Learning through Crowdfunding*

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Abstract

We develop a model where reward-based crowdfunding enables firms to obtain a reliable proof of concept early in their production cycle: they learn about total demand from a limited sample of target consumers pre-ordering a new product. Learning from the crowdfunding sample creates a valuable real option as firms invest only if updated expectations about total demand is sufficiently high. This is particularly valuable for firms facing a high degree of uncertainty about consumer preferences, such as developers of innovative consumer products. Learning also enables firms to overcome moral hazard. The higher the funds raised, the lower the firms' incentives to divert them, provided third-party platforms limit the sample size by restricting campaign length. While the probability of campaign success decreases with sample size, the expected funds raised are maximized at an intermediate sample size. Our results are consistent with stylized facts and lead to new empirical implications.

JEL codes: D80, G30, L14, L26, O30

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1 Introduction

Reward-based crowdfunding platforms enable firms to raise funds directly from future consumers, typically in exchange for the promise to deliver a new product in the future. While they have been seen as means for artists and credit constrained firms to seek support for their projects, primary beneficiaries of these platforms appear to be firms that develop innovative consumer products.

Data we have obtained from Kickstarter, arguably the largest reward-based crowdfunding platform, show that as of August 8, 2018, *i)* 65% of the $3.4 billion successfully raised have gone to firms selling technology, design or gaming products, and *ii)* an average successfully funded project raised \$93K, \$63K and \$55K in technology, design and games, respectively, but only \$10K across all other categories, and *iii)* 284 out of the 310 projects that have raised over $1 million ($4,147$ out of the $5,265$ projects that have raised over \$100K) were in the three categories aforementioned.

In this paper we develop a theoretical model to understand prime sources of value creation in reward-based crowdfunding, and why it is particularly attractive for innovative projects. We argue that reward-based crowdfunding platforms play an important role in enabling firms to test out their market at an early stage of product development. Pre-selling a product through these platforms acts as a credible consumer survey where firms learn about consumer preferences before making their investment decisions. We show that this creates a substantial real option value of learning: observing the decisions of a random sub-sample of consumers (backers) enables the firm to update its beliefs about the preferences of *all* their future consumers, including those outside the crowdfunding sample. There is value in both success and failure: regardless of whether the project seems profitable ex-ante, firms either benefit from learning that demand for their product is sufficiently high or save on investment costs. We show that this real option value of learning is maximized at an intermediate level of the investment cost and that it increases with uncertainty about consumer preferences. This can explain why innovative products with most demand uncertainty are likely to benefit most from reward-based crowdfunding.

While there is value in learning, crowdfunding may be hampered by a well-known moral hazard problem (Tirole, 2006): firms may be tempted to divert the funds they have raised instead of delivering the products and if anything, innovative firms may be particularly prone to moral hazard and informational frictions (Bussgang 2014, Lerner et. al. 2012). Further, reward-based crowdfunding platforms are not legally responsible for guaranteeing the delivery of rewards and proving that a firm has committed a fraud is difficult.[1] Yet, the vast majority of projects do deliver the promised

[1] Kickstarter makes it clear that legal protection is limited and that the relationship relies primarily on interactions between the firm and backers "Backers must understand that Kickstarter is not a store. When you back a project, you're helping to create something new — not ordering something that already exists.", see

rewards. For example, Mollick (2014) finds that only 3.6% of successful Kickstarter projects have failed to deliver them.

We show that the real option value of learning is a powerful force that enables firms to endogenously overcome moral hazard. After its crowdfunding campaign, a firm that expects there to be high future consumer demand will choose to not divert funds, even if it is costless to do so. Since a firm that faces greater demand uncertainty has a greater option value, it can also overcome moral hazard more easily. We allow the firm to set a target of funds to be raised and to solicit the platform to return the funds to backers if that target is not met. If the firm sets a low or zero target, it can invest and deliver the product in some states and divert funds in other states. We prove that the firm chooses an "All-or-Nothing" (AoN) crowdfunding scheme with a sufficiently high target so that it has incentives to invest after a successful campaign. AoN dominates "Keep-it-All" (KiA) where all funds raised are passed on to the firm regardless of the success of the campaign. Under KiA, the discount the firm needs to offer is too high compared to the expected benefit of diverting funds. We also discuss the robustness of our main findings to variable costs and uncertainty about the firm's ability to develop its product.

We further investigate the relationship between crowdfunding outcomes and the crowdfunding sample size, which can be proxied by the campaign length. Under moral hazard, it is neither feasible nor optimal to pre-sell the product to all potential consumers. It then benefits the firm to have a third-party platform that ensures that the campaign targets a limited size sample. We show that in the presence of moral hazard, shorter campaigns are more likely to succeed and the expected funds raised are maximized at an intermediate sample size. This implies that proportional platform fees (charged by Kickstarter and many other platforms) are also maximized at intermediate sample size, and may explain why Kickstarter has set a limit to maximum campaign length. We further show that whether or not a firm is constrained by such limit or prefers an even smaller sample size depends on how costly it is for the firm to fail to meet its crowdfunding target.

Our model can explain a number of stylized facts about reward-based crowdfunding. An important and potentially surprising stylized fact, consistent with our model, is that successful AoN crowdfunding campaigns systematically raise more funds than the target set at the beginning of the campaign. This pattern is present in all categories, but is most pronounced in the case of Technology projects. As illustrated on Figure 1, which plots Kickstarter projects in the first three quarters of 2015, around 40% of successful projects raised at least twice the target, around 20% raised at least four times the target, etc. Appendix A.1 confirms this pattern with another sample period

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https://www.kickstarter.com/help/faq/kickstarter%20basics.

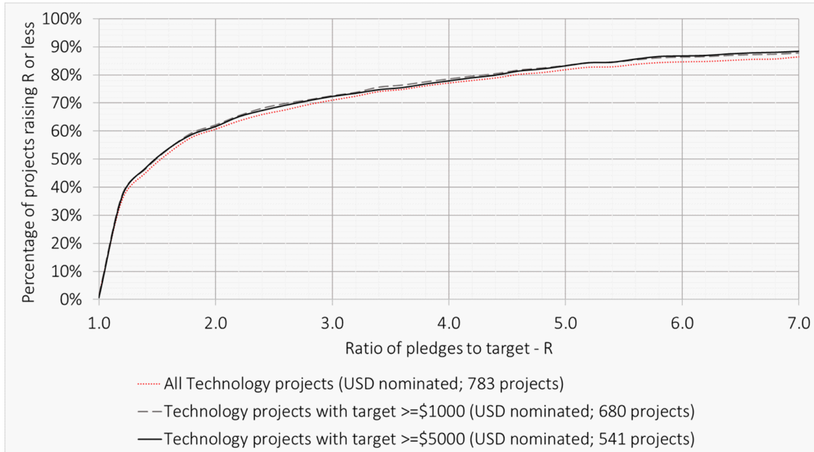

Figure 1: "Oversubscribed" technology projects. The figure includes all successful USD denominated technology projects on Kickstarter between 1 January, 2015 and 17 September, 2015.

for all categories, illustrating that projects that are likely to involve high demand uncertainty such as technology, design or gaming products are most "overfunded".

Earlier theoretical explanations of reward-based crowdfunding have focused on backer preferences rather than learning and moral hazard. For example, Belleflamme, Lambert, and Schwienbacher (2014) assume that participation in crowdfunding provides backers with an additional utility compared to their valuation for the product, which enables firms to raise funds and to price-discriminate. Varian (2013) endogenizes this additional utility by deriving an equilibrium in which seemingly altruistic backer preferences are due to each of them having a pivotal role in ensuring that the firm has enough funds to invest and to produce the product that the backer values. Yet, these important consumer side effects alone cannot explain some important patterns of successful crowdfunding campaigns, such as products being sold at par or at a discount, and the fact that many products are oversubscribed multiple times over the target.

A few contemporaneous theoretical papers also consider producer side effects of reward-based crowdfunding. Ellman and Hurkens (2016) focuses on price discrimination which can arise when the firm commits to invest only if the number of backers who bid in excess of the crowdfunding price is sufficiently high. There is no learning about consumer preferences except in their Section 5.2, in which learning affects price dynamics during the crowdfunding campaign rather than investment decisions. Further, their setting rules out moral hazard by assuming high reputation costs. We suggest tests for the presence of moral hazard by deriving empirical patterns that should only be observed under moral hazard, some of which have indeed been documented. Strausz (2017) does consider moral hazard and argues that it needs to be mitigated by deferred payments and/or by conditional pledging where backers stop making contributions after the firm meets its target. Yet Kickstarter does not impose deferred payments and the systematic overpledging observed on this platform (Figure 1 and Appendix A1) is broadly inconsistent with conditional pledging where backers stop contributing after the target is met. This is important given that "overpledging" is most pronounced in innovative categories that raise most funds. Strausz (2017) also assumes credit constraints and that the distribution of consumer preferences is known, which implies that learning from a sample of backers does not reveal information about consumer preferences out of sample. In our model, we show that in the limit case where the distribution of consumer preferences is known, the benefit of learning is significantly reduced and crowdfunding either brings no value added to an unconstrained firm or is not possible under severe moral hazard. Chang (2016) considers a common value setting in which a firm funds its project via crowdfunding. In his model, backers are assumed to cooperate in deciding whether to invest after observing a common noisy signal about the value of the project and the firm will only use crowdfunding to cover the difference between the project cost and other sources of funding. Our paper also differs from these theoretical contributions in that we analyse the relationship between the size of the crowdfunding sample and crowdfunding outcomes. Our model explains why firms selling innovative products benefit from crowdfunding more than sellers of other products. Further, we derive empirical predictions that can be used to test our model against the alternative theoretical models discussed above.

We also contribute to the literature that points out that investing in entrepreneurial projects enables firms to experiment new technologies (Hellmann 2002, Gromb and Scharfstein 2005, Bettignies and Chemla 2008, Manso 2016, Kerr, Nanda, and Rhodes-Kropf 2014). We show that crowdfunding is an efficient mechanism to learn about demand with minimal experimentation costs. This further relates to the trade-off between exploration and exploitation emphasized in Manso (2011, 2016). Indeed, crowdfunding allows for more effective experimentation than outright entrepreneurship for two reasons: first, it provides tolerance for early failure as the cost of a failed AoN campaign is zero to backers and generally low for the firm; second, crowdfunding allows for timely feedback on performance through an early proof of concept.

Our theory of crowdfunding also relates to the industrial organization literature (Crawford and Shum, 2005, Chu and Zhang, 2011), which primarily views the pre-selling of existing products as a price-discrimination mechanism. A notable exception that analyzes pre-selling at an earlier stage of the product cycle is Cornelli (1996) who highlights the value of pre-selling in a pre-crowdfunding environment. In her paper as in our setting, firms that face high enough fixed costs benefit most from such pre-selling. The added benefit of crowdfunding is its role in facilitating commitment to pre-sell the product being developed to a limited sample, which in turn makes it easier for firms to extract surplus.

Our paper can also be related to the strands of corporate finance and monetary economics that view asymmetric information and moral hazard as major sources of financial constraints (Myers 1977, Stiglitz and Weiss 1981, Hart 1995, Tirole 2006, Kiyotaki and Moore 2002). The campaign itself may either provide actual funding or alleviate the root causes of financial constraints. For example, if financial constraints are driven by asymmetric information about demand between investors and the firm, then the crowdfunding campaign that generates public information about demand alleviates these constraints.[2] This may generate a complementarity between reward-based crowdfunding and traditional outside financing. There are indeed reported cases where firms, after succeeding in reward-based crowdfunding, obtain further funding from angels, venture capitalists or investor-based crowdfunding.[3] Relatedly, investor-based crowdfunding highlights the benefits of learning public information from the "crowd" in terms of screening credit-worthiness (see Iyer et al 2015) and the signaling of loan quality (Hildendbrand, Puri, and Rocholl 2016).

2 The Model

We consider a three-date model in which a firm has a new product idea, and can learn about demand after observing consumer decisions at date 0. At date 1, the firm updates its beliefs and decides whether or not to invest $I \geq 0$. At date 2 production takes place and the firm sets the product price p_2. For now, the firm's marginal cost of production is zero. All agents are risk neutral and the discount rate is normalized to 1. We do not impose financial constraints.

The firm's potential market consists of N consumers. Each consumer $i \in \{1, ..., N\}$ has private valuation $v^i \in \{0, 1\}$ for one unit of the product and 0 for any additional unit. We refer to a consumer with valuation $v^i = 1$ (resp. $v^i = 0$) as a 1-consumer (resp. a 0-consumer). Private valuations are *conditionally* i.i.d., which implies that consumer i's valuation is a Bernoulli trial drawn from the true distribution, i.e., $\Pr\left(v^i = 1|\theta\right) = \theta$. The probability θ that consumer i is a 1-consumer (and the aggregate share of 1-consumers) is unknown to the firm and it follows a beta

[2]Further, publicly disclosing information about consumer preferences can alleviate information asymmetry in an unbiased manner. In contrast, any information revealed publicly through the actions of privately informed entrepreneurs may distort both firm value and investment decisions. For example, Myers and Majluf (1984) argue that equity issuance announcements convey negative information to investors, while Tinn (2010) and Angeletos, Lorenzoni and Pavan (2010) show that technology investments can be perceived as a positive public signal.

[3]Brunstein, Joshua, 2014, "How Kickstarter turned into the Venture Capitalist's Best Friend?", Bloomberg Business, August 11, http://www.bloomberg.com/bw/articles/2014-08-11/kickstarter-successes-pivot-from-crowdfunding-to-venture-capital

distribution, the p.d.f. of which is

$$f_\theta(\theta) = \frac{\theta^{\alpha-1}(1-\theta)^{\beta-1}}{B(\alpha,\beta)}, \tag{1}$$

where α, β are positive parameters and $B(\alpha,\beta)$ is the beta function. Beta distributions enable us to capture different distributions of prior beliefs, be they U-shaped, hump-shaped, or uniform. For the sake of clarity, we write $\alpha = \theta_0\lambda$ and $\beta = (1-\theta_0)\lambda$, where $\theta_0 \in (0,1)$ and $\lambda > 0$ and finite, such that

$$\mathbb{E}[\theta] = \frac{\alpha}{\alpha+\beta} = \theta_0; \, Var[\theta] = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} = \frac{\theta_0(1-\theta_0)}{(\lambda+1)}.$$

That is, θ_0 is the prior mean and a higher λ implies a lower level of uncertainty based on prior beliefs. All agents know the prior distribution. Consumer i knows his own valuation for the product.

Importantly, this information structure captures a realistic feature of learning where observing the preferences of a subsample of consumers reveals information about the preferences of other, out of sample, consumers. If the firm observes that consumer i values the product highly, it Bayesian updates its beliefs about the valuation of all other, $j \neq i$, consumers: $\Pr\left(v^j = 1 | v^i = 1\right) = \frac{\lambda\theta_0+1}{\lambda+1} > \theta_0$. The higher uncertainty (lower λ), the more learning. At the limit, when $\lambda \to \infty$, the consumer preference distribution becomes public information, and learning about preferences of one consumer reveals no information about preferences of other consumers, i.e., $\lim_{\lambda\to\infty} \Pr\left(v^j = 1 | v^i = 1\right) = \lim_{\lambda\to\infty} \frac{\lambda\theta_0+1}{\lambda+1} = \theta_0$. While we maintain the assumption that λ is finite, Section 5 considers the special case where $\lambda \to \infty$.

As a benchmark we consider a frictionless consumer survey where $M \leq N$ consumers truthfully and costlessly reveal their preferences at date 0. The firm makes an investment decision based on updated beliefs about consumer preferences at date 1, and provided it invests, it delivers the product to consumers at date 2. Such frictionless consumer survey is difficult in practice because consumers face zero cost when overstating their interest in the product. In contrast, reward-based crowdfunding enables the firm to ensure that consumers who make pledges are not 0-consumers as these would lose money by pre-ordering the product. Sections 3 to 5 consider a fixed M and Section 6 analyzes how crowdfunding outcomes vary with M.

The players are the firm and N potential consumers of the firm's product. If the product is produced, then all consumers can buy it at date 2, while only a random subset of these consumers are potential backers who can pre-order the product through crowdfunding at date 0. Because pre-ordering takes place before the firm decides whether or not to invest, it involves a potential risk that the product will not be produced and the consumer loses his contribution. Without loss of generality, we index consumers who can participate in crowdfunding as $i = \{1, ..., M\}$. Let us

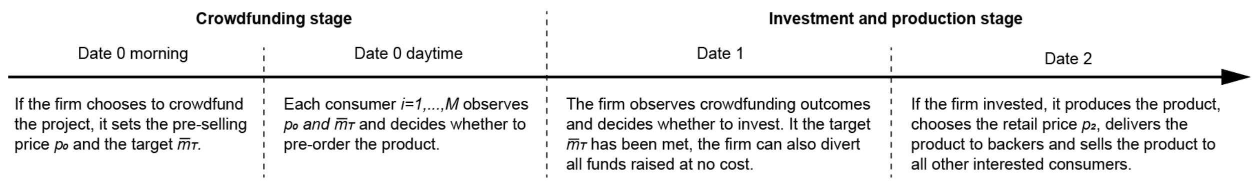

Figure 2: Timing of events under crowdfunding.

denote m the number of 1-consumers in set $\{1, ..., M\}$, and m^B the number of consumers in set $\{1, ..., M\}$ who choose to pre-order the product. We denote the pledging decision of consumer $i = \{1, ..., M\}$ with an indicator function

$$\mathbf{1}_0^i \equiv \begin{cases} 1 \text{ if consumer } i \text{ chooses to pledge} \\ 0 \text{ otherwise.} \end{cases}$$

The crowdfunding platform is an interface that, first, enables the firm to post the project at date 0 in the morning, second, collects pledges during the campaign which takes place at date 0 during daytime, and, third, passes on the funds collected to the firm at date 0 in the evening. The platform also enables the firm to commit to a target number of backers $\bar{m}_T$, which implies that any funds raised during the campaign get automatically returned to the consumers if the firm fails to meet the target.

