1} - a\alpha I)\delta X_k(\tau) + (\hat{A}_{m2} - \alpha I)\delta \dot{X}_k(\tau) \\ &+ \alpha \delta y_k(\tau) + \delta f + \delta M^{-1} m_e]\, d\tau \\ = 2 \int_0^t & \{\delta \dot{X}_k^T(\tau) M A_2 \delta \dot{X}_k(\tau) + \delta X_k^T(\tau) A_2 M A_1 \delta X_k(\tau) \\ &- \alpha \xi_k^T(\tau) A_1 M A_1 \xi_k(\tau) + \delta \dot{X}_k^T(\tau) M A_1 \delta X_k(\tau) \\ &+ \delta X_k^T(\tau) A_2 M A_2 \delta \dot{X}_k(\tau) - \alpha \delta \dot{X}_k^T(\tau) M A_1 \xi_k(\tau) \\ &+ \xi_k^T(\tau) A_1 M A_2 \delta \dot{X}_k(\tau) + \xi_k^T(\tau) A_1 M A_1 \delta X_k(\tau) \\ &- \alpha \delta X_k^T(\tau) A_2 M A_1 \xi_k(\tau) + \delta W_k^1(\tau)\}\, d\tau \qquad (62)\end{aligned}$$

where

$$\delta W_k^1 = 2(\delta \dot{X}_k + A_2 \delta X_k + A_1 \xi_k)^T M(\delta f + \delta M^{-1} m_e). \quad (63)$$

Adding (61) and (62) with each corresponding terms and substitute back into (58), we have

$$\Delta V_k = -z_k^T M z_k - \int_0^t [\delta W_k^2(\tau) - \delta W_k^1(\tau)]\, d\tau \qquad (64)$$

where

$$\begin{aligned}\delta W_k^2(t) = & \delta \dot{X}_k^T (L - \dot{M} - 2MA_2) \delta \dot{X}_k \\ &+ \delta X_k^T (LA_2^2 - A_2 M A_2 - 2A_2 M A_1) \delta X_k \\ &+ \xi_k^T (LA_1^2 - A_1 \dot{M} A_1 + \alpha A_1 M A_1) \xi_k \\ &+ \delta \dot{X}_k^T (LA_2 - MA_2 - 2MA_1) \delta X_k \\ &+ \delta X_k^T (LA_2 - A_2 \dot{M} - 2A_2 M A_2) \delta \dot{X}_k \\ &+ \delta \dot{X}_k^T (LA_1 - \dot{M} A_1 + 2\alpha M A_1) \xi_k \\ &+ \xi_k^T (LA_1 - A_1 \dot{M} - 2A_1 M A_2) \delta \dot{X}_k \\ &+ \delta X_k^T (LA_2 A_1 - A_2 \dot{M} A_1 + 2\alpha A_2 M A_1) \xi_k \\ &+ \xi_k^T (LA_1 A_2 - A_1 \dot{M} A_2 - 2A_1 M A_1) \delta X_k. \qquad (65)\end{aligned}$$

From (64) and (65), by integrating by parts, we note that

$$\begin{aligned}2 \int_0^t & \delta X_k^T(\tau) L A_2 \delta \dot{X}_k(\tau)\, d\tau \\ &= \delta X_k^T(t) L A_2 \delta X_k(t), \qquad (66)\end{aligned}$$

$$\begin{aligned}2 \int_0^t & \xi_k^T(\tau) L A_1 A_2 \delta X_k(\tau)\, d\tau \\ &= \xi_k^T(t) L A_1 A_2 \xi_k(t) + 2\alpha \int_0^t \xi_k^T(\tau) L A_1 A_2 \xi_k(\tau)\, d\tau \qquad (67)\end{aligned}$$

