

## Learning Subjective Functions with Large Margins

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### Abstract

In many optimization and decision problems the objective function can be expressed as a linear combination of competing criteria, the weights of which specify the relative importance of the criteria for the user. We consider the problem of learning such a “subjective” function from preference judgments collected from traces of user interactions. We propose a new algorithm for that task based on the theory of Support Vector Machines. One advantage of the algorithm is that prior knowledge about the domain can easily be included to constrain the solution. We demonstrate the algorithm in a route recommendation system that adapts to the driver’s route preferences. We present experimental results on real users that show that the algorithm performs well in practice.

### Introduction

In many optimization and decision problems the desirability of a particular solution depends on a number of competing factors or criteria. The solution can be rated along a number of dimensions, and the overall quality of the solution depends on its combined score on all dimension simultaneously.

Most algorithms to solve optimization problems, however, depend on the existence of a single objective function that specify how good each potential solution is. Multi-criteria problems are typically handled by computing a cost or rating for each criteria independently and combining these costs into a single function.

A particularly simple way of combining the costs is to use a linear combination. In that case the weights in the combination associated with the different costs specify the relative importance of the criteria for the user. A domain expert can sometimes fix these weights, but often the importance of the different criteria is subjective and varies from user to user. We therefore think of the weights as forming a model of the user preferences, and we call such an objective function a *subjective function*.<sup>1</sup>

It is often difficult or inconvenient to ask the user to explicitly specify the weights for a subjective function.

In many cases, the user might not even be consciously aware of the importance he or she gives to the different criteria. One approach to address this problem is to apply machine learning techniques to learn those weights from traces of interactions with the user. In particular, the subjective function can be inferred from user’s *preference judgments* that specify, either implicitly or explicitly, that one solution should be ranked higher than another (Cohen, Schapire, & Singer 1999). It is often easy to collect these preference judgments unobtrusively, by observing the choices the user makes while interacting with the system.

Gervasio, Iba and Langley (Gervasio, Iba, & Langley 1999) describe a crisis response scheduling assistant that takes this approach to infer the most appropriate schedule evaluation function for its user. Similarly, in (Rogers, Fiechter, & Langley 1999) we describe a route advice system that unobtrusively learns a driver’s route preferences through interaction with that driver. In this paper we extend that work and describe a new learning algorithm that seems particularly well-suited to learn subjective functions from traces of user interactions.

The learning algorithm relies on the basic ideas of Support Vector Machines (SVM) (Vapnik 1995; 1999; Burges 1998) but it takes advantage of the particular structure of the problem. It is consequently much simpler than a general SVM. In particular, computing the coefficients of the subjective function only requires solving a linear program, instead of the quadratic programming problem that a general SVM entails. One advantage of that particular algorithm over other possible machine learning techniques for learning a subjective function is that prior knowledge about admissible coefficients and functions can easily be included.

Below, we first briefly describe our adaptive route advice system, focusing on the route generation component and on the interface that presents the route options to the user and gathers preference feedback. We then formalize the problem of learning the user preference model and describe the learning algorithm. The following section describes some experimental results on real (human) subjects and the conclusion briefly discusses some directions for future research.

<sup>1</sup>The term was coined by Justin Boyan.

## The Adaptive Route Advisor

The Adaptive Route Advisor is an adaptive user interface (Langley 1997) that recommends routes on a road network and adapts to a driver's route preferences through interaction with that driver. Given a routing task, the Route Advisor interacts with the driver to generate a route that he or she finds satisfactory. Initially, the agent proposes a small number of possible routes, taking into account the driver's preferences if known. The driver can then request solutions that differ from the initial options along some dimension. For instance, the driver may request a route that is simpler, even if it means a longer route. The driver and the Route Advisor continue to work together, generating and evaluating different solutions, until the driver is satisfied. During this process, the agent unobtrusively collects information about the driver's choices and uses these data to refine its model of the driver's preferences for the next task.