We denote the event that the firm meets its target with an indicator function

$$T_{\bar{m}_T} \equiv \begin{cases} 1 \text{ if } m^B \geq \bar{m}_T \\ 0 \text{ if } m^B < \bar{m}_T \end{cases}.$$

The target choice is also, de facto, a choice between AoN and KiA, as the firm can set any finite target $\bar{m}_T$ under AoN and by construction KiA boils down to $\bar{m}_T = 0$. Indeed, under KiA a "target" has no economic meaning as the funds raised are passed on to the firm regardless of whether or not this "target" is met.[4]

The firm decides whether to invest at date 1 and its decision to invest is denoted with

$$\mathbf{1}_1^F \equiv \begin{cases} 1 \text{ if the firm invests } I \\ 0 \text{ otherwise.} \end{cases}$$

If the firm invested at date 1, it sets a price p_2 for the product at date 2, and consumers choose whether to buy the product. Denote 1_2^i the choice of consumer $i = 1, ..., N$, at date 2, with $1_2^i = 1$ if the consumer buys the product and 0 otherwise. The timing of events is illustrated on Figure 2

Since a consumer will not buy the product twice, we must have

$$T_{\bar{m}_T} \cdot \mathbf{1}_0^i + \mathbf{1}_2^i \leq 1 \text{ for any } i \in \{1, ..., M\}.$$

[4] In our model, the firm does not invest in its own campaign. In practice, the firm could set a positive target and contribute its own funds to meet its campaign target with probability 1. This would boil down to setting a target of funds raised from backers other than the firm to $\bar{m}_T = 0$.

Given these actions the payoff to a firm that chooses to participate in crowdfunding is

$$\pi^F = T_{\bar{m}_T} \cdot p_0 m^B + \mathbf{1}_1^F \cdot \left(p_2 \sum_{i=0}^{N} \mathbf{1}_2^i - I \right) - (1 - T_{\bar{m}_T}) \cdot \varsigma \cdot \mathbf{1}_1^F, \tag{2}$$

where $\varsigma > 0$ is a cost of investing after a campaign that failed to meet its pre-set target. Our main model in Sections 3-4 considers a parametric restriction $\varsigma \leq \bar{\varsigma}_M \equiv \frac{\lambda+M}{N-M}\left(I - \frac{\lambda\theta_0(N-M)}{\lambda+M}\right)$ whenever $I > \frac{\lambda\theta_0(N-M)}{\lambda+M}$. A positive ς guarantees that the firm strictly prefers to meet its crowdfunding target to failing to meet it.[5] A small ς is realistic for two reasons. First, platforms like Kickstarter take active steps to make it more difficult to find information about failed campaigns, which is likely to greatly limit any pure reputation costs of failure. Second, empirical evidence suggests that many firms complete their projects after a failed campaign (see Section 7), which we will show is consistent with small values of ς. Sections 5 and 6 further consider the effect of $\varsigma \geq \bar{\varsigma}_M$. The payoff function (2) captures the moral hazard problem: if the firm does not invest at date 1, i.e., $\mathbf{1}_1^F = 0$, it diverts $T_{\bar{m}_T} \cdot p_0 m^B \geq 0$ at zero cost.

The payoff to a consumer who belongs to a set of potential backers, $i \in \{1, ..., M\}$, is

$$u^i = T_{\bar{m}_T} \cdot \mathbf{1}_0^i \cdot \left(\mathbf{1}_1^F \cdot v^i - p_0\right) + \mathbf{1}_1^F \cdot \mathbf{1}_2^i \cdot \left(v^i - p_2\right), \tag{3}$$

where $\mathbf{1}_0^i \cdot \left(\mathbf{1}_1^F \cdot v^i - p_0\right)$ means that pledging, $\mathbf{1}_0^i = 1$, comes with the risk of paying p_0 and not getting the product if the firm does not invest after the firm meets its target. The consumer can choose to not pledge at date 0, i.e., he can set $\mathbf{1}_0^i = 0$, and wait until date 2 to decide whether or not to purchase the product. Whether there is a product to purchase at date 2 depends on $\mathbf{1}_1^F$. However, the consumer faces zero risk of no delivery at date 2. The payoff to consumer $i = \{M+1, ..., N\}$ who cannot participate in crowdfunding in date 0 is

$$u^i = \mathbf{1}_F^i \cdot \mathbf{1}_2^i \cdot \left(v^i - p_2\right). \tag{4}$$

In order to derive a Perfect Bayesian Equilibrium (PBE) of this game, we need to specify the information sets to all players at different dates. Both the firm and consumers know the parameters of the model and the distribution of v^i. When choosing p_0 and $\bar{m}_T$ at date 0 in the morning, the firm only knows the prior distribution (1) and parameters of the model. During daytime at date 0, consumer i's information set is $\Omega_0^i = \left\{p_0, \bar{m}_T, v^i\right\}$. When deciding whether to invest at date 1 the firm has the information set $\Omega_1^F = \left\{p_0, \bar{m}_T, m^B\right\}$. At date 2 uncertainty is resolved.

Denote $b^i\left(\Omega_0^i\right)$ the probability with which a consumer $i \in \{1, ..., M\}$ expects to receive the product at date 2 if he pledges at date 0. His beliefs depend on $\bar{m}_T$ and his own valuation v^i.

[5] This assumption rules out the deviating strategy whereby the firm sets an infinite target, perfectly learns consumer preferences and proceeds without raising any funds. Such possibility would make it impossible for crowdfunding platforms to have a viable business model.

On (and off) the equilibrium path, he may also need to form expectations about the strategies of other consumers which depend on their valuation v^j, $j \neq i$. If so, his beliefs must be consistent with Bayes' rule, i.e., the consumer assigns $\Pr\left(v^j = 1|v^i\right) = \frac{\lambda\theta_0 + v^i}{\lambda+1}$ and takes the strategies of other consumers as given. We denote $b^F\left(m|m^B\right)$ the firm's beliefs about the total number of 1-consumers in set $i \in \{1, ..., M\}$ conditional on consumers who backed the project.

Denote the equilibrium quantities with "$*$" and alternative strategies without "$*$". To shorten the notation, we express the players' expected payoffs as a function of their actions only. In any PBE where the firm chooses to participate in crowdfunding, its equilibrium pricing and target $\{p_0^*, \bar{m}_T^*\}$ choice must satisfy

$$\mathbb{E}\left[\pi^F\left(p_0^*, \bar{m}_T^*, \mathbf{1}_1^{F*}, p_2^*\right)\right] \geq \mathbb{E}\left[\pi^F\left(p_0, \bar{m}_T, \mathbf{1}_1^{F*}, p_2^*\right)\right], \tag{5}$$

its equilibrium investment decision, $\mathbf{1}_1^{F*}$, must satisfy

$$\mathbb{E}\left[\pi^F\left(p_0^*, \bar{m}_T^*, \mathbf{1}_1^{F*}, p_2^*\right)|\Omega_1^F\right] \geq \mathbb{E}\left[\pi^F\left(p_0^*, \bar{m}_T^*, \mathbf{1}_1^{F}, p_2^*\right)|\Omega_1^F\right] \tag{6}$$

and its date 2 pricing choice, p_2^*, must satisfy:

$$\pi^F\left(p_0^*, \bar{m}_T^*, \mathbf{1}_1^{F*}, p_2^*\right) \geq \pi^F\left(p_0^*, \bar{m}_T^*, \mathbf{1}_1^{F*}, p_2\right) \tag{7}$$

The pledging strategy of consumer $i \in \{1, ..., M\}$, $\mathbf{1}_0^{i*}$, must satisfy

$$\mathbb{E}\left[u^i\left(\mathbf{1}_0^{i*}, \mathbf{1}_2^{i*}\right)|\Omega_0^i\right] \geq \mathbb{E}\left[u^i\left(\mathbf{1}_0^{i}, \mathbf{1}_2^{i*}\right)|\Omega_0^i\right] \tag{8}$$

and the date 2 buying strategy of consumer $i \in \{1, ..., N\}$, $\mathbf{1}_2^{i*}$, must satisfy

$$\begin{aligned} u^i\left(\mathbf{1}_0^{i*}, \mathbf{1}_2^{i*}\right) &\geq u^i\left(\mathbf{1}_0^{i*}, \mathbf{1}_2^{i}\right) \text{ if } i \in \{1, ..., M\} \\ u^i\left(\mathbf{1}_2^{i*}\right) &\geq u^i\left(\mathbf{1}_2^{i}\right) \text{ if } i \in \{M+1, ..., N\}. \end{aligned} \tag{9}$$

The firm may also choose not to participate in crowdfunding. It is easy to see that all 1-consumers then buy the product at date 2 as long as $0 < p_2 \leq 1$, and it is optimal for the firm to set $p_2 = 1$. This implies that the NPV of the project is $-I + N\mathbb{E}[\theta] = -I + N\theta_0$, and the firm invests if, and only if, $I \leq N\theta_0$. We will refer to this benchmark firm as *the reference firm.* Its expected value is

$$\pi_{ref}^F \equiv \max\left[0, (N\theta_0 - I)\right].$$

This implies that the firm benefits from crowdfunding if its expected payoff satisfies

$$U^F \equiv \mathbb{E}\left[\pi^F\left(p_0^*, \bar{m}_T^*, \mathbf{1}_1^{F*}, p_2^*|\Omega_0\right)\right] - \pi_{ref}^F > 0. \tag{10}$$

In any PBE, the strategies and beliefs must satisfy both sequential rationality and the consistency of beliefs, i.e., at date 0 the firm anticipates that each consumer $i \in \{1, ..., M\}$ will follow the equilibrium strategy given his type v^i, and each consumer $i \in \{1, ..., M\}$ anticipates the firm's optimal investment decision given m^B. We focus on deriving a fully revealing PBE in pure strategies. Since all consumers of the same type are identical, we expect there to be a symmetric equilibrium. If the symmetric PBE is fully revealing, then consistency of beliefs requires that the firm's beliefs on the equilibrium path are *skeptical*, which means that observing m^B, the firm considers that all the backers who did not pledge were 0-consumers, i.e., $b^F\left(m|m^B\right) = \Pr\left(m = m^B\right) = 1$. We consider the off-equilibrium beliefs to be the same.

3 Benchmark and the value of learning

3.1 Updated beliefs

Before analyzing the crowdfunding game described in Section 2, we consider the benchmark of the frictionless survey, where at date 0 the firm observes sample M and learns the preferences of each consumer $i \in \{0, 1, ..., M\}$. After the firm observes the number m of 1-consumers in this sample, it updates its expectations about the share of 1-consumers in the entire target market, N. Since $v^i|\theta$ is a Bernoulli trial, $m|\theta$ follows the binomial distribution. Bayes' rule implies that the posterior distribution is also Beta, with $\theta|m \sim Be\left(\lambda\theta_0 + m, \lambda\left(1-\theta_0\right) + M - m\right)$. Therefore, the posterior expectations of the share of 1-consumers out of sample is

$$\mathbb{E}\left[\theta|m\right] = \frac{\lambda\theta_0 + m}{\lambda + M}, \tag{11}$$

which is monotonically increasing in m. We derive the distribution of m

$$\begin{aligned} q_m &\equiv \Pr\left(m\right) = \mathbb{E}\left[\Pr\left(m|\theta\right)\right] = \int_0^1 \Pr\left(m|\theta\right) f\left(\theta\right) d\theta \\ &= \binom{M}{m} \frac{B\left(\lambda\theta_0 + m, \lambda\left(1-\theta_0\right) + M - m\right)}{B\left(\lambda\theta_0, \lambda\left(1-\theta_0\right)\right)}, \end{aligned} \tag{12}$$

which means that m is a beta-binomial random variable. The shape of the beta-binomial distribution replicates the shape of the underlying prior beta distribution. Unconditional central moments of the beta-binomial variable can be written

$$\mathbb{E}\left[m\right] = M\theta_0 \text{ and } Var\left[m\right] = \frac{M\left(\lambda + M\right)\theta_0\left(1-\theta_0\right)}{\left(\lambda+1\right)} = M\left(\lambda + M\right) Var\left[\theta\right]. \tag{13}$$

The expected m is proportional to the prior mean of the share of 1-consumers, θ_0, and for any $M > 1$, an increase in λ leads to a decrease in uncertainty about both θ and m.

3.2 Benchmark investment decision

The profit of a firm is given by $\mathbf{1}_1^F \left(p_2 \sum_{i=0}^{N} 1_2^i - I \right)$. At date 2, the firm can only sell its product at price $p_2 > 0$ to 1-consumers. Each of these consumers then obtains a surplus $1 - p_2$. If the firm invests at date 1, its expected profit is $p_2 \left(m + (N - M) \mathbb{E} [\theta|m] \right) - I$, where m consumers are known to value the product and $(N - M) \mathbb{E} [\theta|m]$ other future consumers are expected to be 1-consumers. Both the firm value and the joint surplus, i.e. the sum of the payoff to the firm and of the consumer surplus, are maximized when $p_2 = 1$. The firm extracts all consumer surplus and invests in all projects that are non-negative NPV based on updated beliefs.

Denoting $\lceil . \rceil$ the ceiling function, i.e., the nearest integer rounded up, we obtain:

Proposition 1 *At date* 1*, the first best investment decision is as follows: If* $I \leq I_0 \equiv \frac{\lambda \theta_0 (N-M)}{\lambda + M}$*, then the firm invests regardless of the realization of* m*. If* $I > I_0$*, the firm invests if, and only if,*

$$m \geq \tilde{m} \equiv \left\lceil \frac{\lambda + M}{\lambda + N} \left(I - I_0 \right) \right\rceil . \tag{14}$$

When the firm invests, its NPV at date 1 is

$$D(m) = m + (N - M) \mathbb{E} [\theta|m] - I = \frac{\lambda + N}{\lambda + M} m + I_0 - I \tag{15}$$

Proof. Follows from (11), the NPV being non-negative at date 1 and m being an integer. ■

The first part of Proposition 1 highlights that if $N > M$, then the firm with an investment cost lower than I_0 does not benefit from learning about consumer preferences as it invests even if $m = 0$. However, when the fixed cost is sufficiently high, learning affects investment.

3.3 The real option value of learning under the benchmark

At date 0, the firm's expected utility equals its value of learning, i.e., its expected profits with learning minus the profit of the reference firm. From (13) and (15) the unconditional expectation is $\mathbb{E} [D(m)] = N\theta_0 - I$ and the value of learning about demand is

$$U_B^F = \begin{cases} U_{B,I}^F \text{ if } I < N\theta_0 \\ U_{B,NI}^F \text{ if } I \geq N\theta_0 \end{cases} \text{, where} \tag{16}$$

$$U_{B,I}^F \equiv \mathbb{E} [D(m) | m \geq \tilde{m}] \Pr(m \geq \tilde{m}) - (N\theta_0 - I) = -\mathbb{E} [D(m) | m < \tilde{m}] \Pr(m < \tilde{m}) \tag{17}$$

$$U_{B,NI}^F \equiv \mathbb{E} [D(m) | m \geq \tilde{m}] \Pr(m \geq \tilde{m}), \tag{18}$$

and the subscripts "I" and "NI" denote whether or not the reference firm invests. Since $D(m) < (>) \, 0$ for any $m < (>) \, \tilde{m}$ and $D(m) \geq 0$ if $m = \tilde{m}$, both $U_{B,I}^F$ and $U_{B,NI}^F$ are positive. Further, $U_{B,I}^F$

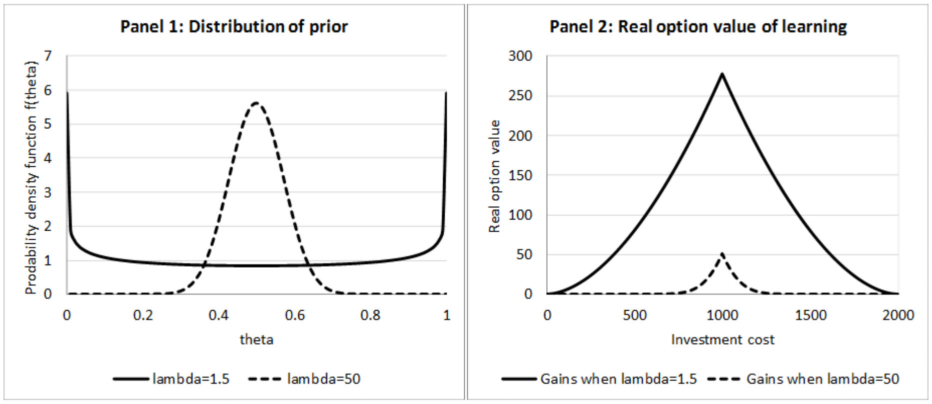

Figure 3: First-best value of learning

and $U^F_{B,NI}$ represent both the value of learning for the firm and the joint surplus from learning for both the firm and consumers. From (14), the value of learning is positive if

$$I_0 < I < I_0 + M\frac{\lambda + N}{\lambda + M}.$$

This guarantees that the investment threshold $\tilde{m} \in \{1, 2, ..., M\}$.