$$\begin{aligned}2 \int_0^t & \delta \dot{X}_k^T(\tau) L A_1 \xi_k(\tau)\, d\tau \\ &= 2\delta X_k^T(t) L A_1 \xi_k(t) - 2 \int_0^t \delta X_k^T(\tau) L A_1 \delta X_k(\tau)\, d\tau \\ &\quad + 2\alpha \int_0^t \delta X_k^T(\tau) L A_1 \xi_k(\tau)\, d\tau \\ &= \alpha \xi_k^T(t) L A_1 \xi_k(t) + 2\delta X_k^T(t) L A_1 \xi_k(t) \\ &\quad - 2 \int_0^t \delta X_k^T(\tau) L A_1 \delta X_k(\tau)\, d\tau \\ &\quad + 2\alpha^2 \int_0^t \xi_k^T(\tau) L A_1 \xi_k(\tau)\, d\tau \qquad (68)\end{aligned}$$

since A_1, A_2, and $A_1 A_2$ are symmetric matrices. Note the fact of the following inequality:

$$\begin{aligned}-2(1-\gamma)\delta \dot{X}_k^T (LA_1) \xi_k \le\ & (1-\gamma) \delta \dot{X}_k^T L \delta \dot{X}_k \\ &+ (1-\gamma) \xi_k^T (LA_1^2) \xi_k \qquad (69)\end{aligned}$$

where $\gamma \in (0, 1]$ is a constant to be defined. Therefore, by partitioning the term $2\delta \dot{X}_k^T (LA_1) \xi_k$ in (65) into $2(1-\gamma)\delta \dot{X}_k^T (LA_1) \xi_k + 2\gamma \delta \dot{X}_k^T (LA_1) \xi_k$, substituting the inequality