### The Routing Algorithm and User Model

The generative component of the Adaptive Route Advisor is a routing algorithm that plans a path through a digital map from a starting point to a destination. The planner represents the digital map as a graph, where the nodes are intersections and the edges are parts of roads between intersections. Our digital maps provide four attributes for each edge: length, estimated driving time, turn angle to connected edges, and road class (e.g., highway, freeway, arterial road, local road). Based on these attributes 14 features of a route are computed, including its estimated driving time, distance, number of left, right, and U-turns, number of intersections, and distance on each road class. The cost of a route is computed as a weighted sum of its features and the system uses an optimized version of Dijkstra's shortest path algorithm (Dijkstra 1959) to find the path with the minimum cost.

As discussed above, the weights in the objective function play the role of a user model. The system is initialized with a default user model and the model is refined with feedback from interaction with the planner. More specifically, we define an interaction with the planner to be the presentation of a set of  $N$  generated routes for a particular routing task and feedback from the user indicating which route is preferable. This is completely unobtrusive to the user, because he or she evaluates a set of routes and selects one as part of the route advice process. We expand the interaction into  $N - 1$  preference judgments, representing the fact that the selected route is preferable to each of the presented alternatives, and the learning algorithm processes a sequence of interactions with the planner to produce a weight vector that models the preferences expressed. We discuss the learning algorithm itself in the next section.

In later interactions with that particular user the routing algorithm uses the weights thus computed in its cost function. In this way, as the driver uses the interface, it adapts itself to his or her preferences. Note



Figure 1: The trip request panel.

that since the routing algorithm is optimal on the cost function, the resulting route is guaranteed to have the lowest cost for that user model among all routes between the same two nodes. In other words, the routes computed are always Pareto optimal, in that there can be routes that are better along each of the dimensions (features) independently, but none that can be better simultaneously on all dimensions.

### The Interaction Module

When started, the Route Advisor displays its trip request panel as pictured in Figure 1. In the current implementation, the user specifies origin and destination in a postal address style, and identifies him or herself for the purpose of loading the user model.

After requesting a route, the route summary panel appears, as displayed in Figure 2(a), providing a list of route options. The routes are presented in terms of seven attributes: total time, number of intersections, number of left turns, right turns and U-turns, total distance and distance on highways.

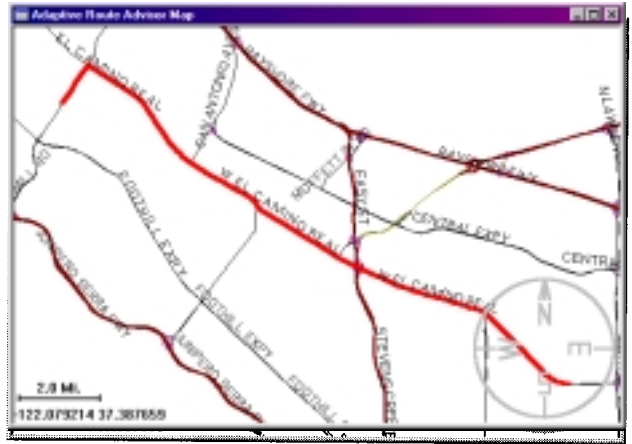
Initially the system computes and presents two routes to the user. The first is computed using the current preference model as the weight vector for the routing cost function. The second route uses novel weights, selected from a small set of prototypical user models, in an attempt to explore new directions in the space of preference models. Presenting at least two route options forces the user to make a choice and provide some feedback to the system.

The map displays the selected route, as shown in Figure 2(b), and the turn directions for the route are available in a separate tabbed panel. Clicking "Select" indicates that the highlighted route is satisfactory and returns to the trip panel. The route advisor assumes that the highlighted route is preferable to the alternative routes and updates the user model. Clicking "Cancel" returns to the trip panel but does not update the model.

The "Modify" panel lets the user generate a new route that is faster, shorter, has fewer turns, has fewer intersections, or has less or more highway than the se-

Time	Inters.	Turns			Hwy	Distance Total
		L	R	U		
16:12	78	2	1	1	-	9.9 mi
15:00	38	2	3	0	-	10.8 mi

(a) The route selection panel.



(b) The map window.