Proposition 2 *The value of learning is maximized when $I = N\theta_0$. Further, $U^F_{B,I}$ is increasing in I and $U^F_{B,NI}$ is decreasing in I.*

Proof. See Online Appendix B.2. ■

Proposition 2 shows that a firm that expects to break even based on prior beliefs has most to gain from learning, while the overall relationship between I and the value of learning is hump-shaped. If $I < N\theta_0$, the firm benefits from avoiding a sub-optimal investment. This benefit increases with the investment cost that it expects to save. If $I > N\theta_0$, the firm can learn that investment is worth undertaking and thus the higher I the lower the returns from the investment. The opportunity to learn about demand provides the firm with a real option.

Proposition 3 *The value of learning increases with the degree of prior uncertainty about demand, i.e., it decreases with λ.*

Proof. See Online Appendix B.3. ■

Proposition 3 shows that the more uncertain the firm about the preferences of its target consumers, the higher its expected gain from learning. Since innovative consumer products (e.g., new technology gadgets) are more likely to be characterized by high demand uncertainty (low λ), we expect innovative firms to benefit most from learning. Uncertainty affects the firm value through three channels. First, an increase in uncertainty increases the difference between the firm prior and updated beliefs about the share of 1-consumers, i.e., $|\mathbb{E}[\theta|m] - \mathbb{E}[\theta]| = \frac{|m-M\theta_0|}{M+\lambda}$. Second, from (14), higher uncertainty may affect the threshold, $\tilde{m}$, at which the firm finds it optimal to invest. Third, the distribution of m with a higher λ second order stochastically dominates the one with a lower λ. Overall, these effects ensure that the effect of an increase in uncertainty increases the value of learning.

Figure 3 illustrates these comparative statics by plotting the value of learning about demand as a function of λ and I. We consider two possible prior distributions with the same mean ($\theta_0 = \frac{1}{2}$) and different values of λ: $\lambda_1 = 1.5 < \lambda_2 = 50$ so that the distribution with λ_1 is U-shaped and the one with λ_2 is hump-shaped, as illustrated on Panel 1. The figure assumes $\theta_0 = \frac{1}{2}$, $M = 200$, and $N = 2000$. The firm breaks even if $I = 1000$ for both $\lambda = \lambda_1$ and $\lambda = \lambda_2$. The value of learning is positive if $I \in (7, 1993)$ for $\lambda = \lambda_1$, and if $I \in (180, 1820)$ for $\lambda = \lambda_2$. Panel 2 plots the value of learning about demand as a function of I for both λ_1 and λ_2.

Overall, this analysis shows that both fixed costs and the degree of uncertainty about consumer preferences are important drivers of the value of learning. These features are likely to be characteristic to technology, design and gaming products.

4 Reward-based crowdfunding

While a survey may not induce consumers to truthfully reveal their valuation, pre-ordering decisions may generate credible information as only 1-consumers might pre-order the product at $p_0 > 0$. As Proposition 1 showed that a firm with a low investment cost does not benefit from learning, we consider only the interesting case where $I > I_{\mathbf{0}} = \frac{\lambda\theta_0(N-M)}{\lambda+M}$ in this Section. This section explores the existence of a fully revealing PBE of the crowdfunding game specified in Section 2.

4.1 Firm's investment decision.

At date 2, it clearly remains optimal for the firm to extract all consumer surplus and to set $p_2 = 1$. Given *skeptical* beliefs $b\left(m|m^B\right) = \Pr\left(m = m^B\right) = 1$, and (6), the firm chooses to invest at date 1 if, and only if,

$$\begin{aligned} m^B &\geq \bar{m}_T \text{ and } p_0 m^B + (N-M)\, E\left[\theta|m^B\right] - I \geq p_0 m^B, \text{ or} \\ m^B &< \bar{m}_T \text{ and } m^B + (N-M)\, E\left[\theta|m^B\right] - I - \varsigma \geq 0 \end{aligned} \tag{19}$$

If $m^B \geq \bar{m}_T$ there is a moral hazard problem because the firm has already received an amount $p_0 m^B$, which it can divert at no extra cost. The firm's incentives to invest in these states are preserved when expected future demand by the $(N - M)$ consumers that did not have the chance to participate in crowdfunding is sufficiently high, i.e., (19) implies that if $m^B \geq \bar{m}_T$, then the incentive compatibility constraint can be written

$$(N - M)\frac{\lambda\theta_0 + m^B}{\lambda + M} - I \geq 0. \tag{20}$$

Learning about future consumer preferences is essential here as the firm's expected future demand is proportional to $E\left[\theta|m^B\right] = \frac{\lambda\theta_0 + m^B}{\lambda + M}$, which is increasing in m^B as long as λ is finite.

Whether or not the firm always collects funds is the crucial difference between AoN and KiA. Under KiA ($\bar{m}_T = 0$) the firm's investment decision is always distorted by the moral hazard problem. In contrast, under AoN, the firm may still choose to invest if its campaign fails, as $m^B < \bar{m}_T$ implies that it will invest if, and only if,

$$m^B + (N - M)\frac{\lambda\theta_0 + m^B}{\lambda + M} - I - \varsigma \geq 0, \tag{21}$$

which is a less restrictive condition than (20) whenever $\varsigma \leq \bar{\varsigma}_M$.

Lemma 4 *The firm invests if, and only if*

$$m^B \geq \begin{cases} \tilde{m}_Y & \text{if } m^B \geq \bar{m}_T \\ \tilde{m}_N & \text{if } m^B < \bar{m}_T \end{cases}, \tag{22}$$

where

$$\tilde{m}_Y \equiv \left\lceil \frac{\lambda + M}{(N - M)}(I - I_0) \right\rceil = \left\lceil \frac{\lambda + M}{(N - M)} \cdot I - \lambda\theta_0 \right\rceil. \tag{23}$$

and

$$\tilde{m}_N = \left\lceil \frac{\lambda + M}{\lambda + N}(I + \varsigma - I_0) \right\rceil \tag{24}$$

We have $\tilde{m}_Y \geq \tilde{m}_N \geq \tilde{m}$.

Proof. Follows from (20) and (21), $E\left[\theta|m^B\right] = \frac{\lambda\theta_0 + m^B}{\lambda + M}$, $I_0 = \frac{\lambda\theta_0(N-M)}{\lambda + M}$ and the constraint that any threshold needs to be a integer. The claim $\tilde{m}_Y \geq \tilde{m}_N$ follows from the condition $\varsigma \leq \bar{\varsigma}_M$ defined in Section 2, and the comparison of (23), (24), and $\tilde{m}_N \geq \tilde{m}$ from (14), (24), and $\varsigma > 0$. ■

Lemma 4 shows that if the firm meets its target $\bar{m}_T$ it invests if $m \geq \tilde{m}_Y$. The threshold $\tilde{m}_Y$ is typically higher than the optimal investment threshold. Yet, this does not necessarily imply suboptimal investment decisions. A firm that fails to meet its target will invest if the NPV based on posterior beliefs is positive. That firm received no funding and faces no moral hazard, its investment threshold is $\tilde{m}_N$, which converges to the first best threshold when $\varsigma \to 0$.

Lemma 4 also shows that if $M \to N$ raising funds via crowdfunding becomes impossible, which suggests that limiting the campaign length is essential to ensure that crowdfunding is feasible.

4.2 Backer pledging decisions

From (3), the expected utility to consumer $i = \{1, ..., M\}$ is

$$\mathbb{E}\left[u^i|\Omega_0^i\right] = \mathbb{E}\left[T_{\bar{m}_T} \cdot \mathbf{1}_0^i \cdot \left(\mathbf{1}_1^F \cdot v^i - p_0\right)|\Omega_0^i\right] + \mathbb{E}\left[\left(1 - 1_0^i\right) \cdot \mathbf{1}_1^F \cdot \mathbf{1}_2^i \cdot \left(v^i - p_2\right)|\Omega_0^i\right].$$

Since $p_2 = 1$ and $\mathbf{1}_2^i = 0$ for any 0-consumer, the second term is zero for all consumers. Hence 1-consumer $i = \{1, ..., M\}$ sets $\mathbf{1}_0^i = 1$ if, and only if, $\mathbb{E}\left[T_{\bar{m}_T}\left(\mathbf{1}_1^F \cdot v^i - p_0\right)|\Omega_0^i\right] \geq 0$, which by the law of iterated expectations becomes

$$\Pr\left(T_{\bar{m}_T} = 1\&\mathbf{1}_1^F = 1|\Omega_0^i\right) v^i - p_0 \Pr\left(T_{\bar{m}_T} = 1|\Omega_0^i\right) \geq 0 \tag{25}$$

As consumer i must have consistent beliefs, he correctly anticipates the firm's decisions as described in Lemma 4 and Bayes's rule requires that consumer i's beliefs satisfy

$$b^i\left(\Omega_0^i\right) = \Pr\left(T_{\bar{m}_T} = 1\&\mathbf{1}_1^F = 1|\Omega_0^i\right) = \begin{cases} \Pr\left(T_{\bar{m}_T} = 1|\Omega_0^i\right) \text{ if } \bar{m}_T \geq \tilde{m}_Y \\ \Pr\left(T_{\bar{m}_T} = 1|\Omega_0^i\right) \Pr\left(\mathbf{1}_1^F = 1|\Omega_0^i, T_{\bar{m}_T} = 1\right) \text{ if } \bar{m}_T < \tilde{m}_Y \end{cases}$$

In a fully revealing equilibrium, each 1-consumer expects all other 1-consumers to pledge and all 0-consumers not to pledge. We denote the total pledges of all consumers except i with $m^{-i} = \sum_{j=1, j\neq i}^{M} v^j$. Consumer i expects the total pledges to be $m = m^B = m^{-i} + \mathbf{1}_0^i$. Bayes' rule implies that conditional on his own type, i's beliefs about θ are $\theta|v^i \sim Be\left(\lambda\theta_0 + v^i, \lambda\left(1 - \theta_0\right) + 1 - v^i\right)$ and his beliefs about other consumer pledges, $m^{-i}|v^i$, follow a beta-binomial distribution with parameters $M - 1$, $\lambda\theta_0 + v^i$ and $\lambda\left(1 - \theta_0\right) + 1 - v^i$.

Lemma 5 *Each consumer $i \in \{1, ..., M\}$ with $v^i = 0$ chooses $1_0^i = 0$.*
Each consumer $i \in \{1, ..., M\}$ with $v^i = 1$ sets $1_0^i = 1$ either if

$$\bar{m}_T \geq \tilde{m}_Y \text{ and } p_0 \leq 1 \tag{26}$$

or if

$$\bar{m}_T < \tilde{m}_Y \text{ and } p_0 \leq \frac{\Pr\left(m^{-i} \geq \tilde{m}_Y - 1|v^i = 1\right)}{\Pr\left(m^{-i} \geq \bar{m}_T - 1|v^i = 1\right)} \tag{27}$$

and sets $1_0^i = 0$ otherwise.

Proof. See Online Appendix B.4. ■

Lemma 5 shows that if the firm has set a target that is sufficiently high to ensure that the firm has incentives to invest at date 1, then each consumer is insured against the risk of no investment and is willing to make a pledge as long as the pre-selling price is not above his private valuation for the product. This case requires the firm to set a positive target as $\bar{m}_T \geq \tilde{m}_Y > 0$, which is only possible with an AoN crowdfunding campaign. If the firm has set a target that is lower than $\tilde{m}_Y$, the consumer is still willing to participate as long as he can pre-order the product at a large enough discount that compensates for the risk of the firm failing to invest. Notice that under a KiA scheme, we would always have $\bar{m}_T = 0 < \tilde{m}_Y$.

4.3 The fully revealing equilibrium

The firm faces a trade-off between target and discount. On the one hand, the firm can set a target $\bar{m}_T \geq \tilde{m}_Y$ and pre-sell the product at no discount, $p_0 = 1$. On the other hand, it can set a lower target, sell the product at a larger discount and divert some funds raised following some outcomes of its crowdfunding campaign.

Lemma 6 *The firm optimally sets $p_0 = 1$ and $\bar{m}_T = \tilde{m}_Y$. The date 0 expected value of participation in AoN crowdfunding is*

$$U^F = U_B^F - \mathbb{E}\left[D\left(m\right)|\tilde{m} < m \leq \tilde{m}_N\right]\Pr\left(\tilde{m} < m \leq \tilde{m}_N\right) - \Pr\left(\tilde{m}_N \leq m < \tilde{m}_Y\right)\varsigma \qquad (28)$$

where U_B^F is the first best utility defined in (16) and $\tilde{m}_N$ and $\tilde{m}_Y$ are defined in (23) and (24). When $\varsigma \to 0$ the investment target after a failed campaign is the same as the optimal target $\tilde{m}$ defined in (14) and the gains from crowdfunding are $\lim_{\varsigma \to 0} U^F = U_B^F$.

Proof. See Appendix A.2. ■

Lemma 6 shows that the firm's utility from crowdfunding is maximized when it chooses AoN and sets a target that is high enough to not have incentives to divert funds after a successful campaign. The reason why the optimal target is not lower, allowing for some fraud to arise in some states in equilibrium, is that backers would then back the project only when there is a high enough discount (Lemma 5). We prove that the discount that the firm needs to offer is too large to outweigh the benefits of diverting funds. The same argument explains why KiA is not optimal in our framework. The reason why the optimal target is no higher than $\tilde{m}_Y$ is that setting the target at $\tilde{m}_Y$ minimizes the risk of crowdfunding campaign failure, which matters for the firm as long as there is any, even arbitrarily small, costs associated with such failure. Indeed, Lemma 6 highlights that the cost of investment under a failed campaign, ς, is the only reason why there is a wedge between the firm's utility under crowdfunding and under the benchmark of a frictionless consumer survey. When ς is small then the firm's utility from crowdfunding is almost as high as under the benchmark case. The derivation of the PBE then follows from consolidating lemmas 4, 5 and 6.

Proposition 7 *There is a fully revealing PBE of the crowdfunding game where*
1) all 1*-consumers in set* $\{1, ..., M\}$ *pledge at date* 0 *and purchase the product at date* 2 *if the firm fails to meet its crowdfunding target and invests. All* 1*-consumers in set* $\{M+1, ..., N\}$ *purchase the product at date* 2 *if the firm invests.* 0*-consumers neither pledge nor purchase the product;*
2) The firm with $U^F > 0$ *participates in crowdfunding and chooses an AoN campaign with crowdfunding target* $\bar{m}_T = \tilde{m}_Y$*, sets prices* $\{p_0, p_2\} = \{1, 1\}$ *and invests as long as* $m^B \geq \tilde{m}_N$*.*

5 Discussion of the main model

Limit case with $\boldsymbol{\lambda} \to \infty$ As highlighted in Section 2 at this limit the firm does not learn from sample M about out of sample consumer preferences, i.e., $\lim_{\lambda \to \infty} E[\theta|m] = \theta_0$ and the distribution q_m (12) converges to a binomial distribution with parameters M and θ_0.[6]

Proposition 3 states that the value of learning is lower if there is less uncertainty (higher λ), which implies that the value of learning is minimized when $\lambda \to \infty$. Further, our results in Sections 2-4 imply that a firm with $\lambda \to \infty$ that can finance its project without crowdfunding either does not benefit from crowdfunding or cannot crowdfund under moral hazard. To see this recall from Proposition 1 that firms with $I \leq I_0 \equiv \frac{\lambda \theta_0 (N-M)}{\lambda + M}$ have zero value of learning and note that $\lim_{\lambda \to \infty} I_0 = \theta_0 (N - M)$. Hence any firm with $I \leq \theta_0 (N - M)$ does not benefit from crowdfunding and is better off not crowdfunding its project if there are any, albeit small, costs associated with running a campaign (e.g., preparing its crowdfunding page). At the same time consider a firm with $I > \theta_0 (N - M)$. As we showed in Section 3 and 4 moral hazard implies that the firm would need to set a target $\bar{m}_T = \tilde{m}_Y$ and from (23) we have $\lim_{\lambda \to \infty} \tilde{m}_Y \to \infty$ when $I > \theta_0 (N - M)$. Such target can clearly never be met, which renders crowdfunding impossible for such firm. Crowdfunding in such an environment would require credit constraints (as firms with $I \leq I_0 \equiv \frac{\lambda \theta_0 (N-M)}{\lambda + M}$ can commit to deliver), high enough reputation costs as in Ellman and Hurkens (2016), and/or conditional pledging behavior whereby backers coordinate not to give more than target funds to the firm as advocated by Strausz (2017). As further discussed in Section 7, these assumptions would not explain a number of documented empirical patterns. In contrast, sufficiently high uncertainty and a sufficiently small sample size M relative to N, guarantee that crowdfunding is possible and close to efficient without imposing these assumptions (Section 4). In particular, when λ is finite, then $I_0 < \theta_0 (N - M)$ and firms with $I_0 < I \leq \theta_0 (N - M)$ benefit from learning and set an achievable crowdfunding target as $\tilde{m}_Y \leq \lceil M \theta_0 \rceil \leq M$. Firms with $\theta_0 (N - M) < I < N - M$ set a target

[6] This distribution is assumed in the baseline setting of Ellman and Hurkens' (2016) and in Strausz's (2015) working paper version. Additionally, Strausz (2017) considers other distributions with this property, and Section 5.2 in Ellmann and Hurkens (2016) allow θ to take two values.