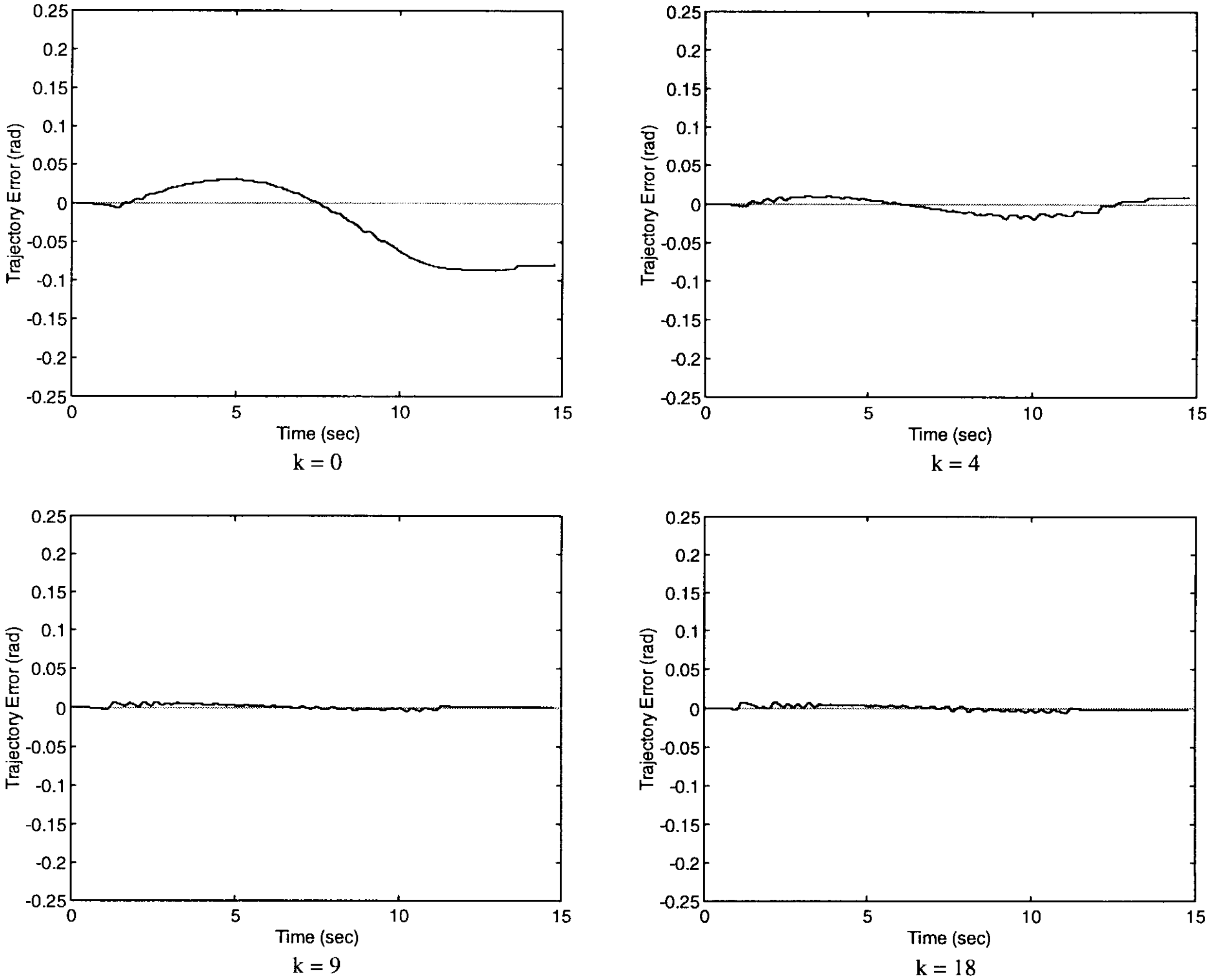

Fig. 10. The reference trajectory error of joint three.

(69) and (66)–(68) into it, we arrive at

$$\begin{aligned}\Delta V_k \leq & -z_k^T M z_k - \xi_k^T L A_2 A_1 \xi_k - \delta X_k^T L A_2 \delta X_k \\ & - 2\gamma \delta X_k^T L A_1 \xi_k - \alpha\gamma \xi_k^T L A_1 \xi_k \\ & - \int_0^t [\delta W_k^3(\tau) - \delta W_k^1(\tau)]\, d\tau \end{aligned} \tag{70}$$

where

$$\begin{aligned}\delta W_k^3(t) = & \delta \dot{X}_k^T (\gamma L - \dot{M} - 2MA_2)\delta \dot{X}_k + \delta X_k^T [L(A_2^2 - 2\gamma A_1) \\ & - A_2 \dot{M} A_2 - 2A_2 M A_1]\delta X_k \\ & + \xi_k^T [L(\gamma A_1^2 + 2\alpha A_1 A_2 + 2\alpha^2 A_1) \\ & - A_1 \dot{M} A_1 + 2\alpha A_1 M A_1]\xi_k \\ & - \delta \dot{X}_k^T (\dot{M} A_2 + 2MA_1)\delta X_k \\ & - \delta X_k^T (A_2 \dot{M} + 2A_2 M A_2)\delta \dot{X}_k \\ & - \delta \dot{X}_k^T (\dot{M} A_1 + 2\alpha M A_1)\xi_k \\ & - \xi_k^T (A_1 \dot{M} + 2A_1 M A_2)\delta \dot{X}_k \\ & - \xi_k^T (A_1 \dot{M} A_2 + 2A_1 M A_1)\delta X_k \\ & - \delta X_k^T (A_2 \dot{M} A_1 - 2\alpha A_2 M A_1)\xi_k. \end{aligned} \tag{71}$$

Now, let us rewrite the second to fourth terms of (70) as

$$\begin{aligned} & -\xi_k^T (L A_2 A_1)\xi_k - \delta X_k^T (L A_2)\delta X_k - 2\gamma \delta X_k^T (L A_1)\xi_k \\ & \leq -(2-\beta)k_1 \begin{bmatrix} \|\xi_k\| \\ \|\delta X_k\| \end{bmatrix}^T \begin{bmatrix} \lambda_{21} & -\gamma b_{a1} \\ -\gamma b_{a1} & \lambda_2 \end{bmatrix} \begin{bmatrix} \|\xi_k\| \\ \|\delta X_k\| \end{bmatrix} \end{aligned}$$