Figure 2: Initially, the Adaptive Route Advisor presents two alternative routes to the user. The best route according to the current user-model is highlighted.

lected route. The implicit assumption is that the driver is willing to accept routes that are somewhat worse on other attributes if he or she can find one that is better on the selected attribute. This approach to navigating through the space of possible solutions is similar to “tweaking” in Burke et al.’s RENTME system (Burke, Hammond, & Young 1996). In that system, the user can navigate a database of apartments for rent by asking for an apartment that is either cheaper, bigger, nicer, or safer than the one currently displayed.

The interface described above simultaneously and seamlessly fulfills two functions in the Adaptive Route Advisor. First, it lets the users easily find routes they like by giving them choices and letting them interactively modify the routes proposed. Second, it unobtrusively collects the preference judgements that the learning algorithm needs to refine the user model and adapt to a particular driver.

### Learning Algorithm

As discussed above, the learning algorithm in the Adaptive Route Advisor processes a sequence of interactions with the planner to produce a weight vector that models the user preferences. These preferences are expressed in the form of pairs of routes  $\langle R_1, R_2 \rangle$  for which the user ranked route  $R_1$  as preferable to route  $R_2$ .

In general we formalize the problem of learning a subjective function from preference judgements as follows:

We assume that each potential solution is characterized by a  $n$ -dimensional vector of real numbers that specify the cost of the solution along  $n$ , not necessarily independent, dimensions. In general, these costs can be simple numeric attributes of the solution or complex functions computed from several features of the solution.

We are given a set  $D$  of  $m$  preference judgments

$\langle x_d^g, x_d^b \rangle$  ( $d = 1, \dots, m$ ), with  $x_d^g, x_d^b \in \mathfrak{R}^n$ , indicating that the user prefers a solution with cost  $x_d^g$  over one with cost  $x_d^b$ . Our goal is to determine a subjective function  $f : \mathfrak{R}^n \mapsto \mathfrak{R}$  that is consistent with those judgements. Specifically, we would like that  $f(x_d^g) > f(x_d^b)$  for  $d = 1, \dots, m$ .

Here we restrict ourselves to subjective functions that are linear combinations of the individual costs, i.e.,  $f(x) = w \cdot x$ , with  $w, x \in \mathfrak{R}^n$ . The vector  $w = (w_1, w_2, \dots, w_n)$  forms our user model and specifies the relative importance of the competing criteria for the user. Since, other things being equal, a rational user should always prefer a solution with lower individual costs, we assume that the  $w_i$ ’s are non-negative.

The restriction to linear models is not as stringent as it might appear at first, since the cost components themselves can be non-linear functions of the attributes of the solutions. Moreover, we believe that a simple representation, with a limited number of parameters, is generally preferable when learning a user model. Indeed, in those applications, it is often more important to quickly acquire an approximate model of the user preferences from a limited number of user interactions than to achieve the highest possible asymptotical accuracy. The experimental results described in the next section illustrate this point.

With this representation, every preference judgment  $\langle x_d^g, x_d^b \rangle$  corresponds to a linear constraint on  $w$ , namely  $w \cdot (x_d^g - x_d^b) > 0$ . We can interpret these constraints as training instances for an induction algorithm that learns a linear classifier  $w$ . Specifically,  $x_d = (x_d^g - x_d^b)$  represents a positive instance and  $-x_d$  represents a negative instance (see figure 3).

Any supervised learning technique that is capable of representing a linear classifier can thus be applied

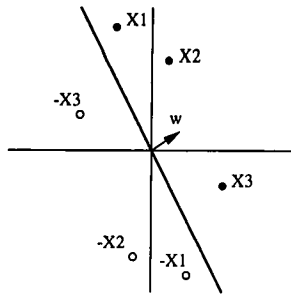


Figure 3: Each preference judgement  $\langle x_d^a, x_d^b \rangle$  defines a positive instance  $x_d$  and a negative instance  $-x_d$  for the linear classifier.