$\tilde{m}_Y \leq M$ as long as $\lambda \leq M \frac{N-M-I}{I-(N-M)\theta_0}$.

High cost of failure In our main model we assume that the cost of a failed campaign is small, i.e., $\varsigma \leq \bar{\varsigma}_M = \frac{\lambda+M}{N-M}\left(I - \frac{\lambda\theta_0(N-M)}{\lambda+M}\right) = \frac{\lambda+M}{N-M}(I - I_0)$. This guarantees that $\tilde{m}_N \leq \tilde{m}_Y$, so that the firm may be willing to complete the project even after failing to meet its crowdfunding target $\bar{m}_T = \tilde{m}_Y$. While we consider this setting more realistic, our model also enables us to analyze the case where $\varsigma \geq \bar{\varsigma}_M$. Then the investment thresholds (23) and (24) remain the same, and the only change to Lemma 4 is that $\tilde{m}_N \geq \tilde{m}_Y$. It then follows that the firm never invests after a failed campaign. As in our main model, it remains optimal to set $\bar{m}_T = \tilde{m}_Y$ (see Appendix A.3 for a formal proof in this environment). This implies that the firm's utility from crowdfunding is

$$U^F = U_B^F - \mathbb{E}\left[D\left(m\right)|\tilde{m} < m \leq \tilde{m}_Y\right]\Pr\left(\tilde{m} < m \leq \tilde{m}_Y\right), \tag{29}$$

which is lower than in our main model.

Communication In the main model, consumers only know their own preferences and have beliefs about other consumer preferences and strategies when deciding whether or not to pledge. In reality, platforms such as Kickstarter and Indiegogo make it possible for consumers to see pledges that have been made on a running basis. However, each backer can make and withdraw his pledge any time during the campaign and he becomes committed to his pledge only at the end of the campaign when funds are taken from his account. One may view the interaction during the campaign as "cheap talk", viewing pledges during the campaign as "messages" followed by a pledging decision at the last moment of the campaign (see Farrell and Rabin, 1996). The equilibrium derived in our main model then corresponds to a "babbling" equilibrium of the messaging game where all potential backers disregard everyone else's messages. Clearly, the PBE described in Proposition 7 is also consistent with a fully revealing messaging game as consumers do not benefit from hiding their type. What can change as a result of communication are the backer strategies on an off-equilibrium path where the firm has set a target $\bar{m}_T < \tilde{m}_Y$ (see Lemma 5 in Section 4.2). Revealing communication enables consumers to coordinate and not back a project with $m < \tilde{m}_Y$.

Variable costs and uncertainty In the main model we considered a zero variable cost and no uncertainty about the firm's ability to successfully develop the product following an investment. Suppose that the crowdfunding campaign takes place as in the main setting, and that after observing the outcome of the campaign, the firm chooses whether to pay a fixed cost I_F at date 1. Further assume that the product development success is uncertain, i.e., conditional on investment at date 1, there will be a product to sell at date 2 with probability $\gamma \in (0, 1]$, which is assumed to be independent of other variables. Furhermore, if there is a product, producing one unit requires paying a variable cost $I_V \in (0, 1)$ at date 2. Realistically, we assume that there is sufficient consumer protection such that a firm, that has invested I_F and successfully developed the product, cannot avoid delivering it to backers while selling the product to the consumers who purchase at $t = 2$. Writing $I \equiv \frac{I_F}{\gamma(1-I_V)}$, the date 1 expected profit under the frictioness consumer survey is $\gamma(1 - I_V) D(m)$, where $D(m)$ is as in (15). This implies that the first best investment threshold as a function of I is the same as the one derived in Section 3 and related comparative statics remain valid. Since I is increasing in I_V and decreasing in γ, variable costs and uncertainty about product development both increase the optimal investment threshold $\tilde{m}$.

Provided the firm invests, the date 1 profit under crowdfunding is now

$$-I_F + p_0 m + \gamma(-I_V m + (1 - I_V)(N - M)\mathbb{E}[\theta|m]) = (p_0 - \gamma) m + \gamma(1 - I_V) D(m)$$

Both variable costs and uncertainty worsen moral hazard. The firm will now invest after a successful campaign if

$$m \geq \tilde{m}'_Y = \left\lceil \left(\frac{N-M}{\lambda+M} - \frac{I_V}{(1-I_V)}\right)^{-1} (I - I_0) \right\rceil.$$

and after a failed campaign if

$$m \geq \tilde{m}'_N = \left\lceil \frac{\lambda+M}{\lambda+N} \left(I - I_0 + \frac{\varsigma}{\gamma(1-I_V)}\right) \right\rceil. \tag{30}$$

As in the main model, it remains optimal for the firm to pre-sell the product at the highest possible price and to set a sufficiently high target. As backers expect product development to succeed only with probability γ, the highest possible pre-selling price is $p_0 = \gamma$. Our results will remain qualitatively similar, with higher thesholds and a quantitatively different effect of the cost of campaign failure ς. Namely, if $\varsigma \leq \bar{\varsigma}'_M \equiv \left(\frac{N-M}{\lambda+M} - \frac{I_V}{(1-I_V)}\right)^{-1} \gamma \left(I - \frac{\lambda\theta_0(N-M)}{\lambda+M}\right)$ then the firm invests after the failed campaign as long as (30) holds and if $\varsigma \geq \bar{\varsigma}'_M$ the firm only invests after a campagn success. In both cases the target is $\bar{m}_T = \tilde{m}'_Y$. Both uncertainty about the firm's ability to develop the product and variable costs make successful crowdfunding more difficult. Uncertainty about product development additionally implies that the firm must offer a discount at the crowdfunding stage.

6 Crowdfunding sample size and outcomes

This section extends the main setting and explores how the crowdfunding sample size, M, affects the observable crowdfunding outcomes and the firm's expected utility. An empirical proxy for the sample size is campaign length. Real world AoN crowdfunding sites such as Kickstarter impose a maximum limit to the campaign length and firms can commit to a shorter campaign length before the campaign starts.

We allow for both low and high values of ς. Note that the parametric restriction on $\bar{\varsigma}_M = \frac{\lambda+M}{N-M}I - \lambda\theta_0$ adopted in Sections 2-4 is increasing in M: from (23) and (24) we have $\tilde{m}_Y \leq (\geq)\, \tilde{m}_N$ if $\varsigma \leq (\geq)\, \bar{\varsigma}_M$. Empirically observable variables of interest are the probability of crowdfunding success $\Pr(m \geq \bar{m}_T)$, and the expected funds raised $\mathbb{E}[p_0 m | m \geq \bar{m}_T] \Pr(m \geq \bar{m}_T)$. When analyzing these variables we consider the PBE derived in Section 4.3, i.e., $p_0 = 1$ and the firm sets the target $\bar{m}_T = \tilde{m}_Y$. As shown in Section 5, the same holds when $\varsigma \geq \bar{\varsigma}_M$.

We consider the choice of an optimal sample size for both the platform and the firm. We assume that the platform's objective is to maximize the expected funds raised, which is consistent with proportional fee structures observed on platforms like Kickstarter. As $p_0 = 1$ the optimal sample size for platform then maximizes

$$\mathbb{E}[m | m \geq \tilde{m}_Y] \Pr(m \geq \tilde{m}_Y),$$

subject to $\tilde{m}_Y$ defined in (23).

The firm chooses M to maximize its expected utility from crowdfunding anticipating its future decisions according to the PBE derived, i.e., from (16)-(18), (28) and (29)

$$\begin{aligned} U^F &= \mathbf{1}_{\tilde{m}_Y \geq \tilde{m}_N} \left(\mathbb{E}[D(m) | m \geq \tilde{m}_N] \Pr(m \geq \tilde{m}_N) - \varsigma \Pr(\tilde{m}_N \leq m < \tilde{m}_Y) \right) \\ &\quad + \left(1 - \mathbf{1}_{\tilde{m}_Y \geq \tilde{m}_N}\right) \mathbb{E}[D(m) | m \geq \tilde{m}_Y] \Pr(m \geq \tilde{m}_Y) - \pi^F_{ref}, \end{aligned} \tag{31}$$

subject to $\tilde{m}_Y$ and $\tilde{m}_N$ defined in (23) and (24) respectively and where $\mathbf{1}_{\tilde{m}_Y \geq \tilde{m}_N}$ is defined as an indicator function that takes value 1 if, and only if, $\tilde{m}_Y \geq \tilde{m}_N$ ($\varsigma \leq \bar{\varsigma}_M$) and zero otherwise.

Online Appendix B.5 proves that when M increases, the probability of meeting a fixed target is higher, but the probability of meeting a target that is at least one unit higher is lower. It is important to emphasize that $\tilde{m}_Y$ increases in M at an increasing rate because the term inside the ceiling function in (23) is increasing and convex in M

$$\frac{\partial \left(\frac{\lambda+M}{(N-M)} \cdot I - \lambda\theta_0 \right)}{\partial M} = \frac{I(\lambda+N)}{(N-M)^2} > 0; \quad \frac{\partial^2 \left(\frac{\lambda+M}{(N-M)} \cdot I - \lambda\theta_0 \right)}{\partial M \partial M} = \frac{2I(\lambda+N)}{(N-M)^3} > 0.$$

This implies that the probability of crowdfunding success must be decreasing in sample size when M is high enough. Online Appendix B.5 shows that this is a symptom of moral hazard: if there was no moral hazard, the firm would set the first best target $\tilde{m}$ under which we would not observe a systematic pattern between sample size and success probability. Online Appendix B.5 further highlights the main forces through which a higher sample size affects the funds raised, and the firm's utility. Since this problem involves endogenous discrete variables, it is more intuitive to present the results graphically.

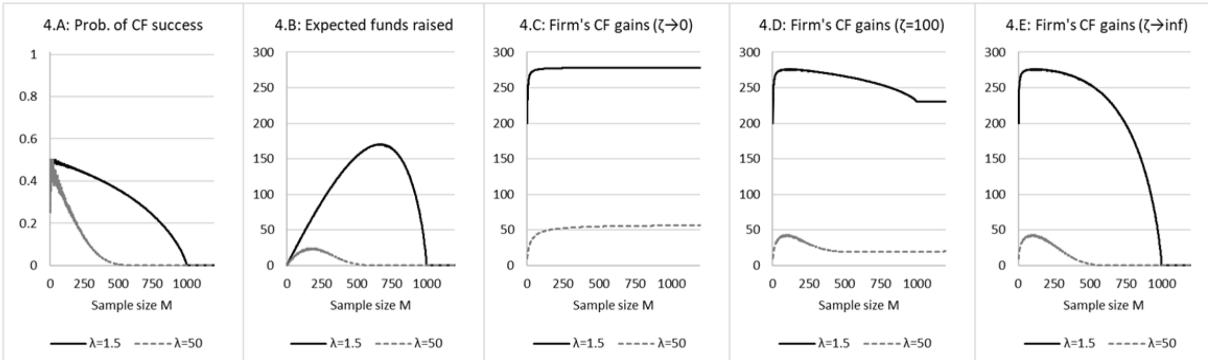

Figure 4: Probability of crowdfunding success, platform's expected fee income and firm's expected utility from crowdfunding.

Recall that a firm with a break-even investment cost $I = N\theta_0$ has most to gain from learning about demand and therefore represents a typical crowdfunding participant. Figure 4 considers such a firm under the same parameter values as Figure 3 and considers three examples of cost ς: $\varsigma \to 0$, an intermediate value of ς and $\varsigma \to \infty$. Panel 4.A of Figure 4 highlights the cost of increasing the sample size as the probability of crowdfunding success decreases with the sample size. As a result panel 4.B shows that the expected funds raised via crowdfunding are maximized at an intermediate sample size. Intuitively, an increase in sample size increases the expected fee income conditional on the firm meeting the target, but reduces the probability of crowdfunding success. If ς is small then the firm's utility does not depend much on whether it meets the target and its investment threshold converges to the first best target, $\tilde{m}_N \to \tilde{m}$. Panel 4.C of Figure 4 shows that when $\varsigma \to 0$, the firm's utility increases with M, but the marginal benefit of a higher M quickly becomes negligible. Higher sample size enables the firm to learn more precise information. However, learning from a small sample is amplified because the firm also learns noisy information about the preferences of $N - M$ out of sample consumers, and this amplification effect diminishes when the sample becomes a large fraction of the firm's entire target market. In contrast, when ς is non-negligible, then the probability of meeting the target $\tilde{m}_Y$ is important to the firm, which creates an additional negative force which leads the firm to prefer a smaller sample size. Panel 4.D and 4.E of Figure 4 shows that the firm prefers a sample size that is sufficiently large to learn about demand but no larger than that. Appendix A.4 provides further confirmation of these patterns and additional comparative statics by further considering higher and lower values of investment cost I and prior mean θ_0.

Figure 4 also suggests that if the platform were to limit the maximum campaign length to the level that maximizes its fee income then a low ς would imply that firms are bound by this target and may invest also after a failed campaign; and a high ς would imply that the firm prefers a smaller

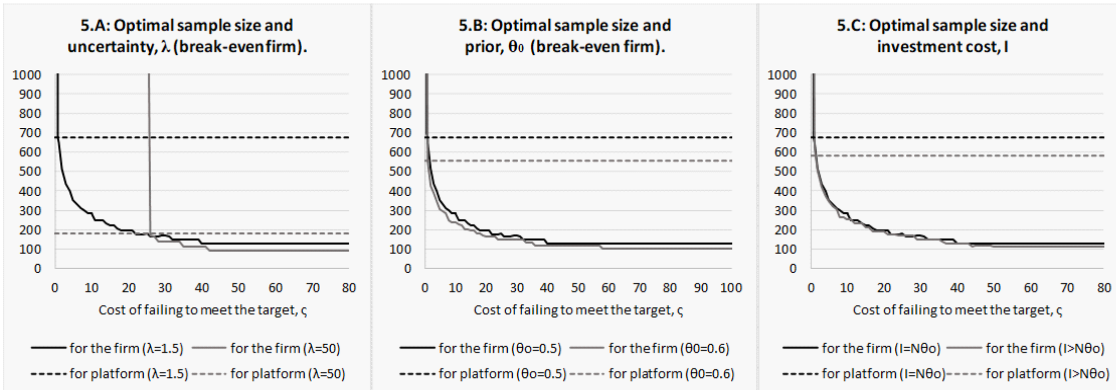

Figure 5: Optimal sample size for the firm and platform at different parameter values.

sample size compared to the platform. Figure 5 confirms this intuition by showing the relationship between the sample sizes that maximize fee income and firm's utility under different values on ς. The baseline assumptions for this figure (black lines) are the same as the high uncertainty case ($\lambda = 1.5$) in Figure 4. Panels 5.A and 5.B additionally consider the effect of distributional parameters λ and θ_0. Both lower uncertainty and higher prior mean reduce the sample size that maximizes funds raised and the platform's fee income. Furthermore, low uncertainty widens the range of cost ς under which the firm prefers a larger sample than the platform. If the cost ς is high enough then the break-even firm benefits from a smaller sample than the platform and its preferred sample size tends to be smaller if there is less uncertainty and the prior mean θ_0 is higher. Panel 5.C in Figure 4 keeps distributional parameters fixed and considers a higher investment cost, i.e., a firm that would not invest without crowdfunding. Then both the firm (provided ς is high enough) and the platform prefer a smaller sample size. The intuition for this is that under moral hazard, the endogenous crowdfunding target $\tilde{m}_Y$ increases faster when the investment cost is higher, which makes it increasingly likely that the firm will fail to meet its crowdfunding target when M increases.

While the sample size that maximizes the firm's utility is highly sensitive to the cost ς, a low value of ς is consistent with the empirical observation regarding firms that complete the project after failing to meet their crowdfunding targets.

7 Empirical implications

As we discuss below, our model of crowdfunding is consistent with existing empirical evidence and our findings suggest some new avenues for further empirical research.

Existing empirical evidence: Mollick (2014) shows that very few successful Kickstarter projects (3.6%) fail to deliver their promised rewards. Our analysis highlights the reasons why the existing crowdfunding mechanisms are efficient enough to endogenously overcome (even an extreme form of) moral hazard. We also argue that the real option value of learning, rather than credit constraints, is the main value driver of crowdfunding. Mollick and Kuppuswamy (2014) survey Kickstarter participants who had completed their campaign before mid 2012 in the technology, project design, and video games categories. They present evidence that the number one reason why both successful and unsuccessful firms in sought crowdfunding was "to see if there is demand for the project" (68% and 60% of successful and unsuccessful projects agreeing, respectively). Financing the project was only the fourth reason cited, while marketing and connecting with the firm's community of fans and supporters were ranked second and third, respectively. Mollick and Kuppuswamy (2014) also find that 30% of firms continue to pursue their projects after failing to meet their target. Xu (2017) finds further evidence of Bayesian learning among Kickstarter participants. As our model predicts, Cumming et. al. (2015) finds evidence that AoN crowdfunding dominates KiA. Mollick (2014) find that shorter campaigns succeed with a higher probability than longer campaigns. In fact, Kickstarter itself shortened its maximum campaign duration from 90 to 60 days in 2011 referring to the observation that shorter campaigns are more likely to succeed.[7]

New empirical avenues: Uncertainty about target consumer preferences plays a central role in our model, and would warrant further empirical investigation. We predict that more uncertain projects should be relatively more "overfunded" compared to less uncertain projects as the wedge $\mathbb{E}\left[\frac{p_0 m}{p_0 \bar{m}_T} | p_0 m \geq p_0 \bar{m}_T\right] - p_0 \bar{m}_T$ is increasing in the degree of uncertainty. This is consistent with stylized facts presented in our Introduction. We also analyzed all successfully funded Kickstarter projects between January 1, 2015 and September 17, 2015 in extreme opposite categories: Technology and Theatre, and constructed unconditional distributions. Online Appendix B.6 confirms that Technology projects are more uncertain than Theatre ones in the sense of second order stochastic dominance, and our data indicates that the average successful US-based technology project raised 5.8 times its target, while the average successful US-based theatre project raised 1.3 times its target. Our model also predicts that firms with uncertain projects should participate more often.