which is nonpositive if γ is chosen such that $\gamma < (\lambda_{21}\lambda_2)^{1/2}/b_{a1}$; where b_{a1} denotes the norm bounds for A_1, $\lambda_2 = \lambda_{\min}[A_2]$, and $\lambda_{21} = \lambda_{\min}[A_2 A_1]$. Now, since the first to fourth terms of (70) are nonpositive, therefore $\Delta V_k(t)$ is negative semi-definite if $\delta W_k^3(t) - \delta W_k^1(t)$ is nonnegative. From (63) and (71), we have (72), shown at the bottom of

$$\delta W_k^3(t) - \delta W_k^1(t) \geq y^T \begin{bmatrix} \gamma(2-\beta)k_1 - c_2 & -c_4 & -c_5 \\ -c_4 & c_7(2-\beta)k_1 - c_3 & -c_6 \\ -c_5 & -c_6 & \gamma\lambda_1^2(2-\beta)k_1 - c_8 \end{bmatrix} y \tag{72}$$

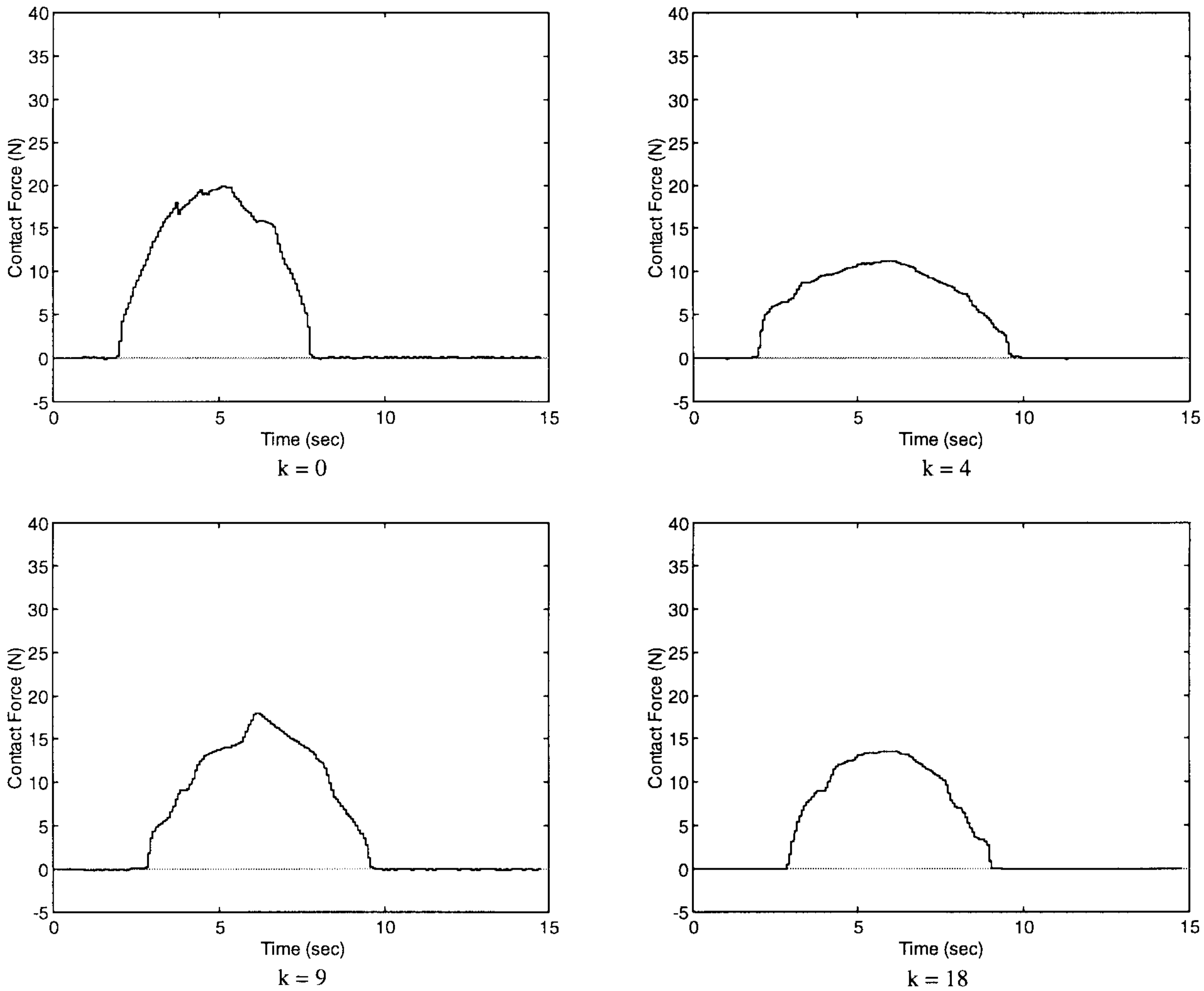

Fig. 11. The contact force.

the previous page, where

$$y = \begin{bmatrix} \|\delta \dot{X}_k\| \\ \|\delta X_k\| \\ \|\xi_k\| \end{bmatrix} \quad \gamma < \min\left\{ \frac{(\lambda_{21}\lambda_2)^{1/2}}{b_{a1}}, \frac{\lambda_2^2}{2\lambda_1}, 1 \right\}$$

$$\begin{aligned} c_1 &= c_f + c_M b_{me}, \ c_2 = b_{g1} + \frac{2b_{a2} + 2c_1}{\overline{\kappa}_1} \\ c_3 &= b_{a2}^2 b_M + \frac{2b_{a1}b_{a2} + 2c_1 b_{a2}}{\overline{\kappa}_1} \\ c_4 &= b_{a2}b_{g1} + \frac{b_{a2}^2 + b_{a1} + c_1 + c_1 b_{a2}}{\overline{\kappa}_2} \\ c_5 &= b_M b_{a1} + \frac{b_{a1}b_{a2} + c_1 b_{a1}}{\overline{\kappa}_1} \\ c_6 &= b_M b_{a1} b_{a2} + \frac{b_{a1}^2 + c_1 b_{a1}}{\overline{\kappa}_1}, \ c_7 = \lambda_2^2 - 2\gamma\lambda_1 \\ c_8 &= b_{a1}^2 b_M \end{aligned}$$

