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Least Mean-Phase Adaptive Filters With Application to Communications Systems

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***Abstract*—The mean-squared-error criterion is widely used in the literature. However, there are applications where the squared-error is not the primary parameter affecting the performance of a system. In many communication systems, for instance, the information bits are carried over the phase of the transmitted signal. In this letter, we introduce a cost function that is based on both the error magnitude and the phase error. The criterion is useful for applications where the performance depends primarily on the phase of the estimated (recovered) signal. An adaptive filter is then developed using the proposed criterion with essentially the same complexity as the standard least mean squared (LMS) algorithm. The filter outperforms LMS specially in situations with fast channel variations. Bit error rate (BER) simulations for two communication systems using the proposed algorithm support the claims.**

***Index Terms*—Adaptive channel estimation, adaptive filtering, phase error minimization.**

I. Introduction

THE MEAN-squared-error criterion is widely used in linear estimation theory and adaptive filtering [1], and it has been applied successfully in many different contexts [1], [2]. However, the criterion is only a function of the error magnitude, and it does not depend on the phase error. In many communications systems, the parameter affecting the performance is primarily the error in phase. For instance, M-PSK and QPSK modulation schemes carry the information in the phase of the transmitted signal [2]. Therefore, performance degradation in PSK, QPSK, and even QAM systems is mostly due to the error in the phase of the estimated signal rather than in the magnitude of the error. In such systems, a symbol error occurs whenever the error in the phase of the estimated signal is more than a threshold value, depending on the constellation. For such applications, we propose a criterion that involves both the estimated phase error and the magnitude of the error. An adaptive filter is developed based on this criterion and simulation results indicate its superior performance over a standard LMS implementation. The improvement is more significant when the adaptive filter is required to track fast channel variations, since the proposed structure can at least track the channel phase variations, even if it fails to properly track the channel magnitude variations.

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II. Problem Formulation

Let d and $\mathbf{u}$ be zero-mean random variables, with d being a scaler and $\mathbf{u}$ a $1 \times M$ vector. Introduce the phase-error cost function

$$\begin{aligned} J_{\rm pe}(\mathbf{w}) &= {\rm E}\,|{\rm phase}(d) - {\rm phase}(\mathbf{uw})|^m \\ &= {\rm E}|\angle d - \angle \mathbf{uw}|^m \end{aligned} \tag{1}$$

where $m = 1, 2$ is discussed in this letter, and $\mathbf{w}$ is an unknown weight vector to be estimated. Moreover, the letter E denotes expectation. Consider further the well-known squared-error cost function

$$J_{\rm se}(\mathbf{w}) = {\rm E}|d - \mathbf{uw}|^2 \tag{2}$$

and introduce the weighted cost function

$$J(\mathbf{w}) = k_1 J_{\rm se}(\mathbf{w}) + k_2 J_{\rm pe}(\mathbf{w}) \tag{3}$$

where k_1 and k_2 define the contribution of each term to the overall cost function. The proper choices of k_1 and k_2 depend on the application and the environment in which the adaptive filter is being used. For instance, setting $k_2 = 0$ leads us to the standard LMS algorithm. The proper ratio between k_1 and k_2 for the scenarios presented in this letter is discussed later. Our goal is to develop an adaptive filter to minimize $J(\mathbf{w})$.

III. Stochastic Gradient Derivation

The update equation for a steepest-descent implementation is given by

$$\mathbf{w}_i = \mathbf{w}_{i-1} - \mu \left[\nabla_{\mathbf{w}} J(\mathbf{w}_{i-1})\right]^*, \quad i \geq 0 \tag{4}$$

where $\nabla_{\mathbf{w}}$ is the complex gradient of $J(\mathbf{w})$ with respect to $\mathbf{w}$. The gradient of $J_{\rm se}(\mathbf{w})$ is given by

$$\nabla_{\mathbf{w}} J_{\rm se}(\mathbf{w}) = -{\rm E}(\mathbf{u}^* d - \mathbf{u}^* \mathbf{uw})^* \tag{5}$$

while the gradient of $J_{\rm pe}(\mathbf{w})$ is given by (for $m = 1$)

$$\nabla_{\mathbf{w}} J_{\rm pe}(\mathbf{w}) = -{\rm E}\left[{\rm sign}(\angle d - \angle \mathbf{uw}) \times \nabla_{\mathbf{w}}(\angle \mathbf{uw})\right] \tag{6}$$