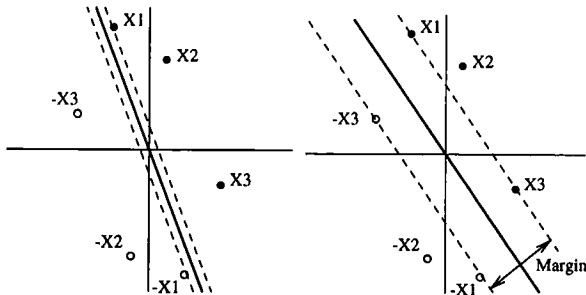


Figure 4: Small margin versus large margin linear classifier.

to learn the weights of the subjective function from the preference judgments. The previous version of the Adaptive Route Advisor (Rogers, Fiechter, & Langley 1999), for instance, used a perceptron algorithm.

The algorithm that we describe here is based on the basic ideas of Support Vector Machines (SVMs) and attempts to maximize the margin of the classifier, which is the minimum distance between the hyperplane defined by the classifier and the training points. There is a strong theoretical motivation for maximizing the margin of a classifier that stems from a result in statistical learning theory (Vapnik 1995; 1999) that relates a bound on the generalization error of a classifier to the size of its margin. It also has a very intuitive interpretation: classifiers with larger margins are more robust. Both classifiers in Figure 4 correctly classify the training instances, but the second one has a larger margin and has a higher probability of correctly classifying a new instance.

In our setting the margin of a classifier  $w$  with respect to a training instance  $x_d$  for which  $x \cdot w = b$  ( $b > 0$ ) is given by  $2 \cdot b / \|w\|$ . Hence, one way to find a classifier that maximizes the margin is to find a vector  $w$  with minimum norm that keeps the separation ( $b$ ) of all training instances above a fixed threshold. In a general SVM this results in a quadratic programming problem. Here, however, since we know that the  $w$ 's components must be non-negative we can minimize the 1-norm of  $w$

instead of its Euclidean norm to achieve a similar result while solving only a *linear* program.<sup>2</sup> Specifically, we can find a classifier with large margin by solving the following problem:

$$\text{minimize} \quad \sum_{j=1}^n w_j$$

$$\text{subject to} \quad w \cdot x_d \geq 1, \quad d = 1, \dots, m \quad (1)$$

$$w_j \geq 0, \quad j = 1, \dots, n \quad (2)$$

Note that the above linear program only has a solution when the data are linearly separable. We can easily extend the formulation to the general, non-separable case by transforming the constraints (1) into “soft” constraints and penalizing the constraint violations (Burges 1998). More precisely, we introduce positive slack variables  $\xi_d$  ( $d = 1, \dots, m$ ) in the constraints and add a penalty on those variables in the objective function:

$$\text{minimize} \quad \sum_{j=1}^n w_j + c \cdot \sum_{d=1}^m \xi_d$$

$$\text{subject to} \quad w \cdot x_d \geq 1 - \xi_d, \quad d = 1, \dots, m$$

$$w_j \geq 0, \quad j = 1, \dots, n$$

$$\xi_d \geq 0, \quad d = 1, \dots, m$$

Here,  $c$  is a parameter that specifies the importance of minimizing the training error relative to maximizing the margin.

We can apply a linear programming algorithm, like the Simplex algorithm (see Papadimitriou & Steiglitz, 1982; Chvatal, 1983) to efficiently solve the above linear program and compute the coefficients  $w$  of the subjective function. One advantage of this approach over other supervised learning algorithms for this problem is that it can be implemented in a truly on-line fashion. Every time the user provides a new preference judgment the system can update the subjective function without having to retrain the learning algorithm on all the data. The system simply adds the constraint associated with the new preference judgment to the linear program and revise the current solution to take new constraint into account. This can be done efficiently within the Simplex algorithm, without having to reconsider all the constraints (Chvatal 1983). In particular, if the new constraint is satisfied by the current solution  $w$  (i.e., if its associated slack variable is null) then  $w$  is still optimal and no work is necessary.