Our model also provides a structure to test the severity of moral hazard and/or the importance of reputation costs and legal enforcement. We have shown that firms that benefit the most from crowdfunding are those with a break-even investment cost without crowdfunding. Hence these firms are likely to represent the typical crowdfunding participant. In the absence of moral hazard, these firms set the optimal target $\bar{m}_T = \tilde{m}$ such that $\mathbb{E}\left[\frac{p_0 m}{p_0 \bar{m}_T}\right] = \mathbb{E}\left[\frac{p_0 m}{p_0 \tilde{m}}\right] = 1$, i.e., the average completion ratio, $\frac{pledges}{target}$, across all projects should be one as well. Instead, under moral hazard

[7] https://www.kickstarter.com/blog/shortening-the-maximum-project-length

such typical firm would set a target $p_0\bar{m}_T = p_0\tilde{m}_Y > p_0\tilde{m}$, which implies a completion ratio below one. Indeed, Cumming et. al. (2015) analyze AoN and KiA projects at Indiegogo, and report an average completion ratio, $\frac{pledges}{target} = \frac{p_0 m}{p_0 \bar{m}_T}$, of 0.403, ranging from 0.337 to 0.617 across innovative, creative, and social categories and with both KiA and AoN schemes.

In addition to distinguishing projects according to preference uncertainty, it would also be interesting to distinguish projects according to variable costs. Our model suggests that firms with higher variable costs face a higher degree of moral hazard, and are thus less likely to participate, set a higher target if they participate, and fail to meet their target with a higher probability.

Our off-equilibrium path results further suggest that projects that do not set ambitious targets are more likely to be fraudulent, unless they offer a bigger discount, and non-fraudulent KiA projects should offer larger discounts than comparable AoN projects.

Empirical patterns that distinguish our paper from other theoretical models: Ellman and Hurkens (2016) argue that reputation costs are high enough to overcome moral hazard. However, data suggests that the average completion ratio is below one and there is a positive correlation between the success rate and shorter campaign length, which we have shown to be symptoms of moral hazard. Strausz (2017) and Varian (2013) predict that successful campaigns should not exhibit large overpledging. Strausz (2017) argues that backers pursue a conditional pledging strategy to overcome moral hazard, Varian (2013) allows for some over-pledging, but only to the degree that each pivotal individual wants to obtain the full gift for his contribution. The magnitude of the over-pledging observed, especially for technology projects, suggests that backers do not become less willing to participate after the firm has met its target. Rather, our model suggests that backers face less risk and are more willing to participate if a firm raises more funds during a limited length campaign. The argument that successful campaigns "tend to succeed by a small margin" often refers to Mollick's (2014) exploratory study of Kickstarter projects from 2009 to 2012. However, a more careful look at the data that this argument builds upon reconciles his findings with our setting: Figure 1 in Mollick (2014) does not show an absence of substantial overpledging. Rather, little overpledging is more likely than large overpledging (e.g., pledges to target ratios of 100-120% are observed more frequently than 120-140%). From a purely statistical perspective, the distribution of pledges to target among successful projects is proportional to the right tail of the distribution of pledges, q_m. Thus our model generates this pattern whenever the prior, and thus also the distribution of pledges q_m, is hump-shaped. As Mollick's finding uses Kickstarter projects across all categories, it is indeed likely that an average project involves noticeable but not extremely high uncertainty about demand, consistently with a hump-shaped distribution on average. Our analysis further suggests that the degree of overpledging should differ across Kickstarter categories and be more pronounced in the case of innovative projects, as we show on Figure A.1. Indeed, Mollick (2014) further highlights that projects that do overpledge most frequently belong to hardware, software, games and product design categories, which is consistent with our predictions. Overall, our model provides a framework that is consistent with both overpledging and empirical patterns characteristic to moral hazard. We have also highlighted more nuanced patterns which arise from considering the role of uncertainty.

Our model predicts that products are sold at par or at a discount at the crowdfunding stage compared to the retail price. While we do not have enough data to test this, anecdotal evidence based on firms' announced retail prices or on examples such as Pebble Watch suggests that this may be the case. With more systematic data, our predictions could be tested as alternatives to those in Belleflamme et. al. (2013) who explain crowdfunding as a means to price-discriminate individuals who enjoy the crowdfunding experience. In contrast to our model, their results imply that the prices would be set at the crowdfunding stage at a premium over the retail price.

8 Concluding remarks

In our model, the firm can learn about its total demand from a limited sample. We show that firms that face highly uncertain demand benefit most from reward-based crowdfunding. We have shown that these firms can also overcome moral hazard most easily. In reality, the benefits can extend to learning about consumer preferences about the specifications of the product, e.g., the color of new widget or the features of a new game.[8] We argue that to fully understand the success of reward-based crowdfunding, it is important to consider its role as a learning device, rather than focusing on a mere funding scheme.

Our model can explain the following stylized facts: 1) a noticeable proportion of firms that fail to meet their target in a crowdfunding campaign still complete the project, 2) many successful projects, in particular those belonging to innovative categories, receive amounts significantly higher than the target amount, 3) firms that develop riskier products that involve sufficiently high investment costs and have high demand uncertainty, e.g. technology gadgets, appear to seek reward-based crowdfunding most often. We also predict that firms offer products at par or at a discount rather than premium during crowdfunding campaigns. These features are specific to our model and would enable to test our predictions relative to alternative papers.

[8]Interestingly, new projects can then lead to the development of new products that can be either spawned or retained (Habib, Hege, and Mella-Barral 2013).

Some established firms may incur high reputation costs. Indeed Sony and Apple routinely pre-sell products on their own websites rather than third-party platforms. However, most firms are largely unknown, do not have much reputation to lose, and cannot pre-commit to money-back guarantees. We have shown that third-party crowdfunding platforms are valuable to those firms because they facilitate commitment to features. In particular, crowdfunding targets and the limited campaign length help them to overcome moral hazard. We have shown that with the currently prevailing fee structure where platforms charge fees from successful campaigns only that are proportional to the funds raised, platforms do indeed have an incentive to limit the campaign length and set it at an intermediate level. We identify conditions under which the firm is bound by this limit or benefits from an even smaller sample size. This helps a firm to overcome the commitment problem that would prevail if it were to crowdfund a project on its own website. Without commitment, the firm may be tempted to keep the campaign running with the purpose of diverting funds should the product fail to attract enough consumer interest.

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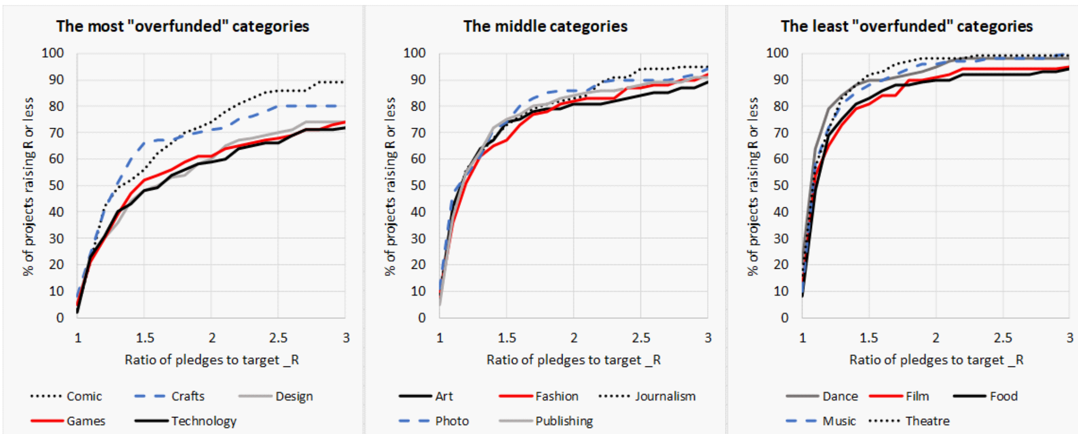
A Appendix

A.1 "Oversubscription" across Kickstarter categories

The figures use a sample of 100 successful projects in each category completed by October 30, 2014. The classification to "most", "middle" and "least" oversubscribed projects is based on median values conditional on success.



A.2 Proof of Lemma 6

On the equilibrium path the firm anticipates that all 1-consumers in set $\{1, ..., M\}$ choose to pledge and $\mathbb{E}\left[\pi^F|\Omega_1^F\right] = \mathbf{1}_1^F \cdot D(m) + T_{\bar{m}_T}\left(p_0 - \mathbf{1}_1^F\right) m - (1 - T_{\bar{m}_T})\,\varsigma \cdot \mathbf{1}_1^F$. From the law of total expectations the expected value of $\mathbb{E}\left[\pi^F|\Omega_1^F\right]$ is

$$\begin{aligned} \mathbb{E}\left[\pi^F\right] \quad = \quad & \Pr\left(\mathbf{1}_1^F = 1\&T_{\bar{m}_T} = 1\right)\mathbb{E}\left[D(m) + (p_0 - 1)\,m|\mathbf{1}_1^F = 1\&T_{\bar{m}_T} = 1\right] \\ & + \Pr\left(\mathbf{1}_1^F = 1\&T_{\bar{m}_T} = 0\right)\mathbb{E}\left[D(m) - \varsigma|\mathbf{1}_1^F = 1\&T_{\bar{m}_T} = 0\right] \\ & + \Pr\left(\mathbf{1}_1^F = 0\&T_{\bar{m}_T} = 1\right)\mathbb{E}\left[p_0 m|\mathbf{1}_1^F = 0\&T_{\bar{m}_T} = 1\right] \end{aligned} \tag{32}$$

Since $\varsigma \leq \bar{\varsigma}_M$, $\tilde{m}_N \leq \tilde{m}_Y$ (see Section 2 and 4). This implies that the target set by the firm can only be in the following intervals: $\bar{m}_T \in [\tilde{m}_Y, \infty)$, $\bar{m}_T \in [\tilde{m}_N, \tilde{m}_Y)$ and $\bar{m}_T \in [0, \tilde{m}_N)$.

Consider the interval $\bar{m}_T \in [\tilde{m}_Y, \infty)$. Within this interval, Lemma 4 implies that the event $\left\{\mathbf{1}_1^F = 1\&T_{\bar{m}_T} = 1\right\}$ occurs when $m \geq \bar{m}_T$, the event $\left\{\mathbf{1}_1^F = 1\&T_{\bar{m}_T} = 0\right\}$ occurs when $\tilde{m}_N \leq m < \bar{m}_T$, and the event $\left\{\mathbf{1}_1^F = 0\&T_{\bar{m}_T} = 1\right\}$ never occurs. From (32), we then obtain that the firm must set $\{p_0, \bar{m}_T\}$ such that it maximizes

$$\Pr(m \geq \tilde{m}_N)\,\mathbb{E}\left[D(m)\,|m \geq \tilde{m}_N\right] + (p_0 - 1)\Pr(m \geq \bar{m}_T)\,\mathbb{E}\left[m|m \geq \bar{m}_T\right] - \varsigma\Pr(\tilde{m}_N \leq m < \bar{m}_T)$$

subject to the consumer participation constraint $p_0 \leq 1$. The first and the last term of this expression do not depend on p_0 and $\bar{m}_T$. The price p_0 only affects the second term, which is maximized when $p_0 = 1$ for any $\bar{m}_T$, and the maximized value is zero. Further, the third term decreases with $\bar{m}_T$ and is maximized when $\bar{m}_T = \tilde{m}_Y$. The firm's payoff with $\bar{m}_T = \tilde{m}_Y$ is

$$\mathbb{E}\left[\pi^F\right]|_{\bar{m}_T=\tilde{m}_Y} = \Pr\left(m \geq \tilde{m}_N\right)\mathbb{E}\left[D\left(m\right)|m \geq \tilde{m}_N\right] - \varsigma \Pr\left(\tilde{m}_N \leq m < \tilde{m}_Y\right) \tag{33}$$

When $\bar{m}_T \in [\tilde{m}_N, \tilde{m}_Y)$ and $\bar{m}_T \in [0, \tilde{m}_N)$, the pre-selling price must satisfy the constraint $p_0 \leq \frac{\Pr\left(m^{-i} \geq \tilde{m}_Y - 1|v^i=1\right)}{\Pr\left(m^{-i} \geq \bar{m}_T - 1|v^i=1\right)}$ (see Lemma 5), which can be expressed as[9]

$$p_0 \leq \left(\frac{\lambda\theta_0 + \mathbb{E}\left[m|m \geq \tilde{m}_Y\right]}{\lambda\theta_0 + \mathbb{E}\left[m|m \geq \bar{m}_T\right]}\right) \cdot \frac{\Pr\left(m \geq \tilde{m}_Y\right)}{\Pr\left(m \geq \bar{m}_T\right)}. \tag{34}$$

Since now $\bar{m}_T \leq \tilde{m}_Y$, we know from Lemma 8 in Online Appendix B.1 that $\mathbb{E}\left[m|m \geq \bar{m}_T\right] \leq \mathbb{E}\left[m|m \geq \tilde{m}_Y\right] \Leftrightarrow \frac{\lambda\theta_0+\mathbb{E}[m|m\geq\tilde{m}_Y]}{\lambda\theta_0+\mathbb{E}[m|m\geq\bar{m}_T]} \leq \frac{\mathbb{E}[m|m\geq\tilde{m}_Y]}{\mathbb{E}[m|m\geq\bar{m}_T]}$, which combined with (34) gives

$$p_0 \leq \left(\frac{\lambda\theta_0 + \mathbb{E}\left[m|m \geq \tilde{m}_Y\right]}{\lambda\theta_0 + \mathbb{E}\left[m|m \geq \bar{m}_T\right]}\right) \cdot \frac{\Pr\left(m \geq \tilde{m}_Y\right)}{\Pr\left(m \geq \bar{m}_T\right)} \leq \frac{\mathbb{E}\left[m|m \geq \tilde{m}_Y\right]}{\mathbb{E}\left[m|m \geq \bar{m}_T\right]} \cdot \frac{\Pr\left(m \geq \tilde{m}_Y\right)}{\Pr\left(m \geq \bar{m}_T\right)}. \tag{35}$$

Suppose that the firm deviates from $\bar{m}_T = \tilde{m}_Y$ and sets $\bar{m}_T \in [\tilde{m}_N, \tilde{m}_Y)$, and denote the firm's profit under such deviating strategy with $\mathbb{E}\left[\pi^F\right]|_{\bar{m}_T\in[\tilde{m}_N,\tilde{m}_Y)}$. Then Lemma 4 implies that the event $\left\{\mathbf{1}_1^F = 1\&T_{\bar{m}_T} = 1\right\}$ occurs when $m \geq \tilde{m}_Y$, the event $\left\{\mathbf{1}_1^F = 1\&T_{\bar{m}_T} = 0\right\}$ occurs when $\tilde{m}_N \leq m < \bar{m}_T$, and the event $\left\{\mathbf{1}_1^F = 0\&T_{\bar{m}_T} = 1\right\}$ occurs when $\bar{m}_T \leq m < \tilde{m}_Y$. From (32) and (33), we obtain

$$\begin{aligned}&\mathbb{E}\left[\pi^F\right]|_{\bar{m}_T\in[\tilde{m}_N,\tilde{m}_Y)} - \mathbb{E}\left[\pi^F\right]|_{\bar{m}_T=\tilde{m}_Y} = -\Pr\left(\bar{m}_T \leq m < \tilde{m}_Y\right)\mathbb{E}\left[D\left(m\right) - \varsigma|\bar{m}_T \leq m < \tilde{m}_Y\right]\\&+\Pr\left(m \geq \bar{m}_T\right)\mathbb{E}\left[m|m \geq \bar{m}_T\right]\left(p_0 - \frac{\mathbb{E}\left[m|m \geq \tilde{m}_Y\right]\Pr\left(m \geq \tilde{m}_Y\right)}{\mathbb{E}\left[m|m \geq \bar{m}_T\right]\Pr\left(m \geq \bar{m}_T\right)}\right)\end{aligned}$$

As the firm's pricing choice must satisfy the constraints (34) and (35), the second term is non-positive. From (15), $D\left(m\right) - \varsigma = \frac{\lambda+N}{\lambda+M}m + I_0 - I$, and from $\bar{m}_T \in [\tilde{m}_N, \tilde{m}_Y)$ and (24) $\mathbb{E}\left[\left(D\left(m\right) - \varsigma\right)|\bar{m}_T \leq m < \tilde{m}_Y\right] \geq \frac{\lambda+N}{\lambda+M}\bar{m}_T + I_0 - I - \varsigma \geq \frac{\lambda+N}{\lambda+M}\tilde{m}_N + I_0 - I - \varsigma \geq 0$, which implies that the first term is non-positive (and typically negative) too. This proves that setting $\bar{m}_T \in [\tilde{m}_N, \tilde{m}_Y)$ instead of $\bar{m}_T = \tilde{m}_Y$ is not profitable to the firm.