where

$$\begin{aligned} &\nabla_{\mathbf{w}}(\angle \mathbf{uw}) \\ &\quad = \nabla_{\mathbf{w}} \left(\arctan \frac{{\rm Im}(\mathbf{uw})}{{\rm Re}(\mathbf{uw})} \pm \pi \right) \\ &\quad = \frac{\nabla_{\mathbf{w}} \left[{\rm Im}(\mathbf{uw})\right] {\rm Re}(\mathbf{uw}) - {\rm Im}(\mathbf{uw}) \nabla_{\mathbf{w}} \left[{\rm Re}(\mathbf{uw})\right]}{\left[{\rm Im}(\mathbf{uw})\right]^2 + \left[{\rm Re}(\mathbf{uw})\right]^2}. \end{aligned} \tag{7}$$

Using the following relations

$$\mathrm{Re}(\mathbf{uw})=\frac{[(\mathbf{uw})+(\mathbf{uw})^*]}{2}\quad \mathrm{Im}(\mathbf{uw})=\frac{[(\mathbf{uw})-(\mathbf{uw})^*]}{2j} \tag{8}$$

(7) becomes

$$\nabla_{\mathbf{w}}(\angle\mathbf{uw}) = \frac{\frac{\mathbf{u}}{2j}\mathrm{Re}(\mathbf{uw}) - \frac{\mathbf{u}}{2}\mathrm{Im}(\mathbf{uw})}{|\mathbf{uw}|^2} = -\frac{j}{2}\frac{(\mathbf{uw})^*}{|\mathbf{uw}|^2}\mathbf{u}. \tag{9}$$

Substituting into (6) gives

$$\nabla_{\mathbf{w}} J_{\mathrm{pe}}(\mathbf{w}) = \mathrm{E}\left(\frac{j}{2}\mathrm{sign}(\angle d - \angle\mathbf{uw})\frac{1}{\mathbf{uw}}\mathbf{u}\right). \tag{10}$$

Using instantaneous approximations leads to the adaptive implementation

$$\mathbf{w}_i = \mathbf{w}_{i-1} + \mu_1\left(d(i) - \mathbf{u}_i\mathbf{w}_{i-1}\right)\mathbf{u}_i^* + \mu_2\mathrm{sign}\left(\angle d(i) - \angle\mathbf{u}_i\mathbf{w}_{i-1}\right)\frac{j\mathbf{u}_i^*}{(\mathbf{u}_i\mathbf{w}_{i-1})^*} \tag{11}$$

where μ_1 and μ_2 are step-size parameters. Similarly, we can show that for $m = 2$ in the phase-error cost function (1), the adaptive update equation will become

$$\mathbf{w}_i = \mathbf{w}_{i-1} + \mu_1\left(d(i) - \mathbf{u}_i\mathbf{w}_{i-1}\right)\mathbf{u}_i^* + \mu_2\left(\angle d(i) - \angle\mathbf{u}_i\mathbf{w}_{i-1}\right)\frac{j\mathbf{u}_i^*}{(\mathbf{u}_i\mathbf{w}_{i-1})^*}. \tag{12}$$

We will refer to the adaptive algorithm defined by (12) as the joint least-mean-phase least-mean-squares algorithm or simply LMP–LMS. Similarly the algorithm defined by (11) will be referred to as sign-LMP–LMS. Note that both algorithms collapse to the standard least-mean-squares algorithm (LMS) by setting $\mu_2 = 0$. By properly choosing the step-size parameters μ_1 and μ_2, we can control the relative weight of the squared-error and phase-error terms in the overall update direction.