c_f and c_M are the Lipschitz constants of the functions $f(\cdot, \cdot)$ and $M^{-1}(\cdot)$, respectively, b_M, $\overline{\kappa}_1$, and b_{a2} are the norm bounds for $\dot{M}(\cdot)$, $M(\cdot)$, and A_2, respectively, $\lambda_1 = \lambda_{\min}[A_1]$ and $b_{me} = \sup_{t \in [0, t_f]} \|m_e(t)\|$. Using Sylvester's criterion, taking determinant of the matrix in (72) and its successive principle minors, we can show that if k_1 is chosen to satisfy condition (23) of the Theorem, where $c_9 = 2c_4c_5c_6 - c_3c_5^2$, then $\delta W_k^3(t) - \delta W_k^1(t) \geq 0$ and hence $\Delta V_k(t) \leq 0$. This implies that $V_k(t)$ converges to a nonnegative constant because $V_0(t)$ is bounded. Therefore $\Delta V_k(t) \to 0$ as $k \to 0$. Furthermore, $z_k(t) \to 0$ for all $t \in [0, t_f]$ because

$$\Delta V_k \leq -z_k^T M z_k \leq 0. \tag{73}$$

This implies that $\dot{z}_k \to 0$ for all $t \in [0, t_f]$ because $z_k(t) \to 0$ for all $t \in [0, t_f]$. From (9) and (18), we have

$$\begin{aligned} w_k &= M_m(\ddot{X}_d - \ddot{X}_k) + C_m(\dot{X}_d - \dot{X}_k) \\ &\quad + K_m(X_d - X_k) + F_k = M_m(\dot{z}_k + \alpha z_k). \end{aligned} \tag{74}$$