Finally, suppose that the firm sets $\bar{m}_T \in [0, \tilde{m}_N)$. Lemma 4 now implies that the event $\left\{\mathbf{1}_1^F = 1\&T_{\bar{m}_T} = 1\right\}$ occurs when $m \geq \tilde{m}_Y$, the event $\left\{\mathbf{1}_1^F = 1\&T_{\bar{m}_T} = 0\right\}$ never occurs, and the event $\left\{\mathbf{1}_1^F = 0\&T_{\bar{m}_T} = 1\right\}$ occurs when $\bar{m}_T \leq m < \tilde{m}_Y$. Denoting the firm's profit under this

[9] In particular, $\Pr\left(m^{-i} \geq \tilde{m}_Y - 1|v^i = 1\right) = \Pr\left(m^{-i} + v^i \geq \tilde{m}_Y|v^i = 1\right) = \Pr\left(m \geq \tilde{m}_Y|v^i = 1\right)$. From Bayes' rule $\Pr\left(m \geq \tilde{m}_Y|v^i = 1\right) = \frac{\Pr\left(v^i=1|m\geq\tilde{m}_Y\right)\Pr\left(m\geq\tilde{m}_Y\right)}{\Pr\left(v^i=1\right)}$ and from the law of iterated expectations $\Pr\left(v^i = 1|m \geq \tilde{m}_Y\right) = \mathbb{E}\left[\Pr\left(v^i = 1|\theta, m \geq \tilde{m}_Y\right)|m \geq \tilde{m}_Y\right] = \mathbb{E}\left[\theta|m \geq \tilde{m}_Y\right] = \mathbb{E}\left[\mathbb{E}\left[\theta|m, m \geq \tilde{m}_Y\right]|m \geq \tilde{m}_Y\right] = \mathbb{E}\left[\frac{\lambda\theta_0+m}{\lambda+M}|m \geq \tilde{m}_Y\right] = \frac{\lambda\theta_0+\mathbb{E}[m|m\geq\tilde{m}_Y]}{\lambda+M}$. Hence, $\Pr\left(m^{-i} \geq \tilde{m}_Y - 1|v^i = 1\right) = \frac{\lambda\theta_0+\mathbb{E}[m|m\geq\tilde{m}_Y]}{(\lambda+M)\Pr\left(v^i=1\right)} \cdot \Pr\left(m \geq \tilde{m}_Y\right)$. Similarly, $\Pr\left(m^{-i} \geq \bar{m}_T - 1|v^i = 1\right) = \frac{\lambda\theta_0+\mathbb{E}[m|m\geq\bar{m}_T]}{(\lambda+M)\Pr\left(v^i=1\right)} \cdot \Pr\left(m \geq \bar{m}_T\right)$.

deviating strategy with $\mathbb{E}\left[\pi^F\right]|_{\bar{m}_T\in[0,\tilde{m}_N)}$ and using (32) and (33), we obtain

$$\begin{aligned}&\mathbb{E}\left[\pi^F\right]|_{\bar{m}_T\in[0,\tilde{m}_N)}-\mathbb{E}\left[\pi^F\right]|_{\bar{m}_T=\tilde{m}_Y}=-\Pr\left(\tilde{m}_N\leq m<\tilde{m}_Y\right)\mathbb{E}\left[D\left(m\right)-\varsigma|\tilde{m}_N\leq m<\tilde{m}_Y\right]\\&+\Pr\left(m\geq\bar{m}_T\right)\mathbb{E}\left[m|m\geq\bar{m}_T\right]\left(p_0-\frac{\mathbb{E}\left[m|m\geq\tilde{m}_Y\right]\Pr\left(m\geq\tilde{m}_Y\right)}{\mathbb{E}\left[m|m\geq\bar{m}_T\right]\Pr\left(m\geq\bar{m}_T\right)}\right),\end{aligned}$$

which again is non-positive as (35) must hold, and as $\mathbb{E}\left[D\left(m\right)-\varsigma|\tilde{m}_N\leq m<\tilde{m}_Y\right]\geq\frac{\lambda+N}{\lambda+M}\tilde{m}_N+I_0-I-\varsigma\geq0$. This proves that the firm's optimal strategy is to set $\{p_0,\bar{m}_T\}=\{1,\tilde{m}_Y\}$.

A.3 Optimal target with $\varsigma\geq\bar{\varsigma}_M$

If $\varsigma\geq\bar{\varsigma}_M$, then $\tilde{m}_N\geq\tilde{m}_Y$ and the possible targets can fall into the following intervals: $\bar{m}_T\in[\tilde{m}_N,\infty)$, $\bar{m}_T\in[\tilde{m}_Y,\tilde{m}_N)$ and $\bar{m}_T\in[0,\tilde{m}_Y)$. Equation (32) also holds here. When $\bar{m}_T\in[\tilde{m}_N,\infty)$ then the event $\left\{\mathbf{1}_1^F=1\&T_{\bar{m}_T}=1\right\}$ occurs when $m\geq\bar{m}_T$, the event $\left\{\mathbf{1}_1^F=1\&T_{\bar{m}_T}=0\right\}$ occurs when $\tilde{m}_N\leq m<\bar{m}_T$, and the event $\left\{\mathbf{1}_1^F=0\&T_{\bar{m}_T}=1\right\}$ never occurs. The backer participation constraint is $p_0\leq1$, and the firm's profit

$$\begin{aligned}&\mathbb{E}\left[\pi^F\right]|_{\bar{m}_T\in[\tilde{m}_N,\infty)}=\Pr\left(m\geq\tilde{m}_N\right)\mathbb{E}\left[D\left(m\right)|m\geq\tilde{m}_N\right]\\&+\left(p_0-1\right)\Pr\left(m\geq\bar{m}_T\right)\mathbb{E}\left[m|m\geq\bar{m}_T\right]-\varsigma\Pr\left(\tilde{m}_N\leq m<\bar{m}_T\right),\end{aligned}$$

is maximized when $p_0=1$ and $\bar{m}_T=\tilde{m}_N$. The corresponding payoff is $\mathbb{E}\left[\pi^F\right]|_{\bar{m}_T=\tilde{m}_N}=\Pr\left(m\geq\tilde{m}_N\right)\mathbb{E}\left[D\left(m\right)|m\geq\tilde{m}_N\right]$.

When $\bar{m}_T\in[\tilde{m}_Y,\tilde{m}_N)$ then the event $\left\{\mathbf{1}_1^F=1\&T_{\bar{m}_T}=1\right\}$ occurs when $m\geq\bar{m}_T$, and the events $\left\{\mathbf{1}_1^F=1\&T_{\bar{m}_T}=0\right\}$ and $\left\{\mathbf{1}_1^F=0\&T_{\bar{m}_T}=1\right\}$ never occur. The backer participation constraint remains $p_0\leq1$, and the firm's profit

$$\mathbb{E}\left[\pi^F\right]|_{\bar{m}_T\in[\tilde{m}_Y,\tilde{m}_N)}=\Pr\left(m\geq\bar{m}_T\right)\mathbb{E}\left[D\left(m\right)|m\geq\bar{m}_T\right]+\left(p_0-1\right)\Pr\left(m\geq\bar{m}_T\right)\mathbb{E}\left[m|m\geq\bar{m}_T\right],$$

is maximized when $p_0=1$ and $\bar{m}_T=\tilde{m}_Y$. The latter holds because $\bar{m}_T\geq\tilde{m}_Y>\tilde{m}$, and from Lemma 9 in Online Appendix B.1 $\Pr\left(m\geq\bar{m}_T\right)\mathbb{E}\left[D\left(m\right)|m\geq\bar{m}_T\right]$ is monotonically decreasing in $\bar{m}_T$, and the corresponding utility $\mathbb{E}\left[\pi^F\right]|_{\bar{m}_T=\tilde{m}_Y}=\Pr\left(m\geq\tilde{m}_Y\right)\mathbb{E}\left[D\left(m\right)|m\geq\tilde{m}_Y\right]\geq\mathbb{E}\left[\pi^F\right]|_{\bar{m}_T=\tilde{m}_N}$ as $\tilde{m}_N\geq\tilde{m}_Y$ (strict inequality whenever $\tilde{m}_N>\tilde{m}_Y$).

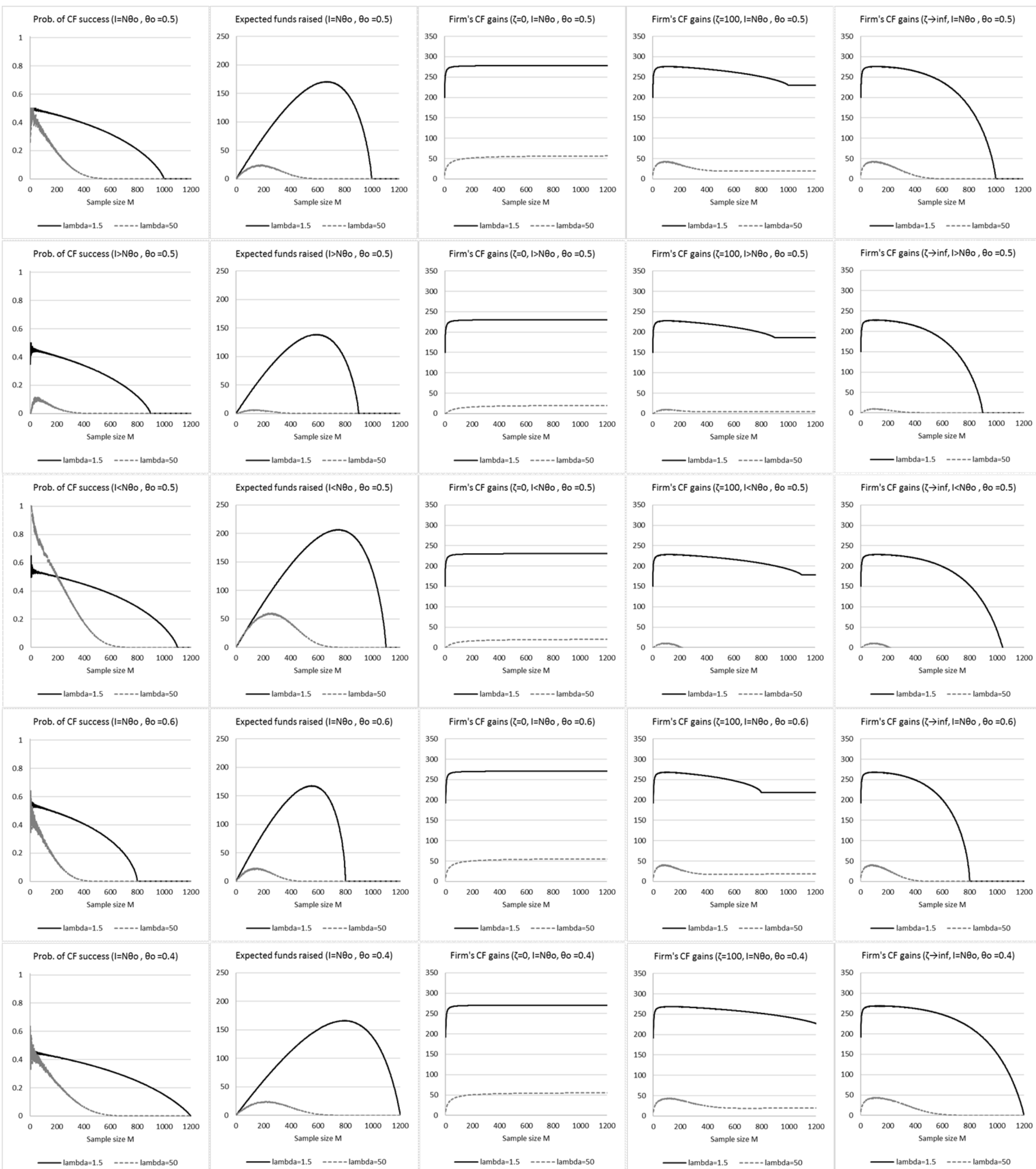
When $\bar{m}_T\in[0,\tilde{m}_Y)$ then the firm may divert funds and as in Appendix A.2, the inequalities (34) and (35) must hold. The event $\left\{\mathbf{1}_1^F=1\&T_{\bar{m}_T}=1\right\}$ occurs when $m\geq\tilde{m}_Y$, the event $\left\{\mathbf{1}_1^F=1\&T_{\bar{m}_T}=0\right\}$ never occurs, and the event $\left\{\mathbf{1}_1^F=0\&T_{\bar{m}_T}=1\right\}$ occurs when $\bar{m}_T\leq m<\tilde{m}_Y$. We obtain

$$\mathbb{E}\left[\pi^F\right]|_{\bar{m}_T\in[0,\tilde{m}_Y)}-\mathbb{E}\left[\pi^F\right]|_{\bar{m}_T=\tilde{m}_Y}=\Pr\left(m\geq\bar{m}_T\right)\mathbb{E}\left[m|m\geq\bar{m}_T\right]\left(p_0-\frac{\mathbb{E}\left[m|m\geq\tilde{m}_Y\right]\Pr\left(m\geq\tilde{m}_Y\right)}{\mathbb{E}\left[m|m\geq\bar{m}_T\right]\Pr\left(m\geq\bar{m}_T\right)}\right),$$

which from (35) is non-positive making this deviation unprofitable compared to $\bar{m}_T=\tilde{m}_Y$.

A.4 Sample size, crowdfunding outcomes and firm's expected utility from crowdfunding under different parameter values

The first row considers the benchmark parameter assumptions as in the main text, i.e., $N = 2000$, $\theta_0 = 0.5$ and $I = \theta_0 N = 1000$, it replicates Figure 4. The second and third row consider higher and lower investment costs, $I = 1100$ and $I = 800$ respectively. The fourth and fifth row consider a break-even firm with higher and lower values of prior mean, $\theta_0 = 0.6$ and $\theta_0 = 0.4$, respectively.



Learning through Crowdfunding
Online Appendix B

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December 6, 2018

B Online appendices

B.1 Useful general results

Lemma 8 *For any* $c \in \{0, 1, ..., M\}$, $\mathbb{E}[m|m \geq c]$ *is increasing in* c.

Proof. Since c is an integer, it is sufficient to show that for any $c \in \{0, 1, ..., M\}$ we have

$$\mathbb{E}[m|m \geq c+1] > \mathbb{E}[m|m \geq c],$$

which can be expressed as

$$\frac{\sum_{m=c+1}^{M} mq_m}{\sum_{m=c+1}^{M} q_m} > \frac{\sum_{m=c}^{M} mq_m}{\sum_{m=c}^{M} q_m} = \frac{cq_c + \sum_{m=c+1}^{M} mq_m}{q_c + \sum_{m=c+1}^{M} q_m}$$

Simplifying, we obtain

$$q_c \sum_{m=c+1}^{M} mq_m > cq_c \sum_{m=c+1}^{M} q_m \Leftrightarrow q_c \sum_{m=c+1}^{M} (m-c)\, q_m > 0,$$

where the inequality holds because $m - c > 0$ for any $c + 1 \leq m \leq M$ and with a beta-binomial distribution, $q_m > 0$ for any m. ■

Lemma 9 *For any* $c \in \{0, 1, ..., M\}$, $\Pr(m \geq c)\, \mathbb{E}[D(m)\,|m \geq c]$ *is monotonically increasing (decreasing) in* c *for any* $c < (>)\, \tilde{m}$; $\Pr(m \geq c)\, \mathbb{E}[D(m)\,|m \geq c]$ *is maximized when* $c = \tilde{m}$, *where* $\tilde{m}$ *is given by (14).*

Proof. As c is an integer, consider the difference

$$\begin{aligned} & \Pr(m \geq c+1)\, \mathbb{E}[D(m)\,|m \geq c+1] - \Pr(m \geq c)\, \mathbb{E}[D(m)\,|m \geq c] \\ = & \sum_{m=c+1}^{M} D(m)\, q_m - \sum_{m=c}^{M} D(m)\, q_m = -D(c)\, q_c. \end{aligned}$$

Since $q_c > 0$ for any c, the difference has the same sign as $-D(c)\, q_c$. From (14) and (15), we obtain

$$\begin{array}{l} D(c) < 0 \text{ if } c < \tilde{m} \\ D(c) \geq 0 \text{ if } c = \tilde{m} \\ D(c) > 0 \text{ if } c > \tilde{m} \end{array},$$

which implies that $\Pr(m \geq c)\, \mathbb{E}[D(m)\,|m \geq c]$ is indeed monotonically increasing (decreasing) in c for any $c < (>)\, \tilde{m}$. This also implies that $\Pr(m \geq c)\, \mathbb{E}[D(m)\,|m \geq c]$ is maximized when $c = \tilde{m}$.

■

B.2 Proof of Proposition 2

We only need to consider $I_0 < I < I_0 + M\frac{\lambda+N}{\lambda+M}$, as the value of learning is zero otherwise. From (14), it is clear that $\tilde{m}$ is weakly increasing in I.