A. Geometric Interpretation

The *a priori* and *a posteriori*-based estimates of $d(i)$ are defined by $\hat{d}(i|i-1) = \mathbf{u}_i\mathbf{w}_{i-1}$ and $\hat{d}(i|i) = \mathbf{u}_i\mathbf{w}_i$. Introduce the following metric as the improvement in the estimate for $d(i)$:

$$\Delta\hat{d} = \hat{d}(i|i) - \hat{d}(i|i-1) = \mathbf{u}_i(\mathbf{w}_i - \mathbf{w}_{i-1}). \tag{13}$$

Using the update (12), the above metric evaluates to

$$\Delta\hat{d} = \mu_1\|\mathbf{u}_i\|^2\left(d(i) - \mathbf{u}_i\mathbf{w}_{i-1}\right) + \mu_2\|\mathbf{u}_i\|^2\mathrm{sign}\left(\angle d(i) - \angle\mathbf{u}_i\mathbf{w}_{i-1}\right)\frac{j}{(\mathbf{u}_i\mathbf{w}_{i-1})^*}. \tag{14}$$

Now consider a communications system with PSK modulation. In such a system, an error in the phase estimate is the only source for symbol error, and we will show how the term due to the phase error in the update (12) can improve the symbol estimates. To show this, an 8-PSK constellation is illustrated in Fig. 1. This figure separately depicts the direction of the improvement in $\hat{d}(i|i)$ given by (14) due to the squared-error and phase-error terms. Fig. 1(a) depicts the first term (squared-error term) in (14), which is along the $(d(i) - \mathbf{u}_i\mathbf{w}_{i-1})$ direction. Fig. 1(b) shows the improvement in $\hat{d}(i|i)$ given by (14) due to the phase-error term. It can be verified that the term $j/(\mathbf{u}_i\mathbf{w}_{i-1})^*$ has a direction perpendicular to the vector $\mathbf{u}_i\mathbf{w}_{i-1}$, as shown in the figure. Therefore, this term corrects the error in the phase of the estimated signal, by rotating the estimated signal toward the true signal. Since an error in phase is the only source for a symbol error, a weighted combination of these two terms can lead to a lower symbol error rate. The relative effect of them can be controlled by the ratio between μ_1 and μ_2.

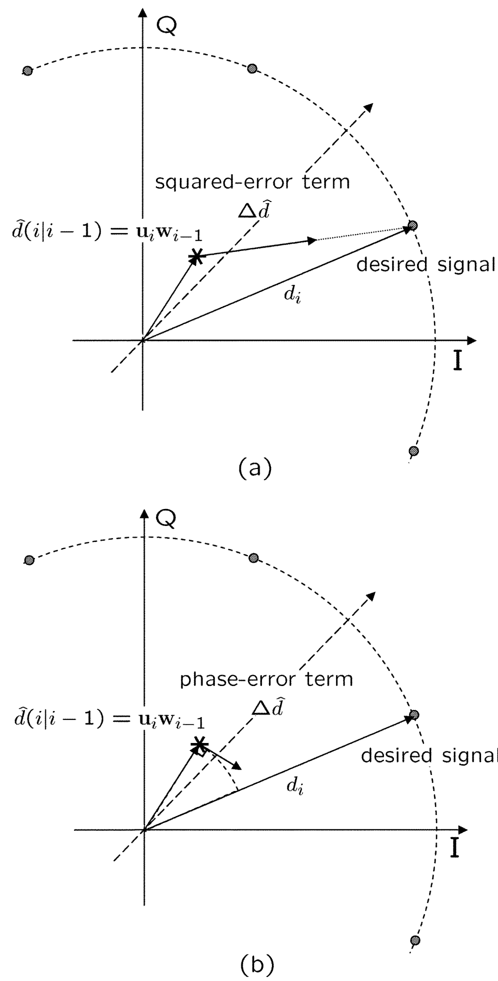

Fig. 1. Direction of the improvement in $\hat{d}(i|i)$ given the new estimate $\mathbf{w}_i$ as defined by (13). (a) Correction due to the squared-error term. (b) Correction due to the phase-error term.

Throughout the simulations, we realized that the phase-error term in the LMP–LMS algorithm enhances the filter ability to track fast channel variations (by that we mean phase variations). This term helps the filter track phase variations even if it cannot track magnitude variations fast enough. Overall, this results in a lower symbol error rate in fast fading wireless channels. Note that the LMS algorithm alone has a limited tracking performance [3], and the step-size parameter μ_1 cannot be chosen unboundedly large in order to track fast channel variations, since it will make the filter unstable. For such a scenario, the phase-error term enhances the phase tracking performance of the filter (and, consequently, the symbol-error-rate), without sacrificing