Therefore, the impedance error w_k converges to zero such that

$$\begin{aligned} w_k &= M_m(\ddot{X}_d - \ddot{X}_k) + C_m(\dot{X}_d - \dot{X}_k) \\ &\quad + K_m(X_d - X_k) + F_k \to 0 \end{aligned} \tag{75}$$

as $k \to \infty$ for all $t \in [0, t_f]$. △△△

REFERENCES

[1] M. Aicardi, G. Cannata, and G. Gasalino, "Hybrid learning control for constrained manipulators," *Adv. Robot.*, vol. 6, no. 1, pp. 69–94, 1992.
[2] S. Arimoto, "Learning control theory for robot motion," *Int. J. Adapt. Contr. Signal Process.*, vol. 4, pp. 543–564, 1990.

[3] ———, "Passivity and learnability for mechanical systems—A learning control theory for skill refinement," *IEICE Trans. Fundamentals,* vol. E75-A, no. 5, pp. 552–560, May 1992.

[4] S. Arimoto, S. Kawamura, and F. Miyazaki, "Bettering operation of robots by learning," *J. Robot. Syst.,* vol. 1, no. 2, pp. 440–447, 1984.

[5] H. Asada, "Teaching and learning of compliance using neural networks: Representation and generation of nonlinear compliance," in *Proc. IEEE Conf. Robot. Automat.,* Cincinnati, OH, 1990, pp. 1237–1244.

[6] P. Bondi, G. Casalino, and L. Gambardella, "On the iterative learning control theory for robotic manipulators," *IEEE J. Robot. Automat.,* vol. 4, pp. 14–22, Feb. 1988.

[7] R. Carelli and R. Kelly, "An adaptive impedance/force controller for robot manipulators," *IEEE Trans. Automat. Contr.,* vol. 36, pp. 967–972, Aug. 1991.

[8] C. C. Cheah and D. Wang, "Discrete-time model reference learning control: Theory and experiment," *Dynam. Contr.,* vol. 8, pp. 145–162, 1998.

[9] ———, "Learning control for a class of nonlinear differential-algebraic systems with application to constrained robots," in *J. Robot. Syst.,* vol. 13, no. 3, pp. 141–151, 1996.

[10] ———, "Learning impedance control for robotic manipulator," in *Proc. IEEE Int. Conf. Robot. Automat.,* Nagoya, Japan, 1995, pp. 2150–2155.

[11] ———, "A model reaching learning control scheme for a class of nonlinear systems," in *Proc. Amer. Contr. Conf.,* Baltimore, MD, 1994, pp. 882–886.

[12] C. C. Cheah, D. Wang, and Y. C. Soh, "Convergence and robustness of a discrete-time learning control scheme for constrained manipulators," *J. Robot. Syst.,* vol. 11, no. 3, pp. 223–238, 1994.

[13] ———, "Learning control of constrained robots," in *Proc. IEEE Int. Symp. Intell. Contr.,* Scotland, U.K., 1992, pp. 91–96.

[14] M. Cohen and T. Flash, "Learning impedance parameters for robot control using an associative search network," *IEEE Trans. Robot. Automat.,* vol. 7, pp. 382–390, 1991.

[15] J. J. Craig, "Adaptive control of manipulators through repeated trials," in *Proc. Amer. Contr. Conf.,* San Diego, CA, 1984, pp. 1566–1573.

[16] C. A. Desoer and M. Vidyasagar, *Feedback Systems: Input–Output Properties.* New York: Academic, 1975.

[17] V. Gullapalli, R. A. Grupen, and A. G. Barto, "Learning reactive admittance control," in *Proc. IEEE Conf. Robot. Automat.,* Nice, France, 1992, pp. 1475–1480.

[18] G. Heinzinger, D. Fenwick, B. Paden, and F. Miyazaki, "Stability of learning control with disturbances and uncertain initial conditions," *IEEE Trans. Automat. Contr.,* vol. 137, pp. 110–114, Jan. 1992.