The main benefit of the Large Margin algorithm, however, is that prior knowledge about the domain can easily be included in the form of additional constraints on the solution. For example, in the Adaptive Route Advisor, we know that regardless of their particular

<sup>2</sup>If  $w_i \geq 0$  ( $i = 1, \dots, n$ ) then  $\sum_{j=1}^n w_j / \sqrt{n} \leq \|w\| \leq \sum_{j=1}^n w_j$ . Therefore, in relatively low-dimensional problems, minimizing  $\sum_{j=1}^n w_j$  is a reasonable approximation to maximizing the margin.

route preferences (e.g., whether they like to drive on highways or not), drivers will always want routes that are reasonably short and fast. We do not want the system to suggest a route that is 50 miles longer because it avoids one mile of local road and previous interactions indicates that the driver dislike those. This is enforced by adding to the linear program a constraint that the combined weight of the “total distance” and “estimated time” features in  $w$  must represent at least 10% of the total weight. The Adaptive Route Advisor includes similar constraints to express other common-sense knowledge about route preferences, like that even a driver that usually prefers route with fewer left turns will not want to do a right turn followed by a U-turn to avoid a left turn.<sup>3</sup>

Those kinds of constraints on the solution are especially useful when the subjective function must be inferred from a limited amount of data. This will often be the case in the context of Adaptive User Interfaces where we do not want to subject the user to a long “training phase” before starting to adapt to his preferences.

## Experimental Results

In order to compare their bias against the route preferences of real users we tested the Large Margin algorithm and several other adaptation algorithms with human participants. We isolated the adaptation component from the rest of the system by simulating a fixed series of interactions on paper. The test consisted of 20 tasks that involved trips between intersections in the Palo Alto area. To compensate for the lack of interactivity, we produced four routes for each task using weight vectors with a unit weight for one attribute and zero for the rest. This created routes optimized for time, distance, number of turns, and number of intersections. We plotted the four routes, labeled randomly  $A$  through  $D$ , on a map of Palo Alto. We presented the tasks in a different random order for each participant. Figure 5 shows an example of one of the tasks and its four route choices.

We asked the participants to evaluate the routes for each task and rank them in preference order. To control for context (e.g., whether the driver is in a hurry or sightseeing), the participants were instructed to imagine that it is Sunday afternoon, and they are going out on errands. Since an ordering of four items gives six independent binary preferences ( $A$  better/worse than  $B$ ,  $C$ ,  $D$ ;  $B$  better/worse than  $C$ ,  $D$ ;  $C$  better/worse than  $D$ ), each participant provided  $6 \cdot 20 = 120$  training instances. The features for each instance were four metrics describing the route: distance, estimated duration, number of turns, and number of intersections.<sup>4</sup>

<sup>3</sup>It is precisely because those types of “aberrant” recommendations were occasionally produced by the previous version of the Adaptive Route Advisor that the Large Margin algorithm was developed.

<sup>4</sup>These metrics were *not* shown to the participant.

More descriptive features would have been helpful, such as lane merges or traffic lights, but they were not available in our digital map.

## Adaptation Algorithms

Besides the Large Margin algorithm, we tested three other learning methods: perceptron training (Nilsson 1965) as in the previous version of the Adaptive Route Advisor, SANE (Moriarty 1997), and a search over lexicographic orderings.

As described in the previous section, both the Large Margin algorithm and the perceptron algorithm implement simple linear classifiers. To test whether a more powerful representation would be beneficial we also implemented a multi-layer neural network. The hidden layer in this network combines the input features into possibly more meaningful features, and next combines these features into a route cost. Instead of training the network by backpropagation, we found a more natural training signal to be a positive feedback if the relative ordering of two routes is correct or negative if the ordering is incorrect. The SANE (Symbiotic, Adaptive Neuro-Evolution) system (Moriarty 1997) evolves a neural network through such weak feedback. SANE is an evolutionary algorithm that manages a population of neurons, constructs three-layer neural networks from the neurons in each generation, and evaluates the networks. The neurons participating in the most effective networks reproduce to the next generation. SANE’s adaptation has been shown to be fast and accurate over several task domains.