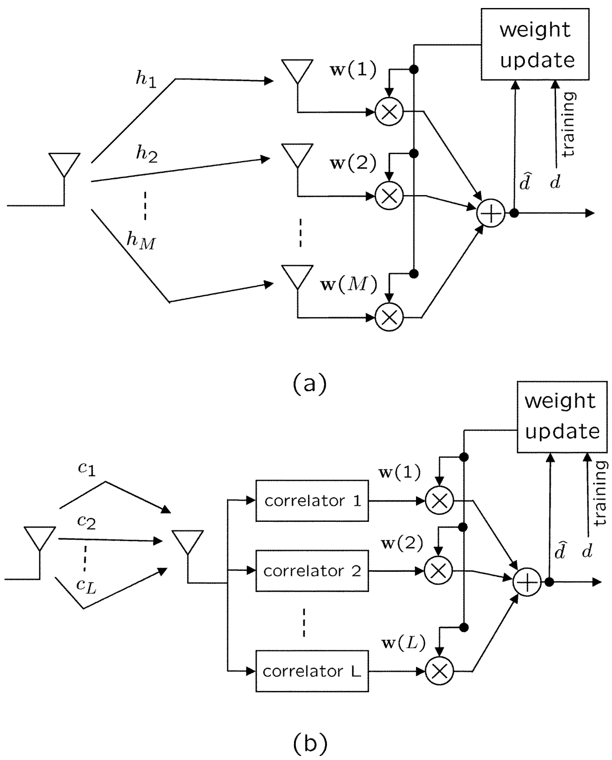

Fig. 2. Two simulated systems. (a) Adaptive antenna diversity combiner. (b) Adaptive CDMA Rake receiver.

the stability of the filter. The proposed LMP–LMS filters outperform the standard LMS filter in phase sensitive scenarios, but they are not significantly superior when the filter is used for interference cancellation.

IV. Simulation Results

The proposed LMP–LMS algorithm is applied to two different scenarios and its performance is compared with the standard LMS algorithm. The two scenarios are depicted in Fig. 2.

A. Adaptive Antenna Combiner

In this simulation, we consider a receiver using multiple antennas to achieve diversity gain. The receiver uses an adaptive filter to combine the received signal on different antennas and estimate the transmitted symbol. Channel taps on multiple antennas are independent and each has a Rayleigh distribution with the same Doppler frequency. To model the channel phase and magnitude variation accurately, we use Jake's model [4] to generate the Rayleigh fading channel taps with certain Doppler frequency. A PSK constellation with Gray labeling is used as the modulation scheme, with a symbol rate of 1 MS/s. One hundred different channel realizations are simulated, each run over 2000 symbols. In this configuration, and referring to (11) and (12), the vector $\mathbf{u}$ contains the received signal on multiple antennas, and d is the transmitted symbol to be estimated. Fig. 3 and 4 show the simulation results for two different set of system parameters with Doppler frequencies of 480 and 320 Hz, respectively. These Doppler frequencies correspond to velocities of 60 and 40 mph at the carrier frequency of 5.4 GHz. The LMS algorithm contains only the squared-error term with step-size parameter μ_1, and the proposed algorithm includes both the terms with step-size parameters μ_1 and μ_2. Realizations of the estimated constellation at the output of the antenna combiner for both algorithms are depicted in Fig. 5.

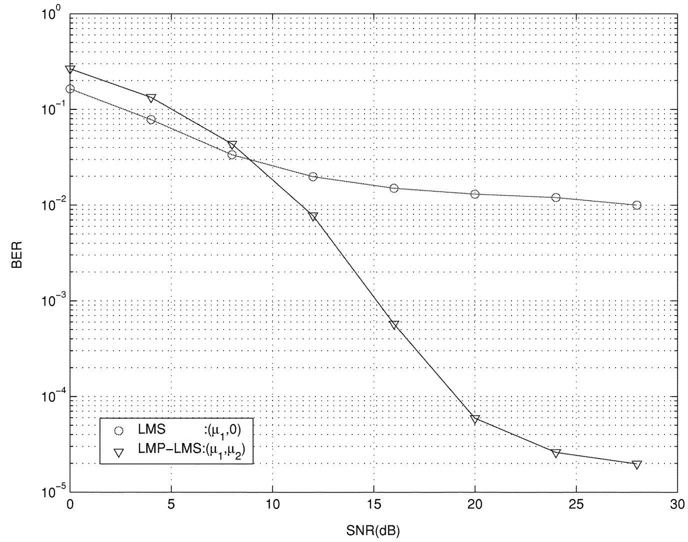

Fig. 3. BER versus SNR for an 8-PSK constellation with five receiver antennas, $\mu_1 = 0.01, \mu_2 = 1$.