[19] N. Hogan, "Impedance control: An approach to manipulation: Part I, part II, part III," *ASME J. Dynam. Syst., Meas., Contr.,* vol. 107, 1985.

[20] S. Jung and T. Hsia, "On neural network application to robust impedance control of robot manipulators," in *IEEE Int. Conf. Robot. Automat.,* Nagoya, Japan, 1995, pp. 869–874.

[21] D. Katic and M. Vukobratovic, "Learning impedance control of manipulation robots by feedforward connectionist structures," in *Proc. IEEE Conf. Robot. Automat.,* 1994, pp. 45–50.

[22] S. Kawamura, F. Miyazaki, and S. Arimoto, "Applications of learning methods for dynamic controls of robotic manipulators," in *Proc. 24th IEEE CDC,* Fort Lauderdale, FL, 1985, pp. 1381–1386.

[23] ———, "Hybrid position/force control of manipulators based on learning method," in *Proc. Int. Conf. Adv. Robot.,* Tokyo, Japan, 1985, pp. 235–242.

[24] T. Y. Kuc, J. S. Lee, and K. Nam, "An iterative learning control theory for a class of nonlinear dynamic systems," *Automatica,* vol. 28, no. 6, pp. 1215–1221, 1992.

[25] F. L. Lewis, C. T. Abdallah, and D. M. Dawson, *Control of Robot Manipulators.* New York: Macmillan, 1993.

[26] W. Li and J. J. E. Slotine, "A unified approach to compliant motion control," in *Proc. Amer. Contr. Conf.,* 1989, pp. 1944–1949.

[27] J. Marsden, *Elementary Classical Analysis.* San Francisco, CA: Freeman, 1974.

[28] N. H. McClamroch and D. Wang, "Feedback stabilization and tracking of constrained robots," *IEEE Trans. Automat. Contr.,* vol. 33, pp. 419–426, 1988.

[29] M. H. Raibert and J. J. Craig, "Hybrid position/force control of manipulators," *ASME J. Dyn. Syst., Meas., Contr.,* vol. 102, p. 126, 1981.

[30] J. J. E. Slotine and W. Li, "Adaptive strategies in constrained manipulators," in *Proc. IEEE Int. Conf. Robot. Automat.,* 1987, pp. 595–602.

[31] M. W. Spong, F. L. Lewis, and C. T. Abdallah, *Robot Control: Dynamics, Motion Planning, and Analysis.* New York: IEEE Press, 1993.

[32] T. Sugie and T. Ono, "An iterative learning control law for dynamical systems," *Automatica,* vol. 27, no. 4, pp. 729–732, 1991.

[33] S. T. Venkatamaran, S. Gulati, and N. Toomarian, "Parameter learning and compliance control of robot manipulators," in *Proc. Conf. Decision Contr.,* 1992, pp. 3475–3480.

[34] D. Wang and C. C. Cheah, "A robust learning control scheme for manipulators with target impedance at end-effectors," *Robotics and Manufacturing: Recent Trends in Research and Applications.* ASME Press, 1996, vol. 6, pp. 851–856.

[35] ———, "An iterative learning control scheme for impedance control of robotic manipulators," *Int. J. Robot. Res.,* 1998.

[36] D. Wang and N. H. McClamroch, "Position/force control design for constrained mechanical systems: Lyapunov's direct method," *IEEE Trans. Robot. Automat.,* vol. 9, pp. 308–313, 1993.

[37] D. Wang, Y. C. Soh, and C. C. Cheah, "Robust motion and force control of constrained manipulators by learning," *Automatica,* vol. 31, no. 2, pp. 257–262, 1995.

[38] D. E. Whitney, "Historical perspective and state of the art in robot force control," *Int. J. Robot. Res.,* vol. 6, no. 1, pp. 3–14, 1987.

[39] T. Yoshikawa, T. Sugie, and M. Tanaka, "Dynamic hybrid position/force control of robot manipulators: Controller design and experiment," *IEEE J. Robot. Automat.,* vol. 4, pp. 699–705, 1988.

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