Another natural approach to evaluating routes is a lexicographic (or *fuzzy-lexicographic*) comparison on the features. This involves generating an ordering on the features, such as [Time, Distance, Turns, Intersections], and comparing the features of two routes in that order. If a route is (significantly) better than another for one feature, it is preferred. Otherwise, the comparison goes to the next feature in the ordering. In the fuzzy version used here, the amount by which a feature has to be better for the whole route to be considered better is computed as the product of a *significance parameter*  $\lambda$  with the standard deviation of the values for that feature. In domains such as this with small feature sets, the adaptation algorithm can find the best ordering by explicitly enumerating all possible orderings.

## Results

Figure 6 presents the testing accuracy of the four adaptation algorithms on each participant. For each algorithm, we measure the accuracy with ten-fold cross validation. The cost  $c$  of misclassification in the large margin algorithm was set to 2, and the perceptron was trained for 100,000 epochs with a learning rate  $\eta = 0.001$ . The lexicographic search estimated the significance parameter  $\lambda$  for each possible ordering via cross validation, and tested the best ordering on a hold-

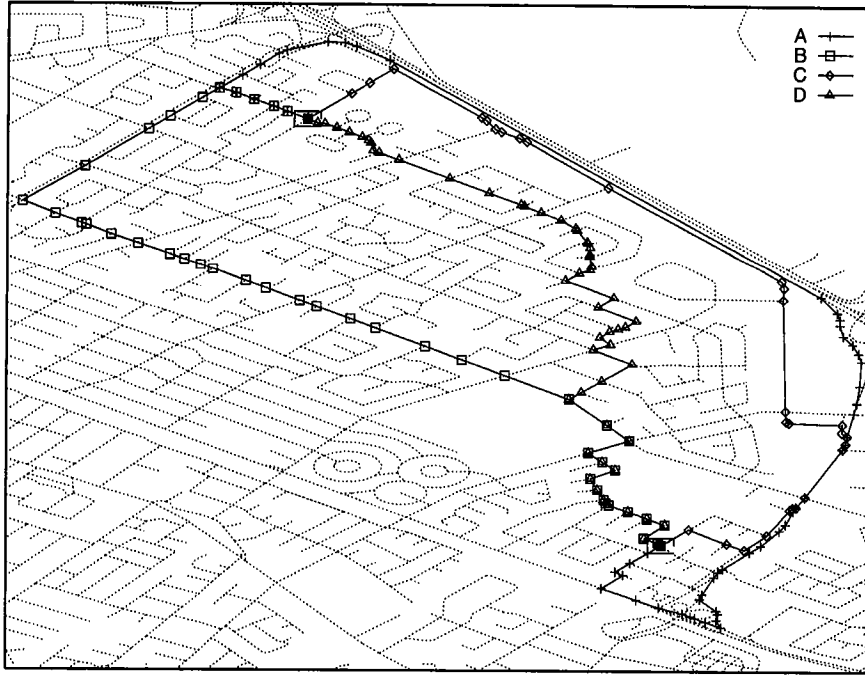


Figure 5: Sample task for the participants. The starting point is the box at the upper left and the ending point is the box at the lower right. *A* is the route with fewest turns, *B* is the fastest route, *C* is the route with fewest intersections, and *D* is the shortest route.

