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Learning Theory: An Approximation Theory Viewpoint

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Felipe Cucker and Ding-Xuan Zhou
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Foreword

This book by Felipe Cucker and Ding-Xuan Zhou provides solid mathematical foundations and new insights into the subject called *learning theory*.

Some years ago, Felipe and I were trying to find something about brain science and artificial intelligence starting from literature on neural nets. It was in this setting that we encountered the beautiful ideas and fast algorithms of learning theory. Eventually we were motivated to write on the mathematical foundations of this new area of science.

I have found this arena to with its new challenges and growing number of application, be exciting. For example, the unification of dynamical systems and learning theory is a major problem. Another problem is to develop a comparative study of the useful algorithms currently available and to give unity to these algorithms. How can one talk about the "best algorithm" or find the most appropriate algorithm for a particular task when there are so many desirable features, with their associated trade-offs? How can one see the working of aspects of the human brain and machine vision in the same framework?

I know both authors well. I visited Felipe in Barcelona more than 13 years ago for several months, and when I took a position in Hong Kong in 1995, I asked him to join me. There Lenore Blum, Mike Shub, Felipe, and I finished a book on real computation and complexity. I returned to the USA in 2001, but Felipe continues his job at the City University of Hong Kong. Despite the distance we have continued to write papers together. I came to know Ding-Xuan as a colleague in the math department at City University. We have written a number of papers together on various aspects of learning theory. It gives me great pleasure to continue to work with both mathematicians. I am proud of our joint accomplishments.

I leave to the authors the task of describing the contents of their book. I will give some personal perspective on and motivation for what they are doing.

Computational science demands an understanding of fast, robust algorithms. The same applies to modern theories of artificial and human intelligence. Part of this understanding is a complexity-theoretic analysis. Here I am not speaking of a literal count of arithmetic operations (although that is a by-product), but rather to the question: What sample size yields a given accuracy? Better yet, describe the error of a computed hypothesis as a function of the number of examples, the desired confidence, the complexity of the task to be learned, and variants of the algorithm. If the answer is given in terms of a mathematical theorem, the practitioner may not find the result useful. On the other hand, it is important for workers in the field or leaders in laboratories to have some background in theory, just as economists depend on knowledge of economic equilibrium theory. Most important, however, is the role of mathematical foundations and analysis of algorithms as a precursor to research into new algorithms, and into old algorithms in new and different settings.

I have great confidence that many learning-theory scientists will profit from this book. Moreover, scientists with some mathematical background will find in this account a fine introduction to the subject of learning theory.

Stephen Smale
Chicago

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Preface

Broadly speaking, the goal of (mainstream) learning theory is to approximate a function (or some function features) from data samples, perhaps perturbed by noise. To attain this goal, learning theory draws on a variety of diverse subjects. It relies on statistics whose purpose is precisely to infer information from random samples. It also relies on approximation theory, since our estimate of the function must belong to a prespecified class, and therefore the ability of this class to approximate the function accurately is of the essence. And algorithmic considerations are critical because our estimate of the function is the outcome of algorithmic procedures, and the efficiency of these procedures is crucial in practice. Ideas from all these areas have blended together to form a subject whose many successful applications have triggered its rapid growth during the past two decades.

This book aims to give a general overview of the theoretical foundations of learning theory. It is not the first to do so. Yet we wish to emphasize a viewpoint that has drawn little attention in other expositions, namely, that of approximation theory. This emphasis fulfills two purposes. First, we believe it provides a balanced view of the subject. Second, we expect to attract mathematicians working on related fields who find the problems raised in learning theory close to their interests.

While writing this book, we faced a dilemma common to the writing of any book in mathematics: to strike a balance between clarity and conciseness. In particular, we faced the problem of finding a suitable degree of self-containment for a book relying on a variety of subjects. Our solution to this problem consists of a number of sections, all called “Reminders,” where several basic notions and results are briefly reviewed using a unified notation.

We are indebted to several friends and colleagues who have helped us in many ways. Steve Smale deserves a special mention. We first became interested in learning theory as a result of his interest in the subject, and much of the

material in this book comes from or evolved from joint papers we wrote with him. Qiang Wu, Yiming Ying, Fangyan Lu, Hongwei Sun, Di-Rong Chen, Song Li, Luoqing Li, Bingzheng Li, Lizhong Peng, and Tiangang Lei regularly attended our weekly seminars on learning theory at City University of Hong Kong, where we exposed early drafts of the contents of this book. They, and José Luis Balcázar, read preliminary versions and were very generous in their feedback. We are indebted also to David Tranah and the staff of Cambridge University Press for their patience and willingness to help. We have also been supported by the University Grants Council of Hong Kong through the grants CityU 1087/02P, 103303, and 103704.