Consider $I < N\theta_0$. From (15) and (17), the expected value of learning is

$$U_{B,I}^F = -\mathbb{E}\left[D\left(m\right)|m < \tilde{m}\right]\Pr\left(m < \tilde{m}\right) = -\sum_{m=0}^{\tilde{m}-1} D\left(m\right) q_m.$$

Since $-D\left(m\right) = \left(I - \frac{\lambda+N}{\lambda+M}m - I_0\right)$ is positive and increasing in I for any $m < \tilde{m}$, and since $\tilde{m}$ is weakly increasing in I, we obtain that $U_{B,I}^F$ is increasing in I.

Consider $I > N\theta_0$. From (15) and (18), the expected value of learning is

$$U_{B,NI}^F = \mathbb{E}\left[D\left(m\right)|m \geq \tilde{m}\right]\Pr\left(m \geq \tilde{m}\right) = \sum_{m=\tilde{m}}^{M} D\left(m\right) q_m.$$

Since $D\left(m\right) = \left(\frac{\lambda+N}{\lambda+M}m + I_0 - I\right)$ is positive and decreasing in I for any $m \geq \tilde{m}$ and since $\tilde{m}$ is weakly increasing in I, $U_{B,NI}^F$ is decreasing in I. This implies that $I = N\theta_0$ maximizes the expected value of learning.

B.3 Proof of Proposition 3

For the sake of exposition, we have limited indexing the variables of interest in the main text and only highlighted the dependence on m. For this proof it is necessary to consider the dependence on λ. We denote

$$\begin{aligned}
q_m\left(\lambda\right) &= \binom{M}{m}\frac{B\left(\lambda\theta_0 + m, \lambda\left(1-\theta_0\right) + M - m\right)}{B\left(\lambda\theta_0, \lambda\left(1-\theta_0\right)\right)} \\
\tilde{m}\left(\lambda\right) &= \left\lceil \frac{\lambda+M}{\lambda+N}\left(I - I_0\left(\lambda\right)\right)\right\rceil \\
D\left(m,\lambda\right) &= \frac{\lambda+N}{\lambda+M}m + I_0\left(\lambda\right) - I,
\end{aligned} \tag{36}$$

where $I_0\left(\lambda\right) = \frac{\lambda\theta_0(N-M)}{\lambda+M}$. The equations in (36) are identical to (12), (14) and (15) respectively. We also denote the cumulative distribution of m

$$Q_m\left(\lambda\right) = \sum_{k=0}^{m} q_k\left(\lambda\right).$$

Since the distribution of m is beta-binomial, for any pair $\lambda \in \{\lambda_1, \lambda_2\}$ such that $\lambda_2 > \lambda_1$, the distribution with higher λ_2 second order stochastically dominates (SOSD) the one with λ_1, i.e.,

$$\sum_{k=0}^{m} Q_k\left(\lambda_2\right) \leq \sum_{k=0}^{m} Q_k\left(\lambda_1\right) \tag{37}$$

for all $m = \{0, 1, ..., M\}$; the inequality is strict for $m < M - 1$.[10]

From (17) and (18), we obtain

$$\begin{aligned} U^F_{B,I}(\lambda) &= -\sum_{m=0}^{\tilde{m}(\lambda)-1} D(m,\lambda)\, q_m(\lambda) \text{ for } I < N\theta_0 \\ U^F_{B,NI}(\lambda) &= (N\theta_0 - I) - \sum_{m=0}^{\tilde{m}(\lambda)\ -1} D(m,\lambda)\, q_m(\lambda) \text{ for } I \geq N\theta_0 \end{aligned} \tag{38}$$

From Abel's Lemma[11], and $D(m+1,\lambda) - D(m,\lambda) = \frac{\lambda+N}{\lambda+M}$,

$$\sum_{m=0}^{\tilde{m}(\lambda)-1} D(m,\lambda)\, q_m(\lambda) = D(\tilde{m}(\lambda), \lambda)\, Q_{\tilde{m}(\lambda)-1}(\lambda) - \frac{\lambda+N}{\lambda+M} \sum_{m=0}^{\tilde{m}(\lambda)-1} Q_m(\lambda) \tag{39}$$

We can express the investment threshold as

$$\tilde{m}(\lambda) = \frac{\lambda+M}{\lambda+N}\left(I - I_0(\lambda)\right) + \varepsilon(\lambda), \tag{40}$$

where $0 \leq \varepsilon(\lambda) < 1$ is the rounding error. Using (36) and (40) we obtain

$$D(\tilde{m}(\lambda), \lambda) = \frac{\lambda+N}{\lambda+M}\tilde{m}(\lambda) + I_0(\lambda) - I = \frac{\lambda+N}{\lambda+M}\varepsilon(\lambda) \tag{41}$$

Replacing (39) and (41) in (38), we express the firm's expected value of learning as

$$U^F_{B,I}(\lambda) = \frac{\lambda+N}{\lambda+M}\left(\varepsilon(\lambda)\sum_{m=0}^{\tilde{m}(\lambda)-2} Q_m(\lambda) + (1-\varepsilon(\lambda))\sum_{m=0}^{\tilde{m}(\lambda)-1} Q_m(\lambda)\right), \tag{42}$$

$$U^F_{B,NI}(\lambda) = (N\theta_0 - I) + \frac{\lambda+N}{\lambda+M}\left(\varepsilon(\lambda)\sum_{m=0}^{\tilde{m}(\lambda)-2} Q_m(\lambda) + (1-\varepsilon(\lambda))\sum_{m=0}^{\tilde{m}(\lambda)-1} Q_m(\lambda)\right). \tag{43}$$

From (36), we obtain

$$\begin{aligned} \tilde{m}(\lambda_2) &\leq \tilde{m}(\lambda_1) \text{ if } I < N\theta_0 \\ \tilde{m}(\lambda_2) &\geq \tilde{m}(\lambda_1) \text{ if } I > N\theta_0, \end{aligned}$$

If $I = N\theta_0$, then $\tilde{m}(\lambda_2) = \tilde{m}(\lambda_1) = \lceil M\theta_0 \rceil = \left\lceil I\frac{M}{N} \right\rceil$ is independent of λ. Since the effect of λ on the firms that would break even without learning is different from its effect on the firms that would not, we analyze these two cases separately.

Case 1: firms with $\mathbf{I < N\boldsymbol{\theta}_0}$

In this case, we have $\tilde{m}(\lambda_2) \leq \tilde{m}(\lambda_1)$. We denote $\Delta\tilde{m} = -(\tilde{m}(\lambda_2) - \tilde{m}(\lambda_1))$, where $\Delta\tilde{m} = \{0, 1, ...\}$. From (40), we obtain

$$\varepsilon(\lambda_2) = \varepsilon(\lambda_1) - \Delta\tilde{m} + \frac{(\lambda_2 - \lambda_1)(N-M)}{(\lambda_2+N)(\lambda_1+N)}(N\theta_0 - I)$$

[10] This can be proved analytically from the fact that beta-binomial distributions have a unimodal likelihood ratio, which implies second order stochastic dominance. See Hopkins, Kornienko, 2003 for the proof of continuous distributions, the proof for discrete distributions is available upon request.

[11] $\sum_{i=0}^{N} a_i b_i = A_N b_N - \sum_{i=0}^{N-1} A_i (b_{i+1} - b_i)$, where $A_n = \sum_{i=0}^{n} a_i$.

This and (42) enable us to decompose the effect of an increase in λ as

$$U_{B,I}^{F}(\lambda_2) - U_{B,I}^{F}(\lambda_1) = -\frac{\lambda_1 + N}{\lambda_1 + M}\left(G_1^I + G_2^I + G_3^I\right) \tag{44}$$
$$-\frac{(\lambda_2 - \lambda_1)(N - M)}{(\lambda_2 + N)(\lambda_1 + M)}\left(U_{B,I}^{F}(\lambda_2) + Q_{\tilde{m}(\lambda_2)-1}(\lambda_2)(N\theta_0 - I)\right),$$

where

$$\begin{aligned} G_1^I &\equiv \varepsilon(\lambda_1)\left(\sum_{m=0}^{\tilde{m}(\lambda_1)-2} Q_m(\lambda_1) - \sum_{m=0}^{\tilde{m}(\lambda_2)-2} Q_m(\lambda_2)\right), \\ G_2^I &\equiv (1-\varepsilon(\lambda_1))\left(\sum_{m=0}^{\tilde{m}(\lambda_1)-1} Q_m(\lambda_1) - \sum_{m=0}^{\tilde{m}(\lambda_2)-1} Q_m(\lambda_2)\right), \\ G_3^I &\equiv -\Delta\tilde{m} Q_{\tilde{m}(\lambda_2)-1}(\lambda_2). \end{aligned}$$

The second term in (44) is non-positive and it is strictly negative if $N < M$, because $U_{B,I}^{F}(\lambda_2) > 0$ and $I < N\theta_0$. The sign of the first term in (44) is determined by $G_1^I + G_2^I + G_3^I$.

If $\Delta\tilde{m} = 0$, i.e., $\tilde{m}(\lambda_1) = \tilde{m}(\lambda_2)$, then $G_1^I + G_2^I + G_3^I \geq 0$ follows from $G_3 = 0$, and $G_1^I, G_2^I \geq 0$ due to second order stochastic dominance (37).

If $\Delta\tilde{m} > 0$, then we can further decompose

$$\begin{aligned} &G_1^I + G_2^I + G_3^I = \varepsilon(\lambda_1)\left(\sum_{m=0}^{\tilde{m}(\lambda_1)-2} Q_m(\lambda_1) - \sum_{m=0}^{\tilde{m}(\lambda_1)-2} Q_m(\lambda_2)\right) \\ &+(1-\varepsilon(\lambda_1))\left(\sum_{m=0}^{\tilde{m}(\lambda_1)-1} Q_m(\lambda_1) - \sum_{m=0}^{\tilde{m}(\lambda_1)-1} Q_m(\lambda_2)\right) \\ &+\sum_{m=\tilde{m}(\lambda_2)\ -1}^{\tilde{m}(\lambda_2)\ +\Delta\tilde{m}-2}\left(Q_m(\lambda_2) - Q_{\tilde{m}(\lambda_2)-1}(\lambda_2)\right) \\ &+(1-\varepsilon(\lambda_1))\left(Q_{\tilde{m}(\lambda_2)+\Delta\tilde{m}-1}(\lambda_2) - Q_{\tilde{m}(\lambda_2)-1}(\lambda_2)\right), \end{aligned}$$

which is non-negative for any $\Delta\tilde{m} > 0$, because of second order stochastic dominance (37) and because the cumulative probability is increasing in m. Hence the value of learning decreases with λ for all firms with $I < N\theta_0$.

Case 2: firms with $\mathbf{I} \geq \mathbf{N}\boldsymbol{\theta}_0$

In this case, we have $\tilde{m}(\lambda_2) \geq \tilde{m}(\lambda_1)$. We again denote $\Delta\tilde{m} = \tilde{m}(\lambda_2) - \tilde{m}(\lambda_1)$, where $\Delta\tilde{m} = \{0, 1, ...\}$ and from (40) we obtain the following relationship between $\varepsilon(\lambda_1)$ and $\varepsilon(\lambda_2)$:

$$\varepsilon(\lambda_1) = \varepsilon(\lambda_2) - \Delta\tilde{m} + \frac{(\lambda_2 - \lambda_1)(N - M)(I - N\theta_0)}{(\lambda_2 + N)(\lambda_1 + N)}. \tag{45}$$

The above and (42) enable us to decompose again the effect of an increase in λ as

$$U^F_{B,NI}(\lambda_2) - U^F_{B,NI}(\lambda_1) = -\frac{\lambda_1 + N}{\lambda_1 + M}\left(G_1^{NI} + G_2^{NI} + G_3^{NI}\right) \tag{46}$$
$$-\frac{(\lambda_2 - \lambda_1)(N - M)}{(\lambda_2 + N)(\lambda_1 + M)}\left(U^F_{B,NI}(\lambda_2) + (I - N\theta_0)\left(1 - Q_{\tilde{m}(\lambda_1)-1}(\lambda_1)\right)\right),$$

where

$$G_1^{NI} \equiv \varepsilon(\lambda_2)\left(\sum_{m=0}^{\tilde{m}(\lambda_1)-2} Q_m(\lambda_1) - \sum_{m=0}^{\tilde{m}(\lambda_2)-2} Q_m(\lambda_2)\right),$$
$$G_2^{NI} \equiv (1 - \varepsilon(\lambda_2))\left(\sum_{m=0}^{\tilde{m}(\lambda_1)-1} Q_m(\lambda_1) - \sum_{m=0}^{\tilde{m}(\lambda_2)-1} Q_m(\lambda_2)\right),$$
$$G_3^{NI} \equiv -\Delta\tilde{m} Q_{\tilde{m}(\lambda_1)-1}(\lambda_1).$$

It is clear that the second term of (46) is non-positive and it is strictly negative if $N < M$, because $\mathbb{E}\left[\pi^F\right]^{B,NI}(\lambda_2) > 0$, and $I \geq N\theta_0$. As in case 1, it is easy to see that if $\Delta\tilde{m} = 0$, then $\tilde{m}(\lambda_1) = \tilde{m}(\lambda_2)$ and $G_1^{NI} + G_2^{NI} + G_3^{NI} \geq 0$. If $\Delta\tilde{m} > 0$, then we can follow the same derivation as before to prove that $G_1^{NI} + G_2^{NI} + G_3^{NI} \geq 0$ (keeping in in mind that now $\tilde{m}(\lambda_2) > \tilde{m}(\lambda_1)$).

This proves that the value of learning increases with the level of uncertainty, i.e., decreases with λ.

B.4 Proof of Lemma 5

From (25) it is immediate that when $v^i = 0$, pledging leads to a strictly negative expected payoff, which proves the first sentence of Lemma 5. When $\bar{m}_T \geq \tilde{m}_Y$, the consumer anticipates the firm will invest with probability 1 if it meets its target, which proves that $1_0^i = 1$ whenever (26) holds.

When $\bar{m}_T < \tilde{m}_Y$, then

$$\Pr\left(T_{\bar{m}_T} = 1|\Omega_0^i\right) = \Pr\left(m^B \geq \bar{m}_T|v^i = 1\right) = \Pr\left(m^{-i} + 1 \geq \bar{m}_T|v^i = 1\right)$$

and

$$\Pr\left(T_{\bar{m}_T} = 1|\Omega_0^i\right)\Pr\left(\mathbf{1}_1^F = 1|\Omega_0^i, T_{\bar{m}_T} = 1\right) = \Pr\left(m^B \geq \tilde{m}_Y|v^i = 1\right) = \Pr\left(m^{-i} \geq \tilde{m}_Y - 1|v^i = 1\right),$$

using (25), we then find 27. The last inequality follows from the fact that $\tilde{m}_Y - 1 > \bar{m}_T - 1 \Longleftrightarrow \Pr\left(m^{-i} \geq \tilde{m}_Y - 1|v^i = 1\right) < \Pr\left(m^{-i} \geq \bar{m}_T - 1|v^i = 1\right)$.

B.5 Analytical results for Section 6

We can highlight the main forces that drive the impact of an increase in M on campaign outcomes and the firm's utility. Because M affects the probability mass function (12), and thus expectations and probabilities, we use here notation "$;M$", "$;M+1$" to highlight under which sample size the distribution of the relevant variable is calculated. For this extension, let us denote $q(m;M)$ the probability mass function of m when the sample size is M, $Q(c;M)=\sum_{m=0}^{c}q(m;M)$ the cumulative distribution function, and $\bar{Q}(c;M)=\sum_{m=c}^{M}q(m;M)$ the probability that m is above some integer $c=\{0,...,M\}$. Clearly $Q(c-1;M)+\bar{Q}(c;M)=1$. The following Lemma is important to understand the main drivers

Lemma 10 *For any integer $c=\{1,...,M-1\}$ we have 1) $\bar{Q}(c;M+1)>\bar{Q}(c;M)$, and 2) $\bar{Q}(c+1;M+1)<\bar{Q}(c;M)$.*

Proof. See B.5.1 below. ■

Lemma 10 provides an explanation for why the probability of meeting an endogenous crowdfunding target $\bar{m}_T=\tilde{m}_Y$ becomes decreasing in M. It states that while the probability of meeting a fixed target under the sample $M+1$ is higher than under the sample M, whenever the target increases at least by one unit[12] then the probability of meeting this target decreases. As explained in the main text when M increases then changes in $\tilde{m}_Y$ become increasingly frequent and larger, hence to the probability of crowdfunding success must decrease when M becomes sufficently high.

This also explains why the probability of meeting the target being decreasing in sample size is a symptom of moral hazard. If there was no moral hazard, the firm could commit to a first best target $\tilde{m}$ defined in (14). It is easy to confirm that the first best target of a breakeven firm with $I=N\theta_0$ is $\tilde{m}=\lceil M\theta_0\rceil$. Consider the example with $\theta_0=0.5$. Due to the ceiling function $\tilde{m}$ remains unchanged whenever M changes from being an odd number to an even one and increases whenever M changes from even to odd. Hence the probability of meeting such target would alternate between being decreasing and increasing leaving the overall probability of the event $m\geq\tilde{m}$ constant on average. A similar reasoning applies for other values of θ_0. Higher (lower) investment costs would make the probability of meeting this target increasing (decreasing) in M, because $\tilde{m}_Y$ is more (less) sensitive to changes in M when I is larger (smaller). In an overall sample with many firms, we should not find a systematic pattern between the sample size and the probability of crowdfunding success if there was no moral hazard.