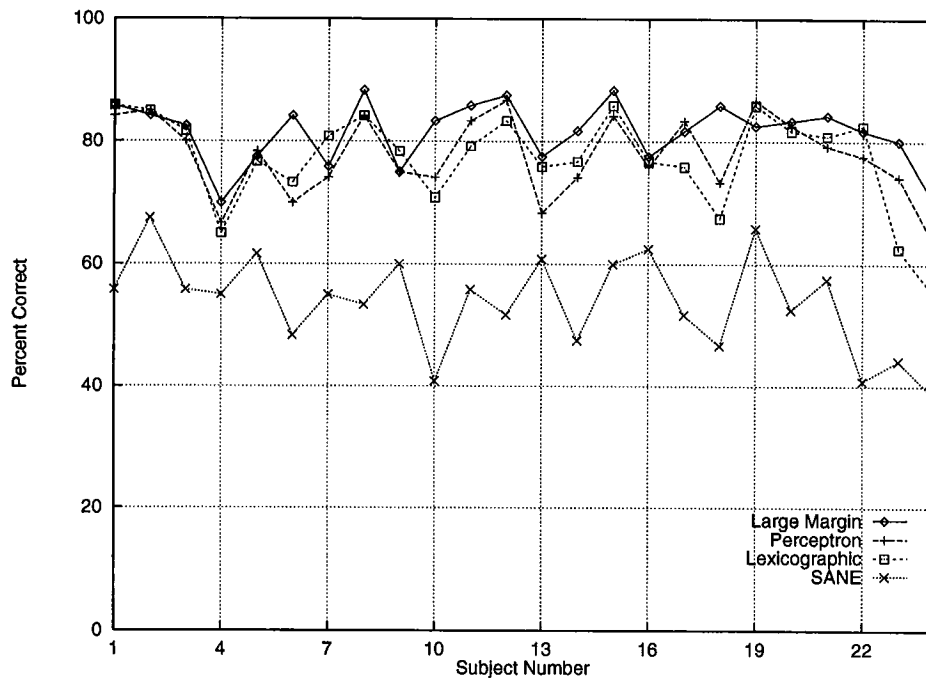


Figure 6: Accuracy of each adaptation technique. The accuracy was computed using ten-fold cross validation.

out set. SANE trained a neural network with five hidden units.

For this data set, the large margin, perceptron, and lexicographic testing accuracies are very similar. They all performed significantly better than chance (50%), but they did not reach 100% performance, even on the training data. For the large margin and perceptron training, this indicates that the human preference data is not linearly separable.

SANE's accuracy, on the other hand, is significantly lower even though its representation is more flexible. There does not appear to be enough training data for SANE to accurately tune the neural network. This seems to corroborate the intuition that in adaptive user interfaces application, where it is important for the system to quickly acquire an approximate model of the user's preferences, simpler representations can learn more accurate models with little data. Other studies of testing accuracy versus representational complexity for a given training set support this finding, such as the comparison of naive Bayes nets to more powerful classifiers (Langley, Iba, & Thompson 1992; Domingos & Pazzani 1997), as well as the comparison of 1-level decision trees (decision "stumps") to full decision-tree induction (Holte 1993).

Although the lexicographic ordering performs well on this data set it is not in general a convenient representation to use in an optimization algorithm. In the route advisor, for instance, it is not practical to generate a lexicographically optimal route, because it is computationally prohibitive to enumerate all possible routes and sort them.

By contrast, both the perceptron training and the large margin algorithm use a linear combination of weights to represent preferences, leading to efficient optimal path calculations. The accuracy of the two algorithms were comparable, but we observed that the model learned by the perceptron for some participants had negative weights on some features. This means that *higher* values for those features makes the route more desirable, which is quite counter-intuitive. This analysis lead us to believe that the large margin approach is a good training algorithm for this domain, because its accuracy is never significantly worse than the best alternative, and its intuitively plausible preference models are more likely to generate better routes.

## Conclusions

To build systems that help their users solve optimization or decision problems it is often desirable to learn a user's subjective function from preference judgments. We have presented a large margin algorithm that we think is particularly well suited for that task. In particular prior knowledge about the domain can easily be used to constraint the models the algorithm generates.

We have demonstrated the algorithm in the Adaptive Route Advisor system and presented here some experimental results on real users that show that the algorithm is competitive. We have also conducted more sys-

tematic experiments on synthetic users that show that the algorithm quickly converges toward good models of the user's route preferences, in a few interactions with the user. These results will be described in a longer version of the paper.

This work can be extended in several ways. We are particularly interested in testing the large margin algorithm in other adaptive recommendation systems to see how it performs in different domains. In the Adaptive Route Advisor, we want to address the problem of the context for the route recommendation. The driver is likely to prefer different routes depending on the time of day or whether he is sightseeing or going to an important business meeting. One simple approach would be to build a separate model for each context. We are more interested in a solution that would automatically "cluster" the preferences into appropriate contexts.

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