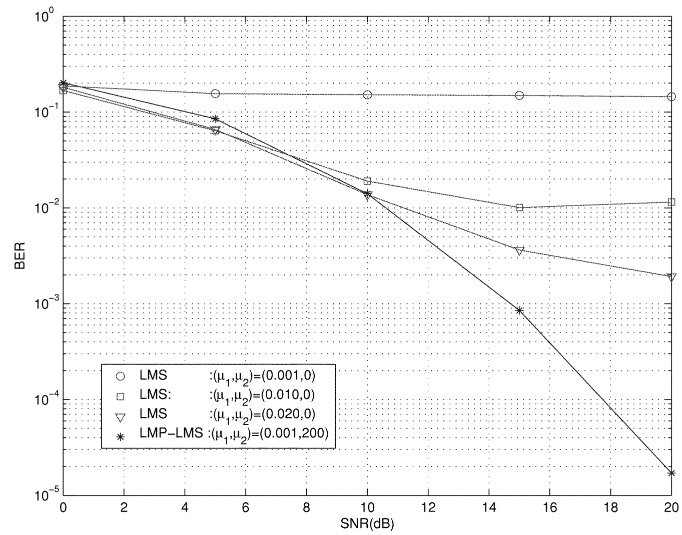

Fig. 4. BER versus SNR for an 8-PSK constellation with five receiver antennas.

The effect of μ_1 and μ_2 on the performance of the proposed algorithm is also evaluated. Fig. 6 shows the ratio between the bit error rates achieved by the standard LMS and by LMP–LMS, i.e., $\text{BER}_{\text{LMS}}/\text{BER}_{\text{LMP}-\text{LMS}}$, for a fixed SNR. A Doppler frequency of 480 Hz is assumed for this simulation. As shown in the plot, certain choices of μ_2 can result in one to two orders of magnitude improvement in the BER over a standard LMS algorithm.

B. Adaptive Rake Receiver

We also simulated a Rake receiver used in DS-CDMA systems [Fig. 2(b)]. The receiver uses an adaptive filter to estimate the Rake coefficients and track the channel variations. A multipath channel with independent taps is used in the simu-

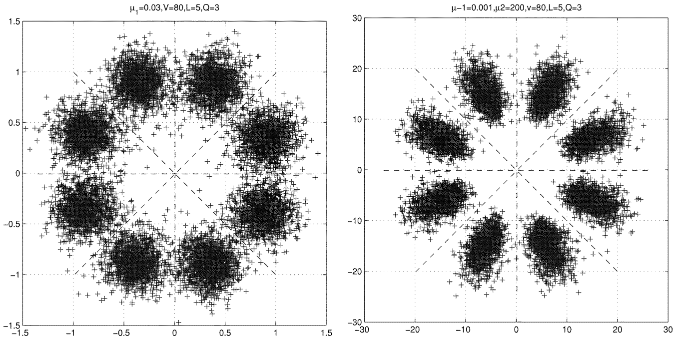

Fig. 5. Recovered constellations at the output of the antenna combiner. The left plot is the result of using LMS, while the right plot is the result of using LMP-LMS.

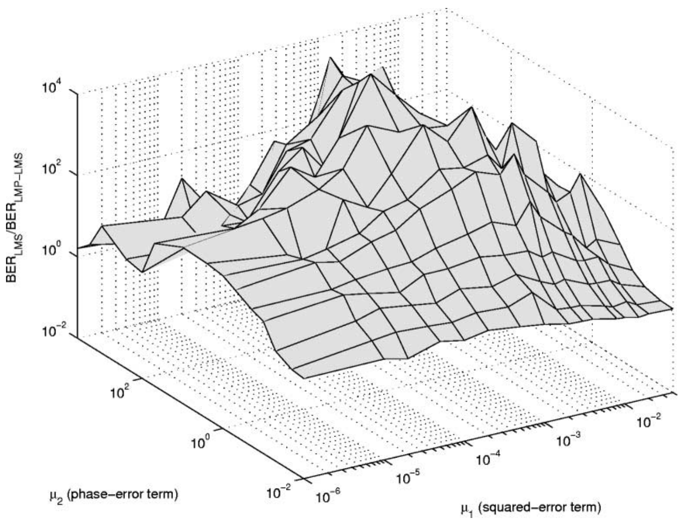

Fig. 6. $\mathrm{BER}_{\mathrm{LMS}}/\mathrm{BER}_{\mathrm{LMP-LMS}}$ for a fixed SNR of 20 dB, 8-PSK constellation, and five receiver antennas.