Lemma 10 further enables to identify the driving forces behind the relationship between the sample size M, funds raised and the firm's utility. Namely, from Abel's Lemma for an integer

[12] Note that $\bar{Q}(c+k;M+1)=\sum_{c+k}^{M}q(c;M)<\sum_{c+1}^{M}q(c;M)$ for any $k>1$, which further implies $\bar{Q}(c+k;M+1)<\bar{Q}(c;M)$.

$c = \{1, ..., M-1\}$ we have

$$\begin{aligned}
& \mathbb{E}[m|m \geq c; M] \Pr(m \geq c; M) = \sum_{m=0}^{M} m \cdot q(m; M) - \sum_{m=0}^{c-1} m \cdot q(m; M) \qquad (47) \\
= \; & M - (c-1) Q(c-1; M) - \sum_{m=0}^{M-1} Q(m; M) + \sum_{m=0}^{c-2} Q(m; M) \\
= \; & M - cQ(c-1; M) - \sum_{m=0}^{M-1} Q(m; M) + \sum_{m=0}^{c-1} Q(m; M) \\
= \; & c\bar{Q}(c; M) + \sum_{m=c}^{M-1} \bar{Q}(m+1; M),
\end{aligned}$$

where the last equality uses $Q(m; M) = 1 - \bar{Q}(m+1; M)$.

This implies that whenever the threshold does not change, i.e., $\tilde{m}_Y(M+1) = \tilde{m}_Y(M)$, the expected funds raised are increasing in M, i.e.,

$$\begin{aligned}
& \Delta Funds|_{\tilde{m}_Y(M+1)=\tilde{m}_Y(M)} \equiv \mathbb{E}[m|m \geq \tilde{m}_Y(M); M+1] \Pr(m \geq \tilde{m}_Y(M); M+1) \\
& -\mathbb{E}[m|m \geq \tilde{m}_Y(M); M] \Pr(m \geq \tilde{m}_Y(M); M) \\
= \; & \bar{Q}(M+1; M+1) + \tilde{m}_Y(M) \left(\bar{Q}(\tilde{m}_Y(M); M+1) - \bar{Q}(\tilde{m}_Y(M); M) \right) \\
& + \sum_{m=\tilde{m}_Y(M)}^{M-1} \left(\bar{Q}(m+1; M+1) - \bar{Q}(m+1; M) \right) > 0
\end{aligned}$$

because the first term is a probability thus positive and the second and third term are positive by Lemma 10. When then threshold does change, i.e., $\tilde{m}_Y(M+1) > \tilde{m}_Y(M)$, then the expected funds raised is affected by a negative force due to the change of threshold, i.e.,

$$\begin{aligned}
& \Delta Funds \equiv \mathbb{E}[m|m \geq \tilde{m}_Y(M+1); M+1] \Pr(m \geq \tilde{m}_Y(M+1); M+1) \\
& -\mathbb{E}[m|m \geq \tilde{m}_Y(M); M] \Pr(m \geq \tilde{m}_Y(M); M) \\
= \; & \Delta Funds|_{\tilde{m}_Y(M+1)=\tilde{m}_Y(M)} - \sum_{\tilde{m}_Y(M)}^{\tilde{m}_Y(M+1)-1} mq(m; M+1)
\end{aligned}$$

This negative force becomes stronger when M is larger as $\tilde{m}_Y(M)$ increases with M at an increasing rate. This explains why the expected funds raised are inverted U-shaped in M.

Let us then analyze the firm's utility. When ς is small, then $\mathbf{1}_{\tilde{m}_Y \geq \tilde{m}_N} = 1$ and from (15), (31) and (47) the firm's utility is given by

$$\begin{aligned}
& U^F(M)|_{low\ \varsigma} = \left(\frac{\lambda+N}{\lambda+M} \tilde{m}_N(M) + I_0(M) - I - \varsigma \right) \bar{Q}(\tilde{m}_N(M); M) \\
& + \frac{\lambda+N}{\lambda+M} \left(\sum_{m=\tilde{m}_N(M)}^{M-1} \bar{Q}(m+1; M) \right) + \varsigma \bar{Q}(\tilde{m}_Y(M); M) - \pi^F_{ref}.
\end{aligned}$$

Furthermore, by (24) $\left(\frac{\lambda+N}{\lambda+M}\tilde{m}_N(M)+I_0(M)-I-\varsigma\right)\approx 0$ as the only difference comes from the rounding error due to the ceiling function and therefore

$$U^F(M)|_{low\ \varsigma}\approx\frac{\lambda+N}{\lambda+M}\left(\sum_{m=\tilde{m}_N(M)}^{M-1}\bar{Q}(m+1;M)\right)+\varsigma\bar{Q}(\tilde{m}_Y(M);M)-\pi^F_{ref}.$$

Because $\tilde{m}_N(M)$ inceases slowly M the term $\sum_{m=\tilde{m}_N(M)}^{M-1}\bar{Q}(m+1;M)$ is increasing in M. There are two negative forces. First, the term $\frac{\lambda+N}{\lambda+M}$ is decreasing in M as a larger sample means that there are fewer consumers who purchase the product after crowdfunding. Essentially the benefits of learning are amplified when the sample size decreases. This explains why even when $\varsigma\to 0$, the benefits of a higher sample size diminish when M becomes large enough. If ς is positive then there is a further negative effect via the last term $\varsigma\bar{Q}(\tilde{m}_Y(M);M)$ as we have argued that meeting the endogenous crowdfunding target becomes increasingly diffficult when the sample size increases. This prompts a firm with high enough ς to also prefer a smaller sample size.

When ς is high then $\mathbf{1}_{\tilde{m}_Y\geq\tilde{m}_N}=0$ and from (15), (31) and (47)

$$U^F(M)|_{high\ \varsigma}=\left(\frac{\lambda+N}{\lambda+M}\tilde{m}_Y(M)+I_0(M)-I\right)\bar{Q}(\tilde{m}_Y(M);M)$$
$$+\frac{\lambda+N}{\lambda+M}\left(\sum_{m=\tilde{m}_Y(M)}^{M-1}\bar{Q}(m+1;M)\right)-\pi^{ref}.$$

From (23) $\left(\frac{\lambda+N}{\lambda+M}\tilde{m}_Y(M)+I_0(M)-I\right)\approx\tilde{m}_Y(M)$ where the only difference comes from the rounding error due to the ceiling function and therefore

$$U^F(M)|_{high\ \varsigma}\approx\tilde{m}_Y(M)\bar{Q}(\tilde{m}_Y(M);M)+\frac{\lambda+N}{\lambda+M}\left(\sum_{m=\tilde{m}_Y(M)}^{M-1}\bar{Q}(m+1;M)\right)-\pi^F_{ref}$$
$$=\mathbb{E}[m|m\geq\tilde{m}_Y(M);M]\Pr(m\geq\tilde{m}_Y(M);M)+\frac{N-M}{\lambda+M}\left(\sum_{m=\tilde{m}_Y(M)}^{M-1}\bar{Q}(m+1;M)\right)-\pi^F_{ref}$$

Now the first term is the funds raised, i.e., the same as the platform's objective, which we argued above to be invested U-shaped in M. As above the term $\sum_{m=\tilde{m}_Y(M)}^{M-1}\bar{Q}(m+1;M)$ is increasing in M if the threshold $\tilde{m}_Y(M)$ does not change, which is true in the case of a very small sample. When $\tilde{m}_Y(M)$ increases due to an increase of M then the term $\sum_{m=\tilde{m}_Y(M)}^{M-1}\bar{Q}(m+1;M)$ is affected by an additional negative force similarly to the funds raised. Additionally, the multiplier $\frac{N-M}{\lambda+M}$ is also decreasing in M . These negative forces lead the firm to prefer a small sample, and according to our numerical results an even smaller sample than the platform.

B.5.1 Proof of Lemma 10

From (12), the definition of the beta function $B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$, and the fact that the gamma function $\Gamma(x)$ satisfies the *recurrence relation* $\Gamma(x+1) = x\Gamma(x)$, the likelihood ratio is

$$\frac{q(m; M+1)}{q(m; M)} = \frac{M+1}{M+1-m} \cdot \frac{\lambda(1-\theta_0) + M - m}{\lambda + M} \tag{48}$$

and the likelihood ratio

$$\frac{q(m+1; M+1)}{q(m; M)} = \frac{M+1}{m+1} \cdot \frac{\lambda\theta_0 + m}{\lambda + M}. \tag{49}$$

Proof of claim 1) of Lemma 10. Proving that $\bar{Q}(c; M+1) > \bar{Q}(c; M)$ is equivalent to proving $Q(c-1; M+1) < Q(c-1; M)$. Differentiating (48) with respect to m, we get $\frac{\partial \frac{q(m;M+1)}{q(m;M)}}{\partial m} = \frac{(M+1)(\lambda(1-\theta_0)-1)}{(\lambda+M)(M-m+1)^2}$. Depending on the parameters of the model, the likelihood ratio is monotonically decreasing ($\lambda(1-\theta_0) < 1$), constant ($\lambda(1-\theta_0) = 1$) or monotonically increasing ($\lambda(1-\theta_0) > 1$). Suppose that $\lambda(1-\theta_0) < 1$. We find

$$Q(c-1; M+1) = \sum_{k=0}^{c-1} \frac{q(m; M+1)}{q(m; M)} q(m; M) \leq \frac{q(0; M+1)}{q(0; M)} \cdot Q(c-1; M),$$

where the inequality holds because $\frac{q(m;M+1)}{q(m;M)}$ is decreasing under these parameters. As $\frac{q(0;M+1)}{q(0;M)} = \frac{\lambda(1-\theta_0)+M}{\lambda+M} < 1 \Leftrightarrow \theta_0 > 0$, $Q(c-1; M+1) < Q(c-1; M)$.

Suppose that $\lambda(1-\theta_0) = 1 \Longrightarrow \lambda = \frac{1}{(1-\theta_0)}$. Then $\frac{q(m;M+1)}{q(m;M)} = \frac{M+1}{M+\lambda} = \frac{M+1}{\frac{1}{1-\theta_0}+M}$. Since

$$Q(c-1; M+1) = \sum_{m=0}^{c-1} \frac{q(m; M+1)}{q(m; M)} q(m; M) = \frac{M+1}{M+\lambda} Q(c-1; M)$$

and $\frac{M+1}{\frac{1}{1-\theta_0}+M} < 1 \Leftrightarrow 0 < \theta_0 < 1$, it follows that $Q(c-1; M+1) < Q(c-1; M)$.

Finally, consider $\lambda(1-\theta_0) > 1$. Because the likelihood ratio is now monotonially increasing, we must have

$$Q(c-1; M+1) = \sum_{k=0}^{c-1} \frac{q(m; M+1)}{q(m; M)} q(m; M) \leq \frac{q(c-1; M+1)}{q(c-1; M)} Q(c-1; M).$$

At the same time

$$\begin{aligned} 1 - Q(c-1; M+1) &= q(M+1; M+1) + \sum_{m=c}^{M} q(m; M+1) \\ &= q(M+1; M+1) + \sum_{m=c}^{M} \frac{q(m; M+1)}{q(m; M)} q(m; M) \geq \\ &\geq q(M+1; M+1) + \frac{q(c; M+1)}{q(c; M)} (1 - Q(c-1; M)), \end{aligned}$$

where again the inequality holds because the likelihood ratio is monotonically increasing. From these two inequalities we obtain that for any $c = \{1, ..., M-1\}$

$$\frac{Q(c-1; M+1)}{Q(c-1; M)} \leq \frac{q(c-1; M+1)}{q(c-1; M)} < \frac{q(M+1; M+1)}{(1 - Q(c-1; M))} + \frac{q(c; M+1)}{q(c; M)} \leq \frac{1 - Q(c-1; M+1)}{1 - Q(c-1; M)},$$

where the middle inequality holds because $\frac{q(c-1;M+1)}{q(c-1;M)} < \frac{q(c;M+1)}{q(c;M)}$ and $\frac{q(M+1;M+1)}{(1-Q(c-1;M))} > 0$. From here,

$$\frac{Q(c-1;M+1)}{Q(c-1;M)} < \frac{1-Q(c-1;M+1)}{1-Q(c-1;M)} \Leftrightarrow Q(c-1;M+1) < Q(c-1;M).$$

Proof of claim 2) of Lemma 10. Differentiating (49) with respect to m, we get $\frac{\partial \frac{q(m+1;M+1)}{q(m;M)}}{\partial m} = \frac{M+1}{\lambda+M} \cdot \frac{1-\lambda\theta_0}{(m+1)^2}$, and we again have three possibilities, the likelihood ratio is monotonically increasing ($\lambda\theta_0 < 1$), constant ($1 = \lambda\theta_0$), or monotonically decreasing ($\lambda\theta_0 > 1$).

Suppose that $\lambda\theta_0 < 1$. We find

$$\bar{Q}(c+1;M+1) = \sum_{m=c}^{M} \frac{q(m+1;M+1)}{q(m;M)} q(m;M) \leq \frac{q(M+1;M+1)}{q(M;M)} \bar{Q}(c;M),$$

where the inequality holds because $\frac{q(m;M+1)}{q(m;M)}$ is increasing. As $\frac{q(M+1;M+1)}{q(M;M)} = \frac{\lambda\theta_0+M}{\lambda+M} < 1 \Leftrightarrow \theta_0 < 1$, $\bar{Q}(c+1;M+1) < \bar{Q}(c;M)$.

Suppose that $\lambda\theta_0 = 1 \Longrightarrow \lambda = \frac{1}{\theta_0}$. Then $\frac{q(m;M+1)}{q(m;M)} = \frac{M+1}{M+\lambda} = \frac{M+1}{\frac{1}{1-\theta_0}+M}$. Since

$$\bar{Q}(c+1;M+1) = \sum_{m=c}^{M} \frac{q(m+1;M+1)}{q(m;M)} q(m;M) = \frac{M+1}{\lambda+M} \sum_{m=c}^{M} q(m;M) = \frac{M+1}{\lambda+M} \bar{Q}(c;M)$$

and $\frac{M+1}{\frac{1}{1-\theta_0}+M} < 1 \Leftrightarrow 0 < \theta_0 < 1$, it follows that $\bar{Q}(c+1;M+1) < \bar{Q}(c;M)$.

Finally, consider $\lambda\theta_0 > 1$. Since the likelihood ratio is now monotonially decreasing

$$\bar{Q}(c+1;M+1) = \sum_{m=c}^{M} \frac{q(m+1;M+1)}{q(m;M)} q(m;M) \leq \frac{q(c+1;M+1)}{q(c;M)} \bar{Q}(c;M).$$

At the same time

$$\begin{aligned} 1-\bar{Q}(c+1;M+1) = \sum_{m=0}^{c} q(m;M+1) &= q(0;M+1) + \sum_{m=0}^{c-1} q(m+1;M+1) \\ &= q(0;M+1) + \sum_{m=0}^{c-1} \frac{q(m+1;M+1)}{q(m;M)} q(m;M) \geq \\ &\geq q(0;M+1) + \frac{q(c;M+1)}{q(c-1;M)} \left(1-\bar{Q}(c;M)\right) \end{aligned}$$

where again the inequality holds because the likelihood ratio is monotonically increasing. From these two inequalities, we obtain

$$\frac{1-\bar{Q}(c+1;M+1)}{\left(1-\bar{Q}(c;M)\right)} \geq \frac{q(0;M+1)}{\left(1-\bar{Q}(c;M)\right)} + \frac{q(c;M+1)}{q(c-1;M)} > \frac{q(c+1;M+1)}{q(c;M)} \geq \frac{\bar{Q}(c+1;M+1)}{\bar{Q}(c;M)}$$

where the middle inequality holds because $\frac{q(c+1;M+1)}{q(c;M)} < \frac{q(c;M+1)}{q(c-1;M)}$ and $\frac{q(0;M+1)}{\left(1-\bar{Q}(c;M)\right)} > 0$. From here

$$\frac{\bar{Q}(c+1;M+1)}{\bar{Q}(c;M)} < \frac{1-\bar{Q}(c+1;M+1)}{\left(1-\bar{Q}(c;M)\right)} \Leftrightarrow \bar{Q}(c+1;M+1) < \bar{Q}(c;M).$$

B.6 Distribution of the ratio of pledges to target

The figures below are based on data from Kickstarter. The upper figure, which plots $\Pr\left(\frac{pm}{p\bar{m}_T} \leq x | \frac{pm}{p_{\bar{m}T}} \geq 1\right)$, is constructed using all 2015 Technology and Theatre projects completed from January 1, 2015 to September 17, 2015. We then constructed the unconditional distribution, $\Pr\left(\frac{pm}{p\bar{m}_T} \leq x\right)$, thanks to the summary statistics available on Kickstarter on September 17, 2015. This statistics cover the full history of Kickstarter, so we assume that the 2015 data is statistically similar to earlier data. We use $\Pr\left(\frac{pm}{p\bar{m}_T} \geq 1\right)$ reported on the site, which is 0.2018 for Technology category and 0.6089 for Theatre, and the frequency of unsuccessful projects raising 0%, $1-20\%$, ..., $81-99\%$ in these categories. We then use the law of total expectations to find $\Pr\left(\frac{pm}{p\bar{m}_T} \leq x\right)$.