lation. Similar to the previous section, each channel tap has a Rayleigh distribution with the same Doppler frequency. The received signal is first correlated with the PN sequence to provide an estimate for the CDMA symbol through each finger and then combined by the Rake coefficients. A QPSK modulation is used in the simulation which is the typical one used in current DS-CDMA systems. A Doppler frequency of 480 Hz and CDMA symbol rate of 32 KS/s is considered for this scenario. In this configuration, the vector $\mathbf{u}$ contains the correlation results provided by the rake fingers and d is the transmitted CDMA symbol. Fig. 7 shows the simulations results for a four-tap channel and three-finger Rake receiver, which is a typical scenario considered for DS-CDMA simulations.

The following are some comments and concluding remarks on the performing behavior of the proposed algorithm. As shown in Fig. 6, the LMP–LMS algorithm can outperform the LMS algorithm for different values of μ_1 and μ_2. One scenario is when the same μ_1 is used for both algorithms and the other scenario is when a different μ_1 is used in LMP–LMS algorithm. In the first approach, the same μ_1 is used for both the LMS and LMP–LMS algorithms, as shown in Fig. 3. In this case, the squared error term has the same step-size

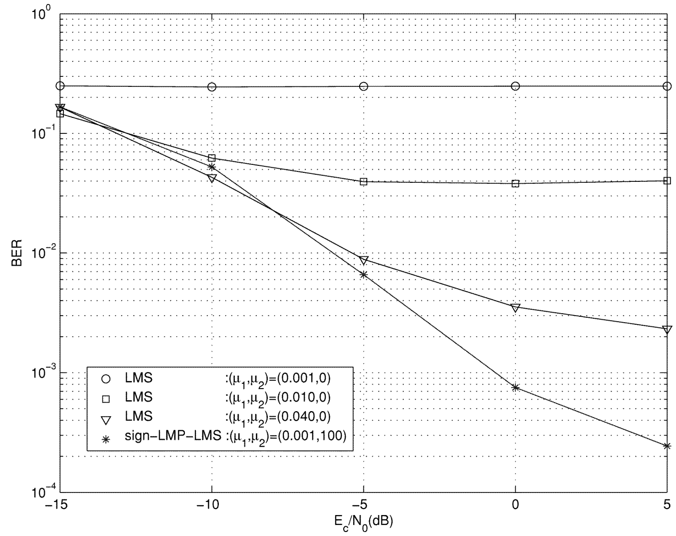

Fig. 7. BER versus E_c/N_0 for a CDMA system with processing gain (PG) of 32, QPSK modulation and three-finger Rake receiver.

in both the algorithms and the claim is that the additional phase error term in the LMP–LMS algorithm can improve the performance. This case proves useful when the LMS step-size μ_1 is not tuned to the optimum value (e.g., when the maximum Doppler frequency is not known) and adding the phase error term helps to improve the performance significantly even if μ_1 is not properly adjusted. The second approach is to use a different μ_1 for the LMP–LMS algorithm. Figs. 4 and 7 show the performance improvement when the LMP–LMS algorithm is used with a different μ_1. In this approach, the μ_1 is tuned for the LMS algorithm separately ($\mu_1 = 0.04$ in Fig. 7) and a different μ_1 is used in the LMP–LMS algorithm along with a μ_2 parameter ($\mu_1 = 0.001$, $\mu_2 = 100$ in Fig. 7). This is consistent with the observation shown in Fig. 6 that a wide range of μ_1 and μ_2 can result in an improvement compared to the LMS algorithm. (Note that the phase term in the LMP–LMS iteration is in radians.)

V. Conclusion

A cost function based on both phase error and magnitude error is introduced, and two adaptive algorithms are developed based on this cost function. In tracking fast channel variations, the new algorithms outperform the standard LMS filter, which is based solely on the squared-error criteria. The purpose of this letter was to describe the filters and to illustrate their superior performance.

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